interaction of a backward-facing step and crossflow instabilities in boundary-layer transition

Jenna L. Eppink*
NASA Langley Research Center, Hampton, Virginia 23681
Richard W. Wlezien†
Iowa State University, Ames, Iowa 50011
and
Rudolph A. King* and Meelan Choudhari‡
NASA Langley Research Center, Hampton, Virginia 23681
DOI: 10.2514/1.J056267

A swept flat plate model with an imposed pressure gradient was experimentally investigated in a low-speed flow to determine the effect of a backward-facing step on transition in a stationary crossflow-dominated flow. Detailed hotwire measurements of boundary-layer flow were performed to investigate the upstream shift in transition due to a step height of 49% of the local unperturbed boundary-layer thickness. Increasing the initial stationary crossflow amplitude caused an upstream movement of the transition front for the backward-facing step case. The step caused a local increase in the growth of the stationary crossflow instabilities, but the stationary crossflow amplitude at transition was sufficiently low (<0.04Uε) so that stationary crossflow was not solely responsible for transition. The unsteady velocity spectra downstream of the step were rich with unsteady disturbances in the 80- to 1500-Hz range. Three distinct families of disturbances were identified based on phase speed and wave angle, namely, a highly oblique instability, the stationary crossflow disturbances caused a modulation of the unsteady disturbances, resulting in spatially concentrated peaks in unsteady disturbance amplitude. This modulation of the unsteady disturbances is believed to be the reason for the upstream movement of the transition front with increasing stationary crossflow amplitude.

Nomenclature

\[ A = \text{amplitude of disturbance, m/s} \]
\[ A_0 = \text{initial amplitude of disturbance, m/s} \]
\[ C_p = \text{nondimensional pressure coefficient; } (p - p_\infty) / (1/2)\rho U_\infty^2 \]
\[ c = \text{chord length, m} \]
\[ c_{ph} = \text{phase speed, m/s} \]
\[ f = \text{frequency, Hz} \]
\[ H(k) = \text{fast Fourier transform components of } U'(z)/U_\epsilon \]
\[ h = \text{step height, mm} \]
\[ k = \text{wavenumber vector, mm}^{-1} \]
\[ k_z = \text{experimental spanwise wavenumber, mm}^{-1} \]
\[ k_{xc} = \text{experimental wavenumber in the direction normal to the leading edge, mm}^{-1} \]
\[ N = \text{N-factor, integrated disturbance growth rate} \]
\[ p = \text{pressure, Pa} \]
\[ Re' = \text{unit Reynolds number; } U_{\infty}/u' \]
\[ Re_h = \text{Reynolds number based on excrescence height and the boundary-layer edge velocity; } U_h/u' \]
\[ Re_k = \text{Reynolds number based on excrescence height and velocity at the excrescence height; } u_h/u' \]
\[ Tu = \text{normalized turbulence intensity; } 1/U_{\infty} \sqrt{1/3(u'^2 + v'^2 + w'^2)} \]
\[ U = \text{mean boundary-layer velocity in the } x \text{ direction, m/s} \]
\[ U' = \text{steady disturbance velocity, m/s} \]
\[ U_\epsilon = \text{boundary-layer edge velocity, m/s} \]
\[ U_{rms} = \text{spanwise root mean square of steady disturbance velocity, } U', \text{ m/s} \]
\[ U_\infty = \text{freestream velocity, m/s} \]
\[ u', v', w' = \text{fluctuating components of velocity, m/s} \]
\[ u'_{rms} = \text{temporal root mean square of } u', \text{ m/s} \]
\[ x = \text{streamwise direction, } \text{m} \]
\[ x_h = \text{direction normal to the leading edge, } \text{m} \]
\[ x_r = \text{streamwise location of step, normalized by } c \]
\[ x_{sh} = \text{number of step heights downstream of step} \]
\[ y = \text{wall-normal direction, mm} \]
\[ z = \text{spanwise direction (parallel to the leading edge), mm} \]
\[ z_n = \text{direction normal to side wall, mm} \]
\[ -\alpha_t = \text{spatial amplification rate, } \text{m}^{-1} \]
\[ \delta = \text{boundary-layer thickness, } \text{mm} \]
\[ \lambda_z = \text{spanwise wavelength, mm} \]
\[ \psi = \text{wave angle, deg} \]
\[ \rho = \text{density, kg/m}^3 \]

Subscript
\[ \infty = \text{freestream quantity} \]

I. Introduction

AMINAR flow remains a promising technique for substantial improvements in the fuel efficiency of aircraft. Although much progress is being made toward achieving laminar flow on wings in the laboratory, putting those techniques into practice presents additional...
difficulties. Aircraft wings will have unavoidable manufacturing defects, such as steps and gaps, and necessary screws and rivets on the surface of the wing. Thus, there will be two-dimensional (2D) and three-dimensional (3D) surface irregularities on the wing that are not typically present in wind tunnel models. It is important to understand the influence of different types of surface nonuniformities on boundary-layer transition so that appropriate manufacturing tolerances can be specified for wing designs with laminar-flow technology. Historically, the effects of 2D and 3D excrescences on transition have been predicted using empirical methods known as the $Re_e$ or $Re_k$ criterion [1–6]. Here, $Re_e$ is defined as the Reynolds number based on the height of the excrescence and the undisturbed streamwise velocity at this roughness height. The $Re_e$ criterion correlates the mean onset of transition with a critical value of $Re_e$ such that the transition location first moves upstream from the nominal location as $Re_e$ increases above this critical number. Below this critical Reynolds number, the transition location is generally unaffected by the presence of the roughness. Nenni and Gliyas [6], proposed the $Re_k$ criterion, where $Re_k$ is the Reynolds number based on the height of the excrescence and the edge velocity. They proposed criterion for backward- and forward-facing steps in 2D flows and found that the critical height of the forward-facing steps may be approximately twice that of the backward-facing steps.

Recent work includes experimental and numerical studies on the effects of surface excrescences on the stability of the boundary-layer disturbances [7–9]. It is assumed that a localized perturbation of the pressure distribution, caused by the surface imperfection, will cause a localized perturbation of the amplification characteristics of linear instability modes. Downstream of this perturbation, the amplification curve will become parallel to the nominal amplification curve, except in the case of forward-facing steps [9]. Hence, linear stability theory is applied to determine a $\Delta N$, that is, the change in the logarithmic amplification ratio (i.e., the so-called $N$-factor) of the instabilities due to the presence of the excrescence. The majority of this work has focused on 2D excrescences in 2D, Tollmien–Schlichting (TS)-dominated boundary layers.

There is considerably less data available for the effect of 2D excrescences in 3D boundary layers. Perraud and Seraudie [9] performed a systematic experimental study of 2D steps on a swept wing with multiple sweep angles. As the angle of attack became more negative (resulting in a more favorable pressure gradient), the increased sweep caused the boundary layer to become more sensitive to the forward-facing step height (i.e., premature transition occurred for lower step heights as the sweep was increased). For a backward-facing step in crossflow, a critical step height was found, below which transition did not move. Above this height, the boundary-layer became more sensitive than in the corresponding 2D case. Recently, Duncan et al. [10] performed flight experiments to determine the effect of 2D steps on transition over a swept wing. Their results indicate that the addition of the crossflow instability caused the transition location to move forward relative to a similar 2D case, in agreement with the results of Perraud and Seraudie [9]. In the companion wind tunnel testing performed by Duncan et al. [11], results indicate that the 3D case is slightly more sensitive than the corresponding 2D case to forward-facing steps but not backward-facing steps. They also performed hotwire measurements downstream of the step for forward- and backward-facing steps to determine the effect of the steps on stationary crossflow instabilities. They found that the backward-facing step caused a small increase in $N$-factor for the stationary crossflow; however, the stationary crossflow amplitudes were very low at the step, and therefore the uncertainty was high. Balakumar et al. [12] performed linear parabolized stability equation (PSE) and direct numerical simulation computations for a supersonic swept-wing case studying the effect of 2D exccrescences on stationary crossflow. Their results indicate a linear effect on the growth of the stationary crossflow modes, but the effect depended on the wavelength of the mode. The shorter wavelength modes were amplified due to the step, but the longer wavelength modes were actually stabilized relative to the baseline case. Tufts et al. [13] recently performed computations to study the effect of 2D step excrescences on stationary crossflow instabilities. They found that the backward-facing step did not amplify the stationary crossflow modes, and they verified the existence of a traveling instability in the recirculation region downstream of the step.

There is still a gap in knowledge of the details of how transition is affected on a swept wing when 2D excrescences are introduced. The above-mentioned research in this area has been concerned primarily with studying the effect of forward- and backward-facing steps on the transition location. Although there has recently been a push toward obtaining more detailed information about the transition process, it is still not clear what mechanism is primarily responsible for transition when 2D excrescences are present. There is very little data that can provide an understanding of how the steps actually affect the boundary-layer instabilities and how the latter ultimately lead to transition. The current research is an attempt to fill this gap. This experiment was performed to enable a detailed look at the effect of a backward-facing step on transition in a swept-wing flow nominally dominated by stationary crossflow. The experimental setup and procedure are discussed in Secs. II and III. A brief description of the data analysis procedures is given in Sec. IV. The results for the baseline (no-step) case are presented in Sec. V, followed by the results for the step case.

II. Experimental Setup

The experiment was performed in the 2 Foot by 3 Foot Low Speed Boundary-Layer Channel at NASA Langley Research Center. The tunnel is a closed circuit facility with a 0.61-m-high by 0.91-m-wide by 6.1-m-long test section. The tunnel can reach speeds up to 45 m/s ($Re'_f = 2.87 \times 10^6$) in the test section. Freestream turbulence intensity levels, $T_u$, were measured using a crosswire in an empty test section and were found to be less than 0.06% for the entire speed range of the tunnel, and less than 0.05% at the test speed of 26.5 m/s. This value represents the total energy across the spectrum (0.25 to 10 kHz) and has not been filtered to remove the low-frequency acoustic component. Based on the criteria outlined by Saric and Reshotko [14], this tunnel can be considered a low-disturbance facility for purposes of conducting transition experiments.

The 0.0127-m-thick flat plate model consists of a 0.41-m-long leading edge piece, swept at 30°, and a larger downstream piece (see Fig. 1). The model is 0.91 m wide (thus spanning the width of the test section) and 2.54 m long on the longest (i.e., inboard) edge. Both the leading-edge piece and the aft section can be adjusted relative to each other using precision shims to create 2D forward-facing or backward-facing steps of different heights at a fixed distance from the leading edge. The leading edge piece was polished to a surface finish of 0.2 $\mu$m root mean square (rms) and the larger downstream plate to a surface finish of 0.4 $\mu$m rms.

The chord $c$ is taken as the longest edge of the plate (2.54 m) and is used to nondimensionalize quantities throughout the paper. Thus, the step is located at $x/c = 0.161$ (see Fig. 2 for a schematic of the coordinate system). A severe suction peak was predicted to occur close to the leading edge on the underside of the plate due to the location of the attachment line on the upper surface and the small leading-edge radius of the plate. Thus, a contoured piece with a large

Fig. 1 Sketch of flat plate model.
radius was designed for the underside of the plate in order to make the suction peak less severe and thus to avoid separation that might have resulted in potential unsteadiness in the attachment line location.

A 3D pressure body along the ceiling was designed to induce a streamwise pressure gradient to promote the growth of stationary crossflow vortices in an approximately infinite swept-wing flow within a midspan measurement region of width 0.3 m (indicated in Fig. 2). This was achieved by ensuring that the $C_p$ contours were parallel with the leading edge within the core region. Two-dimensional computations were performed first in order to determine the necessary streamwise pressure gradient to achieve the desired stationary crossflow growth. Then, the design was modified three-dimensionally, using Fluent to do the 3D inviscid computations, in order to achieve the desired spanwise uniformity. This spanwise uniformity was verified experimentally using pressure belts (see Sec. V.A). More details of the design process are provided by Eppink [15]. The ceiling liner was fabricated out of a hard foam using a computer-controlled milling machine. The curved surface was coated with an epoxy to create a smoother and more durable surface.

All measurements were performed at a freestream velocity of 26.5 m/s ($Re = 1.69 \times 10^6/m$). The data were acquired using a hotwire mounted on a traversing system that could be moved in all three directions. Detailed boundary-layer measurements allowed for tracking of the instability growth and determining the effect of the backward-facing step on the instabilities. Additionally, sublimating chemical flow visualization was performed using naphthalene to determine the transition front. Three different leading-edge roughness configurations were investigated: a clean leading edge and two discrete roughness element (DRE) configurations. The DREs were applied with a spanwise spacing, $\lambda_z$, of 11 mm and were approximately 20 $\mu$m tall. DREs with two different diameters were used: 2.75 mm (small DREs) and 4.4 mm (large DREs). The large-diameter DREs lead to larger initial amplitudes of the stationary crossflow due to the enhanced receptivity. The spacing of the DREs (11 mm) corresponds to the most amplified stationary crossflow wavelength calculated for the baseline case.

III. Experimental Procedure

A. Naphthalene Surface Flow Visualization

Sublimating chemical techniques for surface flow visualization have been used for many years to document transition location in a variety of flows [16]. The sublimating chemical is mixed with a solvent and sprayed onto the model surface to create a thin coating. Naphthalene is usually the sublimating chemical of choice for low-speed flows. The sublimation rate of naphthalene is sensitive to shear stress, and hence the chemical will sublimate faster in regions of higher shear. The transition location on a model can be visualized clearly using this technique because the wall shear stress is much higher in turbulent boundary layers. In fact, the technique is sensitive enough that the high-shear regions of the stationary crossflow vortices are often visible once they reach sufficiently large amplitude [17].

The leading-edge segment of the model was not sprayed in order to prevent an unwanted step or roughness near the leading edge where the boundary layer is thin. The coating typically started several centimeters downstream of the step location and extended about 1.5 m downstream of the leading edge. The naphthalene spray was confined to a limited spanwise region that extended several centimeters wider than the measurement region to conserve time and material. The naphthalene results can be interpreted only qualitatively due to variations in coating thickness and are used primarily to obtain an overall picture of the general location and shape of the transition front. The hotwire results can then be used to verify the interpretation of the global naphthalene results and to gain more insight into the physical mechanisms underlying the transition process.

A camera located above the test section near the leading edge of the model was used to acquire pictures of the transition front. The model surface was visible from just upstream of the step to about 1.4 m downstream of the leading edge. A spatial reference was obtained by drawing a grid on the model and acquiring a baseline picture. The pixels of the grid were extracted from this picture and then used to overlay the grid on the subsequent naphthalene pictures that were acquired. The images have been unswept and undistorted in the spanwise direction in order to aid in visual interpretation of the images.

B. Boundary-Layer Measurements

Several different types of boundary-layer measurements were performed. These included $z$ scans, $yz$ scans, and two-wire scans. The $z$ scans consisted of spanwise scans (parallel to the leading edge) at a constant height in the boundary layer. The spanwise resolution of these scans was typically 0.5 mm, which was sufficient to resolve the primary crossflow wavelength ($\lambda_z = 11$ mm).

The $yz$ scans consisted of closely spaced, full boundary-layer profiles at a sequence of regularly spaced spanwise locations at a constant $x/c$ location. The spanwise spacing of the boundary-layer profiles was typically 0.5 to 1 mm, depending on the desired spanwise resolution. A fouling tip attached to the probe body was used to measure the local wall-location relative to the probe. On average, 30 points were taken per boundary-layer profile, extending to approximately 1.5 boundary-layer thicknesses from the surface. The sampling rate for these scans was typically set to 25 kHz. Typically, 25,000 samples were acquired for each point during the $z$ scans, and 125,000 samples were acquired for each point during the $yz$ scans. The AC-coupled data were low-pass filtered at 10 kHz. Fifty averages were performed with 50% overlap to calculate the velocity spectra. Eppink [15,18] provides more details of the boundary-layer measurement procedure.

C. Surface Pressure Measurements

Surface pressure measurements were performed for the baseline case to determine the effectiveness of the ceiling pressure body at creating the desired pressure distribution. Pressure belts were temporarily installed to measure surface pressures. The belts consisted of ribbon tubing with an outer and inner diameter of 1.6 and 0.71 mm, respectively. One end of each tube was sealed, and the other end was connected to an electronically scanned pressure transducer. A small hole of diameter 0.4 mm was placed at the desired streamwise location on top of each tube. More details of the pressure belts are provided by Eppink [15]. The belts were removed before the boundary-layer measurements, allowing us to avoid the effect of surface protuberances due to the pressure belts. Four pressure belts were used: three with coarse hole spacing and one with finer spacing near the leading edge. One of the coarse belts and the fine belt were placed side-by-side down the middle of the model, and the other two coarse belts were placed on the inboard and outboard edges of the measurement region.

IV. Data Analysis

A. Mode Shape and Amplitude of Stationary Crossflow Vortices

To obtain the stationary crossflow mode shapes, a spanwise averaged wall-normal $U'$ profile was computed and then subtracted from each wall-normal profile to obtain values of $U'$. The $U'$ data at
A. Baseline (No-Step) Results

Results from the surface pressure measurements for the final configuration are shown in Fig. 3. A comparison of the pressure distributions obtained using the various belts verifies very good spanwise uniformity across the measurement region.

Linear PSE calculations were performed using the Langley Stability and Transition Analysis Code (LASTRAC) [20]. A smoothed and interpolated version of the measured midspan pressure distribution was used as input to the boundary-layer code BLSTA [21] to compute the laminar basic state under the assumption of an infinite-span swept airfoil. BLSTA is a boundary-layer code that uses a second-order accurate finite-difference method to solve the compressible laminar boundary-layer equations for different types of flow, including infinite swept-wing flows. Stationary crossflow N-factor predictions are shown in Fig. 4 for the test speed of 26.5 m/s. The goal of the experimental design was to achieve strong crossflow growth resulting in stationary crossflow dominated transition before the suction-peak location at \(x/c \approx 0.6\). Because the stationary crossflow N-factors were predicted to reach values greater than 10 by \(x/c = 0.5\), the design was deemed acceptable. Surface flow visualization was performed using naphthalene as a sublimating chemical to visualize the transition front. The transition front for the large-DRE case is shown in Fig. 5. In this image, the vertical lines indicate the inboard edge of the measurement region, the midspan location, and the outboard edge of the measurement region. The horizontal lines (which are parallel with the leading edge) are also

B. Overview of Uncertainty Analysis

The total uncertainty of a measurement consists of the sum of the round-off error (resulting from the limited accuracy of the measurement technique) and the precision error (i.e., repeatability). The precision limits for all of the instruments used were found to be negligibly small and thus made no contribution to the total uncertainty. The total uncertainty for the hotwire velocity measurements was obtained by taking the root sum square of the contributing uncertainties, which consisted of the uncertainty of the DAQ system (resulting from the uncertainty in the uncertainty of the instantaneous voltage measurement), the uncertainty of the hotwire calibration, the uncertainty of the hotwire calibration, and the uncertainty of the hotwire calibration. The total uncertainty ranged from 7.6% at the low end of the measured velocities (1–2 m/s) down to 0.43% at the larger measured velocities (approximately 20 m/s). The main contributors to the total uncertainty were the uncertainties in freestream velocity and hotwire calibration. The uncertainty in the rms fluctuating velocity was calculated in a similar fashion from the uncertainties associated with the instantaneous voltage measurements and was found to be a maximum of \(\pm 1.5\%\).

A Monte-Carlo simulation was performed to estimate the uncertainty of the stationary crossflow amplitude measurements. For each data set, the uncertainty of the mean hotwire data was first estimated as described above. Random values of streamwise velocity error for each point were then generated in the range bounded by the uncertainty, and the resulting stationary crossflow amplitude was calculated for each iteration. The uncertainty was then estimated by calculating the standard deviation of the simulated \(U'_{\text{rms}}\) values for a 95% confidence level. The uncertainty was found to be fairly constant across all of the runs performed, at approximately 0.012 m/s. For the smallest amplitude stationary crossflow values measured (approximately 0.5% \(U_e\)), this resulted in percent uncertainties near 15%, whereas for the large amplitudes (18% \(U_e\)) the percent uncertainties were as low as 0.2%.

Uncertainties were also estimated for the wave angle, wavenumber, and phase speed. The main source of uncertainty for these results originated from the uncertainty of the slope of the linear regression applied to the phase data. The wave angle uncertainties were fairly large, ranging from \(\pm 4^\circ\) to \(\pm 11^\circ\). The phase speed uncertainties ranged from 4% to 9% of \(c_d\).

The uncertainty in the measurement of the step height was also estimated. This measurement was performed using a surface contact profilometer. Measurements were performed at 13 equally spaced locations across the measurement region of the model. The largest source of uncertainty comes from the variation in the measured step height across the span. Thus, the standard deviation of these measurements was used to estimate the error. The standard deviation for the height measurements was approximately 8.5 \(\mu m\). For a 95% confidence interval, this results in an uncertainty of \(\pm 16.6 \mu m\), which corresponds to percent uncertainties close to 14% of the average step height. A complete description of the uncertainty analysis is provided by Eppink [15].

V. Results and Discussion

Results from the five pressure belts configurations are shown in Fig. 3. A comparison of the pressure distributions obtained using the various belts verifies very good spanwise uniformity across the measurement region.

Linear PSE calculations were performed using the Langley Stability and Transition Analysis Code (LASTRAC) [20]. A smoothed and interpolated version of the measured midspan pressure distribution was used as input to the boundary-layer code BLSTA [21] to compute the laminar basic state under the assumption of an infinite-span swept airfoil. BLSTA is a boundary-layer code that uses a second-order accurate finite-difference method to solve the compressible laminar boundary-layer equations for different types of flow, including infinite swept-wing flows. Stationary crossflow N-factor predictions are shown in Fig. 4 for the test speed of 26.5 m/s. The goal of the experimental design was to achieve strong crossflow growth resulting in stationary crossflow dominated transition before the suction-peak location at \(x/c \approx 0.6\). Because the stationary crossflow N-factors were predicted to reach values greater than 10 by \(x/c = 0.5\), the design was deemed acceptable. Surface flow visualization was performed using naphthalene as a sublimating chemical to visualize the transition front. The transition front for the large-DRE case is shown in Fig. 5. In this image, the vertical lines indicate the inboard edge of the measurement region, the midspan location, and the outboard edge of the measurement region. The horizontal lines (which are parallel with the leading edge) are also
drawn at several streamwise locations. Note that parts of the model surface are not visible due to the limited size of the window (such as the bottom left corner of the image). The sawtooth transition front pattern in the picture is indicative of stationary crossflow–dominated transition, and the sawtooth spacing matches the DRE spacing of 11 mm. Transition across the measurement region occurs between approximately $x/c = 0.42$ and 0.48, confirming that stationary crossflow–dominated transition occurs before the suction peak. The average transition locations obtained from the naphthalene flow-visualization results are $x/c = 0.5, 0.44,$ and 0.42 for the no-DRE, small-DRE, and large-DRE cases, respectively.

Band-limited amplitudes ($\lambda_z = 8$ to 20 mm) of the stationary crossflow disturbances are plotted in Fig. 6a. The discussion in this section will focus solely on the no-step data (open symbols) presented in this figure. The results for the backward-facing step configuration (filled symbols) will be discussed in the following sections. In Fig. 6, the curves for the large-DRE case terminate earlier because data for this case were not acquired near transition due to limited physical access in this region. Sufficient data to determine the amplitude of the stationary crossflow vortices near saturation were taken for the small-DRE case only. For this case, saturation occurs near $x/c = 0.4$ at about 18% $U_e$. The stationary crossflow instability in the no-DRE case appears to saturate near $x/c = 0.46$ at an amplitude of 12% $U_e$. However, the data points are sparse in the region between $x/c = 0.4$ to $x/c = 0.46$, and therefore we cannot say with certainty whether 12% is the largest amplitude reached for this case.

$N$-factors for all three leading-edge roughness cases are plotted in Fig. 6b. The $N$-factors are calculated using Eq. (4), where the stationary crossflow amplitudes are obtained using the peak amplitude of the $U_{rms}$ profile [Eq. (3)]. The amplitude at the most upstream point ($x/c \approx 0.11$) is used as the initial amplitude ($A_0$) for the small and large-DRE cases. These amplitudes are $U_{rms}/U_e = 0.0042$ and 0.0086 for the small and large-DRE cases, respectively. Because of the low crossflow vortex amplitudes in the no-DRE case, the most upstream location for that case ($x/c = 0.18$) is significantly farther downstream than that for the two DRE cases. Therefore, the initial amplitude for this case is calculated such that the $N$-factor at $x/c = 0.18$ matches that for the small and large-DRE cases at the same location. This initial amplitude was found to be $U_{rms}/U_e = 9.34e^{-4}$. The $N$-factors collapse well up until approximately $x/c = 0.3$. At this point, the large-DRE curve begins to depart from the other two, indicating reduced amplification rates. Farther downstream, at approximately $x/c = 0.35$, the small-DRE curve begins to depart from the no-DRE curve as the stationary crossflow amplitude saturates. Thus, the cases with the larger initial amplitude saturate earlier than those with lower initial amplitude, which is consistent with known effects of disturbance nonlinearity.

The growth of the first harmonic of the dominant stationary crossflow mode ($\lambda_z = 5.5$ mm) is shown in Fig. 7, where disturbance amplitude over a wavelength range of 5 to 8 mm is plotted versus $x/c$. This mode starts to grow from $x/c = 0.2$ in the large-DRE case and slightly downstream in the small-DRE case. The amplitudes (Fig. 7a)

---

**Fig. 4** Stationary crossflow $N$-factors calculated using the experimentally measured $C_p$ distribution.

**Fig. 5** Naphthalene flow visualization of baseline case with large DREs.

---

**Fig. 6** Band-limited ($\lambda_z = 8$ to 20 mm) amplitudes and $N$-factors of primary stationary crossflow mode.
stay low (<0.5%U∞) until x/c ≈ 0.375 for the no-DRE and small-DRE cases, and past x/c = 0.3 for the large-DRE case. This smaller wavelength mode grows significantly just before breakdown.

A dominant path for transition in a stationary crossflow-dominated flow involves high-frequency secondary instabilities of the crossflow vortices. These instabilities occur as a result of large spanwise or wall-normal gradients in U, which form due to the deformation of the mean flow [22,23]. The frequencies of the secondary instabilities are typically an order of magnitude higher than the amplified band of traveling crossflow frequencies. For the baseline case, the linear PSE N-factor computations identified amplified traveling crossflow modes within the range of f = 50 to 300 Hz (Fig. 8). Thus, as a rough estimate, the secondary instabilities are expected to occur in the 1- to 3-kHz range.

Power spectral density plots of u’ for each of the leading-edge roughness configurations are shown in Fig. 9. The power spectral densities are calculated from the z-scan data at or near y = 1 mm (y/δ ≈ 0.3 to 0.4) for each of the streamwise locations shown. The secondary modes are known to be concentrated in specific regions of the stationary vortex, and therefore the spectra at different spanwise locations will differ from each other. However, to gain an overall understanding of the spectral peaks, the spectra presented here are averages across all points in the spanwise scan.

The spectra at the upstream locations (until x/c = 0.373) all appear very similar to each other and are typical of the spectra seen in laminar boundary layers. The spectra at locations upstream of x/c = 0.373 are low in amplitude for all three leading-edge roughness configurations. A peak in the spectrum exists close to f = 300 Hz for all three leading-edge roughness configurations. This peak is particularly apparent at x/c = 0.123 in the small-DRE case but is not observed at downstream locations. The theoretical results for N-factor evolution for traveling crossflow (Fig. 8) are based on the experimental Cp distribution obtained at the midspan location. These results show that the higher frequency modes (200–300 Hz) amplify faster near the leading edge but start to decay at x/c ≈ 0.1.

It is possible that the 300-Hz peak in the measured spectra at upstream stations corresponds to a traveling crossflow mode. This mode does not appear to play a role in transition due to its very low amplitude and the fact that the associated peak is no longer prominent at the downstream locations. The traveling crossflow modes that are expected to grow for x/c > 0.13 correspond to the lower frequencies, particularly those near 50–100 Hz.

The spectra for the locations nearing transition for the no-DRE (Fig. 9a, x/c = 0.48) and small-DRE (Fig. 9b, x/c ≥ 0.42) cases exhibit a broad band of fluctuations from 2 to 7 kHz. The maximum amplitude occurs close to 5 kHz for the no-DRE case and 4 kHz for the small-DRE case. We believe that these high-frequency fluctuations are indicative of the presence of high-frequency secondary instabilities of the stationary crossflow vortices. The spectra further downstream continue to increase in amplitude at the higher frequencies and spectral broadening ensues, suggesting the breakdown of secondary instabilities.

Because of the limited range of streamwise locations for which spectral measurements were made for the large-DRE case, no spectral data are available for x/c locations approaching the range of transition locations (0.42 < x/c < 0.48) based on the naphthalene flow visualization. Thus, we did not measure far enough downstream to capture the high-frequency secondary instabilities for this case (Fig. 9c). However, because the amplitudes of the stationary crossflow vortices in these cases were even larger than those in the small-DRE case, the transition mechanism is, again, likely to involve high-frequency secondary instabilities similar to the other two roughness configurations. More detailed measurements of the secondary instabilities are shown by Eppink [15]. The naphthalene results, Urms measurements, along with the high-frequency secondary instabilities evident in the u’ spectra, all verify that stationary crossflow-dominated transition was achieved on the model for the baseline (i.e., no-step) case.
B. Effect of Backward-Facing Step on Transition

1. Naphthalene Flow Visualization Results

Naphthalene flow visualization was used to assess the effect of step height on transition location. These runs were first performed with no DREs on the leading edge. Precision shims were inserted between the splice plate and the bottom of the leading-edge piece to achieve the desired step heights. The actual step height across the span was measured at several locations for the final configuration. However, the step heights for the naphthalene tests were not measured directly but inferred from the total shim thickness used to create the step. These inferred measurements are within ±0.015 mm of the stated value. The boundary-layer thickness at the step location was found to be $\delta = 2.4$ mm from the hotwire measurements for the baseline case.

The transition location moved abruptly forward from $x/c > 0.48$ to $x/c \approx 0.27$ when the step height was increased by just 0.05 mm from 1.16 to 1.21 mm in this no-DRE case. Measurements of the step height were acquired at approximately 25 mm spanwise increments across the measurement region for the $h \approx 1.16$ mm step height in order to determine the step height and its spanwise uniformity. The average step height for these measurements is 1.16 mm, which is about 0.05 mm less than the value expected from the shim thickness (1.219 mm). A 0.025 mm shim was added at the midspan shim location in an attempt to obtain a more uniform step height across the span. This resulted in an increase in step height at the midspan location and an overall increase in step height. The new average step height with the additional shim was 1.184 mm, and a better overall step uniformity was achieved. Therefore, the measurements made throughout the rest of the study (denoted as $h = 1.184$ mm BFS) refer to this configuration. This step height corresponds to an $Re_h$ value of 1555 and an $Re_k$ value of 1232. Naphthalene flow visualization was repeated for this configuration (Fig. 10). The resulting transition location was spatially nonuniform, making it difficult to specify a single transition location. The black line drawn on this figure is added to help delineate the transition front, which can

![Figure 9](image_url)

**Fig. 9** Power spectral density of $u'/U_e$ averaged over $z = 100$ to 200 mm at $y = 1$ mm ($y/\delta \approx 0.3$ to 0.4) for all leading-edge roughness cases, no step configuration.

![Figure 10](image_url)

**Fig. 10** Naphthalene flow visualization of $h \approx 1.184$ mm BFS with no DREs.
be difficult to see due to reflection from the lights. The laminar region that extends farthest back on the model reaches $x/c = 0.42$ before the flow becomes turbulent. On the inboard and outboard sides, the transition location moves significantly upstream. There is also a turbulent wedge originating from close behind the step near the outboard edge of the measurement region.

DREs were applied to the leading edge to study the effect of stationary crossflow amplitude on the transition behavior with a backward-facing step. The result for the small-DRE case is shown in Fig. 11. The transition front for the small DREs is more uniform compared with the no-DRE transition front. The transition front toward the middle of the outboard half of the measurement region has moved forward to $x/c \approx 0.3$. The stationary crossflow amplitudes were increased further by replacing the small DREs with large DREs. A corresponding shift in transition location is observed as transition moves forward to $x/c \approx 0.24$ (Fig. 12). A comparison of the no-DRE, small-DRE, and large-DRE cases shows that increasing the stationary crossflow amplitude causes transition to occur earlier, on average. Thus, stationary crossflow appears to be playing a role in the transition process.

2. Stationary Crossflow

We investigated the effect of the step on the stationary crossflow modes. Note that for the case with no DREs the amplitudes of the stationary crossflow vortices near the step are rather small, and hence the measurement uncertainty is too large to obtain useful data on the growth of these vortex modes. Therefore, the results in this section are restricted to the two cases with DREs.

Mean profiles obtained downstream of the step [24] indicate the presence of a downstream separation bubble. Reattachment occurs approximately 33 step heights after the step. Stationary crossflow amplitudes for the $\lambda_s = 11$ mm stationary mode (which corresponds to the spacing of the DREs) were obtained from $z$ scans for the small- and large-DRE step cases. $N$-factors and amplitudes for these two cases were plotted in Fig. 6 by using filled symbols. The step location ($x_s$) and the reattachment location ($x_r$) are labeled on this plot and are indicated by vertical black lines. The amplitudes were calculated by integrating the wavelength spectra over a range of wavelengths around the primary and harmonic modes. The range used for the fundamental primary mode was $\lambda_s = 8$–20 mm, and the range for the first harmonic of the fundamental mode was $\lambda_s = 5$–8 mm.

The step does have a local effect on the growth of the stationary crossflow modes according to the results displayed in Fig. 6b. The amplitudes are normalized by the same initial amplitudes used to calculate the $N$-factor curves for the no step cases. A $\Delta N$ can be calculated between the $N$-factors measured for the no-step case and the $N$-factors with the step. The maximum $\Delta N$, which occurs at the point at which the stationary crossflow amplitude reaches its largest amplitude in the step case, is close to 0.8 and is approximately the same for both DRE cases. After this point, the $N$-factor relaxes back to and even drops below the baseline $N$-factor curve for the small-DRE case. Insufficient data were acquired to make a similar statement for the large-DRE case. Transition occurs shortly downstream of the location at which the stationary crossflow amplitudes peak.

As discussed previously, a first harmonic of the primary stationary crossflow mode is present in the spanwise spectra at some of the streamwise locations in the baseline case. This mode is also present in the step case. The amplitude and $N$-factors for this mode are plotted in Fig. 7. The step has an even larger effect on the amplitude of the 5.5 mm mode than the 11 mm mode. The maximum $\Delta N$, which occurs at $x/c \approx 0.2$, is approximately 1.8 for both DRE cases.

Figure 6a shows that the peak amplitude of the stationary crossflow disturbances near the transition location reaches rather large values ($>10\% U_e$) in the baseline case but remains below 5\% (and even lower for the small-DRE case) in the presence of the step. The small stationary crossflow amplitudes near the onset of breakdown behind the step imply that, despite the local increase in amplitude behind the step, stationary crossflow is not the dominant transition mechanism.

3. Unsteady Disturbances

Figure 13a shows a comparison of spanwise-averaged spectra from the baseline case and the step case with small DREs at a fixed streamwise location. The velocity spectrum from the step case contains significantly more energy in the range $f \approx 80$–2000 Hz. The evolution of the spanwise-averaged velocity spectra downstream of the step is shown in Fig. 13b for the large-DRE case. The energy across the 80–2000 Hz frequency range grows downstream. Starting at approximately 56 step heights downstream of the step, spectral broadening occurs at frequencies beyond this range, as indicated by the shift in the high-frequency roll-off. Eventually, the spectrum begins to appear increasingly turbulent. However, the spectral levels continue to increase throughout the range of measurement, indicating that a quasi-equilibrium turbulent state may not have been achieved at the last measurement station shown in the figure.

Two sensors were positioned inside the boundary layer and sampled simultaneously in order to obtain phase speed and wave angle information for the unsteady disturbances. One sensor was held at a fixed location, whereas the other wire was traversed in multiple directions. The fixed wire could then be used as a phase reference for frequencies at which the coherence was greater than 0.3. Because of limited time, these measurements were only performed for the large-DRE case. Figure 14 shows the coherence versus frequency for three different streamwise locations. Initially ($x_{sh} = 12.3$), there are three distinct peaks corresponding to frequencies of 110, 250, and 900 Hz at coherence values of 0.3 or greater. Farther downstream, the coherence increases, and the higher frequency peaks merge into one.
broad peak. Even farther downstream ($x_{sh} > 33.1$), the coherence begins to decrease and the three separate peaks become visible once again. The high coherence for frequencies greater than approximately 1500 Hz is believed to be due to correlated noise in the electronics. The energy in the velocity spectra at these locations is very low (i.e., at or near the noise floor).

Scans were performed in multiple directions ($x$, $z$, and $x_c$ directions) to determine the wave angle of each type of disturbance in the $xz$ plane. Figure 15 illustrates the wavenumber vectors for waves traveling close to the spanwise direction and for waves traveling close to the $x_c$ or $x$ direction. The wave angle $\psi$ is drawn relative to the streamwise ($x$) direction. Therefore, a wave angle of $\psi = 0$ deg indicates a wave traveling in the $x$ direction, whereas an angle of $\psi = -30$ deg indicates a wave traveling in the $x_c$ direction. A wave angle of $\psi = 60$ deg would indicate a wave traveling in the spanwise direction from inboard to outboard.

Measurements of disturbance phase relative to the fixed wire, as obtained from several $z$ scans, are shown as functions of $z$ in Fig. 16a. The frequency of interest in this case was 100 Hz (in the L band). The slope of the phase curve can be used to calculate the phase speed and wavelength in the spanwise direction. The slope is approximately linear and nearly the same at the four most upstream stations. Starting at $x_{sh} = 43$, a sinusoidal type of variation begins to appear in the phase data, although the overall slope associated with the underlying trend remains similar to the slope at the upstream stations. The amplitude of the periodic variation becomes larger with increasing downstream distance, and the wavelength of this variation matches the wavelength of the primary stationary crossflow vortices. A similar phase modulation of traveling crossflow modes was observed by Deyhle et al. [25]. This phase modulation is likely to be a result of the distortion of the mode shape due to interactions with the stationary crossflow mode. Similar phase modulations were seen for the M and H bands.

Measurements of disturbance phase relative to the fixed wire, as obtained from several $z$ scans, are shown as functions of $z$ in Fig. 16a. The frequency of interest in this case was 100 Hz (in the L band). The slope of the phase curve can be used to calculate the phase speed and wavelength in the spanwise direction. The slope is approximately linear and nearly the same at the four most upstream stations. Starting at $x_{sh} = 43$, a sinusoidal type of variation begins to appear in the phase data, although the overall slope associated with the underlying trend remains similar to the slope at the upstream stations. The amplitude of the periodic variation becomes larger with increasing downstream distance, and the wavelength of this variation matches the wavelength of the primary stationary crossflow vortices. A similar phase modulation of traveling crossflow modes was observed by Deyhle et al. [25]. This phase modulation is likely to be a result of the distortion of the mode shape due to interactions with the stationary crossflow mode. Similar phase modulations were seen for the M and H bands. The linear best-fit line was used to calculate a spanwise wavelength of approximately 50 mm. This is about 4–5 times larger than the predicted wavelengths of the most amplified traveling crossflow modes for the baseline case. However, scans performed in the direction normal to the leading edge ($x_{hi}$), as in Fig. 16b, reveal that the wave is primarily traveling in the spanwise direction (with a wave angle of 75 deg), which is typical of traveling crossflow modes [25] and also agrees with the linear stability predictions of wave angles for those modes. Based on the PSE computations for the baseline case (see Fig. 8), 100 Hz also falls in the expected frequency range for traveling crossflow. Thus, this mode is likely to be an unstable traveling crossflow mode, though it is unknown why the observed spanwise wavelength is so large.
Wave angle and phase speed results are listed for all three disturbance bands in Table 1. The phase speeds are computed in the direction of propagation of the wave. In the region where these measurements were performed, the computed streamlines were curved only slightly from the streamwise direction (+3 to +5 deg). Thus, to obtain an estimate of the wave angles relative to the external inviscid streamline, one can simply subtract 4 deg from the values of \( \psi \) given in the table.

The H band exhibits an abrupt change in direction and phase speed downstream of reattachment \( (x_r) \). Consequently, the table is divided into two sections. The L and M bands, however, do not exhibit any significant changes downstream of reattachment. The H band is believed to be a shear-layer disturbance upstream of reattachment. This hypothesis is supported by the phase speed and wave angle results, which are typical of a shear-layer type of disturbance. The wave is traveling close to the streamwise direction with a phase speed close to 40% \( U_e \). Additionally, preliminary computations of the shear layer instability for these flow conditions confirmed that the most amplified band of frequencies should fall between 800 to 1500 Hz. Downstream of reattachment, the wave abruptly changes direction and begins traveling in approximately the spanwise direction, with a wave angle and phase speed similar to the L band. It is believed that downstream of reattachment, where shear-layer instabilities should no longer exist, the H band undergoes nonlinear interactions (either with the stationary crossflow modes or with the L band) and thus behaves differently.

The M-band disturbance travels with a phase speed of between 20–30\% of \( U_e \) and travels close to the \( x_c \) direction. This disturbance band is believed to be a TS type of disturbance. Computations for a flat plate with no pressure gradient indicate that this frequency range is approximately what would be expected for these flow conditions, and the mode shapes shortly downstream of reattachment exhibit a second lobe high in the boundary layer, indicative of the TS mode shape. However, farther downstream, this second lobe is no longer apparent. TS disturbances are stable in the baseline case due to the strong favorable pressure gradient, but they can be destabilized downstream of the step in the short adverse pressure gradient region induced by the step.

It was mentioned that the stationary crossflow amplitudes are too low for the stationary crossflow instability to be the dominant transition mechanism. However, increasing the initial amplitude of the stationary vortices does lead to an upstream movement of the transition front. The unsteady disturbance measurements show that the stationary crossflow modes also cause a spanwise modulation of the unsteady disturbance amplitudes. This is illustrated in Fig. 17 in which the rms amplitudes of all three disturbance bands are plotted separately in the \( y-z \) plane. The mean \( U \) contours are also overlaid as solid lines. There is a clear modulation of the M and H bands corresponding to the primary wavelength (\( \lambda_z = 11 \) mm) of the stationary crossflow modes. The L band also shows some modulation, but the pattern appears to have more of a harmonic modulation (5.5 mm) rather than the \( \lambda_z = 11 \) mm wavelength of the dominant fundamental stationary modes.

The streamwise evolution of the three disturbance bands was measured by choosing a peak location of the M band (for instance, \( z \approx 128 \) in Fig. 17) and tracking that peak location downstream. Boundary-layer profiles were acquired at the peak location for numerous streamwise stations. The results from this approach for each of the disturbance bands are shown in Fig. 18. The unsteady disturbance amplitudes at each streamwise station are taken from the rms amplitudes of all three disturbance bands. Figure 18a shows the amplitude of the disturbances normalized by the local edge velocity \( (U_e) \), and Fig. 18b shows the amplitude normalized by the amplitude of the disturbance at the farthest upstream location.

The different growth rates of the disturbances are evident from Fig. 18b. The H band grows the fastest in the upstream (separated) region. This larger growth makes sense given that it is believed to be a shear-layer instability and thus should be highly destabilized in the separated region. The L band grows the slowest in the separated region. The L- and M-band disturbances achieve an amplitude of 3% significantly sooner for the large-DRE case compared with the small-

### Table 1: Wave angle and phase speeds of each frequency band

<table>
<thead>
<tr>
<th>Name</th>
<th>Frequency, Hz</th>
<th>( c_{ph}/U_e ) Upstream of ( x_c )</th>
<th>( \psi ), deg</th>
<th>( c_{ph}/U_e ) Downstream of ( x_c )</th>
<th>( \psi ), deg</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>80–200</td>
<td>0.18</td>
<td>75 deg</td>
<td>0.18</td>
<td>75 deg</td>
</tr>
<tr>
<td>M</td>
<td>200–800</td>
<td>0.2 to 0.3</td>
<td>−40 deg to −20 deg</td>
<td>0.2 to 0.3</td>
<td>−40 deg to −20 deg</td>
</tr>
<tr>
<td>H</td>
<td>800–1500</td>
<td>0.37 to 0.41</td>
<td>−9 deg to −1 deg</td>
<td>0.22</td>
<td>71 deg</td>
</tr>
</tbody>
</table>
The effect of stationary crossflow amplitude on the H-band disturbances is not very obvious until downstream of reattachment, at which point the H-band amplitude increases more rapidly for the large-DRE case. Figure 18a shows that the L-band disturbance level remains similar for the two DRE cases until approximately $x_{sh}/c \approx 0.184$. Beyond this location, the disturbance amplitude in the large-DRE case increases rapidly. The peak amplitude for the L band might not have been tracked well because the peaks for this disturbance band did not align well spatially with the peaks for the M and H bands. Downstream of $x_{sh} = 50$ ($x/c \approx 0.184$), the flow starts to become transitional in the large-DRE case, which may account for the sudden increase in the amplitude in
the L band at this location. Based on all of these results, it appears that the unsteady disturbances in the 80–2000 Hz range are primarily responsible for breakdown in the backward-facing step case, though the increased stationary crossflow amplitude does cause an increase in the peak amplitude of the unsteady disturbances. Of the three distinct types of disturbances, it is not clear which (if any) is the most important.

VI. Conclusions

Detailed measurements of boundary-layer instabilities downstream of a backward-facing step (h/δ ≈ 0.49) on a swept flat plate with induced pressure gradient are presented. The presence of this step leads to a strong upstream shift in transition for all three leading-edge roughness configurations that influence the initial amplitude of the stationary crossflow vortices in the boundary-layer flow. The backward-facing step has a local destabilizing effect on the stationary crossflow modes, but the stationary crossflow vortices have relatively low amplitudes at breakdown (U_{rms}/U_c < 0.04) in the presence of the step and are not directly responsible for transition. However, the stationary crossflow vortices do have an effect on transition because an increase in stationary crossflow amplitude causes the transition front to move farther upstream.

The backward-facing step causes the amplification of three different types of instabilities that were not present in the baseline case. The evidence suggests that these instabilities correspond to a traveling-crossflow instability, a Tollmien–Schlichting (TS) instability, and a shear-layer instability. All three instabilities persist downstream of the reattachment location behind the step. The lower-frequency band of disturbances behaves like a traveling crossflow mode in terms of the frequency and direction of travel. However, the spanwise wavelength of this disturbance is five times larger than the traveling crossflow wavelengths predicted via linear PSE computations for the baseline case. The disturbances in the middle-frequency band display some characteristics of TS waves, such as the mode shape and phase speed. T-S waves, although stable in the baseline case, could achieve large growth rates in the presence of the step as a result of the short adverse pressure gradient caused by the step. The high-frequency disturbances are believed to be shear layer instabilities. The behavior of reattaching boundary layers in swept flows is not well understood. These data show an interesting behavior of the high-frequency disturbances downstream of the reattachment point. Wave angles abruptly and dramatically change from a streamwise to a spanwise direction, very close to the direction of travel of the low-frequency disturbances. It may be that nonlinear interactions begin near the reattachment point and thus the shear-layer mode feeds into some other type of disturbance, because a pure shear-layer instability cannot amplify downstream of reattachment.

The role of stationary crossflow during the transition process can be explained as follows. The mean-flow modulation induced by the stationary crossflow vortices results in the spanwise modulation of the amplitudes of unsteady disturbances. The modulation of the disturbances was visible at relatively low amplitudes of the stationary crossflow vortices in comparison to those in the baseline case. This modulation indicates the strong sensitivity of these disturbances to the local mean-flow profiles. The higher initial stationary crossflow amplitudes associated with the larger diameter DREs result in larger peak amplitudes in the u’_{rms} contours for all three types of disturbances. The larger peak amplitudes in u’_{rms} ultimately lead to earlier transition. Transition occurs downstream of the step, but the amplitudes of all the disturbances involved are low (c<0.02U_c) when large amplitude velocity spikes begin to occur.

The transition location obtained from the naphthalene flow visualization is observed to be highly sensitive to the step height at step heights near h/δ ≈ 0.5 as examined in this study. This sensitivity is reflected in an abrupt upstream movement in transition following a small increase in step height. The measurements described in this paper suggest that transition moves upstream because of a new physical mechanism that is contingent on the disturbance amplitudes becoming large enough to induce breakdown via nonlinear interactions. The region over which these disturbances are linearly unstable does not extend very far downstream of the step. If the unsteady disturbances do not reach sufficiently large amplitudes to interact nonlinearly, then these disturbances will decay before the nonlinear interactions can precipitate a breakdown process, and hence the presence of the step would have only a minimal effect on transition.

Acknowledgments

This work was performed as part of the Revolutionary Computational Aerosciences discipline under the Transformational Tools and Technologies project of the NASA Advanced Air Vehicles Program. The authors would like to thank William T. Jones for his assistance with the design of the ceiling contour, Jackson Johnson for fabrication of the ceiling liner, Charlie Debro for his support of the wind tunnel testing, and the members of the Flow Physics and Control Branch of NASA Langley Research Center for their support and many helpful discussions.

References


L. Ukeiley
Associate Editor