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LAUNCH SERVICES PROGRAM

On the Maximum Expected Electric Field in Electrically Small, Undermoded Enclosures

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ESA Workshop on Aerospace EMC
20-22 May, 2019   Budapest Hungary

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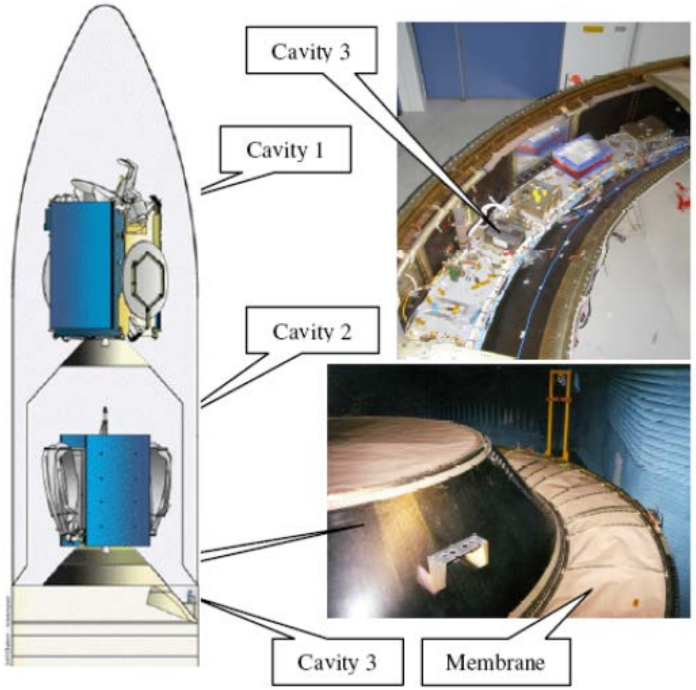
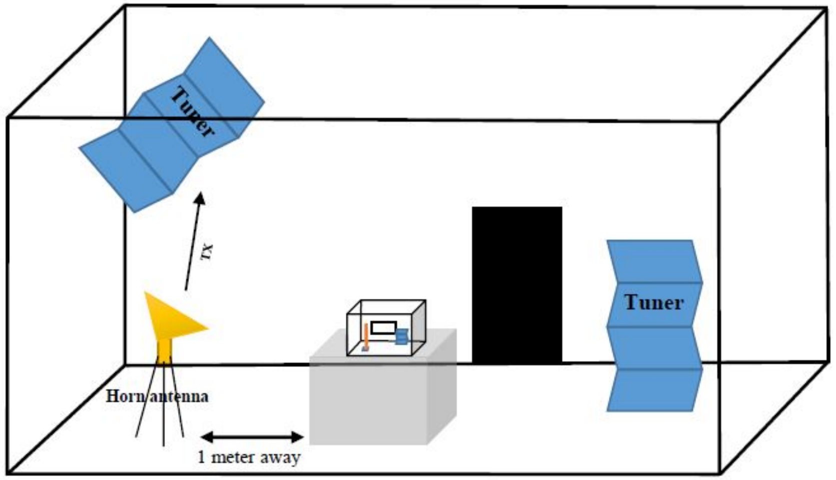
Canonical problem – Wire immision in aperture enclosure

Common mode pick-up of conductors in a spacecraft fairing or avionics bay



– Shaffar & Gineste IEEE EMC 2011

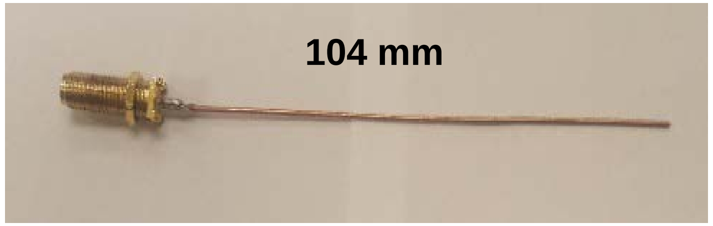
Surrogate system: Wire antenna in aperture box

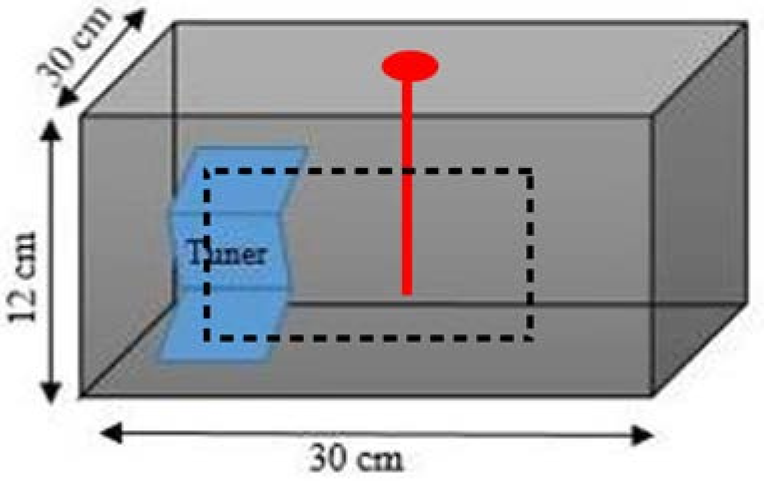


– Hill (NIST) 1994, 1996
– Holland & St.John (AFRL) 1999, 2001
– Tait, Hager (NSWC Dahlgren) 2013
– Rajamani, West & Bunting (OSU) 2014

Enclosure Test Configurations

Wire Antenna in Aperture Box

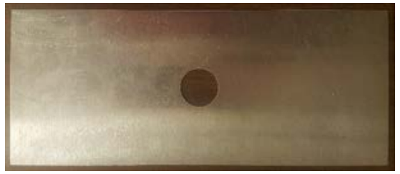




3 different Apertures

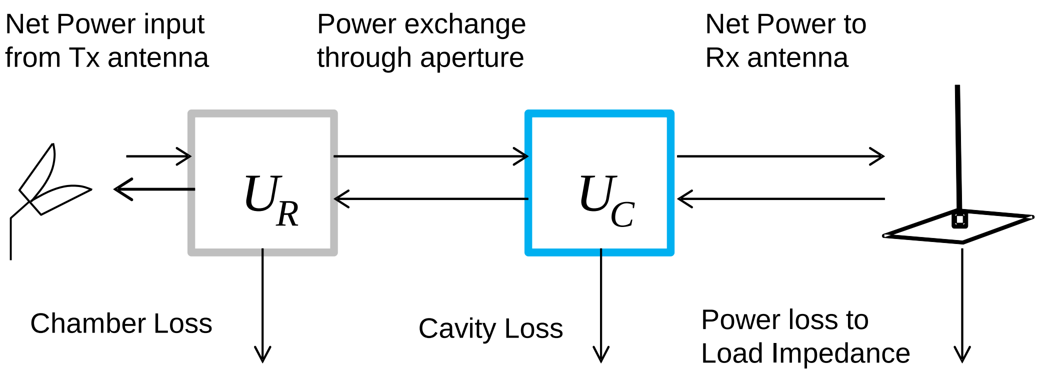
AP1 1cm x 6cm

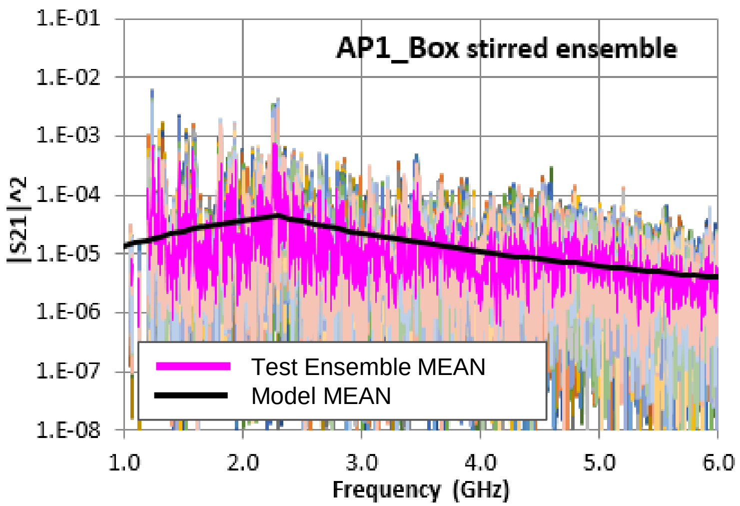
AP2 60cm x 150cm

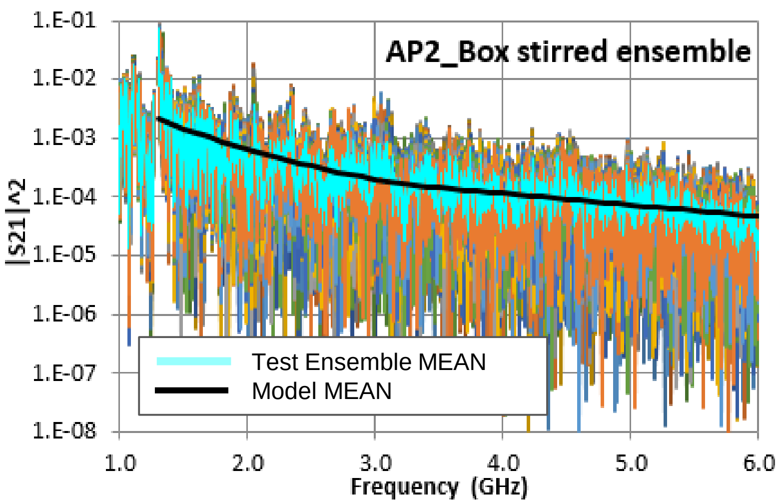


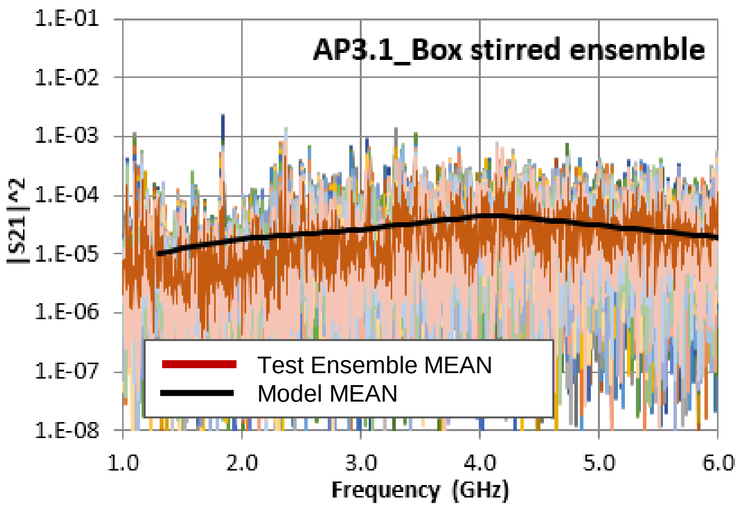
AP3 30cm dia.

Hill's power balance model correctly predicts **statistical mean of mode-stirred E field**









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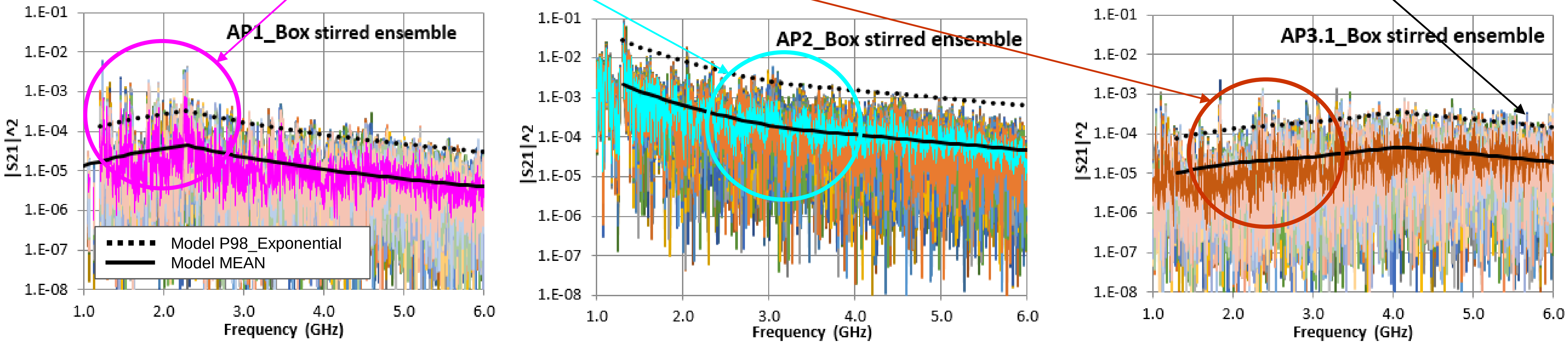
Hill's Chi-sq. 2 dof Exponential distribution **does not predict maximum E field**

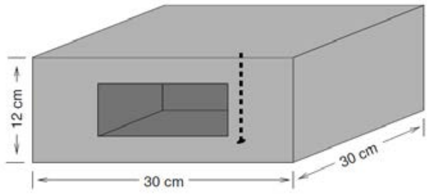
Underpredicts at low frequencies due to "frequency variance" of individual modes in the mean mode-stired mean

Max / Mean converges to Exponential at higher frequencies, lower frequency variance of individual modes

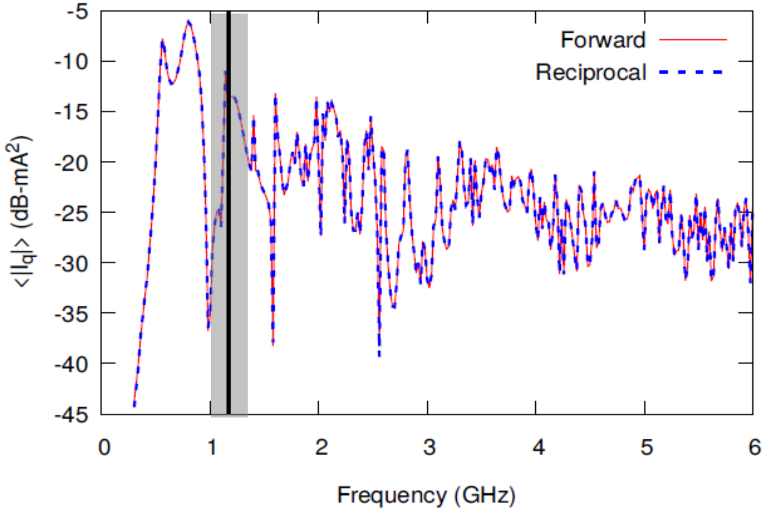


West explains why undermoded cavity enclosure fields are not Rayleigh distributed

Numerical model of current received by wire in aperture box



Resonant modes are widely spaced; response statistics need to be defined over frequency band



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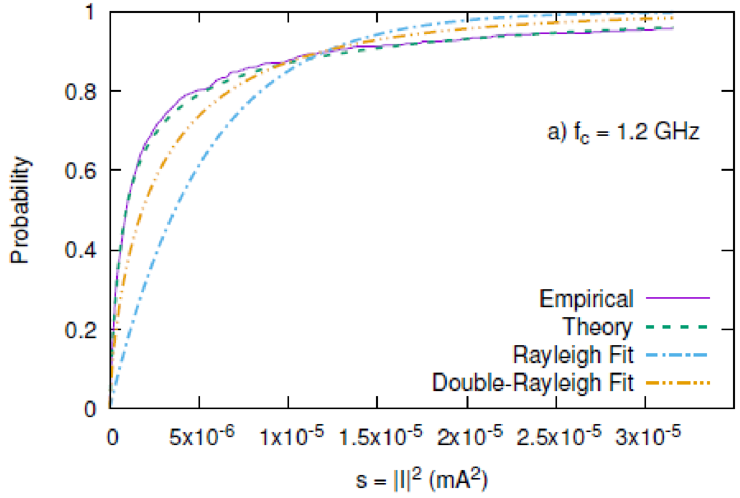
Statistical distribution of wire current is $|\langle S_{11}(f_m)\rangle|^2$ *weighted sum of Exponential PDFs*

The PDF of the frequency-stirred samples is then [14, (4-74)]

$$p_S(s) = \frac{N_1}{N_T} p_{S|f}(s \mid f_1) + \frac{N_2}{N_T} p_{S|f}(s \mid f_2) + \cdots + \frac{N_M}{N_T} p_{S|f}(s \mid f_M), \quad (19)$$

$$= \sum_{m=1}^{M} \frac{N_m}{N_T} p_{S|f}(s \mid f_m), \quad (20)$$

where $N_T = \sum_{m=1}^{M} N_m$ is the total number of samples.



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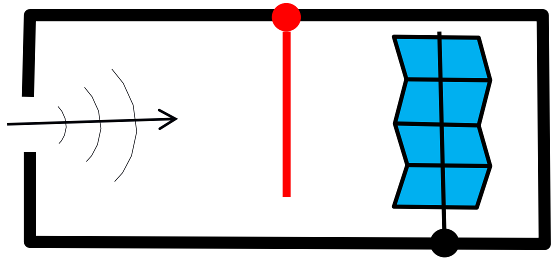
Statistical model alternative
to West's *numerical* $|<S_{11}(f_m)>|^2$ *weighted PDF*

Ensemble of box-stirred modes
(de-coupled from RC modes; no RC mode stirring)



Power input *deterministic for single box stirrer position*:

- Effective source current at aperture (measured)
- Q of box modes (measured)

$$P_{in}\left(\mathbf{x}_i,\omega\right)=\frac{L_d^2}{\varepsilon V}\sum_r \frac{\omega\beta_r\omega_r^2\,\psi_r^2\left(\mathbf{x}_i,\xi\right)I_\xi^2\left(\mathbf{x}_i,\omega\right)}{\left[\left(\omega_r^2-\omega^2\right)^2+\beta_r^2\omega_r^4\right]}$$

$$\equiv\sum_r a_r\left(\mathbf{x}\right)g_r\left(\omega\right)$$

Lyon variance model [JASA 1969] based on statistics of mode spacing

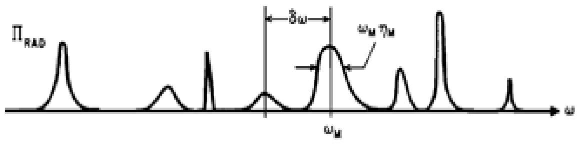
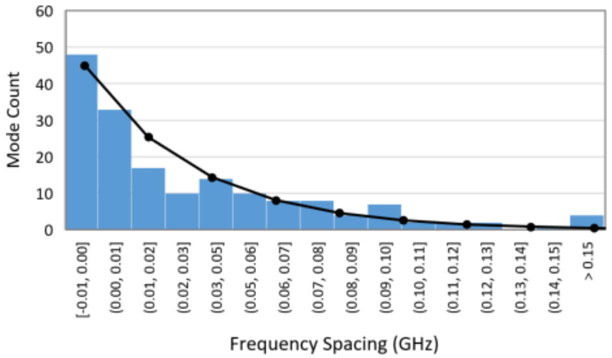


Figure 7. Graph of power received versus frequency ω for point excitation of a multi-modal system



Relative variance of Power input

$$r^2\left(P_{in}\right)=\frac{\sigma_P^2}{\left\langle P_{in}\right\rangle}=\frac{1}{2m}\frac{\left\langle\psi_r^4\right\rangle}{\left\langle\psi_r^2\right\rangle^2}$$

Relative variance on mean-squared Electric field

$$r^2\left(\left|E\left(\mathbf{x},\omega\right)\right|^2\right)=1+\frac{1}{2m}\frac{\left\langle\psi_r^4\right\rangle^2}{\left\langle\psi_r^2\right\rangle^4}=1+\frac{5.7}{m}$$

where m = **Modal Overlap**

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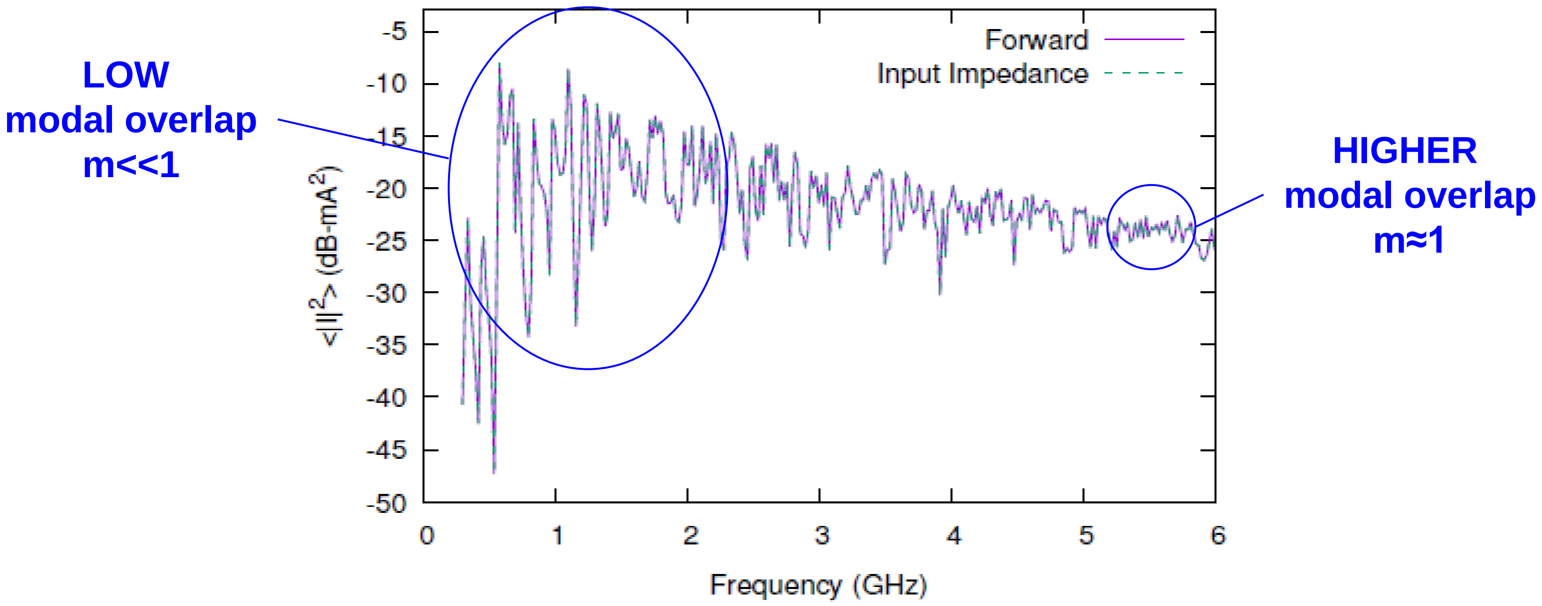
Modal Overlap



Modal Ovelap = Ratio of Modal Q Bandwidth to Average Frequency spacing of Resonant Modes

$$m(\omega) = \pi\omega n / 2Q$$

where $n(\omega) = \omega^2 V / \pi^2 c^3$ is modal density and frequency spacing $df = 1/2\pi n(\omega)$

Validation of Frequency Variance Model
Low Frequency EM Energy of Reverb Chamber

Statistical model for Relative Variance of EM field Energy level

Lyon model for Poisson mode spacing statistics and infinite ensemble

$$r^2\left(P_{in}\right) = \frac{\sigma_P^2}{\langle P_{in}\rangle} = \frac{1}{2m}\frac{\langle \psi_r^4\rangle}{\langle \psi_r^2\rangle^2}$$

Weaver enhancement for Gaussian Orthogonal Ensemble (GOE) statistics and finite sample ensemble

$$r^2\left[\left\langle |E|^2 \right\rangle\right] = \frac{1}{LN} + \frac{1}{2m}\left\{\left(\frac{K}{N}+1-\frac{1}{N}\right)\left(\frac{K}{L}+1-\frac{1}{L}\right)-\frac{2}{LN}-1\right\}$$

where L and N are respectively the number of receiver and source positions used to calculate cavity energy

NIST measurement of LaRC Reverb chamber

Bremner IEEE EMC 2015

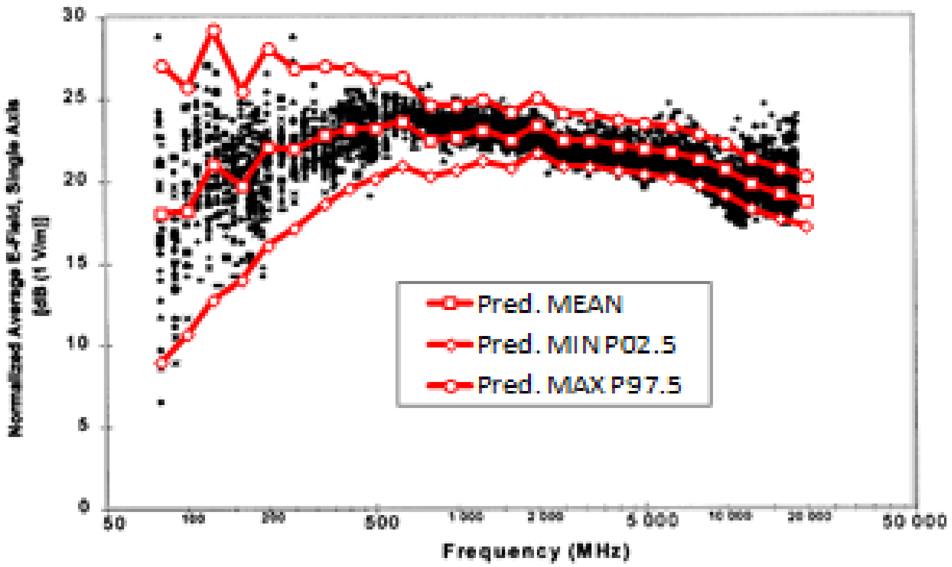


Figure 9. Maximum [P99.5] and minimum [P2.5] of Avg[Er] predicted by (14), compared with measurements in LaRC chamber B (reproduced from [12], [3])

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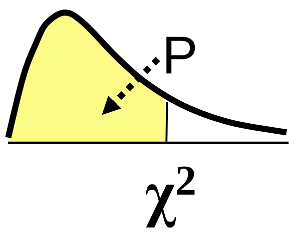
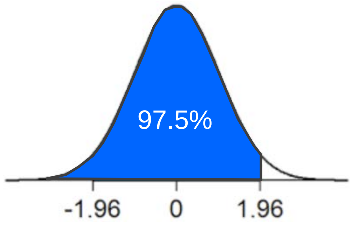
Validation of Frequency Variance Model
Low Frequency E field in Undermoded Enclosure

$$r^2\left(\left|E(\mathbf{x},\omega)\right|^2\right) = 1 + \frac{1}{2m}\frac{\left\langle \psi_r^4 \right\rangle^2}{\left\langle \psi_r^2 \right\rangle^4} = 1 + \frac{5.7}{m}$$



Mode shape PDF
Rayleigh (Exponential)



Modal Frequency statistics
Log Normal PDF

$$Max\left[\left|S_{21}\right|^2\right] = Max_{Freq}\left[\left|S_{21}\right|^2\right] + P98_{Rayleigh}\left[\left|S_{21}\right|^2\right]$$

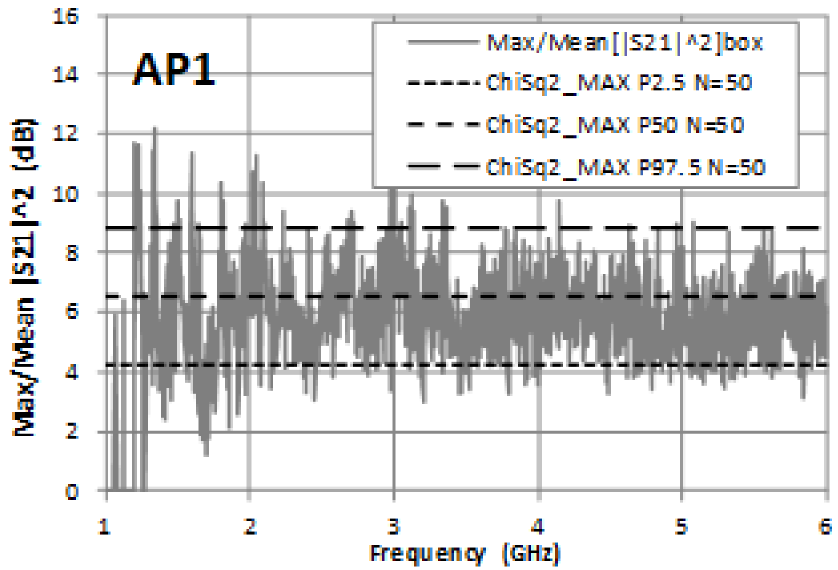


Figure 12. Ratio of maximum to mean $|S_{21}|^2$ at each frequency, band average and expected ratio of exponential distribution with ensemble N=50

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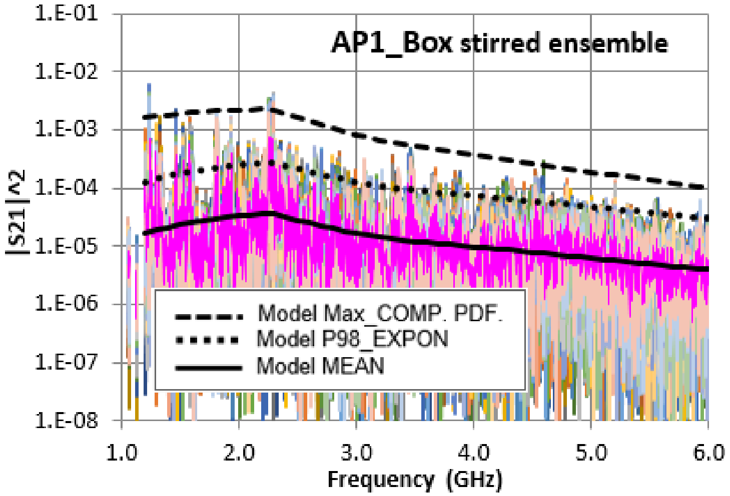
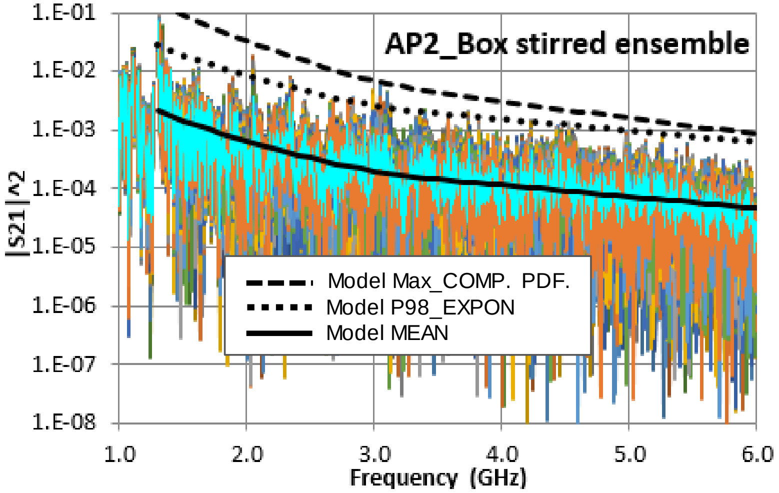
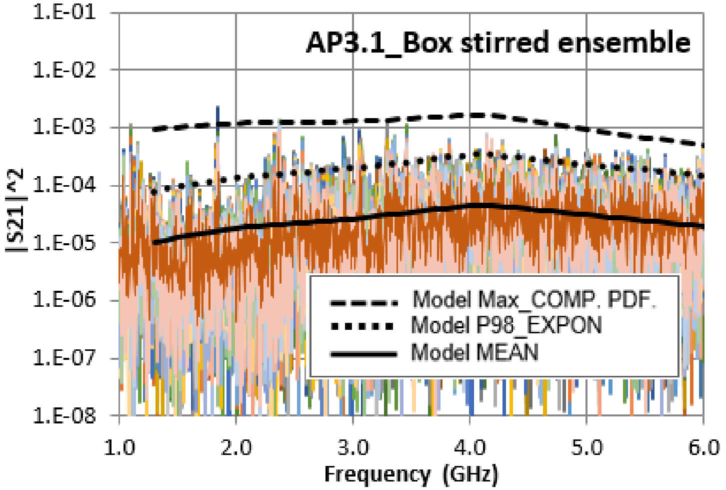
Validation of Frequency Variance Model
Low Frequency E field in Undermoded Enclosure

$$r^2\left(\left|E\left(\mathbf{x},\omega\right)\right|^2\right)=1+\frac{1}{2m}\frac{\left\langle\psi_r^4\right\rangle^2}{\left\langle\psi_r^2\right\rangle^4}=1+\frac{5.7}{m}$$

$$Max\left[\left|S_{21}\right|^2\right]=Max_{Freq}\left[\left|S_{21}\right|^2\right]+P98_{Rayleigh}\left[\left|S_{21}\right|^2\right]$$







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Conclusions

- Hill power balance model is robust for frequency- & space-averaged E field prediction
  - Both undermoded and overmoded enclosures

- Variance and Max Expected E field in undermoded enclosure not predicted by Rayleigh statistics
  - **Additional “frequency variance” due to widely spaced modal responses**
  - **West [IEEE EMC 2018] provides deterministic method to calculate Variance, PDF & Max Expected**
  - **Lyon [JASA 1969] provides an alternative “frequency spacing statistics” model for Variance**

- Max Expected E field response of undermoded enclosures
  - **Frequency variance at low frequencies; Rayleigh statistics at high frequency**
  - **Authors postulate a PDF that transitions from LogNormal to Exponential**
  - **Trends match measured data**

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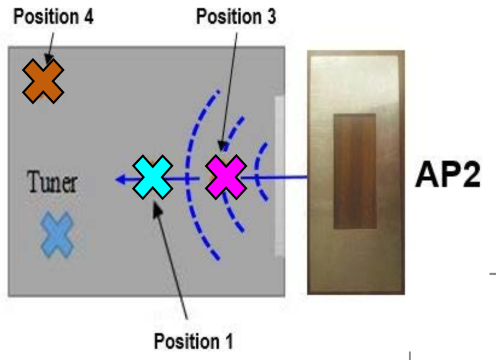
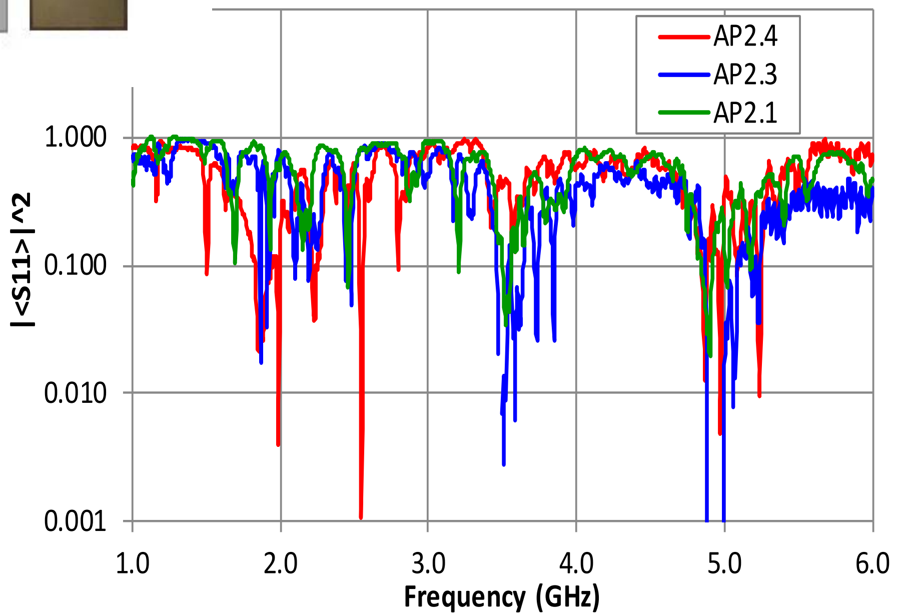
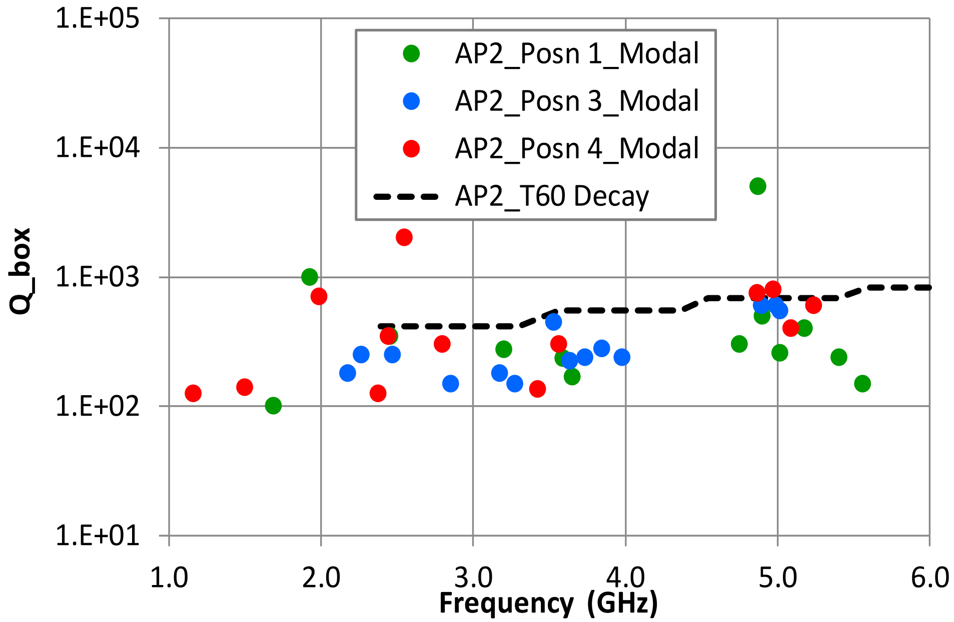
Back-up material

Modal Q estimation
Sensitivity to S_{11} measurement location







Real antenna effects explain modal Q scatter

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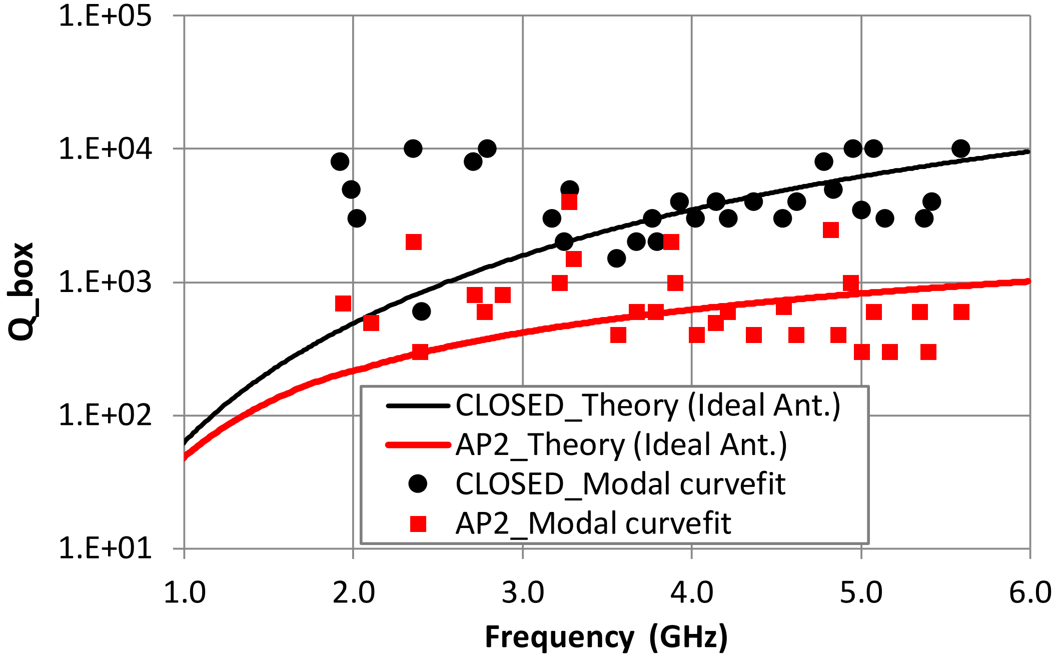
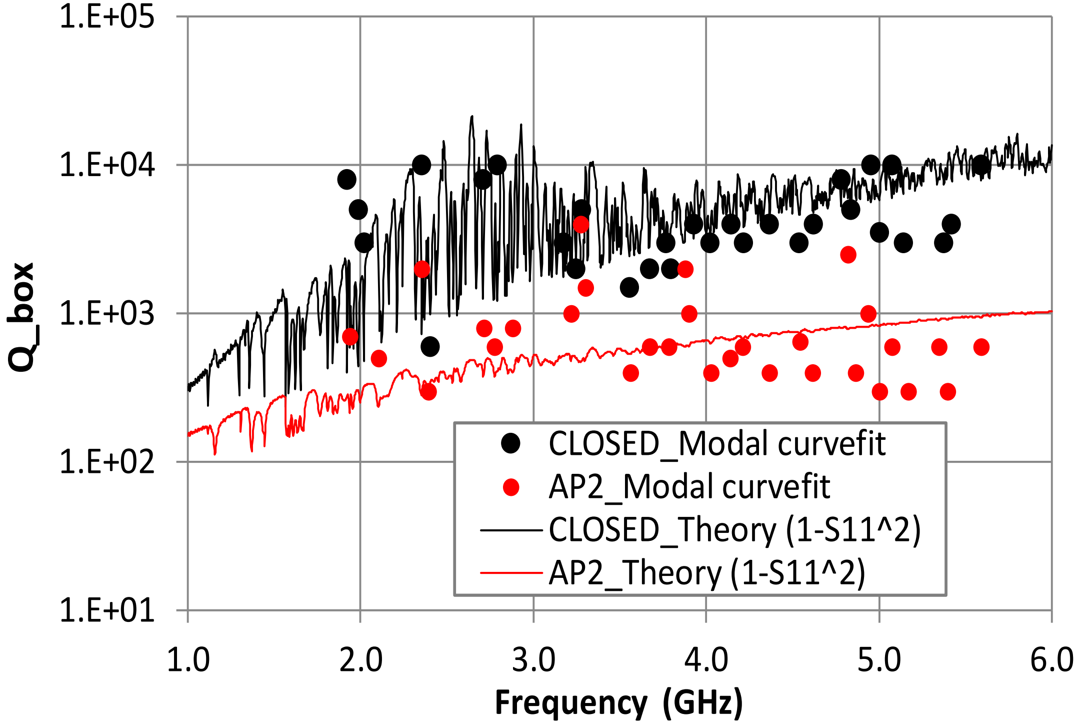
Ideal Antenna

$$Q_{ant} = \frac{16\pi^2 V_c}{\lambda^3}$$



Wire Antenna

$$Q_{ant} = \frac{16\pi^2 V_c}{\left(1 - \left|\langle S_{11} \rangle\right|^2\right)\lambda^3}$$



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