Adjustments and Uncertainty Quantification for SLS Aerodynamic Sectional Loads

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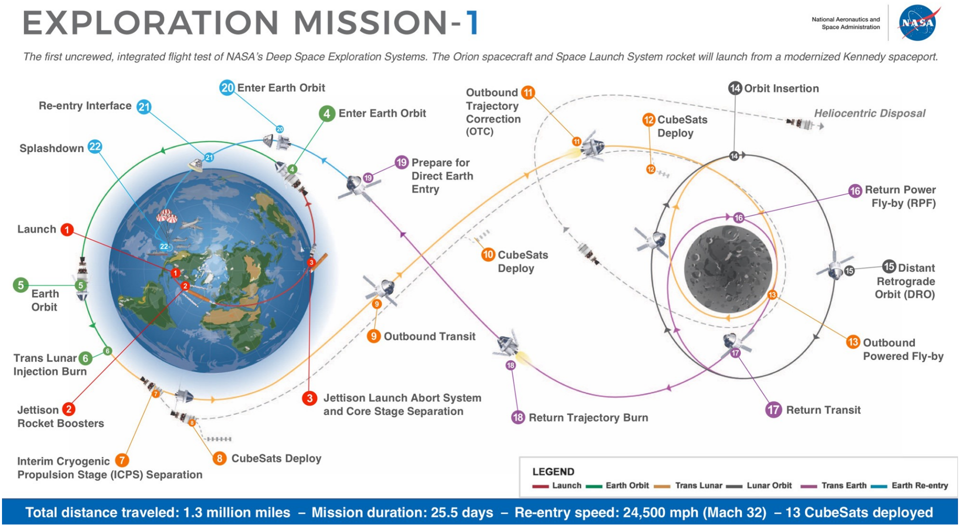
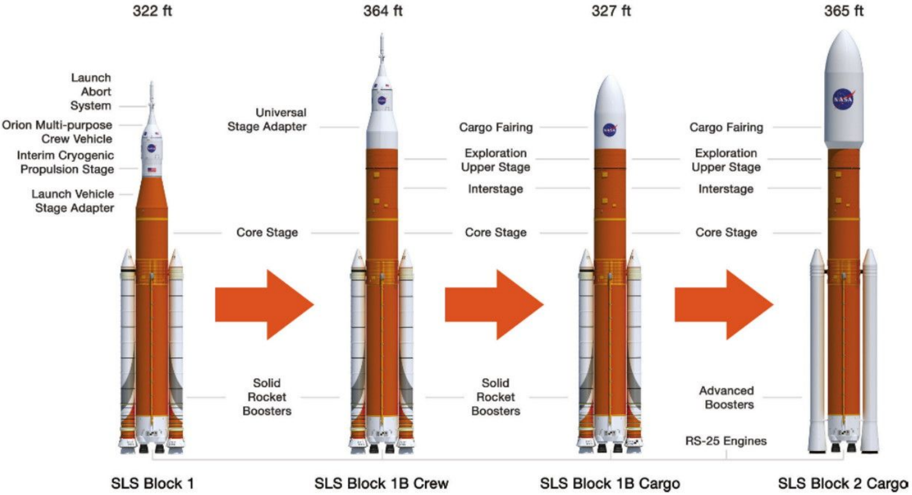
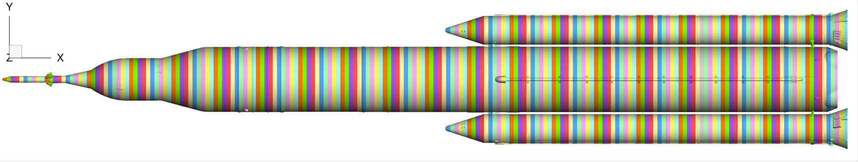
The Space Launch System (SLS): EM-1 mission map



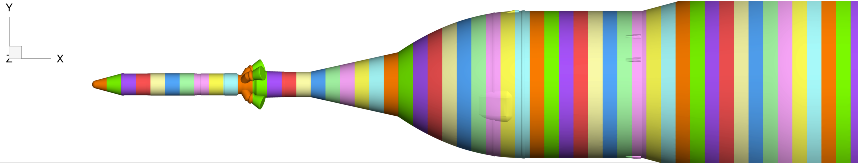
Space Launch System multiple configurations



Sectional Loads/Line Loads

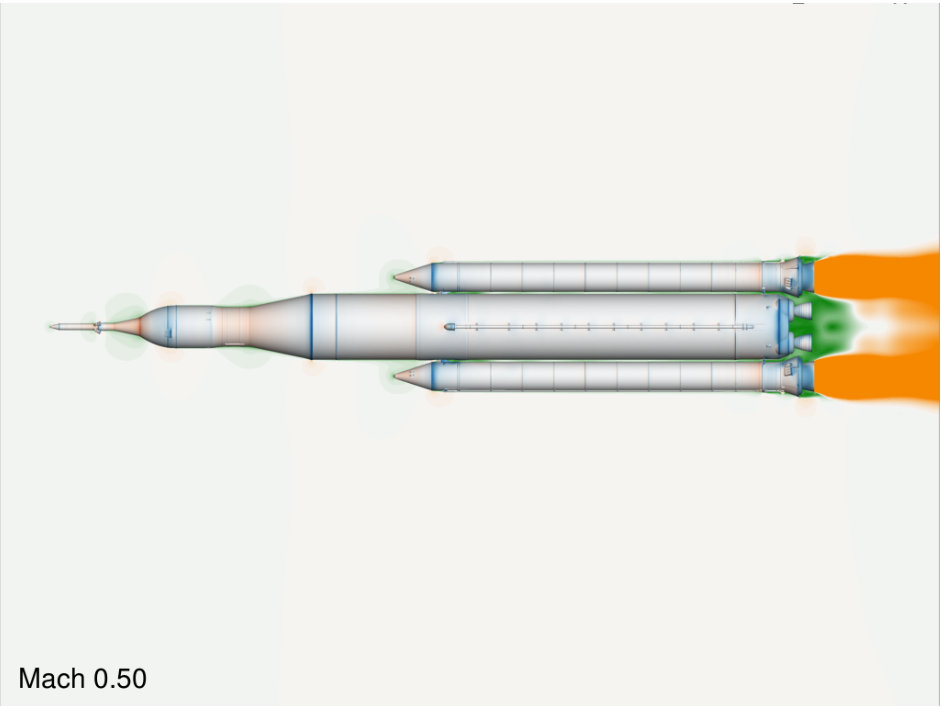


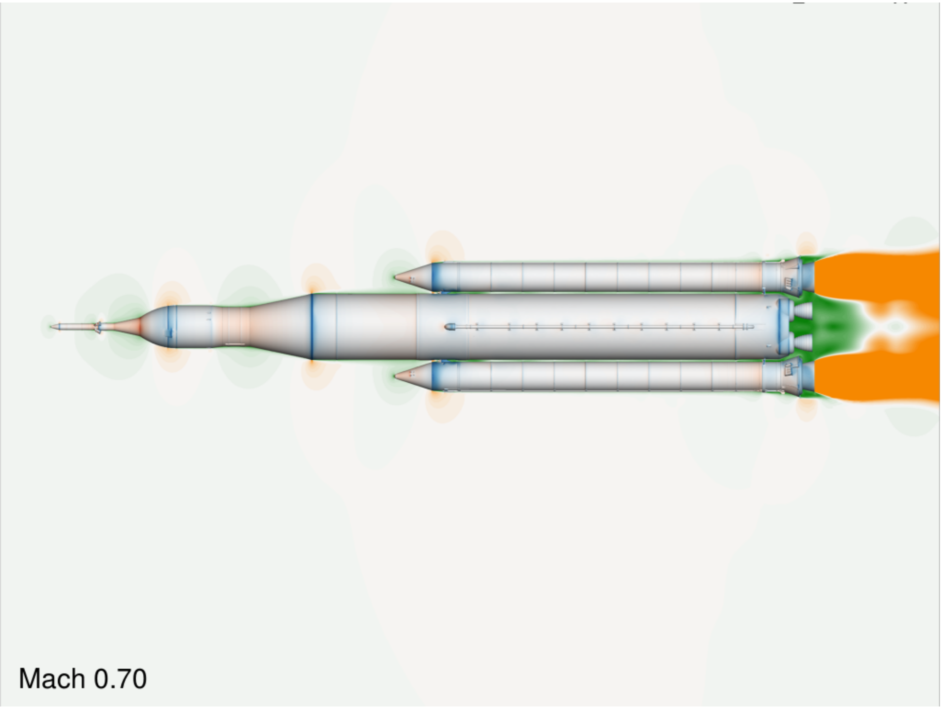
Sectional load slices for SLS Block 1B Crew configuration



Sectional load slices on forward portion of SLS Block 1B

- Line loads are a simple tool to interface aero loads and vehicle structures by dividing vehicle into a number of slices
- Calculate the load on each slice
- Valid for long/skinny vehicles


Mach 0.50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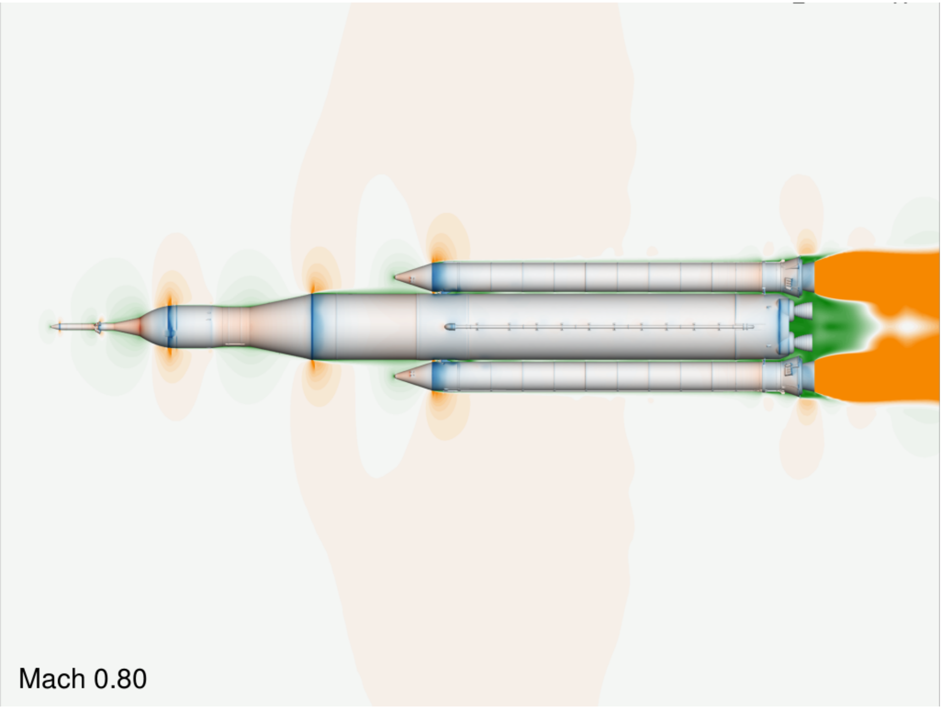
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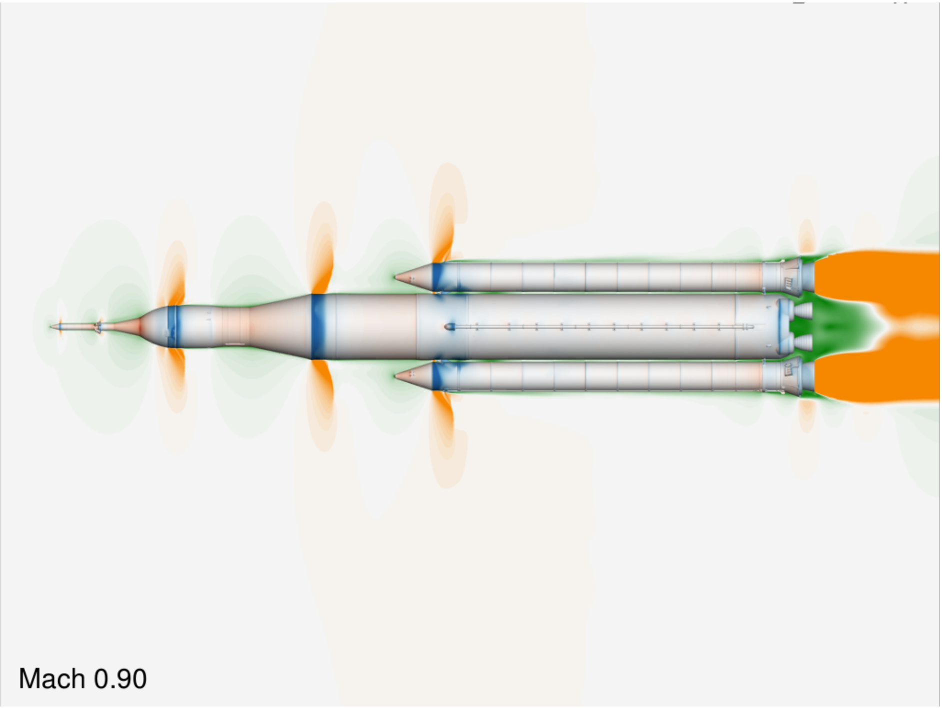
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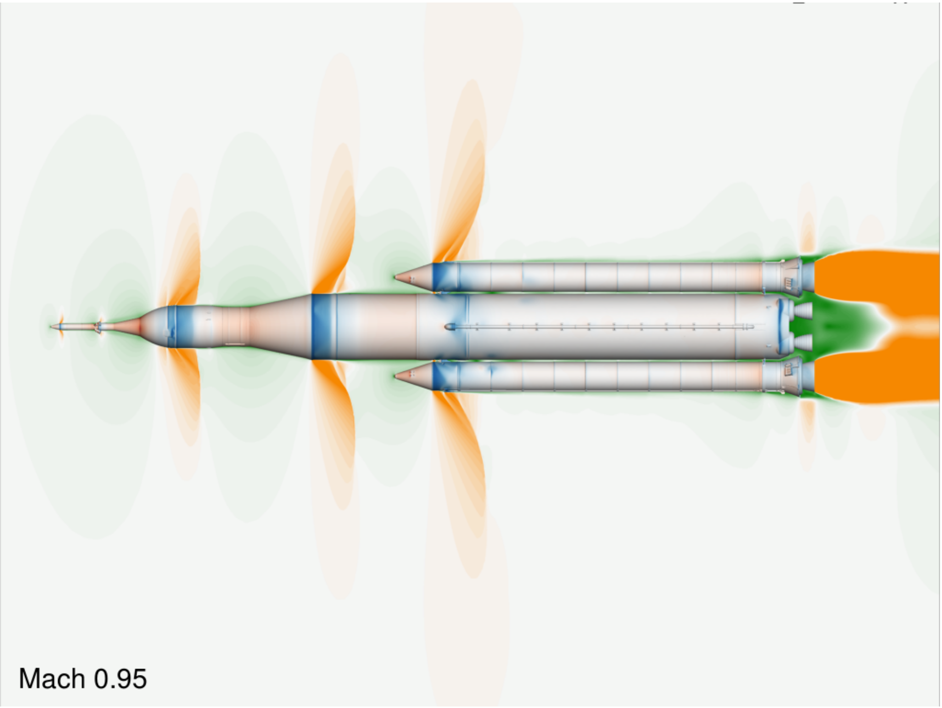
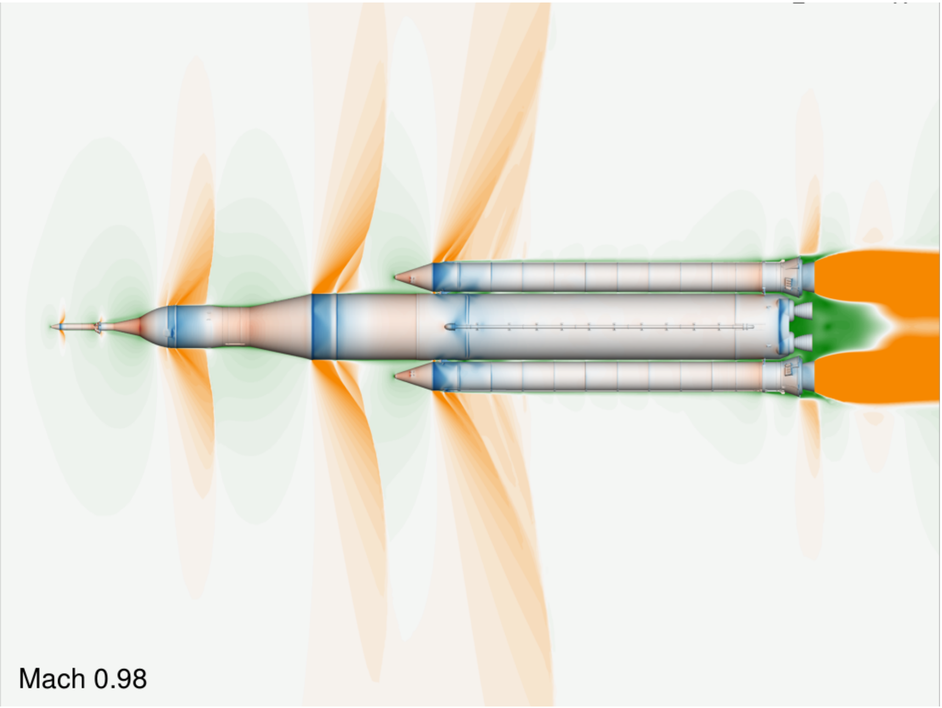

Mach 0.95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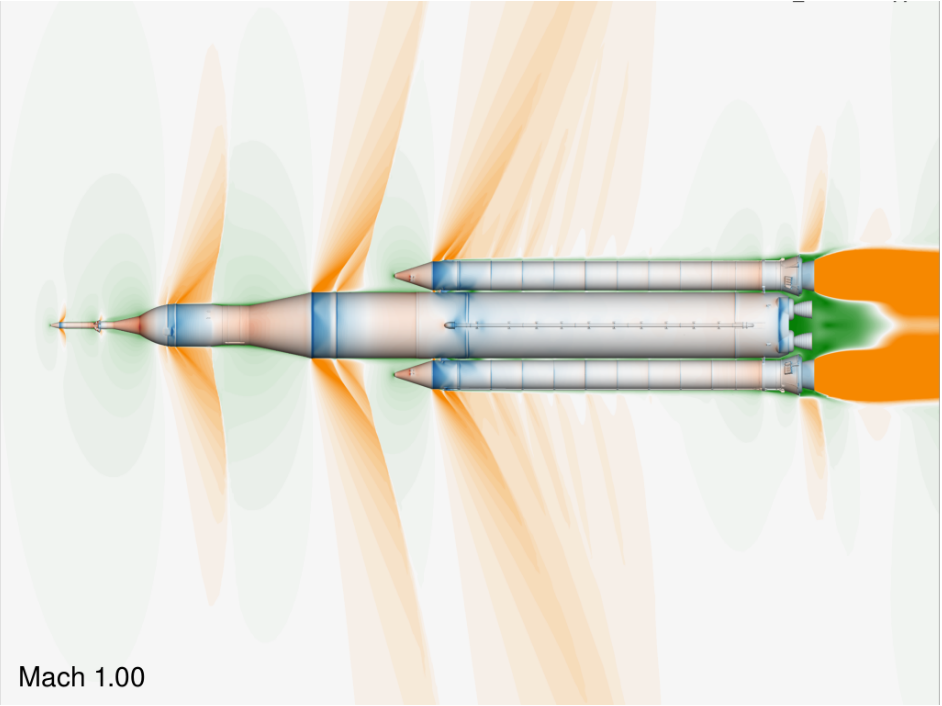
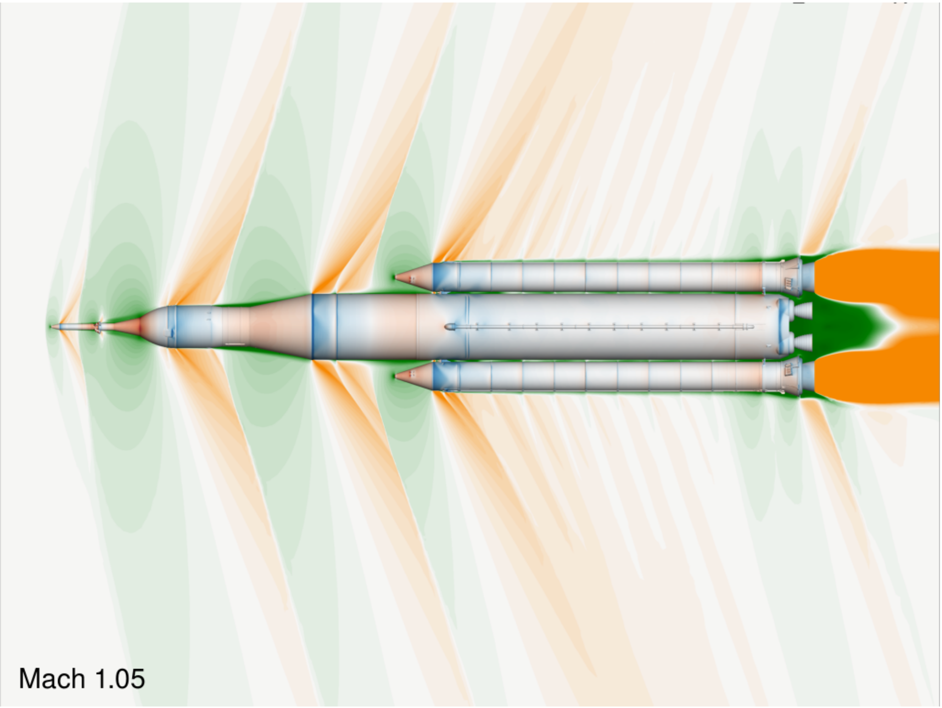

Mach 1.00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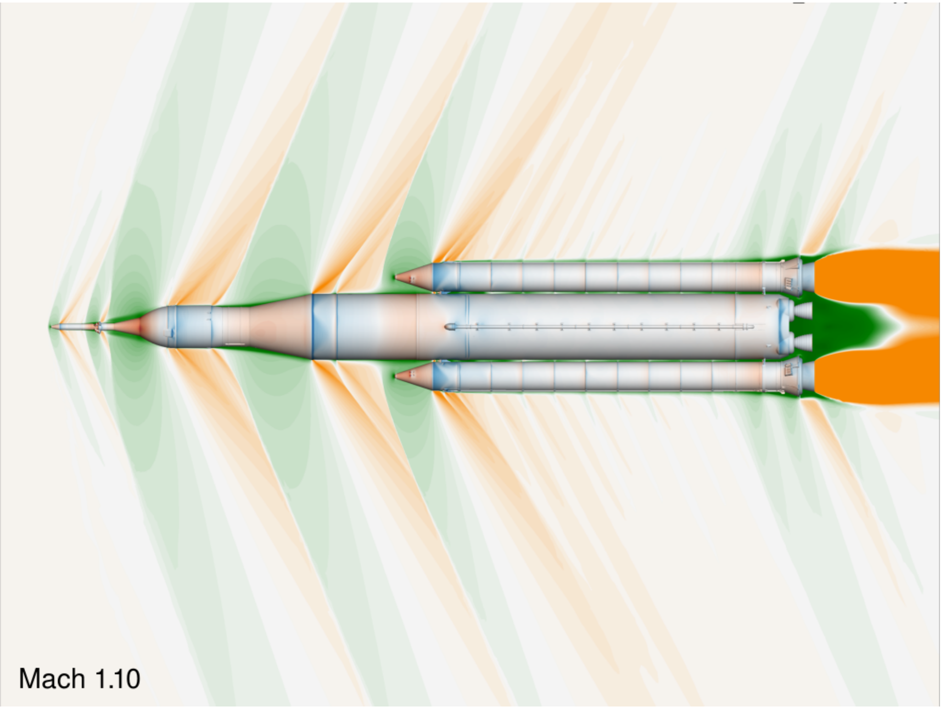
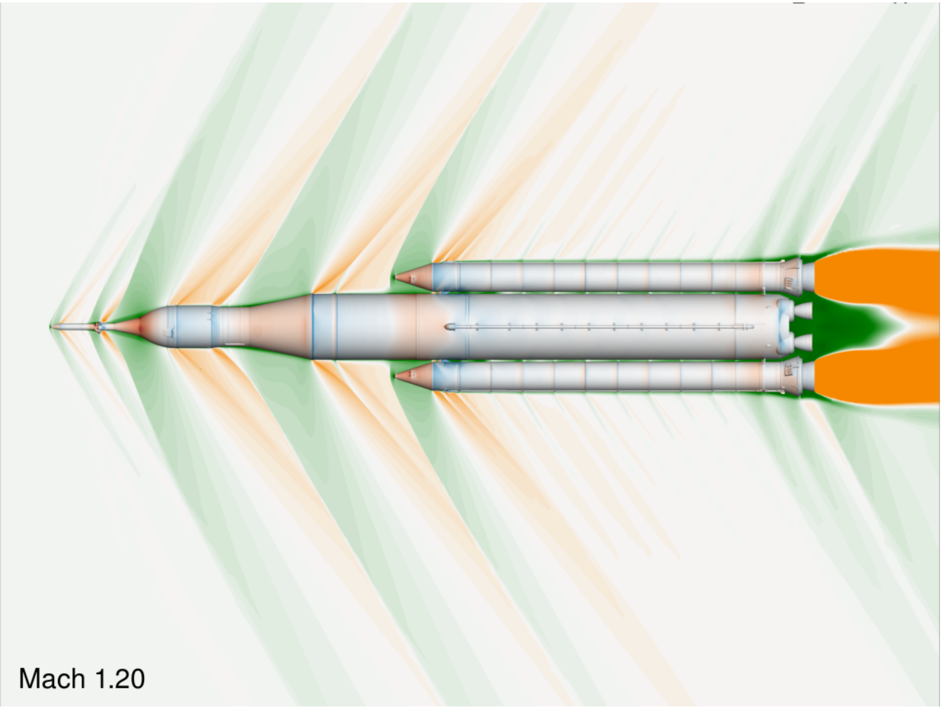

Mach 1.10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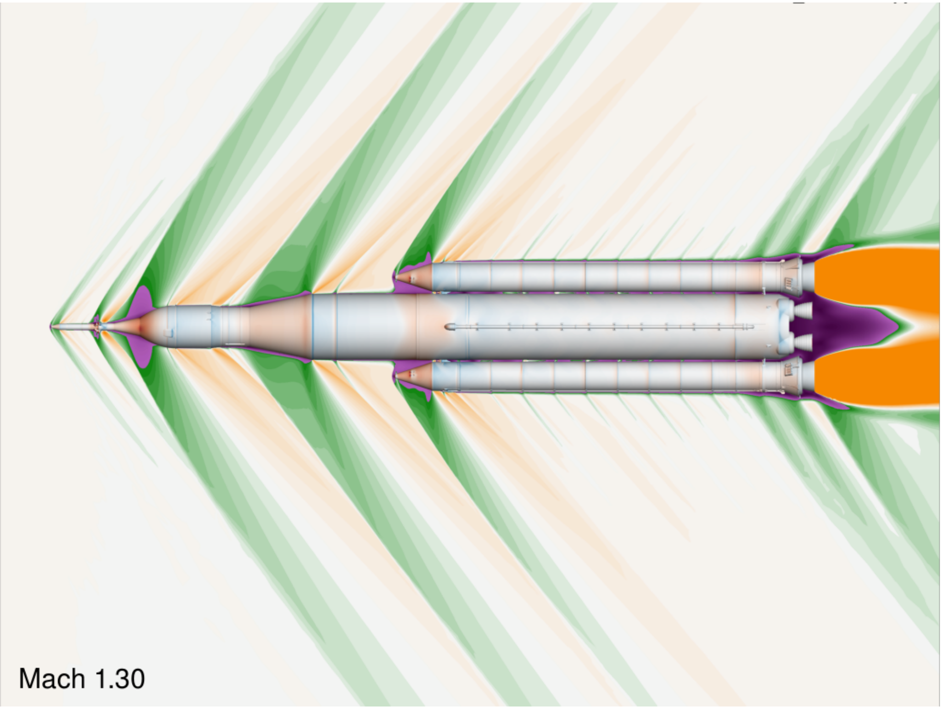
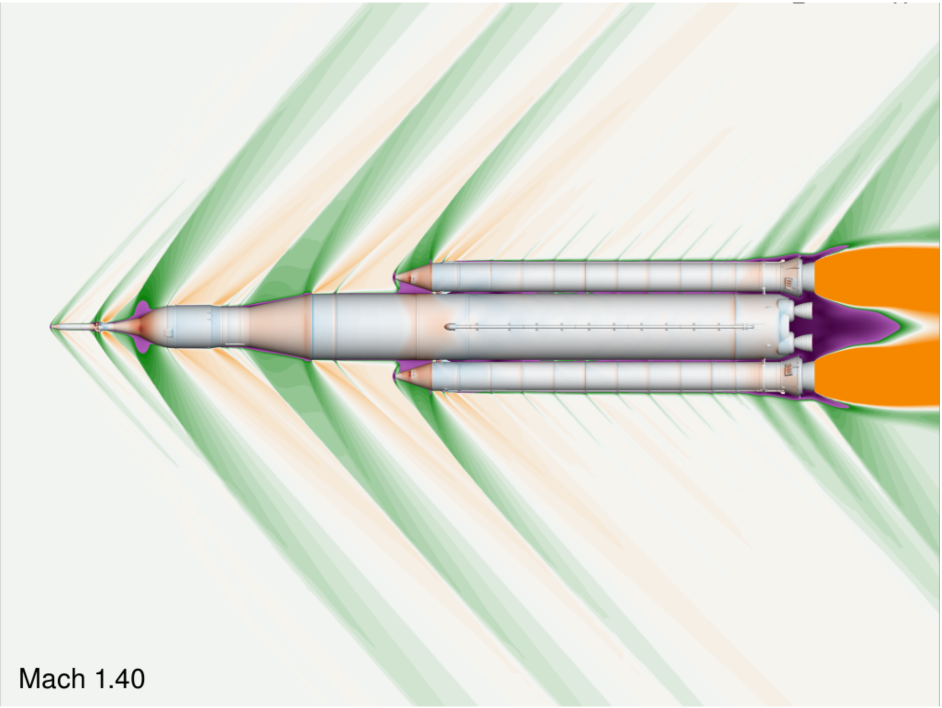

Mach 1.30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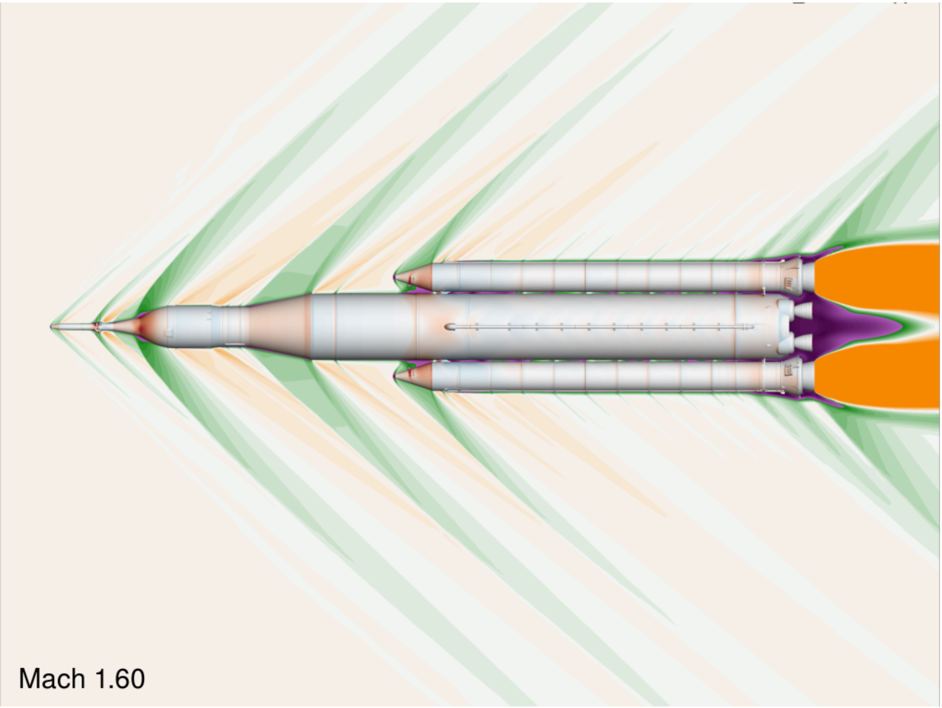
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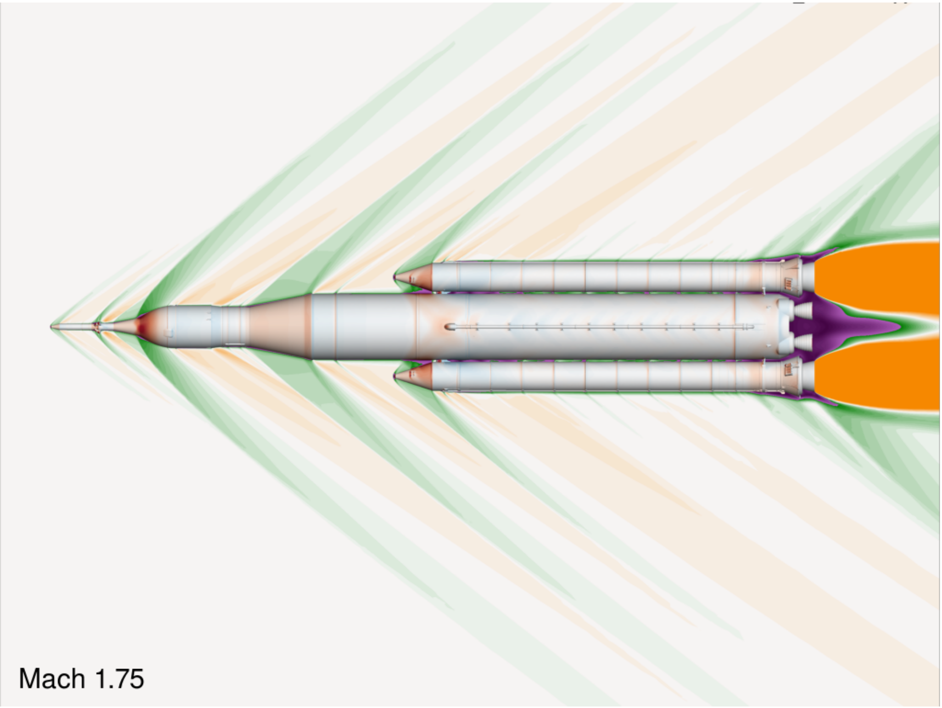
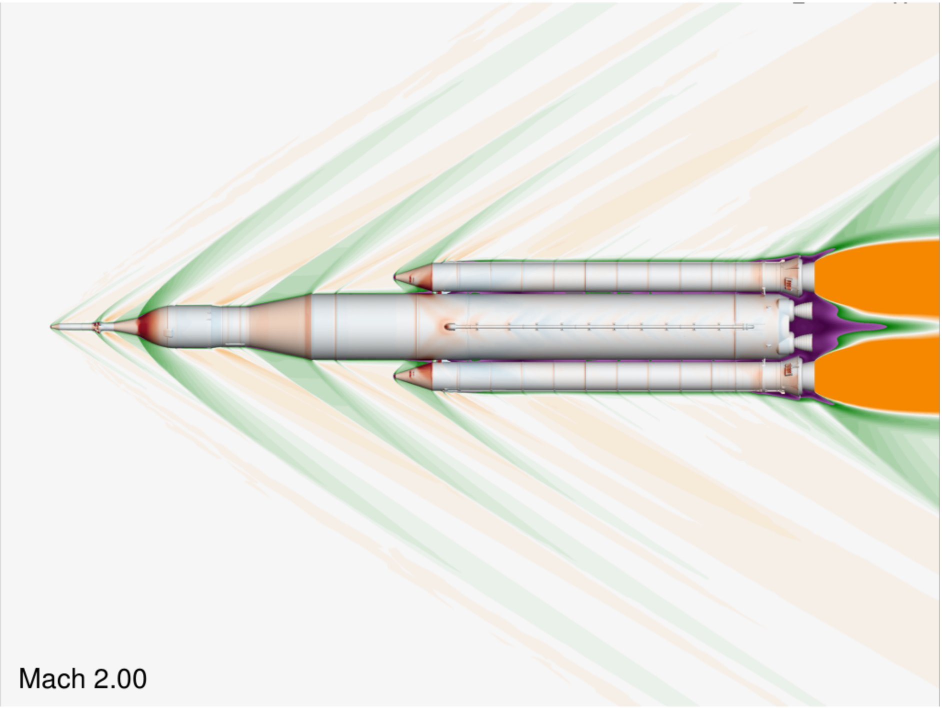
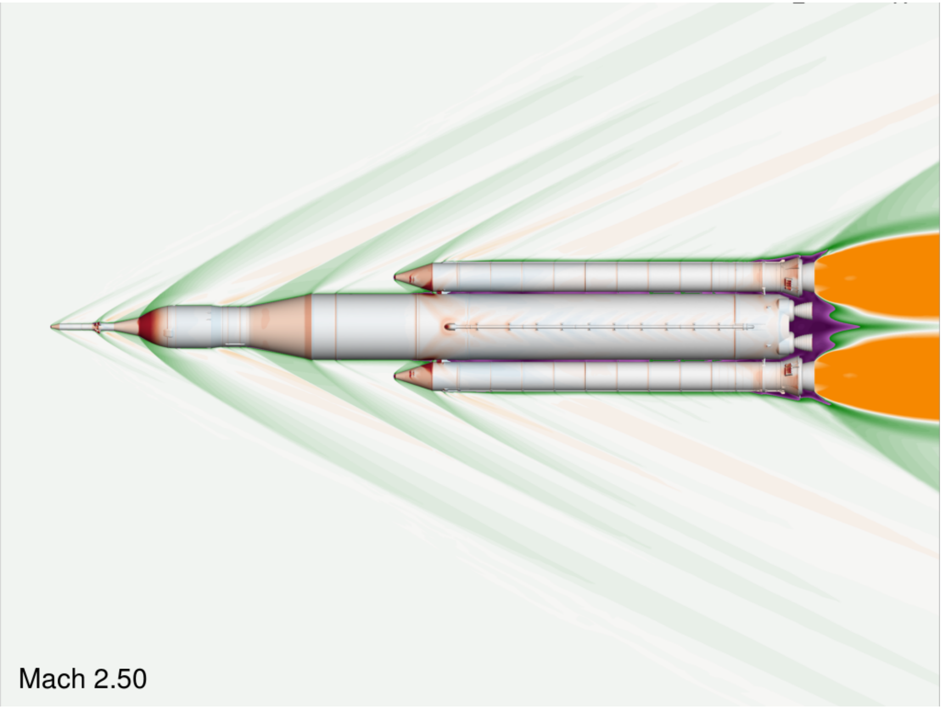

Mach 1.75


Mach 2.00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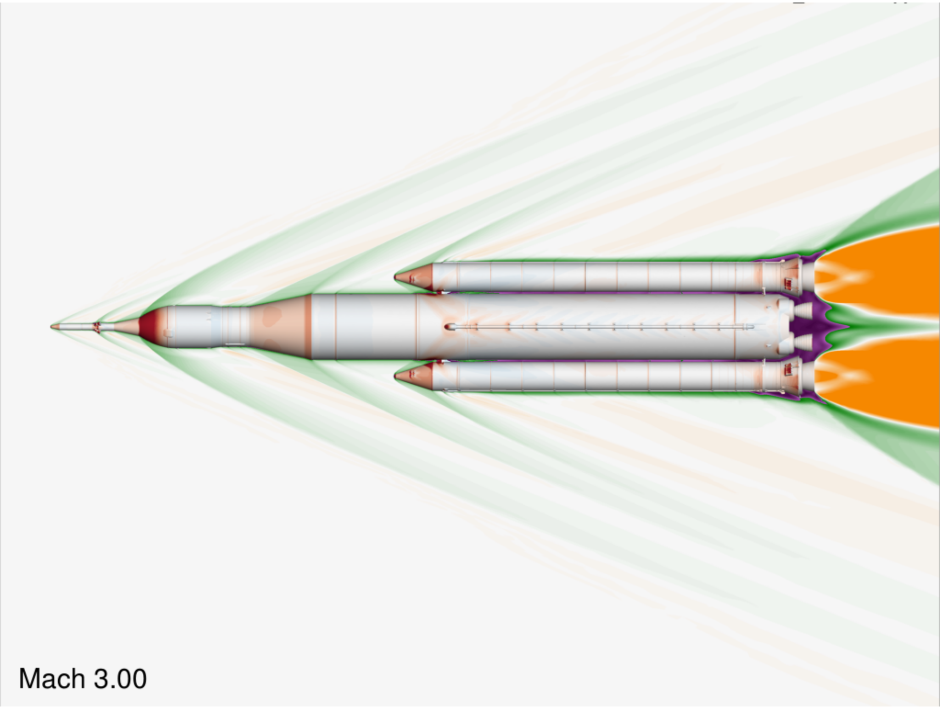
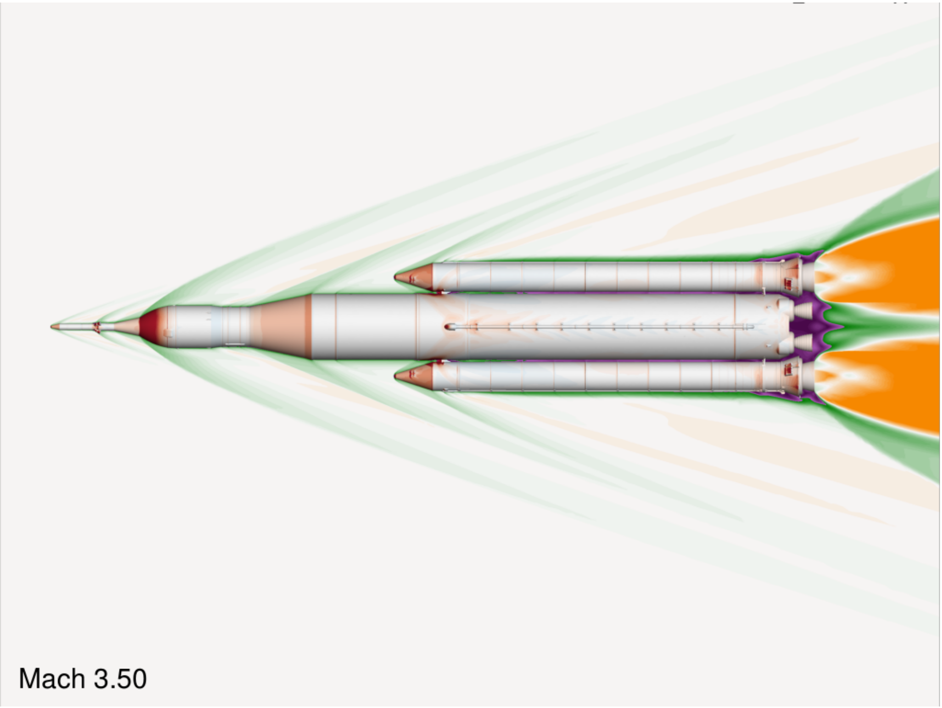
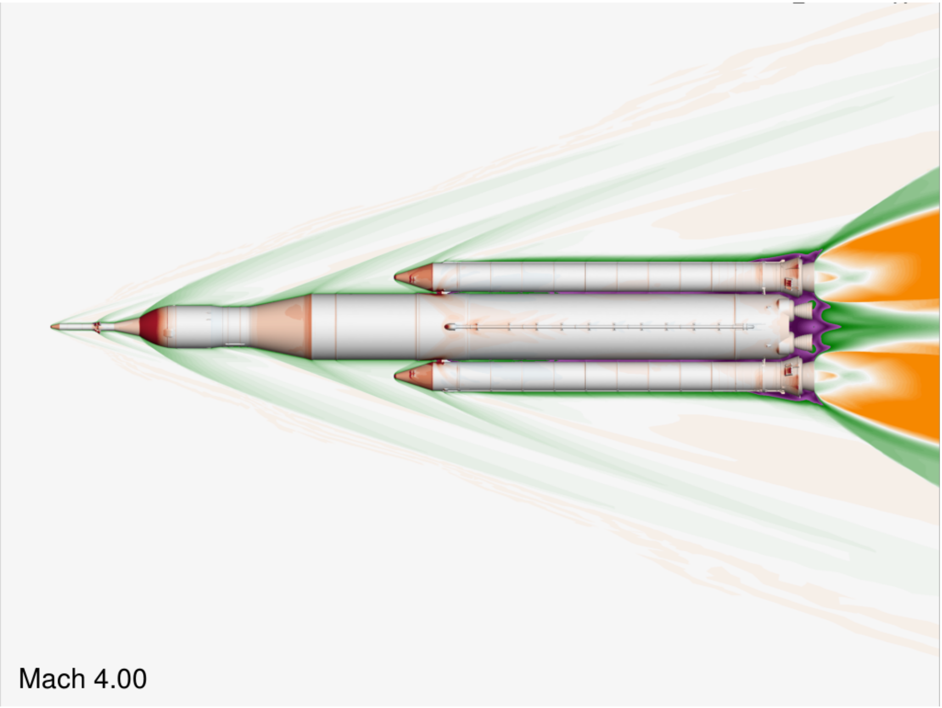
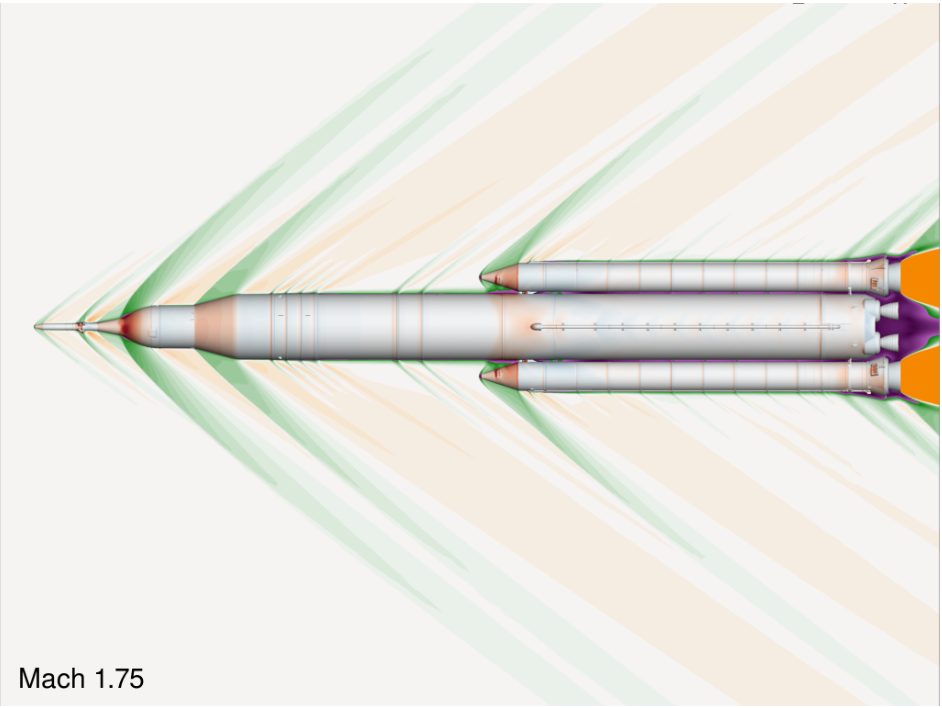

Mach 3.00



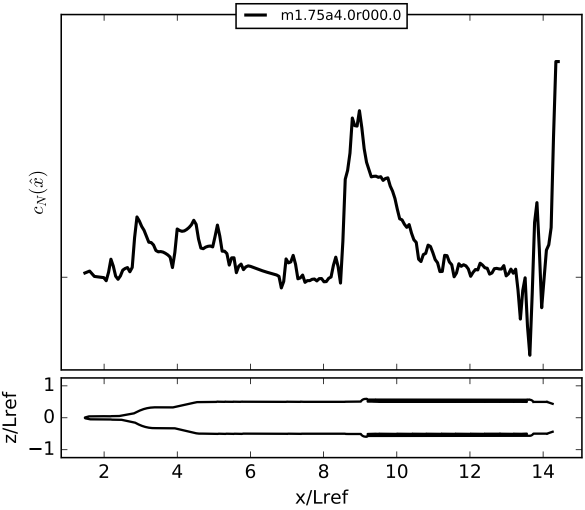
Mach 3.50



Mach 4.00


Mach 1.75

What Does a Sectional Load Look Like?

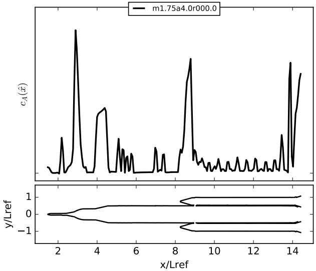


The load is basically a set of 3 discretized curves, $c_A(x)$, $c_Y(x)$, and $c_N(x)$

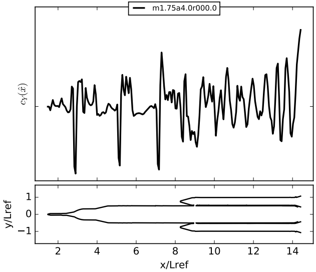
This example is from SLS Block 1B, Mach 1.75, α=4°, β=0°

Example of a Sectional Load

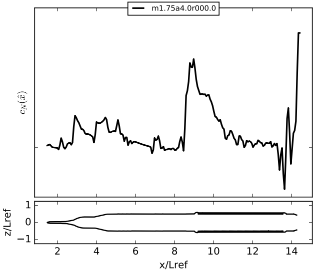
- Three force components each have a profile as a function of axial distance along the rocket
- The dimensional version of this are force per length, e.g. lbf/in
- For SLS, we use 200 slices and deliver line loads on the core, left booster, and right booster all separately



Axial loads: $c_A(x/L_{ref}) = c_A(\hat{x})$

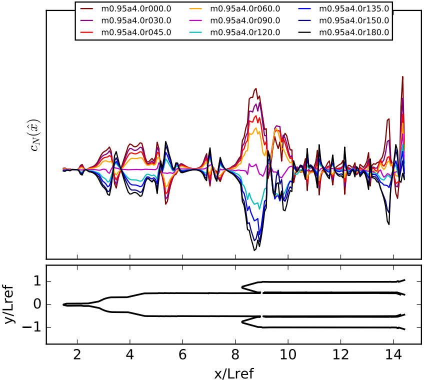


Lateral loads: $c_Y(\hat{x})$

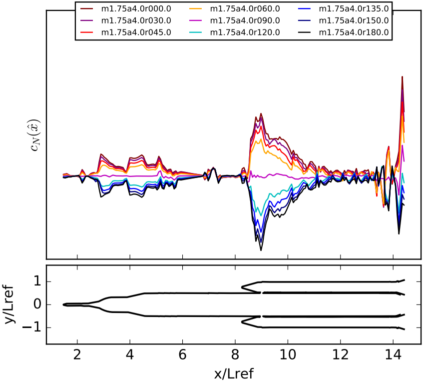


Normal loads: $c_N(\hat{x})$

Sample Sectional Normal Loads on SLS Block 1B



Mach 0.95, $\alpha_t = 4^\circ$, $\beta \geq 0^\circ$



Mach 1.75, $\alpha_t = 4^\circ$, $\beta \geq 0^\circ$

- Each plot contains 9 sectional loads at the maximum angle between the nose and the velocity
- The load profiles change quite a bit with Mach number
- At one Mach number, load is just about proportional to α

Motivation for Adjustments

Inconsistency

The line load profiles should have certain integral properties:

$$C_N = \int_{\hat{x}_1}^{\hat{x}_2} c_N(\hat{x})\,\mathrm{d}\hat{x} \qquad\qquad C_m = \int_{\hat{x}_1}^{\hat{x}_2} (\hat{x} - \hat{x}_{MRP}) c_N(\hat{x})\,\mathrm{d}\hat{x}$$

However, frequently the line loads $c_N(\hat{x})$ come from CFD because of the higher density of data, while the force & moment database is derived from wind tunnel testing

How can we adjust the profile to create $\bar{c}_N(\hat{x})$ that's consistent the integral constraints above?

Uncertainty Quantification

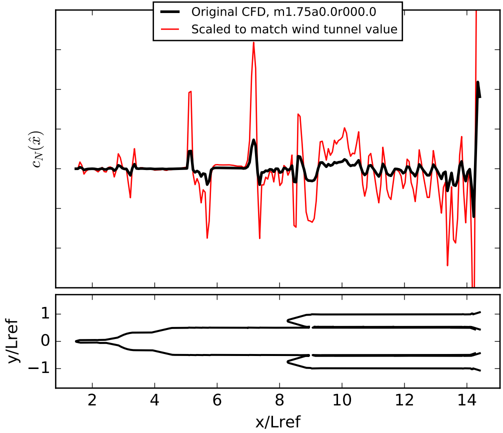
It is easy (and common practice) to disperse the integrated forces and moments:

$$\tilde{C}_N = C_N + \varepsilon_{CN} U_{CN} \qquad\qquad \tilde{C}_m = C_m + \varepsilon_{CLM} U_{CLM} + (\hat{x}_{MRP} - \hat{x}_{cg})\varepsilon_{CN} U_{CN}$$

But once we have dispersed values of $\tilde{C}_N$ and $\tilde{C}_m$, how do we generate a load profile that's consistent?

This is more than an esoteric question; for example, we need dispersed loads if we want to know the UQ on other integral properties like maximum bending moment

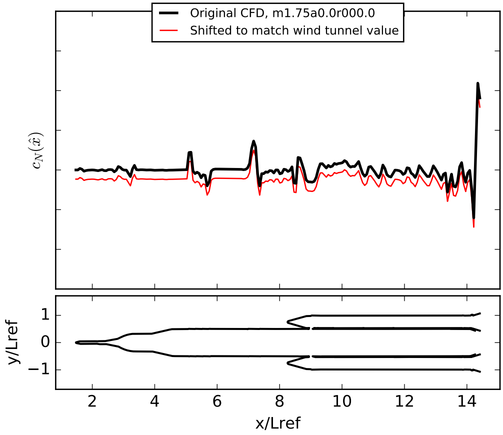
Bad Idea 1: Scaling



Directly scaled *CN* for Mach 1.75, $\alpha = 0^\circ$, $\beta = 0^\circ$

- Scaling the entire profile has huge problems with small integrated loads
- Suppose the CFD value of the database is $C_N = 0.001$ and the value measured in the wind tunnel is $\bar{C}_N = 0.02$
- Scaling the value shifts the black load profile to the red one
- If CFD val. is $C_N = -0.001$, it gets much worse
- Also, doing this eliminates control over the pitching moment, $\bar{C}_m$

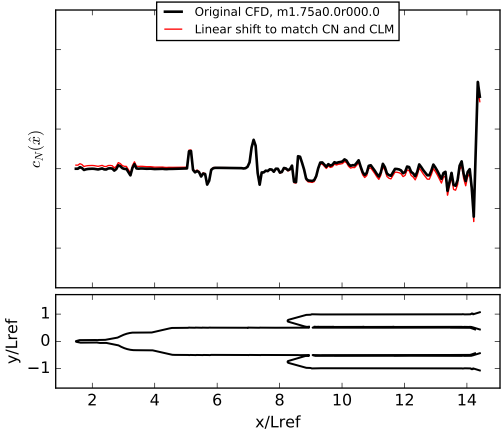
Bad Idea 2: Constant Shift



CN with shift for Mach 1.75, $\alpha = 0°$, $\beta = 0°$

- Shifting the entire profile has similar problems
- Suppose the CFD value of the database is $C_N = 0.001$ and the value measured in the wind tunnel is $\bar{C}_N = 0.02$
- Shifting the value shifts the black load profile to the red one
- Loses track of all the places where the load should be zero
- For UQ, the resulting dispersion is (locally) too small
- If CFD val. is $C_N = -0.001$, no dramatic difference
- Also, doing this eliminates control over the pitching moment, $\bar{C}_m$

Bad (but better) Idea 3: Linear Shift



CN at Mach 1.75, α=0°, β=0°, with linear shift

- Now matching $\bar{C}_N$ and $\bar{C}_m$
- Suppose the CFD value of the database is $C_N = 0.001$ and the value measured in the wind tunnel is $\bar{C}_N = 0.02$
- Wild shifts no longer apparent
- However, largest adjustments are always at the nose and tail
- All zero crossings are shifted
- Better than other two, but this doesn't utilize any specific information about the vehicle or conditions

Proper Orthogonal Decomposition

Concept: Use a family of discretized line loads and use first few POD modes as candidate adjustment functions

$$\mathbf{c}_{N,i} = \begin{bmatrix} c_{N,i,1} \\ c_{N,i,2} \\ \vdots \\ c_{N,i,n} \end{bmatrix} = \begin{bmatrix} c_N(\hat{x}_1, M_i, \alpha_i, \beta_i) \\ c_N(\hat{x}_2, M_i, \alpha_i, \beta_i) \\ \vdots \\ c_N(\hat{x}_n, M_i, \alpha_i, \beta_i) \end{bmatrix}$$

Here i represents the flight condition index

Now take several of these "snapshot" vectors and put them into a matrix

$$\mathbf{C}_N = \begin{bmatrix} \mathbf{c}_{N,1} & \mathbf{c}_{N,2} & \cdots & \mathbf{c}_{N,n} \end{bmatrix}$$

Then perform a singular value decomposition of the $n \times m$ matrix $\mathbf{C}_N$

$$\mathbf{C}_N = \mathbf{\Phi}_N \mathbf{\Sigma}_N \mathbf{V}_N^T$$

Dimensions: $\mathbf{\Phi}_N \in \mathbb{R}^{m \times m}$, $\mathbf{\Sigma}_N \in \mathbb{R}^{m \times n}$, $\mathbf{V}_N \in \mathbb{R}^{n \times n}$

Adjustment Modes from POD

$$\mathbf{C}_N = \mathbf{\Phi}_N \mathbf{\Sigma}_N \mathbf{V}_N^T$$

The columns of $\mathbf{\Phi}_N$ are basically line load profiles

$$\hat{\phi}_{N,k} = \begin{bmatrix} \phi_{N,k,1} \\ \phi_{N,k,2} \\ \vdots \\ \phi_{N,k,n} \end{bmatrix}$$

with some special properties

$$\|\hat{\phi}_{N,k}\| = 1, \quad \hat{\phi}_{N,j} \cdot \hat{\phi}_{N,k} = \begin{cases} 1 & j = k \\ 0 & j \neq k \end{cases}$$

The matrix $\mathbf{\Sigma}_N$ is a rectangular matrix with singular values along its diagonal

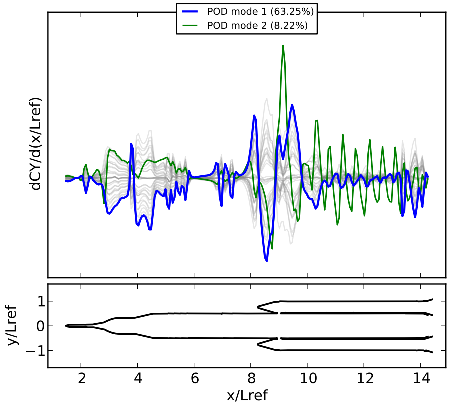
$$\mathbf{\Sigma}_N = \begin{bmatrix} \sigma_{N,1} & 0 & \cdots & 0 \\ 0 & \sigma_{N,2} & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_{N,m} \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}$$

Singular values give relative energy content in each mode, $\sigma_{N,1} \geq \ldots \sigma_{N,m} \geq 0$

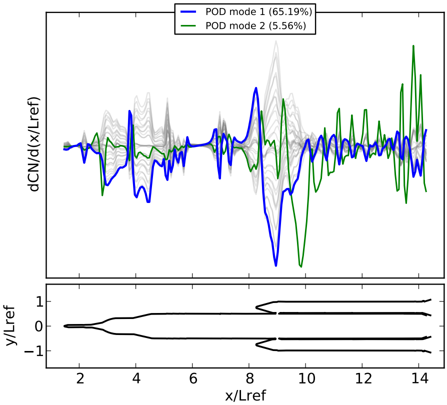
Then we can select the first $K \leq m$ modes and use a linear combination to adjust the line load profile

$$\bar{c}_{N,i}(\hat{x}) = c_{N,i}(\hat{x}) + \sum_{k=1}^{K} a_k \hat{\phi}_{N,k}(\hat{x})$$

POD Snapshots at Mach 1.30



CY snapshots



CN snapshots

Optimal Weights for POD Adjustment

Select the first $K \leq m$ POD modes to adjust the line load profile for case i

$$\bar{c}_{N,i}(\hat{x}) = c_{N,i}(\hat{x}) + \sum_{k=1}^{K} a_k \hat{\phi}_{N,k}(\hat{x})$$

Now we have K degrees of freedom (usually $K \approx 10$ works well) and only two constraints

$$\Delta C_{N,k} = \int_{\hat{x}_1}^{\hat{x}_2} \hat{\phi}_{N,k}(\hat{x})\,\mathrm{d}\hat{x} \qquad \bar{C}_N = C_N + \sum_{k=1}^{K} a_k \Delta C_{N,k}$$

$$\Delta C_{m,k} = \int_{\hat{x}_1}^{\hat{x}_2} (\hat{x} - \hat{x}_{MRP})\hat{\phi}_{N,k}(\hat{x})\,\mathrm{d}\hat{x} \qquad \bar{C}_m = C_m + \sum_{k=1}^{K} a_k \Delta C_{m,k}$$

Our solution is to minimize a weighted L_2 norm of the total adjustment

$$\min_{\mathbf{a} \in \mathbb{R}^K} f(\mathbf{a}) = \sum_{k=1}^{K} w_k a_k^2$$

Using maximum absolute value (questionable) and singular value (pretty logical) to set the weights

$$v_k = \max_{\hat{x} \in [\hat{x}_1, \hat{x}_2]} |\hat{\phi}_{N,k}(\hat{x})| = \|\hat{\phi}_{N,k}\|_\infty \qquad w_k = v_k / \sigma_{N,k}$$

Optimal Weights for POD Adjustment

One reason for this setup is that it can be easily solved using lagrange multipliers

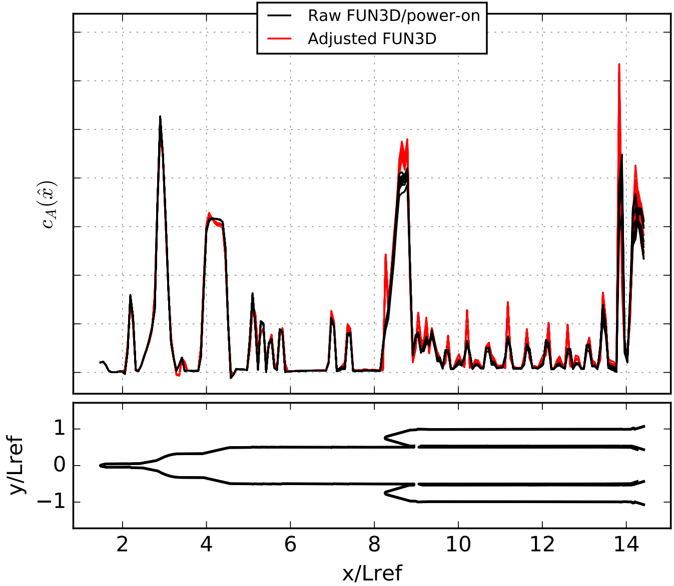
$$F(a_1, \ldots, a_K, \lambda_1, \lambda_2) = \lambda_1 \left(\bar{C}_N - C_N - \sum_{k=1}^{K} a_k \Delta C_{N,k} \right) + \lambda_2 \left(\bar{C}_m - C_m - \sum_{k=1}^{K} a_k \Delta C_{m,k} \right) + \sum_{k=1}^{K} w_k a_k^2$$

This leads to a linear system of equations with a predictable format

$$\begin{bmatrix} \Delta C_{N,1} & \Delta C_{N,2} & \cdots & \Delta C_{N,K} & 0 & 0 \\ \Delta C_{m,1} & \Delta C_{m,2} & \cdots & \Delta C_{m,K} & 0 & 0 \\ -2w_1 & 0 & \cdots & 0 & \Delta C_{N,1} & \Delta C_{m,1} \\ 0 & -2w_2 & \ddots & \vdots & \Delta C_{N,2} & \Delta C_{m,2} \\ \vdots & \ddots & \ddots & 0 & \vdots & \vdots \\ 0 & \cdots & 0 & -2w_K & \Delta C_{N,K} & \Delta C_{m,K} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_K \\ \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} \bar{C}_N - C_N \\ \bar{C}_m - C_m \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

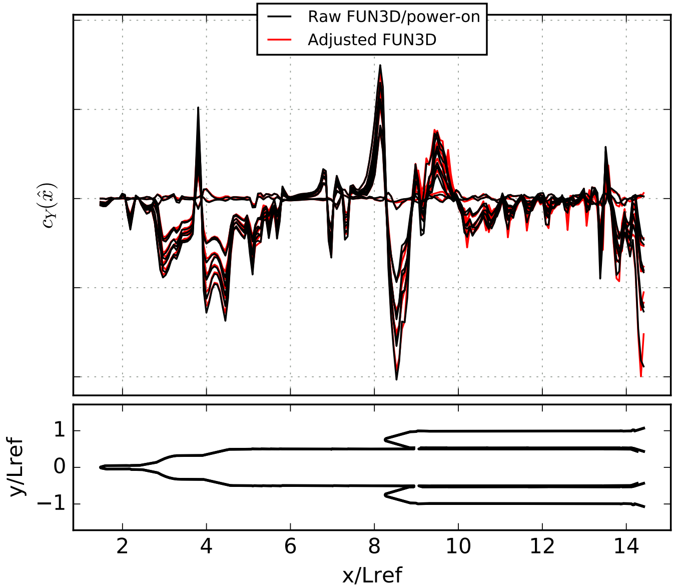
There is a similar system for $c_Y(\hat{x})$ and one with one less row and column for $c_A(\hat{x})$

Sample Adjustments of C_A



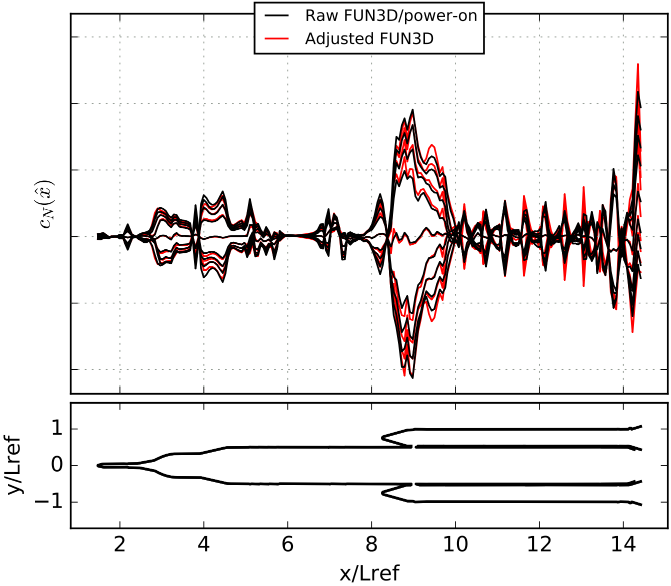
Mach 1.30, α_t=4°, $\beta \geq 0°$ original (black) and adjusted (red) loads

Sample Adjustments of C_Y



Mach 1.30, α_t=4°, $\beta \geq 0°$ original (black) and adjusted (red) loads

Sample Adjustments of C_N



Mach 1.30, α_t=4°, $\beta \geq 0°$ original (black) and adjusted (red) loads

Uncertainty Quantification

The idea is simple: use the UQ from the force & moment database

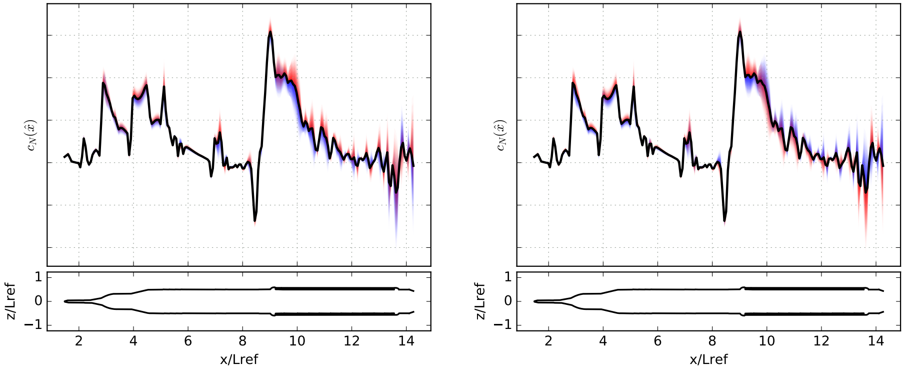
$$\tilde{C}_N = \bar{C}_N + \varepsilon_{CN} U_{CN}$$
$$\tilde{C}_m = \bar{C}_m + \varepsilon_{CLM} U_{CLM} + (\hat{x}_{MRP} - \hat{x}_{cg}) \varepsilon_{CN} U_{CN}$$

Here ε_{CN} and ε_{CLM} are randomly dispersed variables and U_{CN} and U_{CLM} are the quantifications of uncertainty in normal force and pitching moment, respectively

That's basically it; now just readjust the line loads to match $\tilde{C}_N$ and $\tilde{C}_m$.

To use properly, structures team or other customers should really do analysis for each trajectory in the Monte Carlo instead of just once for each flight condition

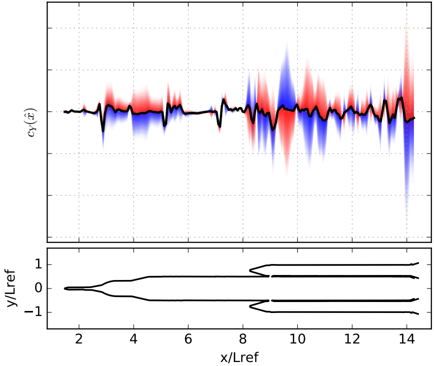
Dispersed $c_N(\hat{x})$ at Mach 1.75, $\alpha = 4^\circ$, $\beta = 0^\circ$



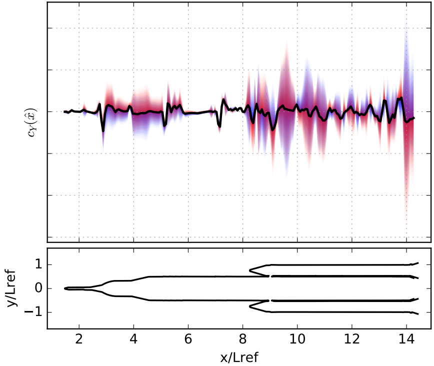
R: Increase in C_N, B: Decrease in C_N R: Increase in C_m, B: Decrease in C_m

- Each curve is the $c_N(\hat{x})$ profile for one combination of C_N and C_m
- Enough curves here to to make it look like a PDF at each x value
- The two charts show the same data but colored in two different ways
- Some regions correlate better to C_N and some to C_m

Dispersed $c_Y(\hat{x})$ at Mach 1.75, $\alpha = 4^\circ$, $\beta = 0^\circ$



R: Increase in C_Y, B: Decrease in C_Y



R: Increase in C_n, B: Decrease in C_n

- Each curve is the $c_N(\hat{x})$ profile for one combination of C_Y and C_n
- Enough curves here to to make it look like a PDF at each x value
- The two charts show the same data but colored in two different ways
- In this case, local loads correlate with C_Y and not C_n

Conclusions

- Relatively simple, very reliable method to adjust CFD-based line loads to be consistent with wind tunnel integrated loads
  - This paper suggests a method, but it has several opportunities to make other decisions
  - No need for additional CFD solutions

- Easily extended to create an uncertainty quantification that is consistent with a force & moment UQ

- Technique easily extended:
  - Adjust/disperse surface pressures (and skin friction) instead of line loads to match all six F&M at once
  - Add more dispersion modes that do not affect integrated F&M

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