

**Advanced Mathematics for Control System Design:
Guidance, Navigation, and Control (GN&C) Studies**

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Applied Mathematics and Physics¹

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NASA Internships and Fellowships Program – Spring Session

04/11/2019

Advanced Mathematics for Control System Design: Guidance, Navigation, and Control (GN&C) Studies

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I. Introduction to LSP and GN&C

Beginning in 1998, NASA's Launch Service Program (LSP) was established in the Kennedy Space Center to oversee launch operations and countdown management ensuring quality and mission assurance. Their principle objectives are providing safe, reliable, cost-effective, and on-schedule processing mission analysis, spacecraft integration and launch services for payloads seeking transportation to space commercial launch vehicles. It operates under the Human Exploration and Operations (HEO) mission directorate of NASA and has produced expendable launch vehicle such as the Atlas V, Delta IV, Pegasus, and the Taurus. Their work is considered earth's bridge to Space!

II. PENNY Robot Overview

The study and control of inverted pendulum dynamics is of interest to NASA to better control the stability of a rocket. The different compartments of a rocket can move within a rocket during launch (i.e., rocket fuel slosh) and can affect its overall trajectory. Focusing on the inverted pendulum robot, PENNY, the dynamics are reduced to a simpler model which can prove to be more insightful for deriving more complex models and control laws. From the inverted pendulum bot, we can incorporate complexity into our physical model and approximate to the dynamics of a rocket (i.e., flexible inverted pendulum, multistage pendulum).

A. Penny Robot Theory Modeling and Simulink implementation

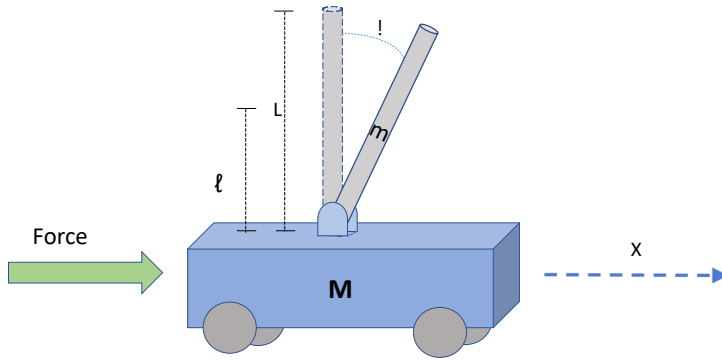
The study and control of inverted pendulum dynamics is of interest to NASA to better control the stability of a rocket. The different compartments of a rocket can move within a rocket during launch (i.e., rocket fuel slosh) and can affect its overall trajectory. Focusing on the PENNY bot the dynamics are reduced to a simpler model which can prove to be more fruitful and pedagogical for deriving more complex models and control laws. From the inverted pendulum bot, we can incorporate complexity into our physical model and approximate to the dynamics of a rocket (i.e., flexible inverted pendulum, multistage pendulum).

B. Penny Robot Theory Modeling and Simulink implementation

An inverted pendulum bot follows dynamics resembling that of a regular pendulum, having angular displacement away from equilibrium θ , however there are additional degrees of freedom including angular velocity, angular acceleration, linear displacement of the bot (for stability), linear velocity, and linear acceleration.

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The model schematic portrayed in figure 1 depicts the different degrees of freedom of the PENNY bot. The approach to modeling the dynamics of the system relies on the physical concept of a LaGrangian equation and implementation of the Euler-LaGrange formula, i.e.,

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_j} \right) - \frac{\partial L}{\partial q_j} + \frac{\partial R}{\partial \dot{q}_j} = Q_j$$

Figure 1. Model of Basic Inverted Pendulum with angular displacement and cart width linear displacement

Where the R term is the Rayleigh dissipation term and the Q term is the external force for the

j^{th} degree of freedom. The LaGrangian takes the form:

$$L = \frac{1}{2} (M + m) \dot{x}^2 - m \dot{x} \ell \dot{\theta} \cos \theta + \frac{1}{2} m \ell^2 \dot{\theta}^2 - m g \ell \cos \theta$$

With the Rayleigh terms:

$$R = \frac{1}{2} b_x \dot{x}^2 + \frac{1}{2} b_\theta \dot{\theta}^2 \approx \frac{1}{2} b_x \dot{x}^2$$

Where the second dissipation term becomes negligible due to the friction on the pendulum being negligible (small coefficient of friction b_θ). After implementing the Euler-LaGrange formula the equations become:

$q_1 = x$:

$$(M + m) \ddot{x} - m \ell \ddot{\theta} \cos \theta + m \ell \dot{\theta}^2 \sin \theta + b_x \dot{x} = u_x$$

$q_2 = \theta$:

$$-\ddot{x} + \ell \ddot{\theta} - g \sin \theta = 0$$

Note the external force has taken the form u_x as the force we can manipulate in the system.

C. PID preliminaries

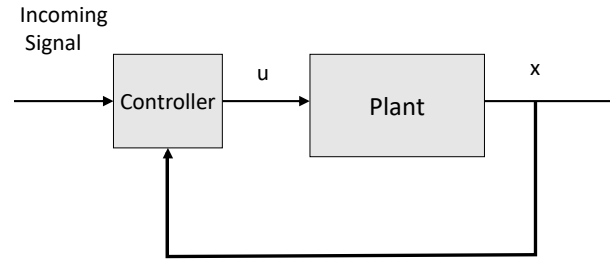
Crucial to the concept of PID (Proportional Integral Derivative) Control is the error variable defined as $e_x = x - x^*$. Here the variable x is the variable in consideration (linear displacement, angular displacement, etc.) and x^* is the desired state of the variable (held constant). Upon applying this transformation the controller can interpret this as an input. It is worth noting the PID controller is a single input single output controller, so for a system with n variables, n PID controllers would be needed.

$$u = - \left[\underbrace{k_p e_x}_{\text{Proportionality Term}} + \underbrace{k_i \int e_x}_{\text{Integral Term}} + \underbrace{k_d \frac{d}{dt} e_x}_{\text{Derivative Term}} \right]$$

Each of these terms are incorporated in the PID controller so the error converges to 0 (otherwise known as x converging to x^*). The proportionality term linearly pushes x to x^* , yet the integral term “excites” convergence at a quicker pace. This can lead to overshooting, which is the purpose of the derivative term on the far right. That term

acts as a dampener in the case of overshoot or steady-state oscillation. In this manner the PID has earned the “gold” standard for general control methods.

The setup of the controller relative to the plant model is shown below:



This way it follows an iterative process and can be easily implemented in MATLAB Simulink.

D. Penny Robot Hardware²

The inverted pendulum robot is built on a metal chassis, covered in the front and back with bumpers to blunt any inadvertent impact. It is powered by a 4-cell 14.8 volt battery, which subsequently powers two voltage regulators. The first, a 12-volt regulator, provides power to the electronic speed controllers (ESCs) and four 12-volt motors. The ESCs and the motors are separated by a bus that facilitates connections in the robot.

A second 5-volt regulator powers an Arduino Mega 2560 and four sensors that act like the central and peripheral nervous system of the inverted pendulum robot. The microcontroller board receives data from the sensors, processes it, and then sends out signals to the ESCs to control motor speed. The on-board sensors consist of three rotary encoders and one 6-DOF inertial measurement unit (IMU). Two encoders measure the motion of the front left and right wheels of the robot. One encoder measures the motion and direction of motion of the inverted pendulum that is mounted on the top-front. The IMU board is mounted on the inverted pendulum and contains an MPU 6050 with an integrated 3-axis accelerometer and 3-axis gyroscope, whose data can be fused for pendulum angle estimation.

The robot uses an encoder to measure angular displacement, but the true vertical shifts over time due to sensor drift. In order to correct for this drift over time, an IMU was introduced to measure true vertical with respect to the local gravity vector. The accelerometer contains a sensitivity of 16384 least-significant-bit (LSB) per unit gravitational force (LSB/g), while the gyroscope’s sensitivity is 131 LSB/°/s.

All of the data interpretation and visualization is done through Matlab/Simulink.

E. Penny Robot Future Steps – Flexible Pendulum and Multistage Pendulum

Future work on the PENNY bot will include introducing complexity into the models currently at hand. This includes modeling an inverted pendulum with a flexible rod and modelling one with a multistage pendulum. This will help to better approximate and control the dynamics of real aerospace structures.

The flexible inverted PENNY bot is represented in the following figure with an additional degree of freedom to the rigid inverted pendulum previously considered:

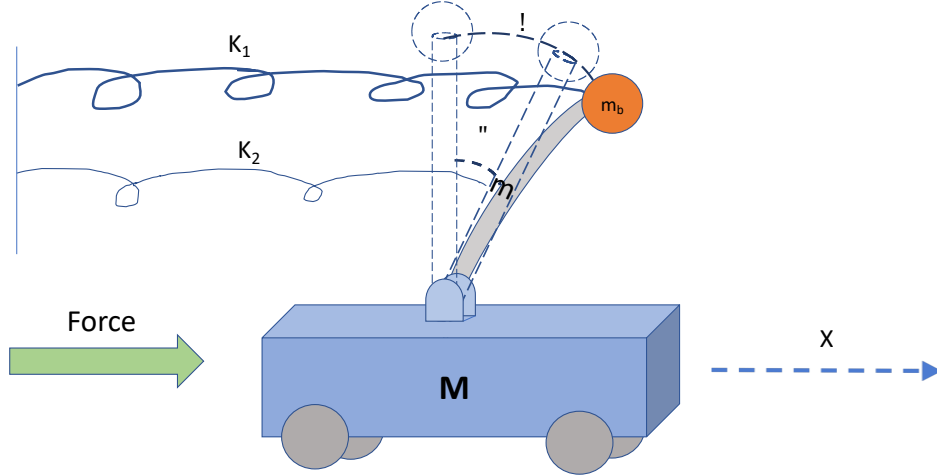


Figure 2. The schematic for Flexible Inverted Pendulum Model

For such a system with more degrees of freedom than previously considered, state space was implemented⁵:

$$\dot{X} = AX + Bu$$

where,

$$X = [x \quad \dot{x} \quad \theta \quad \dot{\theta} \quad \phi \quad \dot{\phi}]^T$$

For the given system the LaGrangian was found to be:

$$L = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}m_b[\dot{x}^2 + 2\ell\dot{x}\dot{\phi}\cos\phi + \ell^2\dot{\phi}^2] + \frac{1}{2}m[\dot{x}^2 + 2\dot{x}(L\dot{\theta}\cos\theta + \ell\dot{\phi}\cos\phi) + L^2\dot{\theta}^2 + \ell^2\dot{\phi}^2 + 2L\ell\dot{\theta}\dot{\phi}\cos(\theta - \phi)] + mgL(1 - \cos\theta) - \frac{1}{2}k_1(L\sin\theta)^2 + m_bgl(1 - \cos\phi) - \frac{1}{2}k_2(\ell\sin\phi)^2$$

With the Rayleigh Damping Terms:

$$R_d = \frac{1}{2}b_3\dot{x}^2 + \frac{1}{2}b_2[(\dot{x} + \ell\dot{\phi}\cos\phi)^2 + (-\ell\dot{\phi}\sin\phi)^2] + \frac{1}{2}b_1[(\dot{x} + L\dot{\theta}\cos\theta + \ell\dot{\phi}\cos\phi)^2 + (-L\dot{\theta}\sin\theta - \ell\dot{\phi}\sin\phi)^2]$$

After implementing Jacobian approximation and small angle approximation, the linearization of the state space model produced the following A and B matrices:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \alpha_1(b_1 + b_2 + b_3) & 0 & \alpha_1 b_1 L & 0 & \alpha_1(b_1 + b_2)\ell \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{1}{mL} \left(\frac{k_1 L}{2} - mg \right) & -\frac{b_1}{m} & 0 & -\frac{b_1}{2m} \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & \alpha_2(b_1 + b_2)\ell & 0 & \frac{\alpha_2 b_1 L^2}{2} & \alpha_2(k_2 \ell^2 + m_b g)\ell & \alpha_2(b_1 + b_2)\ell^2 \end{bmatrix}$$

$$B = -\alpha_1 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

III. Universal Robots UR10 Robotic Arm Overview²

Guidance, navigation and control (GN&C) plays an important role at NASA; it is part of every launch vehicle and spacecraft system. GN&C deals with the design of systems to control the movements of objects in space. It provides multiple capabilities, such as mission planning, guidance, ascent trajectory for design and dynamics simulation, navigation, control, and even sensor hardware. However, present methods used to describe the translation and rotation of objects are defined in matrices and Euler-angles, which can be cumbersome in calculation, processing time, and computer memory space. Here, dual-quaternions are considered to describe a Universal Robots (UR) arm's equations of motion, which includes multiple advantages over previous methods used.

The UR10 robotic arm is part of a Mars lander mock-up that is located in the Granular Mechanics and Regolith Operations (GMRO) laboratory that supports research and development of technologies that are instrumental for in-situ resource utilization (ISRU). The UR10 is located on top of an octagonal lander deck, being one of the multiple stations on deck. The UR10 plays an essential role in the movement and transport of objects and materials needed to support a simulated Martian habitat. The UR10 supports different mechanisms and robotic technologies that make up the integrated system, such as a simulated Atmospheric Processing Module (APM) that separates in-situ resources into volatiles for fuel and water for usage, and a simulated Soil Processing Module that uses carbon dioxide and 500 °C to extract water from soil mined by the low-gravity excavator, Regolith Advanced Surface Systems Operations Robot (RASSOR). Dual-quaternion descriptions of translation and rotation will play an important role in space exploration where size and power limitations are present, such as in GN&C and ISRU.

A. Euler-Angles and Gimbal Lock

The Euler-angle method is one of the most common ways to represent rigid-body rotation by fusing three sequential angles around the principle orthogonal axes (x, y, and z). This application requires the conversion of angles into matrices in which the product of the three angle-matrices produces the Euler-angle set. Euler-angles in 3D do not commute under composition, so order of execution is determined by convention and sequentially executed for the transformations to be correctly computed. One of the advantages of Euler-angles is that they are easy to comprehend, are minimalistic, and require only three parameters.

The disadvantages of Euler-angles include the following. First, converting, combining, and extracting Euler-angles is computationally expensive. Second, Euler-angles suffer from singularities at certain rotation angles, known as “gimbal lock,” which happens when one degree of freedom is lost due to the overlap of another degree of freedom. Finally, when Euler-angles are interpolated linearly, the resulting path may not be the shortest path.

B. Dual-Quaternions

Dual-quaternions are known to solve most of the disadvantages mentioned above. Dual-quaternions are used for multiple reasons, which include the following: singularity free; un-ambiguous; shortest path interpolation; most efficient and compact form for representing rigid transforms (3x4 matrix floats compared to a dual-quaternion 8 floats); unified representation of translation and rotation; can be integrated into a current system with little coding effort; and lastly, the individual translation and rotational information combines to produce a single invariant coordinate frame.

As found in literature, the rigid rotational and translational information for the unit dual-quaternion is:

$$q_r = r \text{ and } q_t = \frac{1}{2}tr$$

where \mathbf{r} is a unit quaternion representing the rotation and \mathbf{t} is the quaternion describing the translation. The dual-quaternion can represent a pure rotation and pure translation, as shown in the following formulae:

$$q_r = \left[\cos\left(\frac{\theta}{2}\right), n_x \sin\left(\frac{\theta}{2}\right), n_y \sin\left(\frac{\theta}{2}\right), n_z \sin\left(\frac{\theta}{2}\right) \right] [0,0,0,0]$$

$$q_t = [1,0,0,0] \left[0, \frac{t_x}{2}, \frac{t_y}{2}, \frac{t_z}{2} \right]$$

In order to represent a rotation followed by a translation, the following transform can be used to convert each part into a single unit quaternion:

$$q = q_t \times q_r$$

which is used to define how to transform a point p , using a single unit quaternion:

$$p' = qpq^*$$

where q and q^* represent a dual-quaternion transform and its conjugate, whereas p and p' represent an initial and rotated orientation, respectively.

C. Universal Robotics – UR10 Internship Work

During my time at this spring internship, I first went through safety training to have unescorted access at the GMRO and ESPL labs. Once the literature review was done, I went on to research robotic arm kinematics, which included forward and inverse kinematics. As part of this assigned project, it was critical to have a basic understanding of the different methods used to describe the rotation and translation of objects in space. Therefore, matrices, axis-angles, Euler-angles, and quaternions reading was necessary. Once that was done, I needed to review the tutorial and user-manual for the UR10 robotic arm located in the GMRO. I then continued to learn how to manipulate the UR10 through a series of prescribed movements using the manufacturer's manual, including the definition of the safety boundaries to prevent the harm of the UR10.

D. UR10 Future Work

Future steps for control systems research using the UR10 robotic arm includes finding a way to bypass the manufacturer's controller and access the UR10 robotic arm's actuators and sensors through Matlab/Simulink. Next, it will be helpful to implement Denavit-Hartenberg (DH) parameters and methods to shore up our understanding of forward and inverse kinematics. Once the DH method has been demonstrated, further research will develop and describe the UR10 kinematics in dual-quaternion space. Lastly, an adaptive control of the UR10 in dual-quaternion space should take place to test the difference of translation and rotation in space between DH transformation matrices and dual-quaternions.

IV. FIRST Robotics Competition Experience²

During the period of the internship, I have been helping mentor the KSC FIRST Robotics Competition (FRC) team, known as the Pink Team, No. 233. FIRST is a 501(c)(3) not-for-profit public charity that mixes innovation and participation in science and technology. This motivates primary and secondary education students pursue Higher Education in the fields of science, technology, engineering, and mathematics. FIRST Robotics is the highest of four levels, where high school students have a six-week time limit to design, build, program, and test industrial-sized robots, meeting specific guidelines and rules to develop a fair competition against other high school robotic teams.

This project has been completely voluntary, requiring as much time needed to reach our goals. As part of this volunteer work, I have witnessed the process of designing a robot, building it, and testing it locally and in competitions, where the team notices improvements needed on the robot. This realization involves the redesigning, rebuilding, and retesting of the robot for consecutive competitions. Each person's skills and background benefits the team. According to this experience, being an expert is not the essence of becoming successful, rather collaboration and diversity in skills, attitudes, and backgrounds that make up a whole have evidently propelled us to perform great during match-playing.

This year's 30th season is themed "Destination: Deep Space," presented by the Boeing Company. The match play demands robots to place hatch panels on rockets and cargo ships, load cargo for transport off the planet, and return to the safety of their habitat. Hatch panels are large disks, while cargo are large inflated balls. The habitat has a flat level, low level, and high level, where a climb would be needed for both the low and high levels. Teams can concentrate on playing offensively or defensively. The Team Pink is able to place hatchels and load cargo dexterously, becoming a main target to robotic teams that play defensively. This was evident at the Orlando Regionals and Wisconsin Regional. At the Orlando Regionals, we not only ranked 14th place, but also won champions along two other teams that formed our alliance. Furthermore, at the Wisconsin Regionals, the Pink Team ranked 6th place, while also winning the Judges' Award.

My responsibilities have been many. The characteristic I find most relevant is the willingness to learn and help. I did not know the name of tools nor how to go about different machinery. I have been fortunate enough to be surrounded all the time by mentor, colleagues, and friends that do not shame me for my lack of knowledge, rather encourage me to pick up a new set of skills, such as tapping, milling, drilling, and putting gearboxes together to name a few. My biggest contribution has been my organizational skills and lightened sense of humor. I believe that a clean and organized workspace provides room for creativity and innovation. At the same time, I find a lightened sense of humor

critical for teamwork; a tense, gloomy environment is not healthy psychologically, which automatically diminishes the motivation for physical tasks at hand.

As a result of this experience, I have grown my understanding of tools and mechanical hardware since my background consists of electrical engineering principles. What I have valued the most is the idea used by FIRST: “coopertition,” which is commonly known as a cooperative competition. I saw this in action firsthand during the Orlando Regionals inside the pits. Teams collaborated, providing materials and tools between each other during times of need. Moreover, I have been able to be a living example to the high school students of what comes after high school and before employment. It gives me the opportunity to advise and encourage the students to pursue higher education on whatever subject they find passionate.

V. Conclusion

As an introduction to the description and control of dynamical systems, an inverted pendulum robot (Penny) was modeled by Newtonian and simple LaGrangian mechanics, resulting in solutions that reasonably agree with previously measured system dynamics. Having produced models to simulate the dynamics of an inverted pendulum robot, we have begun focusing on developing higher-fidelity models through exciting the actual system hardware and measuring output responses. We are now beginning to implement basic control laws starting with the gold standard: PID control, state-space control, and ultimately adaptive control. More of the hardware modeling side of Penny still needs to be determined, such as the conversion of control signal to force. Once complete, increased model complexity will be introduced (i.e., the flexible inverted pendulum model and the multi-stage inverted pendulum model). Additionally, the introduction of dual-quaternion descriptions of rotation and translation has been very enriching, and will eventually be attempted on the UR10 robotic arm, as well as on Penny. Robotic technologies will continue to develop, so being part of a robotics research team has been an invaluable experience that not only will help the interns grow professionally in their careers, but also personally as individuals.

Acknowledgments

These projects have been supported by the NASA Grant and Cooperative Agreement Number NNX13AJ45A. We are also grateful for the “spiels” or invaluable sharing of knowledge and advice our mentor, Michael DuPuis, has been providing us with.

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