NASA/TM—2019-220164

Converging-Diverging Nozzle Thruster Code for Nuclear or Chemical Rocket Performance Computations

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April 2019

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Acknowledgments

The author would like to thank Steve Geng for his help in transferring the APL vector language environment containing the author-generated code and subroutines onto a new laptop machine using a Microsoft Windows 10 Operating System. In addition, the author gratefully acknowledges the many helpful suggestions of Robert Hendricks in reviewing this report.

Level of Review: This material has been technically reviewed by technical management.

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Abstract

This report describes development of a variable area ratio converging-diverging (C-D) nozzle numerical computational code “THRST”, applicable to the evaluation of nuclear or chemical rocket performance. The propellant for the nuclear rockets is parahydrogen with gas properties supplied by a subroutine, while properties for chemical thrusters need to be user input. Detailed documentation of input and output variables and sample output listings enable the report to function as a users’ manual.

1.0 Introduction

The primary purpose of subject computational code (THRST) is to perform Nuclear Thermal Propulsion (NTP) Thrust and specific impulse (I_{SP}) computations, applicable to updated NERVA (Nuclear Energy for Rocket Vehicle Applications) systems. These utilize high thermal power fission reactors, capable of heating supercritical parahydrogen (H_2) propellant from a liquid state at near 20 K, to a high temperature normal hydrogen gas at near 2,700 K chamber temperature. The required propellant properties, including *specific heat ratio* and *molecular weight*, are included in a “properties” subroutine, called by the main program. The THRST code can also compute chemical rocket performance, provided the necessary propellant gas property values, such as *molecular weight* and *specific heat ratio*, are supplied as *user input* to the code.

As an internal check on code computational accuracy, evaluation of all computed equation results are performed, and relevant code output results are presented both in English, i.e., U.S. Customary System of Units and SI (Systeme Internationale d’Unites), i.e., metric units.

Due to computations being performed in two sets of engineering units, the THRST numerical computation code utilizes over 15 subroutines with an aggregate total of over 1,000 lines of code, involving computational relationships in *isentropic flow*, *normal shock flow,* and *flow in a constant area duct with friction,* also referred to as *Fanno line flow.*

Although the primary purpose of the THRST code is to carry out rocket performance calculations for nozzle exit-to-throat area ratios ranging from ~10 to 500, it should be noted, that subsonic and transonic computations, applicable to aircraft propulsion, could also be performed for area ratios ranging from 1.0 to 1.5. Of course, just as for chemical rocket performance computations, the values for propellant gas properties, i.e., *molecular weight* and *specific heat ratio*, need to be specified as *user input* to the code.

After a brief description of the relevant equations for *thrust*, *specific impulse*, the *characteristic velocity* related to the chamber (or combustor for chemical rockets) and the nozzle *thrust coefficient*, the paper lists code *input variables* and *output parameters* shown, including units for both the English and SI systems. Finally, the report shows five sets of code “Output Results”, in tabulated form with relevant discussions.

2.0 Comments on NERVA Derived NTP System

A drawing of a typical NTP system, developed under the NERVA program, also referred to as an NTR (Nuclear Thermal Rocket) Engine system, is shown in Figure 1. It consists of a nuclear fission reactor, sized from several hundred to over 5,000 MW_t to accommodate the design thrust level, a C-D nozzle, internal and external radiation shields, a reactor pressure vessel surrounding the reactor core, control drums and reflector, turbopumps, propellant lines and miscellaneous support hardware. These NTR systems are typically based on the *expander cycle,* which have a flow diagram as shown in Figure 2.

In an expander cycle the thermal energy gained from active cooling by the propellant gas of various engine components, such as the reactor control drums and the high temperature portion of nozzle diverging duct, is used to drive the turbopump assembly (TPA), and thus provide the mechanical power required by the cycle.

As NTR designs evolved to higher chamber pressures, propellant mass flow rates, and increased engine sizes, with reactor thermal power levels approaching 5,000 MW_t additional thermal energy was required to meet the heightened power demands for the turbopump assembly. To meet this requirement, thermal energy was extracted from the core by using propellant to regeneratively cool the tie tube structural support elements and the high temperature region of the nozzle wall. The thermal energy gained from this regenerative cooling provided the power to drive the high pressure turbopumps as illustrated in Figure 2.

Regarding Figure 2, the numbers inside the circles designate the propellant flow path from the LH_2 tank (no. 1) to the exit plane of the nozzle diverging duct (no. 13). Note the path of the regenerative cooling flow, represented by 1 – 2 – 3 – 5 – 6 – 7 – 8, and 9.

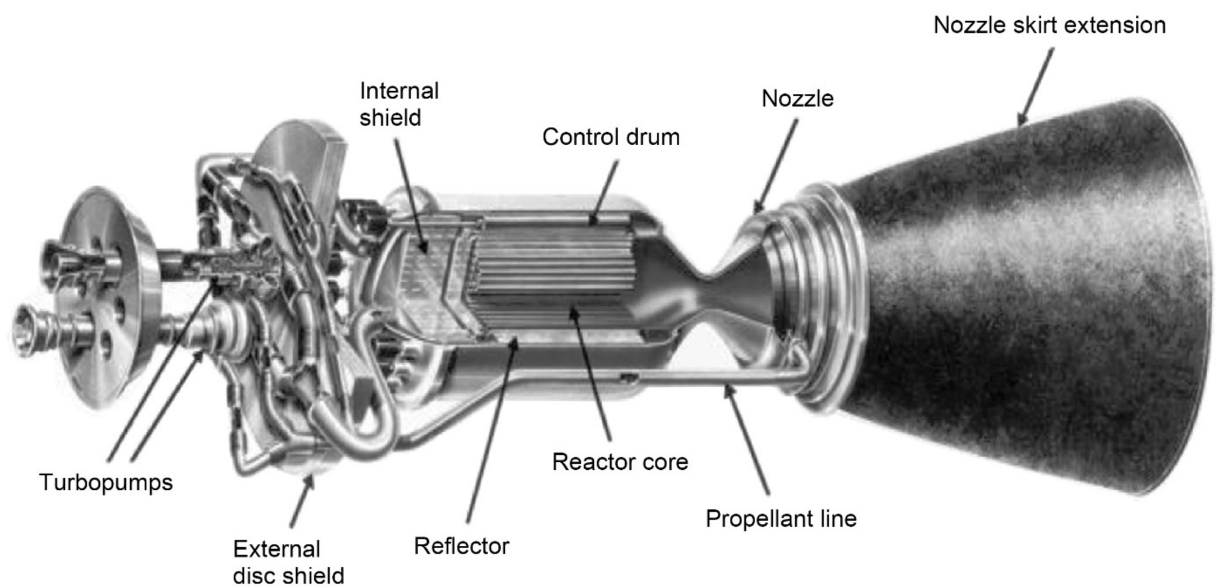

Figure 1.—Typical NTR Engine System.

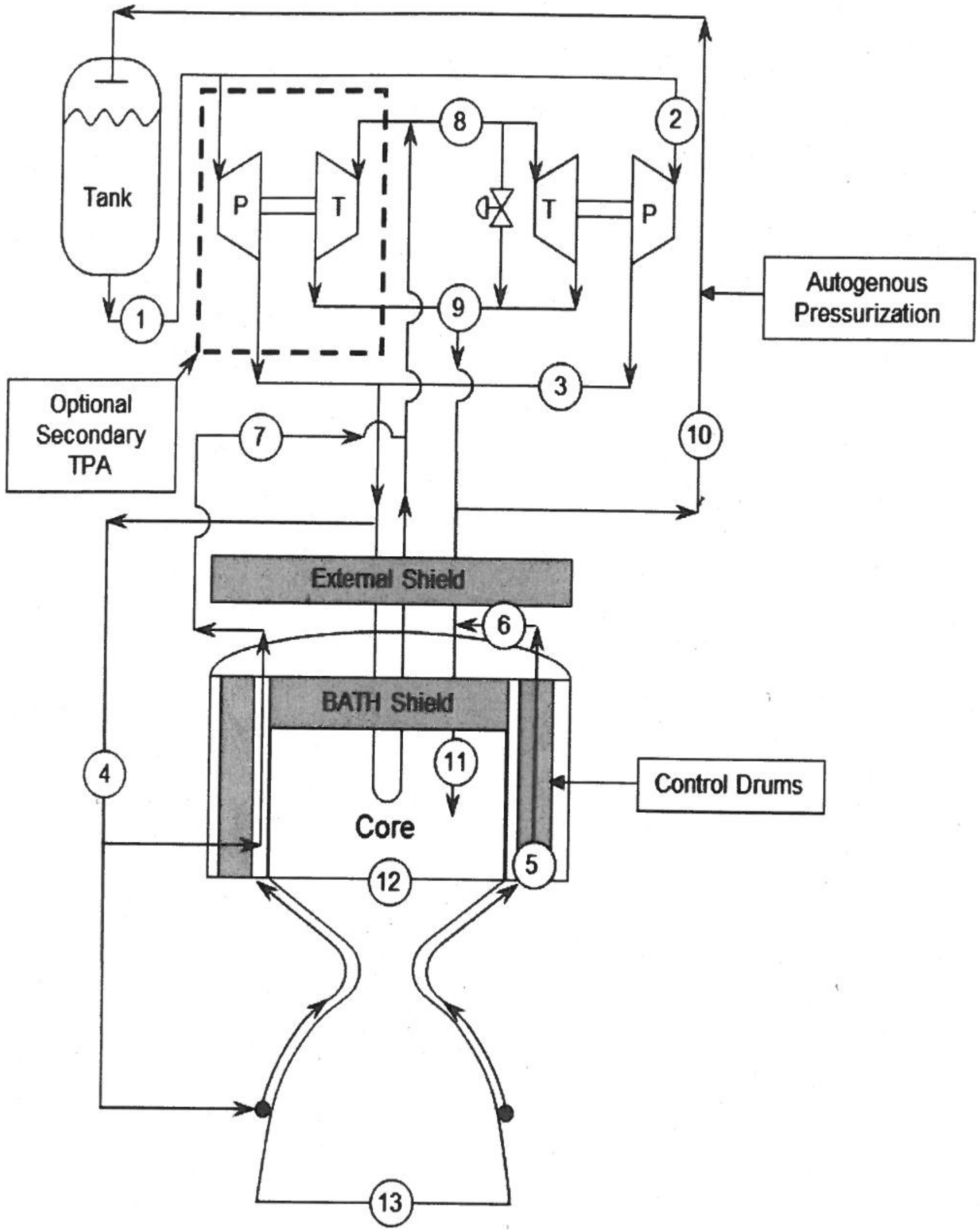

Figure 2.—Typical NTR Flow Diagram Based on Expander Cycle.

3.0 THRST Code Input Variables

AR	nozzle exit to throat area ratio, also referred to as ARGOL or A/A* (values may range from 1 to 1,000)
EFFN; η_n	nozzle efficiency (value range from 0.95 to 0.99)
GAMMA	propellant specific heat ratio at chamber, PO and TO, conditions. (Computed internally for NTP. User Input is required for chemical rocket computations.)
ISEXP	flag indicating isentropic flow computations (1 = true; 0 = not true; set internally)
LBMS	propellant mass flowrate, lb_m/sec, optional, if MSFLO = 0
MSFLO	propellant mass flowrate, kg/sec

MOLW	propellant gas molecular weight
PA	back-pressure downstream of nozzle exit plane, psia
PAI	initial external back-pressure, psia, user specified
PAO	specified external back-pressure, psia
PAF	optional final back-pressure, if other than vacuum, i.e., 0 psia
PE	pressure at nozzle exit plane, psia
PSEA	pressure at sea level (14.696 psia)
PO	chamber pressure, psia
RHOMAT	density of nozzle material, kg/m^3
TO	absolute chamber temperature, °R
TW	nozzle wall thickness, mm

Note: The code will display the numerical value of any of these variables when the name of a variable is typed in. If the numerical value of an *input variable* is to be changed, the name is typed, followed by a left arrow, (i.e., ←) and the new numerical value.

4.0 THRST Code Output Parameters (Displayed in English Units)

AE/ASTR	nozzle area ratio, dimensionless
ASTR	nozzle throat area, also referred to as ATHRT, in.
CSTAR	characteristic velocity, ft/sec
CT	nozzle thrust coefficient, dimensionless
DEX	nozzle exit diameter, ft
DTHRT	nozzle throat diameter, in.
FST	thrust developed for expansion to design back-pressure, lb_f
ISP	specific impulse, $lb_f/(lb_m/sec)$ or sec
MDOT	propellant mass flowrate, lb_m/sec
MEXH	exhaust or exit Mach number, dimensionless
MOLW	propellant gas molecular weight, dimensionless
PA	back-pressure downstream of nozzle exit plane, psia
PAI	initial specified external back-pressure, psia
PAO	specified external back-pressure, psia
PE	pressure at nozzle exit plane, psia
PO	chamber pressure, psia
TO	absolute chamber temperature, °R
TEX	absolute temperature at nozzle exit plane, °R
UEXH	nozzle exit velocity, ft/sec

5.0 THRST Code Output Parameters (Displayed in SI Units)

AE/ASTR	nozzle area ratio, dimensionless
ASTR	nozzle throat area, also referred to as ATHRT, m^2

CSTARSI	characteristic velocity, m/sec
CT	nozzle thrust coefficient, dimensionless
DEX	nozzle exit diameter, m
DTHRT	nozzle throat diameter, m
FST	thrust developed for expansion to design back-pressure, kN
ISP	specific impulse, m/sec
MACH	supersonic Mach number before shock
MACHSUBS	subsonic Mach number after shock
MDOT	propellant mass flowrate, kg/sec
MEXH	exhaust Mach number, dimensionless
LNOZ	nozzle length, m, computed in subroutine NOZWT
MOLW	propellant gas molecular weight, dimensionless
PA	back-pressure downstream of nozzle exit plane, atm
PAI	initial specified external back-pressure, atm
PAO	specified external back-pressure, atm
PE	pressure at nozzle exit plane, atm
PO	chamber pressure, atm; MPa
MWTH	reactor thermal power, MW_t
TEX	absolute temperature at nozzle exit plane, K
TO	absolute chamber temperature, K
UEXH	nozzle exit velocity, m/sec
WNOZ	nozzle weight, kg

6.0 Mathematical Relationships Used in Computations (Hill 1965)

Thrust Equation

$$\tau = \eta_n \left\{ \left[\frac{\left(A^*\right) p_o}{\sqrt{\left(\frac{R}{M}\right) T_o}} \sqrt{\gamma \left(\frac{2}{\gamma+1}\right)^{\frac{(\gamma+1)}{(\gamma-1)}}} \right] \times \left[\sqrt{\frac{2\gamma R T_o}{(\gamma-1)\mathrm{M}} \left\{ 1 - \left(\frac{p_e}{p_o}\right)^{(\gamma-1)/\gamma} \right\}} \right] + \left(p_e - p_a\right) A_e \right\} \tag{1}$$

where

A^*	choked throat area, m^2 or ft^2
A_e	nozzle exit area, m^2 or ft^2
M	molecular weight of propellant
R	universal gas constant, 8,314.34 J $kmol^{-1}K^{-1}$ or 1,545.43 ft-lb_f $lb_m^{-1}R^{-1}$
p_o	chamber pressure, MPa or psia
p_r	reference pressure along nozzle axis
p_r/p_o	critical pressure ratio
p_e	nozzle exit plane pressure, MPa or psia
p_a	external back pressure, MPa or psia

γ specific heat ratio of propellant gas

τ thrust developed by nozzle, N or lbf

η_n nozzle efficiency

Specific Impulse

$$I_{SP} = \tau / m_{dot} \text{ – (m/sec); or } (kg_f/(kg_m/sec)) \text{ – in SI units} \quad (lb_f/lb_m/sec) \text{ i.e., (sec) – in English units} \tag{2}$$

Characteristic Velocity – C^*

$$C^* = p_0 A^* / m_{dot} \tag{3}$$

Thrust Coefficient – C_t

$$C_t = \tau / (p_0 A^*) \tag{4}$$

Note: $\tau = m_{dot}\, C^*\, C_t$

Critical Pressure and Temperature Ratio

$$(p/P),\ (t/T) \tag{5}$$

$$p/P = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \tag{5a}$$

$$t/T = \frac{2}{\gamma+1} \tag{5b}$$

Reactor Thermal Power

$$R_{th} = \tau * I_{SP}\, G_{acc} / (2\times10^6 \times EFFN) \tag{6}$$

where

R_{th} reactor thermal power, MW_t

G_{acc} dimensional constant = 9.806 $kg_m\text{-}m/(kg_f\text{-}sec^2)$

EFFN nozzle efficiency, as defined in Section 3.0

7.0 Pressure Profiles in a C-D Nozzle

The behavior of gas flow in the diverging section of C-D nozzles has been described in quite a number of textbooks, some of which are listed in the “References” section, e.g., Shapiro (1953), Zucrow (1958), John and Keith (2006), and Mattingly (1996). These texts illustrate and discuss the static pressure profile behavior of compressible gas flow in a nozzle, discharging to an external environment at a pressure of p_a, with the aid of an illustration, as show in Figure 3. The horizontal line 1 at the top describes the case when there is no flow in the nozzle, and hence the back pressure p_a is identically equal to the Chamber pressure p_o as shown on the top drawing. As the back-pressure p_a is reduced below the chamber pressure $p_r = p_o$ subsonic flow develops in the nozzle, with pressure decreasing to the throat, and then increasing again to the nozzle exit plane, as indicated by curves (2) and (3). When the back-pressure is lowered to the value shown for curve (a), sonic flow occurs at the throat at the critical pressure ratio p_r / p_o. But in the diverging section of the nozzle the flow is compressed isentropically to the value represented by curve (a).

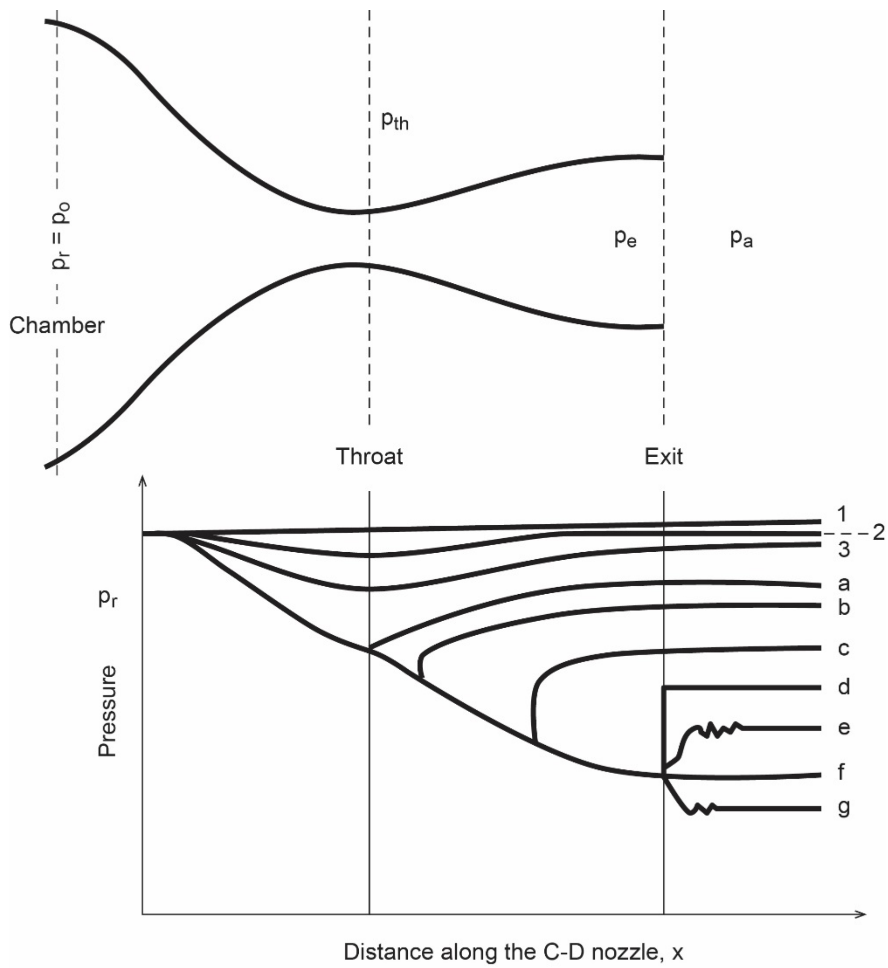

Figure 3.—C-D Nozzle Pressure Profiles Controlled by Downstream External, i.e., Back-Pressure.

Hence the flow is subsonic in both the converging and diverging sections of the nozzle, but sonic at the throat. However, since the flow is choked, meaning that the mass flowrate cannot be increased by decreasing the back-pressure below that of curve (a), sometimes this condition is referred to as the "first critical point".

As the back-pressure is lowered to the values of curve (b) and curve (c), a standing normal shock will form downstream of the throat, with the flow abruptly dropping from supersonic-to-subsonic and a loss in total pressure across the shock front. Such normal shocks are associated with significant entropy change and associated performance losses. The formation of a normal shock wave due to a highly adverse pressure gradient (caused by the shock) results in complete flow separation in the divergent section of the nozzle, downstream of the normal shock.

As the pressure ratio is lowered further, the normal shock moves further downstream toward the nozzle exit plane, until it becomes located at the exit plane, as shown in curve (d). This condition can be referred to as the second critical point. With the normal shock at the nozzle exit plane, the nozzle performance regarding thrust and I_{SP} is still adversely affected.

With the external back-pressure in the region represented by curve (e), oblique compression waves are established which tend to adjust the slightly over-expanded flow in the divergent section of the nozzle to the external pressure environment. The effect of nozzle performance is slightly negative, but the small performance penalty disappears as the back-pressure is reduced to that of curve (f).

When the back-pressure is lowered to that of curve (f), supersonic flow exists throughout the entire diverging section, with sonic flow at Mach 1, at the throat, and subsonic flow in the converging section of the nozzle. This condition is referred to as the “third critical point” in some texts. Since at this condition the external back pressure matches the pressure at the nozzle exit plane, this desirable operating condition is also referred to as the “design condition”.

For back-pressures below the value represented by curve (f), there will be a low pressure ratio expansion taking place downstream of the nozzle exit plane with a weak oblique expansion shock waves forming downstream and external to the nozzle exit plane. Since the final expansion to the existing back-pressure takes place outside the nozzle, the flow within the diverging section of the nozzle is referred to as “under expanded”. With isentropic flow existing throughout the nozzle, and adjustment to the environmental back pressure taking place downstream of the nozzle exit plane via a series of weak expansion waves at Mach > 1, the adverse effect on nozzle performance is relatively small. Thus a high area ratio nozzle with A/A* = 100, which operating at a chamber pressure of 68.05 atm or 6.89 MPa (1,000 psia), the isentropic nozzle exit plane pressure is 0.0282 atm (0.414 psia). As will be shown in Table 3, the thrust at this isentropic exit plane pressure (which matches the environmental back pressure), will be 878.7 kN (197,521 lb_f), with the I_{SP} = 896 sec.

But, as the rocket leaves the atmosphere and encounters the vacuum of Earth orbit and interplanetary space, due to the $(p_e - p_a) * A_e$ term of the Thrust Equation (1), the thrust will rise to 898.9 kN (202,075 lb_f) with an I_{SP} increase to 916.6 sec., as shown in Table 4.

8.0 Discussion of five Output Result Sets for C-D Test Case

The test case is a hypothetical nozzle of area ratio (A/A^*) =100 and a H_2 propellant mass flow of 100 kg/sec (220.46 lb_m/s). The sets of output results are the following:

1. Expansion to a specified back pressure;
2. Expansion to choked throat condition;
3. Expansion to a specified area ratio;
4. Vacuum back-pressure operation at a specified area ratio;
5. Nozzle dimensions, weight, and reactor thermal power.

Starting the discussion with the results shown in Table 1(a), the key point shown, is that a C-D nozzle with an A/A* = 100 requires (as shown in Table 3), a back-pressure of ~ 0.414 psia, for isentropic expansion. For higher pressure values a normal shock will form in the diverging section of the nozzle, as discussed in Section 7.0.

Hence, at sea level pressure a *normal shock* arises in the nozzle diverging duct at a location where the A/A^* <= 7.408 and the cross section diameter is 0.828 m (2.718 ft).

The upstream Mach number (M) is 3.313, which becomes subsonic downstream of the shock at a Mach number value of M = 0.432. As shown in Table 1(b), if one shortens the nozzle area ratio, A/A^*= 7.408 or less, then the sea level back pressure of identically 1.0 atm (14.696 psia) will be the correct isentropic value for a flow expanding from a chamber pressure of 68.05 atm (1,000 psia) and a chamber temperature of 2,650 K (4,770 °R).

Hence, as shown in Table 1(b), no normal shock will occur! Thus shortening the divergent section of the nozzle to ensure that A/A^* is slightly less than 7.408, say A/A^* = 7.407 avoids the normal shock due overexpansion within the nozzle.

However, just to perform sea level testing a high area ratio nozzle, designed to operate at high altitude and space vacuum conditions, there is low cost a way to avoid the shock related losses during sea level testing. The technique for accomplishing this objective will be described in detail during the discussion relating to Table 3 “Expansion to Specified Area Ratio”.

TABLE 1.—CODE OUTPUT (SET 1)

(a) Code output (Set 1) for nozzle area ratio = 100, showing normal shock at diameter 0.828 M (2.718 ft) on expansion from 68.05 atm (1,000 psia) to sea level pressure – 1 atm

```
NTR H2 THRUSTER CODE O U T P U T FOR VARIED C-D NOZZLE BACK PRESSURES -

STARTING BACK-PRESSURE = 14.696 PSIA

NOZZLE EFFICIENCY-(EFFN)= 0.975 ◊ ISEXP= 1 ◊ PAO= 14.696 ◊ MSFLO(KG/S)= 100.00

NOZZLE AREA RATIO (AR=ARGOL) = 100   ◊ GAS MOLECULAR WEIGHT =   2.00

1. EXPANSION TO SPECIFIED BACK PRESSURE - PAO(PSIA) = 14.696

NTR C-D NOZZLE THRUSTER - ENGLISH UNITS
```

PO(PSIA)	TO(R)	GAMMA	PE(PSIA)	PA(PSIA)	TEX(R)	MOLWT	ISP(SEC)
1000.0	4770.0	1.2980	14.696	14.696	1810.33	2.00	773.37

FST (LBF)	CT (THRST COEF)	MDOT (LBM/SEC)	ASTR (IN2)	DTHRT (IN)	MEXH	UEXH (FT/SEC)	AE/ASTR	DEX (FT)
170498.6	1.5118	220.46	112.78	11.98	3.313	25317.7	7.408	2.718

```
NTR C-D H2 NOZZLE THRUSTER - SI UNITS
```

PO(ATM) (MPA)	TO(K)	GAMMA	PE(ATM)	PA(ATM)	TEX(K)	MOLWT	ISP(M/SEC)
68.05 6.89	2650.00	1.2980	1.0000	1.0000	1005.74	2.00	7584.60

FST (KN)	CT THRST COEFF	MDOT (KG/SEC)	ATHRT (M2)	DTHRT (M)	MEXH	UEXH (M/SEC)	AE/ATHRT	DEX (M)
758.5	1.5118	100.00	.0728	.304	3.313	7716.8	7.408	.828

```
NORMAL SHOCK AT DEX(FT) =  2.718; MACH =  3.313; MACHSUBS =  .432
```

TABLE 1.—CODE OUTPUT (SET 1)

(b) Code output (Set 1) for nozzle area ratio = 7.408, showing expansion from 68.05 atm (1,000 psia) chamber pressure to Sea Level pressure of 1.0 atm (14.696 psia) without normal shock

```
NTR H2 THRUSTER CODE O U T P U T FOR VARIED C-D NOZZLE BACK PRESSURES -

STARTING BACK-PRESSURE = 14.696 PSIA

NOZZLE EFFICIENCY-(EFFN)= 0.975 ◊ ISEXP= 1 ◊ PAO= 14.696 ◊ MSFLO(KG/S)= 100.00

NOZZLE AREA RATIO (AR=ARGOL) = 7.407   ◊ GAS MOLECULAR WEIGHT =   2.00

1. EXPANSION TO SPECIFIED BACK PRESSURE - PAO(PSIA) = 14.696

NTR C-D NOZZLE THRUSTER - ENGLISH UNITS
```

PO(PSIA)	TO(R)	GAMMA	PE(PSIA)	PA(PSIA)	TEX(R)	MOLWT	ISP(SEC)
1000.0	4770.0	1.2980	14.696	14.696	1810.33	2.00	773.37

FST (LBF)	CT (THRST COEF)	MDOT (LBM/SEC)	ASTR (IN2)	DTHRT (IN)	MEXH	UEXH (FT/SEC)	AE/ASTR	DEX (FT)
170498.6	1.5118	220.46	112.78	11.98	3.313	25317.7	7.408	2.718

```
NTR C-D H2 NOZZLE THRUSTER - SI UNITS
```

PO(ATM) (MPA)	TO(K)	GAMMA	PE(ATM)	PA(ATM)	TEX(K)	MOLWT	ISP(M/SEC)
68.05 6.89	2650.00	1.2980	1.0000	1.0000	1005.74	2.00	7584.60

FST (KN)	CT THRST COEFF	MDOT (KG/SEC)	ATHRT (M2)	DTHRT (M)	MEXH	UEXH (M/SEC)	AE/ATHRT	DEX (M)
758.5	1.5118	100.00	.0728	.304	3.313	7716.8	7.408	.828

First, let us consider the expansion from chamber to choked throat condition as discussed with reference to Table 2.

As implied, the information presented in this table, deals only with the converging section of the C-D nozzle, namely the expansion from the same chamber pressure, i.e., PO = 6.89 MPa (1,000 psia), and chamber temperature, i.e., TO = 2,650 K (4,770 °R) to the nozzle throat conditions at M = 1. We see that the nozzle exit, now being at the throat and the DEX and DTHRT diameters are the same, namely 0.304 m.

The PE and PA pressures, shown as 37.16 atm (546.09 psia), and TEX = 2,306.37 (4,151.47 °R), which are obtained by multiplying the chamber conditions, PO and TO mentioned above, by the critical pressure and temperature ratio, respectively. Note that the UEXH velocity of 3,527.7 m/sec (11,574 ft/sec), is referred to as the *critical speed of sound*. It is evaluated at the above mentioned the nozzle throat temperature, TEX equal to 2,306.4 K (4,151.5 °R), with choked flow at the throat, with the Mach No. (MEXH) = 1.0.

Note also that the *Characteristic Velocity* (CSTARSI, CSTAR) and the thrust coefficient (CT), computed in Equations (3) and (4), characterize the performance of the thermal chamber, (i.e., reactor or combustor) and the nozzle, respectively.

TABLE 2.—CODE OUTPUT (SET 2) FOR A/A* = 1.0 EXPANSION FROM CHAMBER TO CHOKED THROAT CONDITION

NTR H2 THRUSTER CODE O U T P U T FOR VARIED C-D NOZZLE BACK PRESSURES -

STARTING BACK-PRESSURE = 14.696 PSIA

NOZZLE EFFICIENCY (EFFN)= 0.975 ◇ ISEXP= 1 ◇ PAO= 14.696 ◇ MSFLO(KG/S)= 100.00
NOZZLE AREA RATIO (AR=ARGOL) = 100 ◇ GAS MOLECULAR WEIGHT = 2.00

2. EXPANSION TO CHOKED THROAT CONDITION

NTR C-D NOZZLE THRUSTER - ENGLISH UNITS

PO(PSIA)	TO(R)	GAMMA	PE(PSIA)	PA(PSIA)	TEX(R)	MOLWT	ISP(SEC)
1000.0	4770.0	1.2980	546.093	546.093	4151.47	2.00	353.54

FST (LBF)	CT (THRST COEF)	MDOT (LBM/SEC)	ASTR (IN2)	DTHRT (IN)	MEXH	UEXH (FT/SEC)	AE/ASTR	DEX (FT)
77943.3	.6911	220.46	112.78	11.98	1.000	11574.0	1.000	.999

NTR C-D H2 NOZZLE THRUSTER - SI UNITS

PO(ATM) (MPA)	TO(K)	GAMMA	PE(ATM)	PA(ATM)	TEX(K)	MOLWT	ISP(M/SEC)
68.05 6.89	2650.00	1.2980	37.1593	37.1593	2306.37	2.00	3467.29

FST (KN)	CT THRST COEFF	MDOT (KG/SEC)	ATHRT (M2)	DTHRT (M)	MEXH	UEXH (M/SEC)	AE/ATHRT	DEX (M)
346.7	.6911	100.00	.0728	.304	1.000	3527.7	1.000	.304

2.1 CHARACTERISTIC VELOCITY - CSTAR (FT/SEC) = 16328.6; CSTARSI (M/SEC) = 4976.9

Returning now to Table 3, for sea level testing of a high area ratio nozzle of a nuclear propulsion system. Rather than destroying the divergent section of a C-D nozzle (i.e., with an area ratio of $A/A^* = 100$ for example), by shortening it to $A/A^* = 7.408$, consider the following alternative: The nozzle has a throat diameter of 0.304 m (11.98 in.) for the conditions of subject example, just as does the $A/A^* = 7.408$ nozzle. But the exit diameter for the $A/A^* = 100$ nozzle has increased to $0.304\times10 = 3.044$ m (9.99 ft), compared to 0.828 m (2.718 ft) for the $A/A^* = 7.408$ nozzle. Concentrically inserting a constant area pipe with a constant area pipe with a precision ground internal diameter of 0.828 m (2.718 ft) and a smooth transition seal, into the diverging section of the larger nozzle to match the 0.828 m diameter cross section, would avoid formation of a normal shock.

Using *Fanno-flow* (i.e., flow in a constant area duct with friction) calculations, with assumption of a smooth surface low friction factor ($f = 0.005$), one could determine the corrected sea level thrust based on the exit gas velocity of the constant area duct, i.e., pipe, having an inlet Mach number, M = 3.313. The exit Mach number may drop to M ~ 3.2 or lower, depending on the length and internal surface finish of the constant area pipe. As mentioned above, the exact L/D value can be determined via *Fanno-flow* calculations, also referred to as "frictional flow in a constant area (cylindrical) duct". Knowing that for supersonic flow at the duct inlet (i.e., $M > 1/\gamma^{0.5}$), the velocity and Mach number decreases, a brief demo-calculation to compute the length of the constant area duct for a C-D NTP nozzle flowing H_2 propellant includes the following steps:

1. Assume friction factor $f = 0.005$ as indicated above
2. For $M_1 = 3.313$ at inlet of duct with $\gamma = 1.298$, the $4fL_1/D$ value = 0.68562
3. For $M_2 = 3.200$ assumed at the exit ($\gamma = 1.298$), the $4fL_2/D$ value = 0.66685
4. $fL_1/D - 4fL_2/D = 0.01877$
5. So $\Delta(L/D) = 0.01877/(4*.005) = 0.94$

TABLE 3.—CODE OUTPUT (SET 3)—EXPANSION TO SPECIFIED AREA RATIO

```
NTR H2 THRUSTER CODE O U T P U T FOR VARIED C-D NOZZLE BACK PRESSURES -

STARTING BACK-PRESSURE = 14.696 PSIA

NOZZLE EFFICIENCY (EFFN)= 0.975 ◊ ISEXP= 1 ◊ PAO= 14.696 ◊ MSFLO(KG/S)= 100.00
NOZZLE AREA RATIO (AR=ARGOL) = 100   ◊ GAS MOLECULAR WEIGHT =   2.00

3. EXPANSION TO SPECIFIED AREA RATIO - AR = 100

NTR C-D NOZZLE THRUSTER - ENGLISH UNITS
```

PO(PSIA)	TO(R)	GAMMA	PE(PSIA)	PA(PSIA)	TEX(R)	MOLWT	ISP(SEC)
1000.0	4770.0	1.2980	.414	.414	797.83	2.00	895.94

FST (LBF)	CT (THRST COEF)	MDOT (LBM/SEC)	ASTR (IN2)	DTHRT (IN)	MEXH	UEXH (FT/SEC)	AE/ASTR	DEX (FT)
197521.1	1.7514	220.46	112.78	11.98	5.781	29330.3	100.000	9.986

```
NTR C-D H2 NOZZLE THRUSTER - SI UNITS
```

PO(ATM) (MPA)	TO(K)	GAMMA	PE(ATM)	PA(ATM)	TEX(K)	MOLWT	ISP(M/SEC)
68.05 6.89	2650.00	1.2980	.0282	.0282	443.24	2.00	8786.69

FST (KN)	CT THRST COEFF	MDOT (KG/SEC)	ATHRT (M2)	DTHRT (M)	MEXH	UEXH (M/SEC)	AE/ATHRT	DEX (M)
878.7	1.7514	100.00	.0728	.304	5.781	8939.9	100.000	3.044

Hence, with the duct diameter being 0.828 m (2.718 ft), as shown in Table 1(a), the duct length for the exit Mach number to drop to $M_2 = 3.2$, is 0.94*0.828 m = 0.778 m (2.55 ft). Note that, as the friction factor f is halved, the Δ (L/D) is doubled for the same M_2.

Note also, as the M_2 is lowered at the same f, the Δ (L/D) will increase. A conceptual installation of a constant area duct in the divergent section of a high area ratio nozzle is shown in Figure 4. Note that the cylindrical duct is positioned in the divergent section of the nozzle by four centering fins fastened to the cylinder's outer surface with retainer screws, at 90° intervals.

As noted, the above calculation for the constant area duct L/D is for a typical NTP case as shown in Table 1(a).

But, suppose that we are interested in a chemical propulsion system that is designed to operate from sea level to about 40,000 m (~130,000 ft) altitude where the *Stage 1* thrusters are jettisoned and begin fall back to ground level with sufficient reserve fuel (propellant). With an area ratio designed for operation near sea level, the thruster is capable of decelerating and landing the nearly empty stage in a vertical position, as is done following Space X Falcon Launches. Since the propellant for the Merlin thrusters Falcon thrusters is a combustion products mixture of hydrocarbon fuel and O_2, and the ideal area ratio, A/A*, for sea level operation is 16.0.

Although safety protocols prohibit nuclear propulsion stages from operating in the Earth's atmosphere, the isentropic area ratio (A/A*) for complete expansion of H_2 propellant from chamber pressure to sea level environmental back pressure, would be identical to 7.408 as shown in Table 1(a).

Hence, for the operating conditions assumed in this example, an area ratio of 7.408 would ensure ideal nozzle operation at sea (ground) level with the external back-pressure being equal to 1.0 atm (0.10133 MPa or 14.696 psia).

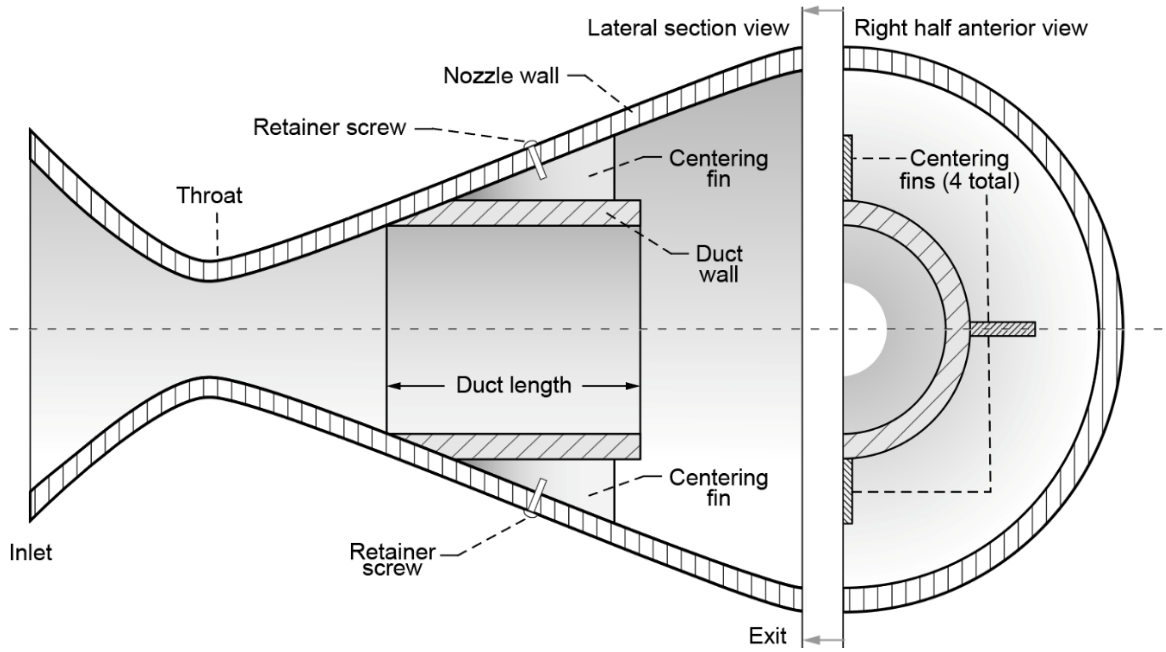

Figure 4.—Location of Constant Area Duct in Nozzle Divergent Section.

Admittedly such a nozzle would operate in a highy *under-expanded* condition at high altitude, but it would not encounter any thrust destroying normal shocks at sea level.

Recall that in the discussion of Table 1(a) it was mentioned that for an area ratio, A/A^* of 100 the nozzle exit pressure or, back pressure, needed to be much less than atmospheric, i.e., 14.696 psia, to avoid formation of a *normal* shock. Table 3 shows that the value of this pressure for ideal expansion at the nozzle exit plane equal to the background pressure is, as shown above, for PE = PA = 0.0282 atm (0.414 psia), which is the pressure at near 79,000 ft altitude in the Earth's atmosphere.

The thrust (FST) is shown as 878.7 kN (197,521 lb_f) and the I_{SP} = 8,786.7 m/sec in SI, and 895.9 lb_f/(lb_m/sec), or sec in English units. The reason for the different form of the ISP units in SI and English is that starting with the English, I_{SP} = 895.9 lb_f/(lb_m/sec), an identical SI value would be 895.9 kg_f/(kg_m/sec). Multiplying this value by the dimensional constant, GACC= 9.8066 kg_m- m/(kg_f - sec^2), the result becomes 8,786.69 m/sec, as shown above for I_{SP} in SI units. The decision to define I_{SP}, as shown above, by the SI community was arbitrary. As a matter of fact, some papers (e.g., Fortini et al. (1959)) quote the value of I_{SP} as lb-sec/lb in the English system.

Included in the output results of Table 3 and Table 4 are values of the thrust (FST) and I_{SP}. These values were obtained under complete expansion to 0.0282 atm (0.414 psia), at the nozzle exit plane. Note that in Table 3 the exit plane pressure, PE, is equal to the back-pressure, PA, at the value 0.0282 atm (0.414 psia), as stated above. In Table 4 the increase in thrust and specific impulse is displayed for the vacuum back pressure condition, with PE still at 0.0282 atm (0.414 psia), but with PA now equal to zero. Note that the thrust has increased from 878.7 kN (197,521 lb_f) to 898.9 kN (202,075 lb_f), due to the $(p_e - p_a)*A_e$ term, as shown in Equation (1) in Section 6.0, "Mathematical Relationships Used in Computations (Hill 1965)."

Code Output Set 5 of Table 4, displays salient data on the nozzle dimensions, including total nozzle weight, based on wall thickness and material density, e.g., for titanium alloy in this case. The value for net thrust (kg_f) or (kN) is determined by deducting the nozzle weight from total thrust. Reactor thermal power (MW_t) is computed, as shown by Equation (6) in Section 6.0.

TABLE 4.—CODE OUTPUT (SET 4 AND 5) EXPANSION TO SPECIFIED AREA RATIO AT VACUUM BACK PRESSURE AND NOZZLE DIMENSIONS AND WEIGHT; REACTOR THERMAL POWER

```
    NTR H2 THRUSTER CODE O U T P U T FOR VARIED C-D NOZZLE BACK PRESSURES -

                      STARTING BACK-PRESSURE = 14.696 PSIA

NOZZLE EFFICIENCY(EFFN)= 0.975 ◊ ISEXP= 1 ◊ PAO= 14.696 ◊ MSFLO(KG/S)= 100.00
    NOZZLE AREA RATIO (AR=ARGOL) = 100   ◊ GAS MOLECULAR WEIGHT =   2.00

        4. VACUUM BACK PRESSURE OPERATION AT AREA RATIO - AR = 100


  NOZZLE EFFICIENCY(EFFN)= 0.975 ◊ ISEXP= 0  ◊ PA= 0  ◊ MSFLO(KG/S)= 100.000

              NTR C-D NOZZLE THRUSTER - ENGLISH UNITS

 PO(PSIA)    TO(R)  GAMMA   PE(PSIA)  PA(PSIA)   TEX(R)   MOLWT     ISP(SEC)
   1000.0   4770.0  1.2980      .414      .000   797.83    2.00      916.60

    FST        CT         MDOT    ASTR  DTHRT   MEXH    UEXH    AE/ASTR     DEX
   (LBF)  (THRST COEF)(LBM/SEC) (IN2)   (IN)          (FT/SEC)              (FT)
  202075.0  1.7917     220.46  112.78  11.98  5.781  29330.3   100.000    9.986

              NTR C-D H2 NOZZLE THRUSTER - SI UNITS

 PO(ATM)    TO(K)  GAMMA    PE(ATM)   PA(ATM)    TEX(K)     MOLWT   ISP(M/SEC)
   (MPA)
   68.05  2650.00 1.2980     .0282     .0000     443.24      2.00     8989.27
   6.89
    FST      CT       MDOT     ATHRT      DTHRT   MEXH     UEXH   AE/ATHRT    DEX
   (KN)     THRST   (KG/SEC)   (M2)       (M)             (M/SEC)             (M)
            COEFF
   898.9  1.7917    100.00    .0728      .304   5.781   8939.9  100.000   3.044
------------------------------------------------------------------------------
              5. C-D NOZZLE DIMENSIONS AND WEIGHT

 C-D NOZZLE WEIGHT (KG) =    286.3; AT MATERIAL DENSITY(KG/M3)=  4000.0
  C-D NOZZLE LENGTH (M) =    4.87;         WALL THICKNESS(MM) =     2.5
C-D EXIT CYL. CONST.-C2EX =   .40;      NET THRUST-KGF;(KN)  = 91384.8;(  896.2)
------------------------------------------------------------------------------

              REACTOR THERMAL POWER(MWT) = 4091.56
```

9.0 THRST Code Results for Alternate NTR Designs

The THRST Code Output results shown in Tables 1 to 4 represented design and performance values for a relatively large NTR, generating approximately 900 kN (200,000 lb_f) of thrust, with thermal power of over 4,000 MW delivered by a full sized NERVA-type reactor.

In Table 5 the reader can study a complete set of results for a smaller NTR propulsion system, which is sized for 75 kN (15,000 lb_f) of thrust, by limiting the the H_2 propellant flow to 8.43 kg/sec (i.e., MSFLO = 8.43). For maximizing specific impulse, I_{SP}, a relatively large area ratio of = 300 (i.e., A/A* = 300) was selected for this demonstration test case.

The reactor thermal power required for this design can be provided by a much smaller H_2 gas reactor, namely in the 360 MW_t range.

TABLE 5.—CODE OUTPUT FOR A 75 kN NTR ENGINE

```
NTR H2 THRUSTER CODE O U T P U T FOR VARIED C-D NOZZLE BACK PRESSURES -

                STARTING BACK-PRESSURE = 14.696 PSIA

NOZZLE EFFICIENCY-(EFFN)= 0.95 ◊ ISEXP= 1 ◊ PAO= 14.696 ◊ MSFLO(KG/S)=   8.43

   NOZZLE AREA RATIO (AR=ARGOL) = 300   ◊ GAS MOLECULAR WEIGHT =   2.00

        1. EXPANSION TO SPECIFIED BACK PRESSURE - PAO(PSIA) = 14.696

           NTR C-D NOZZLE THRUSTER - ENGLISH UNITS

 PO(PSIA)    TO(R)  GAMMA   PE(PSIA)  PA(PSIA)   TEX(R)   MOLWT     ISP(SEC)
  1000.0    4930.2  1.2955   14.696    14.696  1883.00    2.00       767.11

   FST       CT          MDOT     ASTR  DTHRT   MEXH    UEXH    AE/ASTR     DEX
  (LBF) (THRST COEF)(LBM/SEC) (IN2)   (IN)          (FT/SEC)               (FT)
  14256.7  1.4739      18.58     9.67   3.51  3.310  25773.6     7.440     .798

           NTR C-D H2 NOZZLE THRUSTER - SI UNITS

 PO(ATM)    TO(K)   GAMMA     PE(ATM)   PA(ATM)     TEX(K)     MOLWT  ISP(M/SEC)
  (MPA)
  68.05   2739.00 1.2955     1.0000     1.0000     1046.11      2.00     7523.19
   6.89
   FST      CT       MDOT    ATHRT       DTHRT    MEXH    UEXH   AE/ATHRT    DEX
  (KN)     THRST   (KG/SEC)  (M2)         (M)             (M/SEC)             (M)
           COEFF
   63.4  1.4739       8.43   .0062        .089   3.310   7855.8    7.440     .243

NORMAL SHOCK AT DEX(FT) =    .798; MACH =  3.310; MACHSUBS =  .432
--------------------------------------------------------------------------------

        2. EXPANSION TO CHOKED THROAT CONDITION

           NTR C-D NOZZLE THRUSTER - ENGLISH UNITS

 PO(PSIA)    TO(R)  GAMMA   PE(PSIA)  PA(PSIA)   TEX(R)   MOLWT     ISP(SEC)
  1000.0    4930.2  1.2955  546.549   546.549  4295.61    2.00       350.07

   FST       CT          MDOT     ASTR  DTHRT   MEXH    UEXH    AE/ASTR     DEX
  (LBF) (THRST COEF)(LBM/SEC) (IN2)   (IN)          (FT/SEC)               (FT)
  6506.0    .6726      18.58     9.67   3.51  1.000  11761.7     1.000     .292

           NTR C-D H2 NOZZLE THRUSTER - SI UNITS

 PO(ATM)    TO(K)   GAMMA     PE(ATM)   PA(ATM)     TEX(K)     MOLWT  ISP(M/SEC)
  (MPA)
  68.05   2739.00 1.2955    37.1903    37.1903     2386.45      2.00     3433.20
   6.89
   FST      CT       MDOT    ATHRT       DTHRT    MEXH    UEXH   AE/ATHRT    DEX
  (KN)     THRST   (KG/SEC)  (M2)         (M)             (M/SEC)             (M)
           COEFF
   28.9    .6726      8.43   .0062        .089   1.000   3585.0    1.000     .089
--------------------------------------------------------------------------------

2.1 CHARACTERISTIC VELOCITY - CSTAR (FT/SEC) = 16611.8; CSTARSI (M/SEC) = 5063.3
```

TABLE 5.—Concluded.

3. EXPANSION TO SPECIFIED AREA RATIO - AR = 300

NTR C-D NOZZLE THRUSTER - ENGLISH UNITS

PO (PSIA)	TO (R)	GAMMA	PE (PSIA)	PA (PSIA)	TEX (R)	MOLWT	ISP (SEC)
1000.0	4930.2	1.2955	.097	.097	599.67	2.00	914.48

FST (LBF)	CT (THRST COEF)	MDOT (LBM/SEC)	ASTR (IN2)	DTHRT (IN)	MEXH	UEXH (FT/SEC)	AE/ASTR	DEX (FT)
16995.6	1.7571	18.58	9.67	3.51	6.992	30725.2	300.000	5.065

NTR C-D H2 NOZZLE THRUSTER - SI UNITS

PO (ATM) (MPA)	TO (K)	GAMMA	PE (ATM)	PA (ATM)	TEX (K)	MOLWT	ISP (M/SEC)
68.05 6.89	2739.00	1.2955	.0066	.0066	333.15	2.00	8968.53

FST (KN)	CT THRST COEFF	MDOT (KG/SEC)	ATHRT (M2)	DTHRT (M)	MEXH	UEXH (M/SEC)	AE/ATHRT	DEX (M)
75.6	1.7571	8.43	.0062	.089	6.992	9365.0	300.000	1.544

---

4. VACUUM BACK PRESSURE OPERATION AT AREA RATIO - AR = 300

NOZZLE EFFICIENCY(EFFN)= 0.95 ◊ ISEXP= 0 ◊ PA= 0 ◊ MSFLO(KG/S) = 8.430

NTR C-D NOZZLE THRUSTER - ENGLISH UNITS

PO (PSIA)	TO (R)	GAMMA	PE (PSIA)	PA (PSIA)	TEX (R)	MOLWT	ISP (SEC)
1000.0	4930.2	1.2955	.097	.000	599.67	2.00	928.92

FST (LBF)	CT (THRST COEF)	MDOT (LBM/SEC)	ASTR (IN2)	DTHRT (IN)	MEXH	UEXH (FT/SEC)	AE/ASTR	DEX (FT)
17264.0	1.7849	18.58	9.67	3.51	6.992	30725.2	300.000	5.065

NTR C-D H2 NOZZLE THRUSTER - SI UNITS

PO (ATM) (MPA)	TO (K)	GAMMA	PE (ATM)	PA (ATM)	TEX (K)	MOLWT	ISP (M/SEC)
68.05 6.89	2739.00	1.2955	.0066	.0000	333.15	2.00	9110.16

FST (KN)	CT THRST COEFF	MDOT (KG/SEC)	ATHRT (M2)	DTHRT (M)	MEXH	UEXH (M/SEC)	AE/ATHRT	DEX (M)
76.8	1.7849	8.43	.0062	.089	6.992	9365.0	300.000	1.544

---

5. C-D NOZZLE DIMENSIONS AND WEIGHT

```
C-D NOZZLE WEIGHT (KG) =     129.2; AT MATERIAL DENSITY(KG/M3)=  4500.0
 C-D NOZZLE LENGTH (M) =      2.47;        WALL THICKNESS(MM) =     4.0
C-D EXIT CYL. CONST.-C2EX =    .40;     NET THRUST-KGF;(KN)   =  7702.6;(    75.5)
```

REACTOR THERMAL POWER(MWT) = 358.89

Since the mass flow rate for this case is reduced from 100 kg/sec for the 900 kN (200,000 lbf) thruster to a mere 8.43 kg/sec, much smaller thruster dimensions, including reactror, engine system, and C-D nozzle are specified by code output results. While a normal shock will still occur at an area ratio of ~7.44 for operation at sea level back pressure at the same area ratio as for the high thrust case, the nozzle exit diameter has been reduced from 0.83 m (2.718 ft) to 0.243 m (0.798 ft). The sea level thrust at the 7.44 area ratio is 63.4 kN (14,257 lbf), while the thrust at space vacuum at the specified area ratio of 300 (A/A* = 300) will rise to 76.8 kN (17,264 lbf). The reader can confirm these performance values by comparing the Set 1 code output results (i.e., expansion to a specified back pressure – PAO (psia) = 14.696 to those of Output Set 4 (i.e., vacuum back pressure operation at AR = 300).

10.0 Concluding Remarks

A computational code, referred to as THRST, has been developed principally for analysis and conceptual design of high area ratio C-D nozzles, applicable to Nuclear Thermal Propulsion (NTP), utilizing nuclear fission thermal rockets (NTR) with supercritical hydrogen (H_2) propellant, as initially developed for the NERVA-Phoebus program. But the THRST Code can also evaluate chemical rocket performance with propellants being combustion product mixtures of hydrogen or hydrocarbon fuels and oxygen.

These rockets utilize high thermal power fission reactors, capable of heating supercritical parahydrogen (H_2) propellant from a liquid state at near 20 K, to a high temperature gas at near 2,700 K chamber temperature. The required propellant properties, including *specific heat ratio* and *molecular weight*, are included in a properties subroutine, called by the main program. The THRST code can also compute chemical rocket performance, provided the necessary propellant gas property values, such as *propellant molecular weight* and *specific heat ratio*, are supplied as "User Input" to the code.

As an internal check on code computational accuracy, evaluation of all computed equation results are performed, and relevant code output results are presented in both English and SI units. The Code is also applicable for aircraft propulsion applications in the subsonic-to-transonic region, with nozzle area ratios ranging from 1.0 to 1.5, and propellant gas properties changed to air – and hydrocarbon fuel combustion product mixtures.

Code capabilities include performance at vacuum or near vacuum environmental pressures as well as atmospheric pressures down to sea level or higher, applicable for ground testing using bore-holes. With high area ratio nozzles being run at elevated environmental back pressures the code will display at what A/A* location in the divergent section of the nozzle a *normal shock* is established. Using normal shock flow computations the supersonic Mach numbers upstream of the shock and the subsonic values downstream are also listed, along with static pressure values upstream and downstream of the shock front.

A limit on current THRST Code capabilities is that, for under-expanded flows the effect of oblique expansion waves is not included, nor is the effect of oblique compression waves for slightly over-expanded flows. However, the thrust reduction even at values of $1 > p_a / p_e > 0.5$ will be approximately 1 to 2 percent (Sutton, 1956).

The code has been extensively "Beta" tested for the high area ratio nozzle NTP operating mode with high temperature H_2 propellant by essentially reproducing results displayed in the Fittje et al. (2014). Note that for this reference, results were generated by application of the commercially available National Propulsion System Simulation (NPSS) Code.

But excellent validation results were also obtained for chemical rockets with propellants formed by combustion products of hydrogen and oxygen or hydrocarbon fuels and oxygen as documented in reference by Fortini et al. (1959), or the texts by Sutton (1956) and John and Keith's (2006) "Gas Dynamics".

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