National Aeronautics and Space Administration

Engineering Elegant Systems: Systems as Mathematical Categories

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Space Launch System

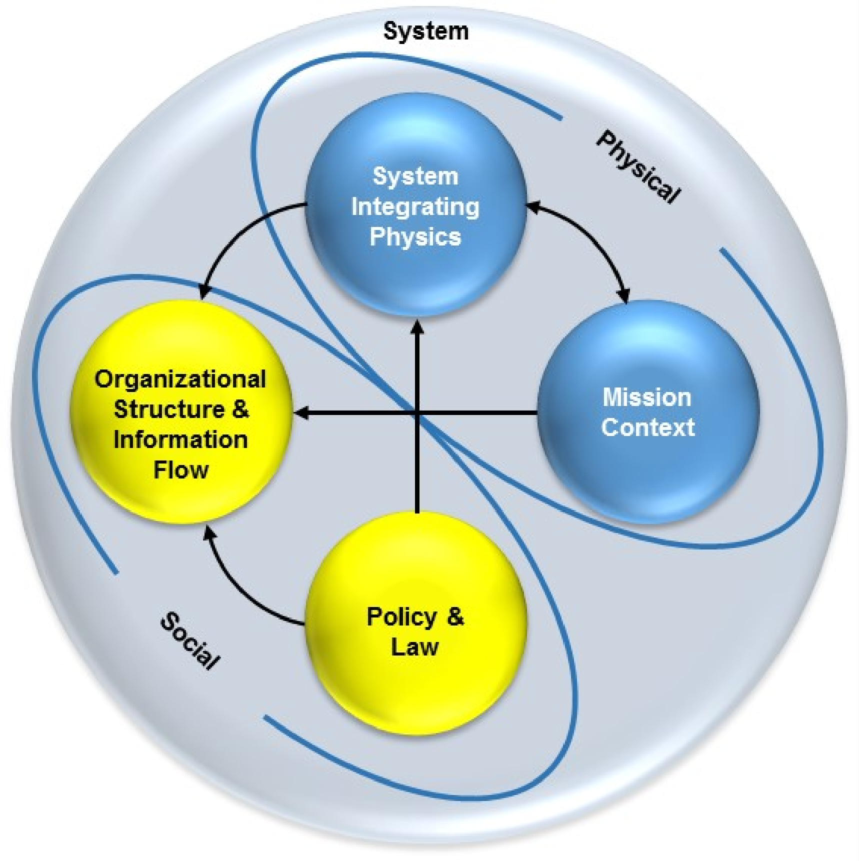
Theoretical Basis of Systems Engineering

Understanding Systems Engineering

- **Definition – System Engineering is the engineering discipline which integrates the system functions, system environment, and the engineering disciplines necessary to produce and/or operate an elegant system.**
  - Elegant System - A system that is robust in application, fully meeting specified and adumbrated intent, is well structured, and is graceful in operation.

- **Primary Focus**
  - System Design and Integration
    - Identify system couplings and interactions
    - Identify system uncertainties and sensitivities
    - Identify emergent properties
    - Manage the effectiveness of the system
  - Engineering Discipline Integration
    - Manage flow of information for system development and/or operations
    - Maintain system activities within budget and schedule

- **Supporting Activities**
  - Process application and execution
    - Processes organize the engineering



Systems Engineering Postulates

System Integration (physical/logical system)
Discipline Integration (social system)
Both System and Discipline Integration

- **Postulate 1: Systems engineering is system specific and context dependent in application**
- **Postulate 2: <u>The Systems Engineering domain consists of subsystems, their interactions among themselves, and their interactions with the system environment</u>**
- **Postulate 3: The function of Systems Engineering is to integrate engineering disciplines in an elegant manner**
- **Postulate 4: Systems engineering influences and is influenced by organizational structure and culture**
- **Postulate 5: Systems engineering influences and is influenced by budget, schedule, policy, and law**
- **Postulate 6: Systems engineering spans the entire system life-cycle**
- **Postulate 7: Understanding of the system evolves as the system development or operation progresses**
- **Postulate 7 Corollary: Understanding of the system degrades during operations if system understanding is not maintained.**

Systems Engineering Principles

- **Principle 1: Systems engineering integrates the system and the disciplines considering the budget and schedule constraints**
- **Principle 2: Complex Systems build Complex Systems**
- **Principle 3: A focus of systems engineering during the development phase is a progressively deeper understanding of the interactions, sensitivities, and behaviors of the system, stakeholder needs, and its operational environment**
  - Sub-Principle 3(a): Mission context is defined based on understanding of the stakeholder needs and constraints
  - Sub-Principle 3(b): Requirements and models reflect the understanding of the system
  - Sub-Principle 3(c): Requirements are specific, agreed to preferences by the developing organization
  - Sub-Principle 3(d): Requirements and design are progressively elaborated as the development progresses
  - Sub-Principle 3(e): Hierarchical structures are not sufficient to fully model system interactions and couplings
  - Sub-Principle 3(f): A Product Breakdown Structure (PBS) provides a structure to integrate cost and schedule with system functions
  - Sub-Principle 3(g): As the system progresses through development, a deeper understanding of the organizational relationships needed to develop the system are gained.
  - Sub-Principle 3(h): Systems engineering achieves an understanding of the system’s value to the system stakeholders
  - Sub-Principle 3(i): Systems engineering seeks a best balance of functions and interactions within the system budget, schedule, technical, and other expectations and constraints.

Systems Engineering Principles

- **Principle 4: Systems engineering has a critical role through the entire system life-cycle**
  - Sub-Principle 4(a): Systems engineering obtains an understanding of the system
  - Sub-Principle 4(b): Systems engineering defines the mission context (system application)
  - Sub-Principle 4(c): Systems engineering models the system
  - Sub-Principle 4(d): Systems engineering designs and analyzes the system
  - Sub-Principle 4(e): Systems engineering tests the system
  - Sub-Principle 4(f): Systems engineering has an essential role in the assembly and manufacturing of the system
  - Sub-Principle 4(g): Systems engineering has an essential role during operations, maintenance, and decommissioning
- **Principle 5: Systems engineering is based on a middle range set of theories**
  - Sub-Principle 5(a): Systems engineering has a physical/logical basis specific to the system
  - Sub-Principle 5(b): Systems engineering has a mathematical basis
  - Sub-Principle 5(c): Systems engineering has a sociological basis specific to the organization(s)
- **Principle 6: Systems engineering maps and manages the discipline interactions within the organization**
- **Principle 7: Decision quality depends on system knowledge present in the decision-making process**
- **Principle 8: Both Policy and Law must be properly understood to not overly constrain or under constrain the system implementation**

Systems Engineering Principles

- **Principle 9: Systems engineering decisions are made under uncertainty accounting for risk**
- **Principle 10: Verification is a demonstrated understanding of all the system functions and interactions in the operational environment**
- **Principle 11: Validation is a demonstrated understanding of the system's value to the system stakeholders**
- **Principle 12: Systems engineering solutions are constrained based on the decision timeframe for the system need**
- **Principle 13: Stakeholder expectations change with advancement in technology and understanding of system application.**
- **Principle 14: The real physical system is the perfect model of the system**
  - Kullback-Liebler Information shows the actual system is the ideal information representation of the system
    - $I(f,g) = \int f(x)\log(f(x))\,dx - \int f(x)\log(g(x|\theta))\,dx = 0$

System Engineering Hypotheses

- **Hypothesis 1: If a solution exists for a specific context, then there exists at least one ideal Systems Engineering solution for that specific context**
  - Hamilton's Principle shows this for a physical system
    - $\int_{t_1}^{t_2}(\delta T - \delta V + \delta W)dt = 0$

- **Hypothesis 2: System complexity is greater than or equal to the ideal system complexity necessary to fulfill all system outputs**

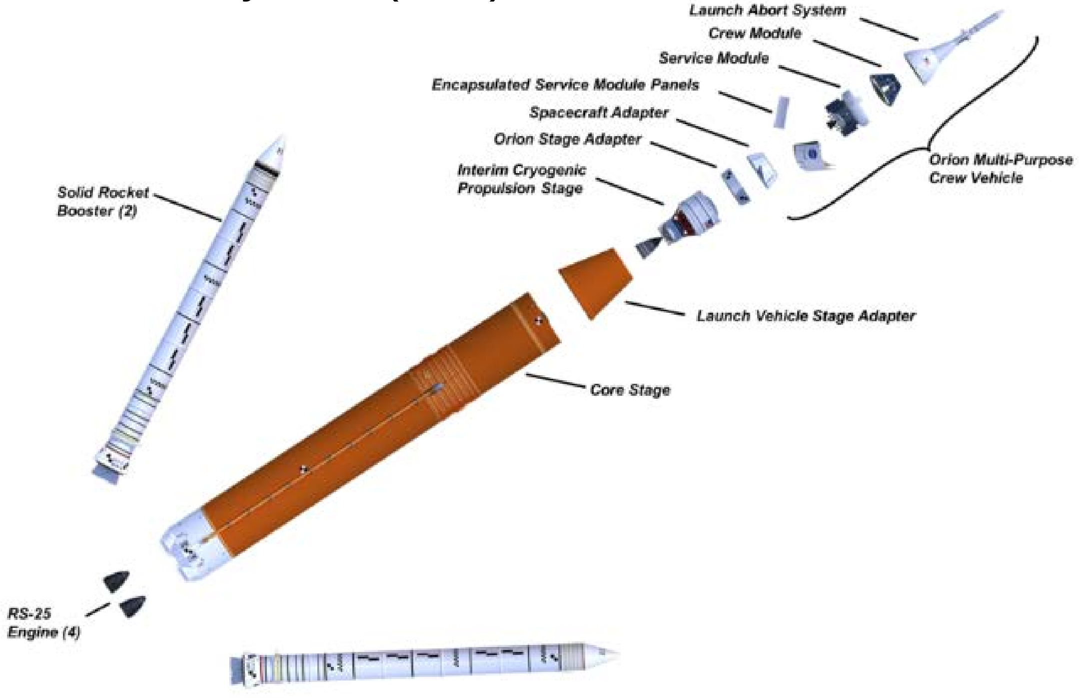
- **Hypothesis 3: Key Stakeholders preferences can be accurately represented mathematically**

Mathematical Basis of Systems Engineering: Mathematical Category Theory

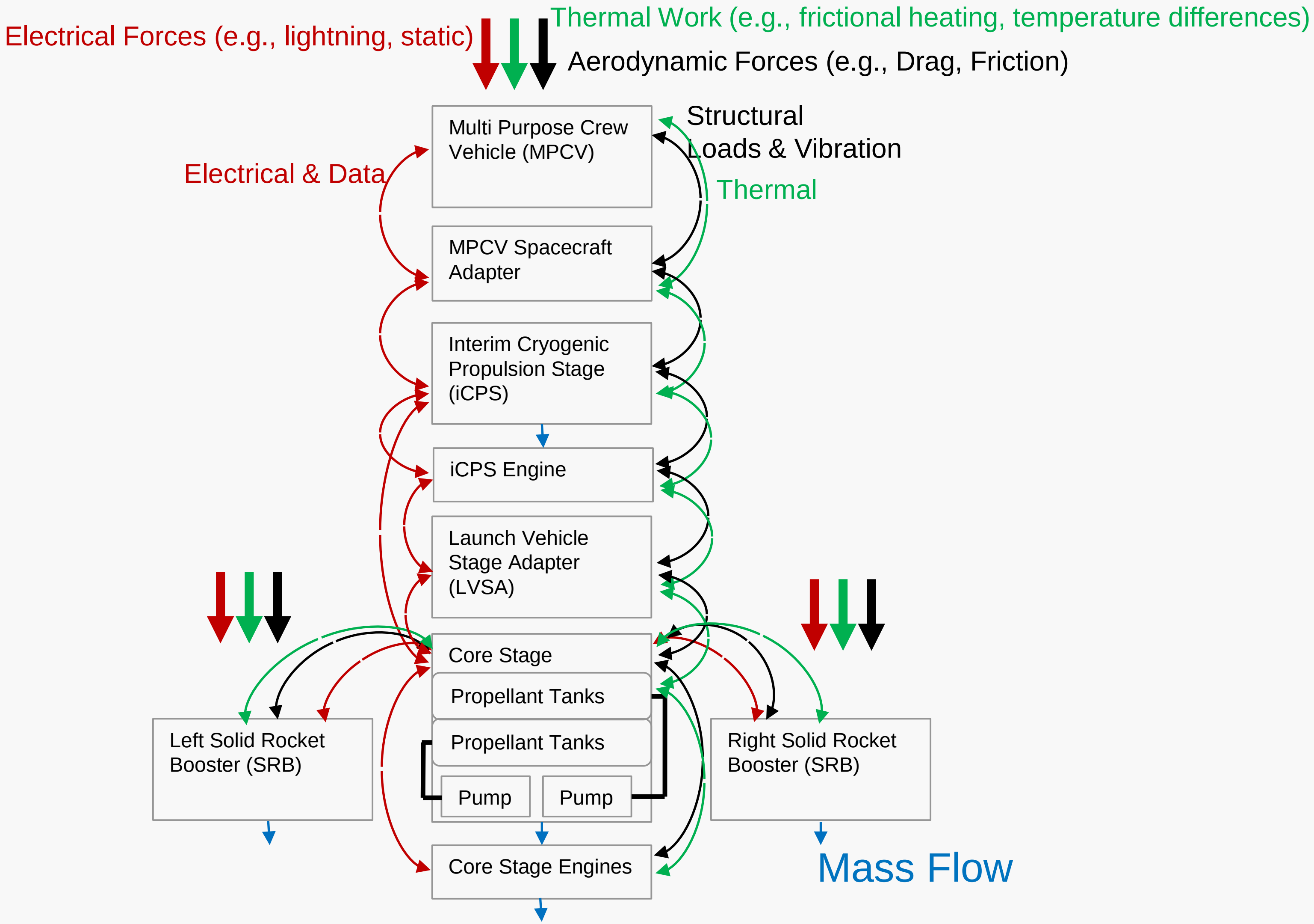
System Representations

- **Systems are comprised of 2 basic structures**
  - Postulate 2: The Systems Engineering domain consists of subsystems, their interactions among themselves, and their interactions with the system environment
  - Components
  - Relationships among components
    - Physical
    - Logical
  - Relationships with the environment
    - Physical

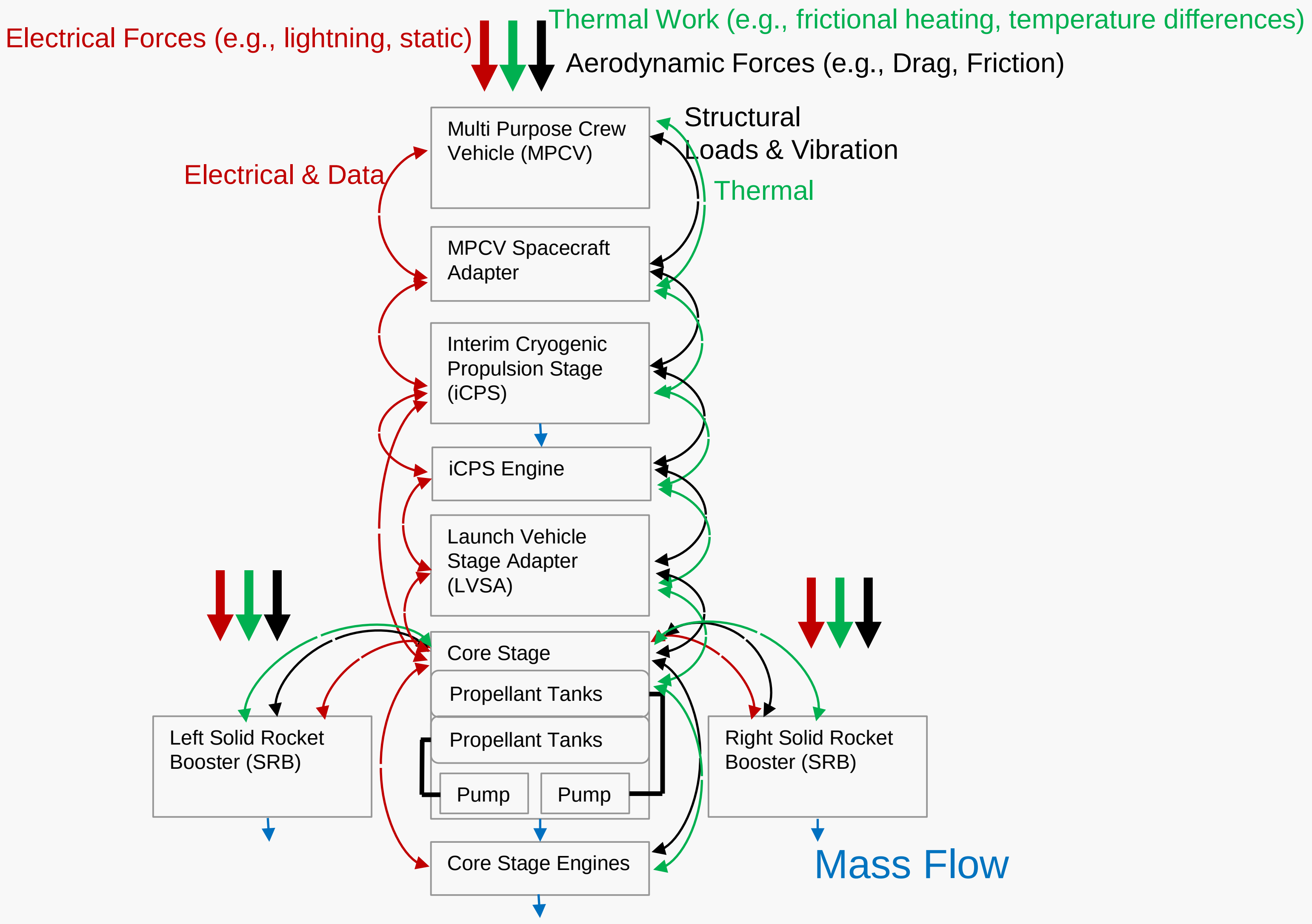
Major Components of the NASA Space Launch System (SLS)



Rocket Physical and Logical Relationships



Rocket as a Mathematical Category



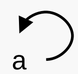
Mathematical Category

A Mathematical Category consists of

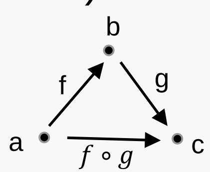
- Objects (i.e., system components): a,b,c,…
- Arrows (i.e., system relationships between components and the environment): f,g,…

A Mathematical Category has properties

- Domain/Codomain
  - f: a → b where a is the domain of f and b is the codomain of f
- Identify Relationship
  - $id_a = 1_a: a \to a$



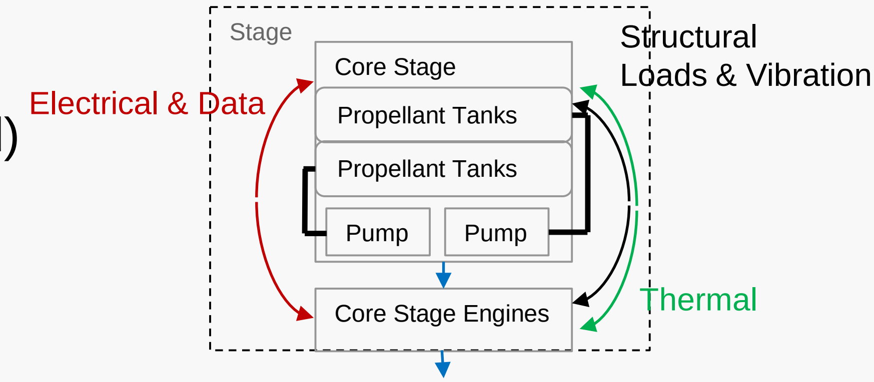
- Associativity
  - $f \circ (g \circ h) = (f \circ g) \circ h$
- Composition
  - Composition can be performed by various mathematical operations (i.e., addition, subtraction, multiplication, division)
  - $a \xrightarrow{f} b \xrightarrow{g} c = a \xrightarrow{f \circ g} c$



Mathematical Category Types

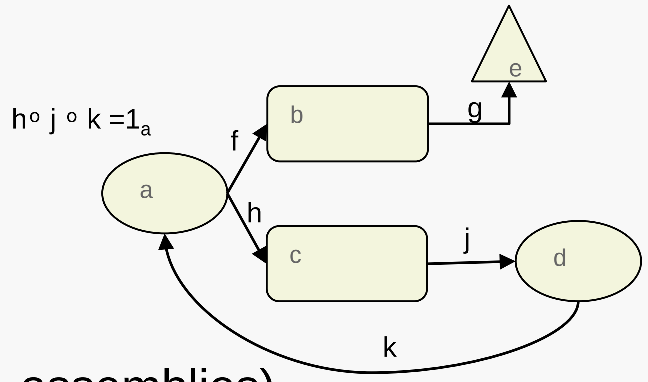
Category Types

- Category of Sets
- Category of Arrows (objects are implied)
- Category of Groups
- Category of Categories
- Universal Category
- Category of Small Categories
- Abelian Categories



Objects within a category can be

- Objects (i.e., individual parts or components)
- Sets (i.e., sets of individual parts)
- Groups
- Smaller Categories (i.e., stages, subsystems, assemblies)



Directed Graphs

- Directed graphs, when they meet the property conditions, are a form a mathematical category

Mathematical Category Transformations

Functors

- Mathematical morphisms between categories, F: $A \xrightarrow{F} C$
- Creates a mapping from one category to another
- Includes composition in the mapping

Natural Transformations

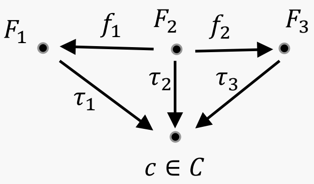
- Transformation is the same among all objects
- Is commutative
- If invertible, then is a ‘natural equivalence’ or ‘isomorphism’

Isomorphism

- If the relationships (arrows) are invertible between two objects, then the objects are isomorphic, $a \cong b$
  - $a \xrightarrow{f} b \xrightarrow{g} a$, f = g’, g = f’
- Categories can be isomorphic, $A \cong B$
  - The objects can be different, but the relationships between the objects of the two categories are preserved
    - i.e., different copies of the same system are isomorphic
    - Or, two different designs of the same system type may be isomorphic (e.g., different automobile makes with similar models)

Mathematical Category Transformations

- **Co-cones/Co-limits**
  - Co-cone
    - A common codomain for Functors operating on Category C



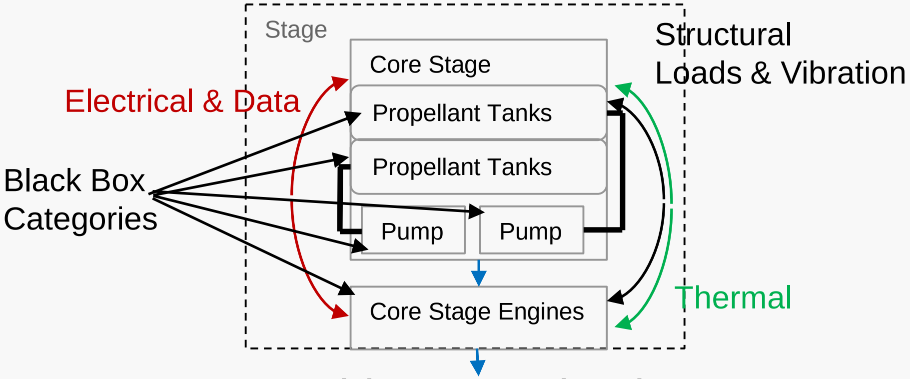
  - Co-limit
    - The limit of the Co-cone defining the conditions where all Functors and mappings to objects of the Category, C, are included

https://www.nasa.gov/content/goddard/nasa-engineer-set-to-complete-first-3-d-printed-space-cameras/

Systems Engineering Application

Black Box

- Since a Category may contain smaller Categories, then an engineering 'black box' is a Category treated as an object within a larger Category



System Completeness

- The mathematical structure of the system Category provides a mechanism to construct a completeness proof for a given system

System Specification

- The System objects and relationships form the basis of the system requirements
- The Category must contain the correct and complete objects and relationships
  - Variations result in a system different than intended

System Assembly

- Co-cones and co-limits define the assembly operations needed to construct the system Category
- The Functors map parts from the parts category(s) to the system category
  - The parts may map to sub-categories (i.e., assemblies and subsystems) within the system category
- The limits define what must be included at each step of the assembly in order to be complete

Summary

- **System Engineering Postulates provides a mathematical definition of a system**
  - Postulate 2: The Systems Engineering domain consists of subsystems, their interactions among themselves, and their interactions with the system environment
- **System are Mathematical Categories**
  - Mathematical Category Theory provides the mathematical structure to fully represent a system: all of its components and all of its physical and logical relationships
- **Mathematical Category Theory provides insight into systems**
  - Category Types
  - Categories as objects within larger Categories
  - Directed Graph Representations
  - Functors
  - Natural Transformations
  - Isomorphism
  - Co-cones and co-limits
- **Mathematical Category Theory supports representations of systems**
  - Engineering Black Box
  - Parts/components, assemblies, subsystems as smaller categories within the system category
  - System Completeness
  - System Specification
  - System Assembly