ON THE MAXIMUM EXPECTED ELECTRIC FIELD IN ELECTRICALLY SMALL, UNDERMODED ENCLOSURES

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This paper reports the experimental validation of an improved statistical model for the prediction of maximum expected electric field in electrically small and under-moded enclosures. The aerospace community is interested in application of Hill's statistical models [1] to design of avionics boxes for shielding effectiveness and for tailoring EMC test requirements for critical applications. However, it is observed [2] that the probability distribution for mean-squared electric field $\left|E_x\right|^2$ *in an electrically small enclosure* differs from the exponential distribution which is widely used in reverberation chamber testing. It is postulated here that the difference is attributable to the under-moded character of the small enclosure. We will define "under-moded" as the condition where a single excitation frequency does not excite enough closely spaced resonant modes to achieve Hill's assumption of an isotropic (or fully diffuse) plane wave field in the enclosure.

In the field of vibro-acoustics, Lyon [3] has shown that the mode spacing of a reverberant acoustic wavefield in a rectangular enclosure follows a Poisson distribution and the variance of total energy $U\left(\omega\right)$ can be calculated from the modal overlap factor $m\left(\omega\right)$. For electromagnetic waves in a small enclosure, modal overlap is defined as the ratio of the modal damping bandwidth to the frequency spacing

$$m = \omega\eta n \equiv \omega n/Q \qquad (1)$$

The modal density n (s/rad) of the enclosure with volume V is [1] $n\left(\omega\right) = V\omega^3 / 2\pi^2 c^3$. It follows that modal overlap greater than unity is a sufficient condition for over-moded cavity statistics.

For a wider class of reverberant wavefields with irregular boundaries, Weaver [4] has shown that the natural frequency spacings are more correctly described by the Gaussian Orthogonal Ensemble (GOE) from random matrix theory. Weaver provides a variance formulation for the energy of a reverberant wave field excited by one or more point excitations. The relative variance $r^2\left[U\right] = \sigma_U^2 / \left\langle U\right\rangle^2$ takes the form

$$r^2\left[U\right] = \frac{1}{LN} + \frac{1}{\pi m}\left\{\left(\frac{K}{N}+1-\frac{1}{N}\right)\left(\frac{K}{L}+1-\frac{1}{L}\right) - \frac{2}{LN} - 1\right\} \qquad (2)$$

where L and N are respectively the number of receiver and source positions used to calculate cavity energy. The modal overlap *m* is the principal term describing the uncertainty in resonance frequency. The parameter *K* is a measure of the spatial variance of the cavity mode shapes $\psi_r\left(x\right)$, which asymptotes to a value close to 3 [3]

$$K = E\left[\psi_r^4\right] / E\left[\psi_r^2\right]^2 \cong 3 \qquad (3)$$

It can be seen that the relative variance of energy – due to the undermoded condition – reduces rapidly with increasing frequency, because modal overlap increases with modal density. It follows that the statistics of the electric field at a point should transition from low modal overlap energy statistics (2) at low frequencies, to Hill's exponential distribution ("Rayleigh statistics") at higher frequencies. Following Cotoni & Langley [5] the variance of the mean squared electric field component $\left|E_x\right|^2$ at any position in the enclosure can be estimated

$$r^2\left(\left|E_x\right|^2\right) = 1 + 2r^2\left(U\right) \qquad (4)$$

Bremner [6] has shown that the energy is log normally distributed, to a good approximation.

Measurements were made in a small "box" enclosure with high Q, with different apertures. The box dimensions shown in **Error! Reference source not found.** are chosen so that its electromagnetic field transitions from low modal overlap to higher modal overlap [7], over the measurement frequency range of 1.0-6.0 GHz. Two box apertures were tested – a slot aperture AP1 (1cm x 6cm) and a circular aperture AP3 (3cm diameter).

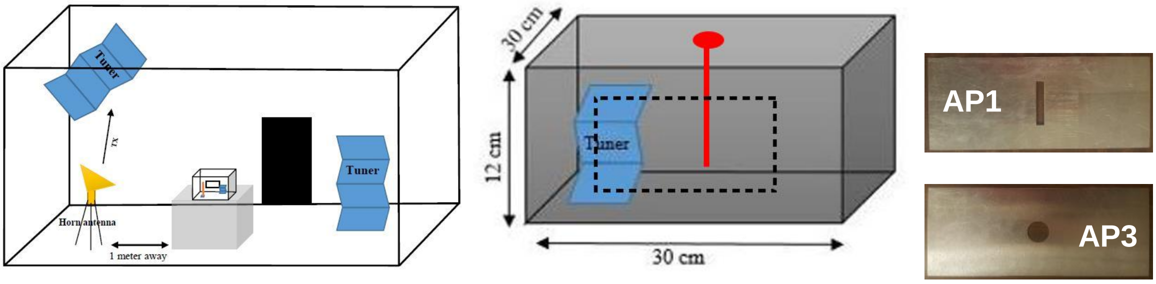

Figure 1. Measurement set-up; Source antenna, mode strirrers and aperture box in reverberation chamber (left), aperture box with receive antenna and strirrer (middle) and box apertures AP1, P2 (right)

An ensemble of S21 measurements from source antenna in the reverberation chamber to receive antenna in the aperture box are shown in Figure 2, for 50 stirrer positions in the box. The Hill power balance model prediction of the mean electric field strength is seen to be in good agreement with the mean of the measurement ensemble, for both aperture cases AP1 and AP3. The difference in the spectrum shape reflects the wave energy transmission characteristics of the different aperture shapes and dimensions. The statistical power balance model will therefore be a good predictor of the mean shielding effectiveness of the enclosure.

The ensemble of S21 measurements in Figure 2 also shows an overlay of the exponential distribution prediction of P98 maximum expected level, which is 5-10 dB lower than the maximum S21 measured. Incorporating the energy variance due to low modal overlap – equation (2) and (4) – and using a log normal distribution can be seen to better predict the maximum expected electric field, for two different apertures.

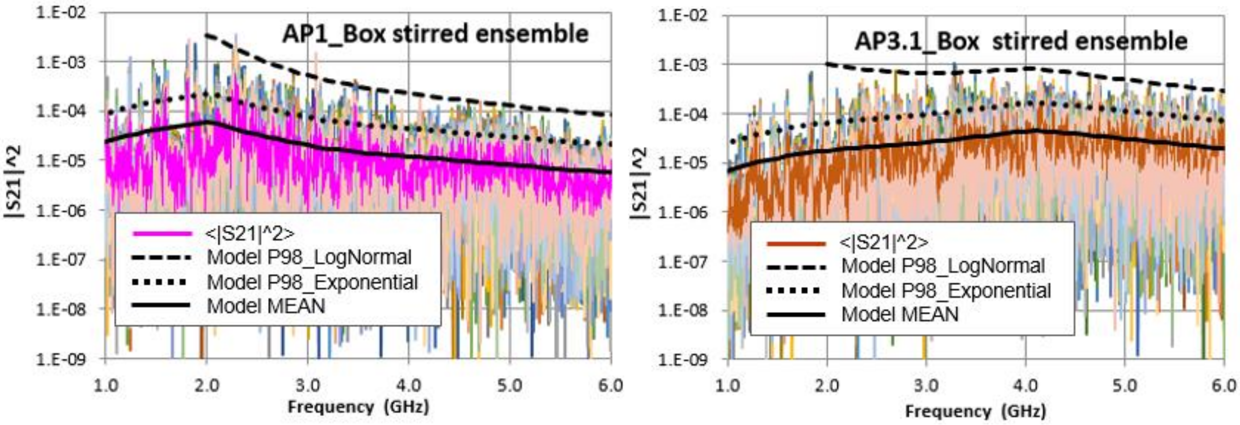

Figure 2. Ensemble of S21 measurements from source antenna to receive antenna in the aperture box for 50 stirrer positions in the box with statistical model predictions of mean and P98; aperture Ap1 (left) and aperture AP3 (right)

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