

SMALL LAUNCH VEHICLE SIZING ANALYSIS WITH SOLID ROCKET EXAMPLES

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ABSTRACT

Small launch vehicles are governed by the same physics as large launch vehicles of course, but due to their small size, some aspects and sensitivities become more important and others less. This paper shows semi-empirical correlations to quantify dry mass fraction for both stage and whole vehicle optimization: mass fraction due to density, mass fraction due to thrust-to-weight, and mass fraction due to size reduction. For single-stage optimizations, a stage performance requirement can be met by a locus of mass fraction vs. specific impulse. Based on the above correlations, this alone can recommend a solid or liquid rocket for a stage.

Rocket designs of similar technology levels are compared, focusing on where stages become less mass-efficient as they get smaller. The Mars Ascent Vehicle is shown to exemplify a trade between a two-stage solids vehicle and a one- or two-stage liquids vehicle.

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NOMENCLATURE

A_{mpref}	global coefficient of mass scaling	LOx	liquid oxygen
AV	pertaining to Atlas V	MAV	Mars ascent vehicle
C_F	coefficient of thrust-to-weight	MMH	monomethyl hydrazine
C_{mpref}	coefficient of mass scaling	N2O4	nitrogen tetroxide
CAD	computer-aided design	λ	propellant mass fraction
DAC	design-analysis cycle	m	mass
ΔIV	pertaining to Delta IV	m_i	inert mass
f_i	inert mass fraction	m_{gross}	gross mass, stage
FW_p	thrust-to-weight ratio (using propellant weight)	m_p	propellant mass
GLOM	gross liftoff mass	m_{pay}	payload mass
H2O2	hydrogen peroxide	OF	oxidizer-to-fuel ratio
I_{sp}	specific impulse	ref	reference
LCH4	liquid methane	ρ_p	propellant bulk density
LH2	liquid hydrogen	v_p	propellant volume
		ΔV	change in velocity

INTRODUCTION

In preliminary launch vehicle concept studies, there is a temptation to focus too early on a single propellant combination, engine, or design point; to over-constrain with requirements that define the design and size of the rocket stages. It may be thought that design definition takes so much work that one must pick something specific and go through the design and analysis cycles until the design closes; that there's not time to consider multiple alternatives. In opposition to this design point mentality is the trade space mentality. The trade space mentality seeks to generate much information for little cost. The trade space mentality helps move past the point of using a table in the Sutton¹ text to match an application to an engine, while also finding answers before building up a CAD model and engaging all the analysis disciplines in a costly DAC cycle.

A trade space mentality might seek to answer one of the following questions:

- Which propellant choice accomplishes the mission in a smaller gross mass, volume, or length?
- Is this different propellant combination able to deliver more performance* than a reference propellant combination, given similar technology level and construction standards?
- Is this technology improvement more impactful for propellant A or propellant B?
- Is this technology improvement more impactful for mission A or mission B?
- Can Stage B be swapped in for Stage A for equal or greater performance?

**For this study, "performance" is either ΔV at constant mass or volume, or change in payload capability at constant ΔV .*

To answer these questions, a model is needed that analyzes and trades these mission-critical variables. This paper proposes a semi-empirical model to do this that is explicit and transparently fits a broad base of historical stage data. When focusing on smaller stages and launch vehicles, there are fewer like stages to compare to, so a model that can robustly assess the effects of salient factors is even more important.

This paper presents the inert mass fraction model of three independent variables, recommends model factors by comparing to launch vehicle data, discusses some similarities and differences between solids and liquids, presents examples comparing single stages for a defined change-in-velocity (ΔV) mission, and presents examples of the vehicle trade space for the Mars Sample Return Ascent Vehicle.

INERT MASS FRACTION MODEL

Understanding how rocket stage inert mass is affected by propulsion choices is of first-order importance for performance-driven trade studies. A vehicle architect using the design point mentality is faced with these choices for estimating mass:

1. Use some global propellant mass fraction assumption, as from a text book
2. Keep designs close to existing stages and use their propellant mass fractions
3. Enlist a subsystem designer or algorithm

Option 1 risks overlooking important effects, even those of propellant choice. Option 2 is likely effective for typical stages, but may fail for smaller stages or novel propellant combinations. Option 3 could succeed if the resources are available, but may suffer from the following dysfunctions:

- definition imbalance: one propulsion system or stage architecture receives deeper study than another (either to its detriment or betterment);
- the algorithm implies more precision than it can deliver, perhaps because the current inquiry changes the assumptions upon which the model was built; or
- mass is underestimated because a component or subsystem was not included.

A good mass model would fill the gaps between the options above in order to provide greater efficacy for mission planning. The model would have to be simple to understand, easy to implement, and adjustably empirical. It also would need to address first-order drivers for beginning trade studies. The

model shown below is an explicit equation that clearly addresses the effect of variables (eq. (1)). For liquid rocket stages,

$$f_i = f_{i,vp,ref} \left(\frac{\rho_p}{\rho_{p,ref}} \right)^{-1} + C_{FW} FW_p + C_{mpref} \left(\frac{m_p}{m_{p,ref}} \right)^{\frac{2}{3}} \quad (1)$$

This model improves upon a previous effort by the authorⁱⁱ that led to a more complicated function that was cumbersome and impossible to validate. Equation (1) has coefficients consistent with a broad database of designed and fielded launch vehicle stages. For instance, knowing the specifications of the Centaur upper stage, one could anchor the coefficients in the equation to predict the mass fraction of, “other stages like Centaur, except for...”. Finally, not all trade studies ask the same questions, but this model includes the effects of common choices that are traded early in launch vehicle and mission concept design studies (eq. (3)).

Defining f_i as the “inert mass fraction,” that is, stage inert mass divided by propellant mass, the model in equation (1) suggests that non-dimensional stage mass is a sum of the effects of three factors:

- Mass due to volume – $f_{i,vp,ref}$ – estimates essential tank-based masses
- Mass due to thrust – C_{FW} – estimates engine mass & other thrust- and loads-driven structure
- Mass due to size – C_{mpref} – estimates how mass efficiency suffers as stages get smaller

The three factors listed above could also be termed technology factors, or existing stage similarity factors. Recommendations that cover most launch vehicle stages generally are shown in Table 1. Equation (1) can also be adjusted to fit a single example or design-style family. If a baseline stage or group of stages is well-represented by these factors, then other stages that share their material choices, safety factor, manufacturing or design choices (e.g., stainless steel balloon tanks vs. aluminum-lithium orthogrid, or pump-fed vs. pressure-fed) can be reasonably expected to follow this trend.

For general modeling of different propellant combinations, let

$$C_{mpref} = A_{mpref} \left(\frac{\rho_p}{\rho_{p,ref}} \right)^{-\frac{3}{4}} \quad (2)$$

where A_{mpref} is the C_{mpref} for liquid oxygen (LOx)/Kerosene at a 2.7 kg/kg oxidizer to fuel (OF) ratio. Table 1 lists the A_{mpref} for the range of technology levels. Table 2 shows how this calculates out to C_{mpref} for the three common liquid propellant combinations, along with the appropriate density ratios to use throughout. The correlation of these values and equation (2) are discussed below.

Table 1. Useful Values for the Technology Parameters of Equation (1) & (2)

MODEL PARAMETER	HIGH ESTIMATE	MEDIUM	LOW ESTIMATE
$f_{i,vp,ref}$ (ref is at typical LOx/Kerosene density, 2.7 OF)	0.030	0.022	0.015
C_{FW}	0.040	0.031	0.025
A_{mpref} (ref is at 10,000 lbm)	0.186	0.127	0.077

Table 2. Density Ratioⁱ and Recommended C_{mpref} for Storables, LOx/Kerosene, and LOx/LH2.

	DENSITY RATIO $\frac{\rho_p}{\rho_{p,ref}}$	C_{mpref} LOW	C_{mpref} MEDIUM	C_{mpref} HIGH
STORABLES	1.17	0.165	0.113	0.068
LOX/KEROSENE	1	0.186	0.127	0.077
LOX/LH2	0.35	0.407	0.278	0.168

The first technology factor captures mass due to propellant volume as a function of the independent variable propellant density ρ . Throughout, density for bipropellant liquid (or hybrid) rocket stages means the bulk density of a propellant combination. The reference density $\rho_{p,ref}$ and reference inert fraction for propellant volume $f_{i,vp,ref}$ go together. Another way to think of this ratio is the propellant tank's specific volume. For example, consider a propellant combination whose density is less than the reference propellant combination. This term scales inert mass fraction to answer either 1) how much less new propellant can fit in a stage of the same volume as the reference stage? or 2) how much more volume must envelope the same mass of new propellant as the reference stage? For convenience, the reference is LOX and Kerosene at a 2.7 oxidizer to fuel ratio.

The second technology factor captures mass due to thrust. To keep the equations explicit, thrust is non-dimensionalized by propellant weight instead of by stage weight, as might be more common. To illustrate mass fraction due to thrust-to-weight, consider a comparison between the Atlas V core stage and the Delta IV core stageⁱⁱⁱ. By appearances, they are similar design and construction technology levels, but reflect two different propellant combinations. Assuming they are both large enough to not suffer a significant loss of efficiency due to size, equation (1) can be re-written as equation (3) to solve for the Delta IV core mass fraction by considering the Atlas V core as reference stage:

$$f_{i,\Delta IV} = f_{i,AV} \frac{\rho_{p,ref}}{\rho_{p,\Delta IV}} + C_F \left(FW_{p,\Delta IV} - \frac{\rho_{p,ref}}{\rho_{p,\Delta IV}} FW_{p,AV} \right) \quad (3)$$

Given Atlas V's inert mass fraction of 0.079 kg/kg, and a density ratio of 2.84, the first term alone computes an increase in inert fraction to 0.224, whereas the catalog shows only a 0.134 for the Delta IV core. But the second technology factor shows that there's a difference in mass due to thrust, even if the stages both had the same thrust-to-weight ratio. One way to think of this is to recall the question that the mass-due-to-volume factor can answer: "How much less new propellant can fit in a stage of the same volume as the reference stage?" If LOX/liquid hydrogen (LH2) was put in the same stage as Atlas V's LOX/Kerosene without changing anything else – particularly the engines – the same thrust as Atlas V would result, but with only 35% as much propellant by mass. That would increase thrust-to-weight from Atlas V's 1.49 to 4.54! Since the Delta IV core's FW_p is only 1.69, it's like cutting mass off by removing engine and associated thrust structures.

To exactly fit the values above, the value required for C_F is 0.036, and the resulting $f_{i,vp,ref}$ comes out to 0.026. If an engine had a thrust-to-engine-weight of 75, that would be a C_F of approximately 0.0133. So, since the stage thrust drives more masses than just the engine (like thrust take-out structure and perhaps tank skin thickness), a value for C_F representing up to 2x or 3x of engine weight would seem reasonable, as seen here.

This result suggests some general values for these factors, but use a broader dataset to see what ranges on them might cover more stages.^{iv,v,vi,vii} Unsurprisingly, there is a large spread in the data, as these cover very broad use cases as well as never-built design studies at various stages of maturity. For each plot below, the trend calculated above is shown for high and lower inert mass tracks. All the high trend lines use the same C_F and $f_{i,vp,ref}$ across propellants; same for the low trend lines.

On the LOX/Kerosene plot (Figure 1), notice the data's agreement with a slope and intercept between the "low trend" and the "Atlas – Delta" trend, nicely anchored at the high and low thrust-to-weight extremes. On the Storable propellants plot (Figure 2), it looks like a low and a high family of points are represented. This might be due to some stages being designed as launch boosters and others as auxiliary upper stages like a service module. These datasets only included the designs considered large enough for dimensional scale to not matter significantly. For completeness, the LOX/LH2 version (Figure 3) is given. Although it does not shed much light, for all its noise, neither does it dispute the result.

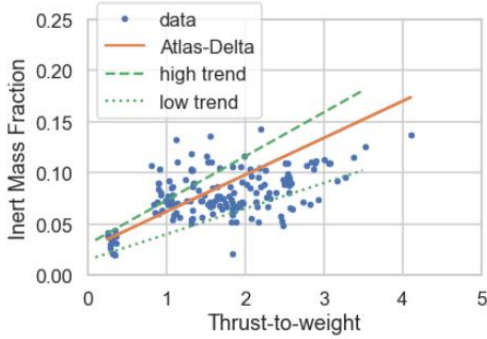


Figure 1. LOX/Kerosene mass-to-thrust data.

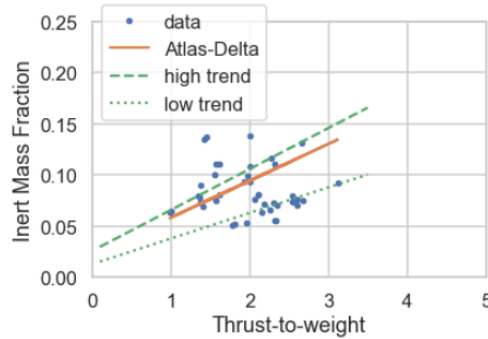


Figure 2. Storable propellant mass-due-to-thrust data.

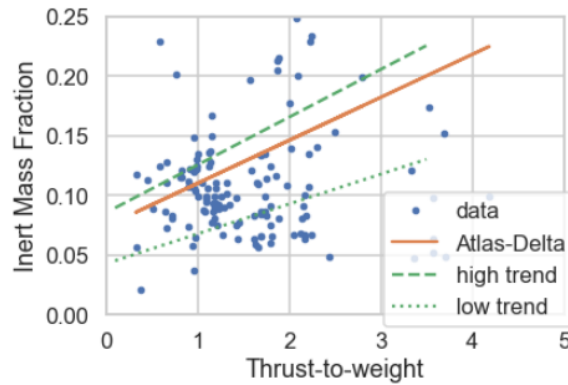


Figure 3. LOX/LH2 mass-to-thrust data.

Equations (4), (5), and (6) define and relate the terms inert mass fraction f_i propellant mass fraction λ and bulk density ρ_p in terms of inert mass m_i propellant mass m_p oxidizer and fuel densities, $\rho_{oxidizer}$ and ρ_{fuel} , respectively, and oxidizer-to-fuel ratio OF.

$$f_i = \frac{\text{inert mass}}{\text{propellant mass}} = \frac{m_i}{m_p} = \frac{1}{\lambda} - 1 \quad (4)$$

$$\lambda = \frac{m_p}{m_p + m_i} = \frac{1}{1 + f_i} \quad (5)$$

$$\rho_p = \frac{OF + 1}{\frac{OF}{\rho_{oxidizer}} + \frac{1}{\rho_{fuel}}} \quad (6)$$

The best dataset by which to begin addressing the mass-due-to-size technology factor is solid rocket motors (SRM), which have the healthiest population of real stage designs down to very small sizes.

The distribution of SRM data points has long been a curiosity to the author. Initially, a piecewise trend was tried. It had no physical reference, and it led to optimizations that over-selected the hinge of the piecewise function. The next supposition was that some mass may be constant, whereas the rest is scalable with propellant. But that begged the question of how much constant mass there should be, and made for steeper curves that did not fit the whole range of data well. Asking what the constant mass should be a function of led to supposing that some mass is a function of diameter and the rest is a function of propellant mass. Assuming that classes of motors follow constant length-to-diameter ratios, this relationship can be put in terms of propellant mass. Again, it does not have to actually be so that the user consider only ranges with constant length-to-diameter; rather it shows the way to a useful approximating model.

For solid rocket motors,

$$f_i = f_{i,max,solids} + C_{mpref} \left(\frac{m_p}{m_{p,ref}} \right)^{-\frac{2}{3}} \quad (7)$$

This equation is plotted below (Figure 4) with solid rocket motors over several orders of magnitude of propellant mass.^{viii} For typical solid rocket motors, this equation is complete without a thrust-to-weight dependency factor: for center-perforated, outwardly-burning propellant grains (as opposed to end-burners), the amount of thrust doesn't drive inert mass, as long as it is commensurate with the typical range of propellant burning rates. That one gets thrust for free is a key benefit of solid rocket motors.

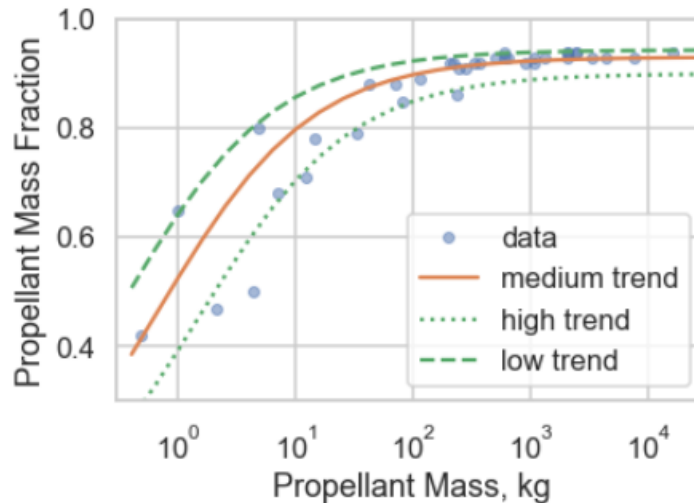


Figure 4. Solid rocket motors within typical outwardly-burning burn time ranges.

Is Equation (7)'s minus two-thirds exponent mass scaling valid for liquids as well? The astronautix.com datasets^{iv,v,vi,vii} show that it is. In the first plot of Figure 5 (A), the Storable propellants have some examples of stages pretty far down the curve, at low propellant mass and consequently, low propellant mass fraction (equivalent to high inert mass fraction). These help to suggest possible values of C_{mpref} . LOx/Kerosene and LOx/LH2 show similar breaking trends, in Figure 5 (B) and (C), respectively. A single A_{mpref} of 0.127 along with Equation (2) generated the medium trend curves for all 3 propellant combinations. The high trend represents stages with higher inert mass fraction—high A_{mpref} are 1.5 times the medium A_{mpref} . The low trend represents stages with lower inert mass fraction—low A_{mpref} are 0.6 times the medium A_{mpref} .

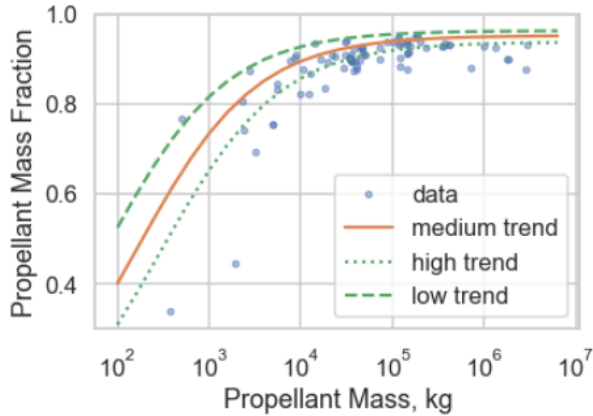
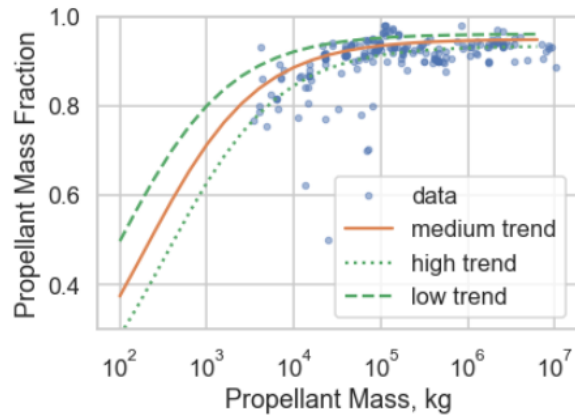
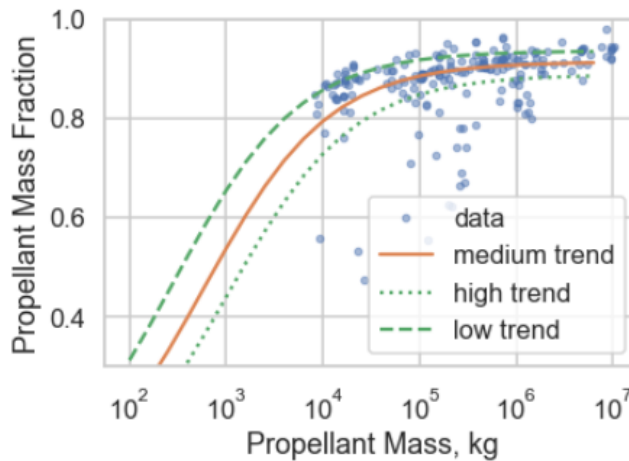


Figure 5. A) Storables Size Effect



B) LOx/Kerosene Size Effect



C) LOx/LH2 Size Effect

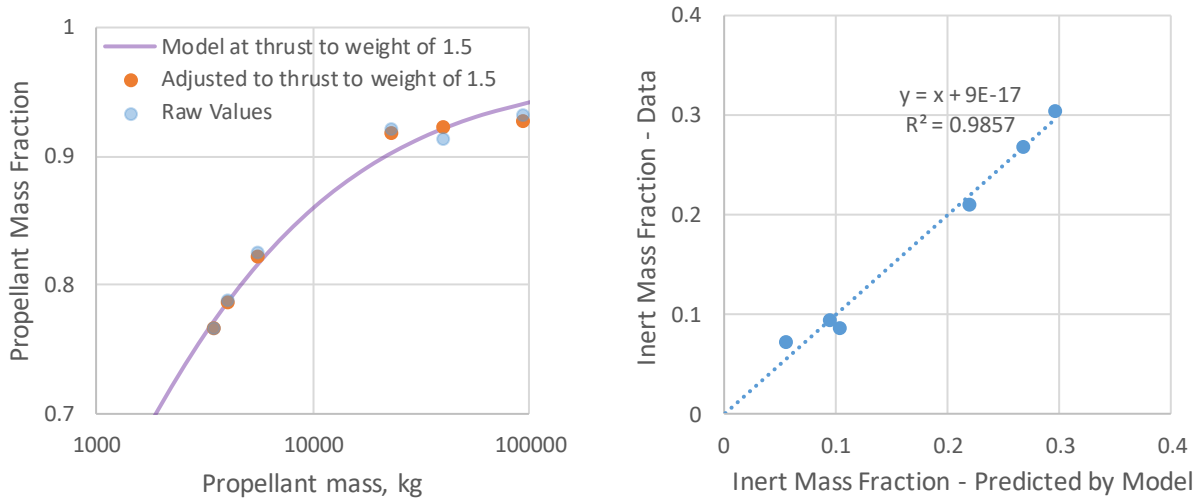
Of course, there are lower efficiency points below the lowest curve, but the same shape as the solids data is unmistakable. Initially, C_{mpref} was set for each propellant by trying to best characterize the trend. For the liquids, the numbers were then found to be consistent with equation (2). The curves shown have been updated for that. The controlling parameters for all the analysis so far are shown in Table 1, Table 2, and Table 3.

Table 3. Solids: Useful Values of the Technology Parameters of Equation (7)

	HIGH $f_{i,min}$	MEDIUM	LOW $f_{i,min}$
λ_{max}	0.9	0.93	0.943
$f_{i,min}$	0.111	0.075	0.06
C_{mpref}	0.0052	0.003	0.0018

Finally, a family of stages was found among the LOx/Kerosene data: 6 stages of similar characteristics to each other, except for stage scale. This is the Molniya family, shown in Figure 6 (A) and (B). This chart shows the benefits of using this correlation as a first approximation, as it captures the first-order changes over a propellant mass range of 30 X. After correcting for thrust-to-weight, a residual

difference can be seen over the larger trio of masses. Perhaps a model at the next level of fidelity would predict the refined differences. But the goodness-of-prediction shows this model explains 98.6% of the variation between these six stages.



SINGLE STAGE PERFORMANCE COMPARISONS

Beyond the values in the above tables, the parameters can be adjusted to match specific sets of stage examples. Of course, if there are only two data points and three parameters, the solution can be speculative, but interpreting the values within the rest of this context could be useful. Consider the two Electron stages, and the two Falcon 9 stages.^{ix,x} All four stages utilize the LOx/Kerosene propellant combination. Supposing the coefficient on thrust-to-weight is near the medium value from Table 1, both these sets of stages can be fit with the same size parameter and just a different propellant volume parameter. Observing these numbers in Table 4, note:

1. C_{mpref} is lower than the low parameter developed from the larger dataset. Because Electron was pushing to the lower end of the mass spectrum for LOx/Kerosene stages, it would make sense that they pushed technology in that area.
2. The propellant volume components are low, even lower for Falcon 9. This is reasonable based on the Falcon's flight-derived heritage of block upgrades to get more performance by reducing mass.

Table 4. Values for Recently-Developed LOx/Kerosene Stages

TARGETED VALUES	ELECTRON	FALCON 9
$f_{i,pv,ref}$	0.0142	0.01
C_{FW}	0.03	0.03
C_{mpref} (ref is at 10,000 lbm)	0.044	0.044

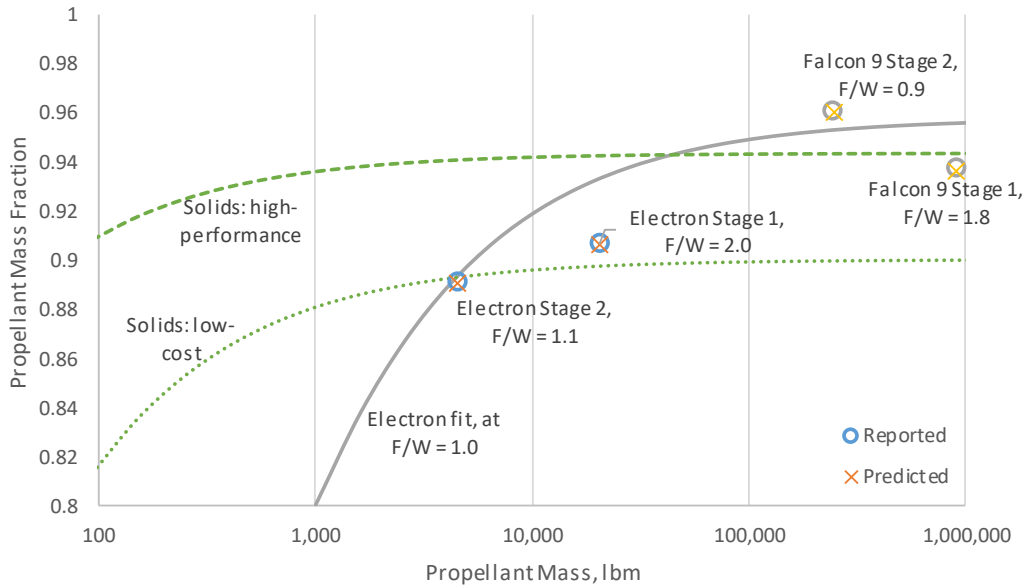


Figure 7. Falcon 9 and Electron Stages: Data with Model

Now to illustrate, stages that are similar to Electron at different sizes and with different propellants can be defined. A payload near the lightweight end of the stated Electron capability (138 kg) would get 6,000 m/s of ΔV from the stage. Look for stages that deliver the same ΔV to payloads one order of magnitude smaller and larger, respectively, for each of the propellant options in Table 5. The liquid rocket options use the mass scaling parameters listed above for Electron, with the C_{mpref} scaled for density according to equation (2). For Solids, the “low $f_{i,min}$ ”, i.e., “high-performance” parameter set is used.

Table 5. Specific impulse and density ratios.

INPUTS	LOX/KEROSENE	LOX/LH2	SOLIDS	LOX/LCH4
I_{sp}	333	450	290	360
Density ratio	1	0.35	1.76	0.81

For single-stage performance comparisons, a useful first-order metric is the stage gross mass. Again, consider a reference launch vehicle and note how, for an upper stage trade, any increase in gross mass would require the stages below to resize in order to deliver the same ΔV as the reference. This is derived from the ideal rocket equation in the following equation (8):

$$e^{-\frac{\Delta V}{v_e}} = 1 - \lambda \left(1 - \frac{m_{pay}}{m_{gross} + m_{pay}} \right) \quad (8)$$

The propellant mass fraction is a function of propellant mass, so the equation iterates to find the gross stage mass m_{gross} that delivers a defined payload m_{pay} to the target ΔV . Note that exit velocity v_e is just the specific impulse in seconds multiplied by the earth reference acceleration due to gravity – this amounts to a units conversion to m/s.

Figure 8 shows the results for four propellant combinations and the three sizes. At the target Medium payload of the Electron stage, LOx/kerosene is unsurprisingly one of the best choices. Notice the solids, kerosene, and hydrogen options are all about the same stage gross mass for that point (top plot).

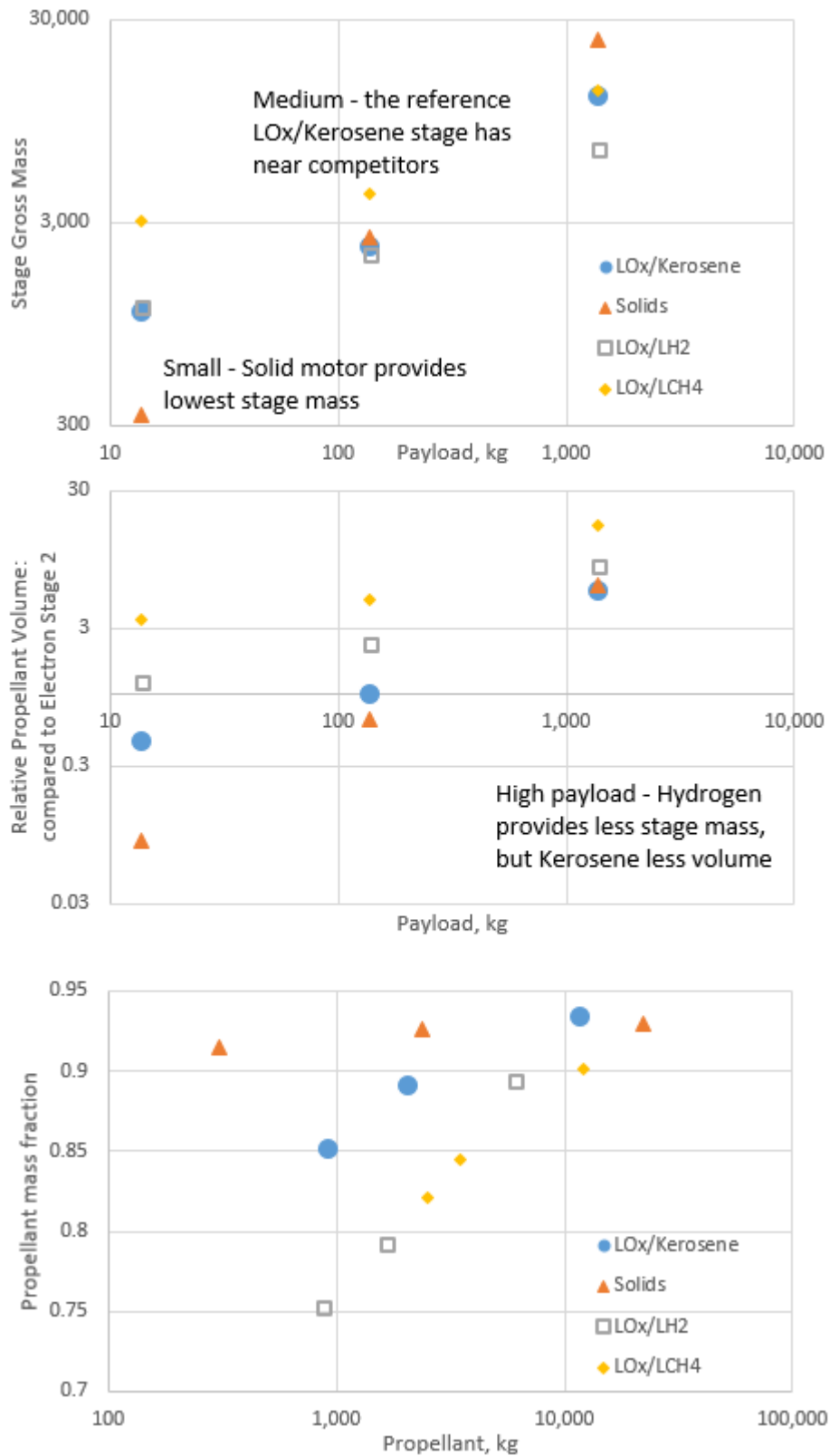


Figure 8. Effect of Propellant Mass upon Optimum Stage Choice for a $\Delta V=6,000$ m/s mission

For getting smaller, the power of the scaling term is evident: the solid stage required to impart 6,000 m/s to the payload is much less massive than the liquid stages. As the bottom plot shows, the solid stage is able to maintain a propellant mass fraction above 0.9 for small stages, where the liquids would all be dropping off precipitously. This mass fraction difference is able to more than offset the solid's I_{sp} deficit.

Notice in the middle plot what may tip the balance toward Kerosene for the medium mission: though the hydrogen predicts a slightly lower mass, it requires a much larger volume. This could be an important effect for some missions. Jumping to the larger payload, hydrogen is predicted to win the gross mass trade, but still require a larger volume.

Finally, note that methane (LCH4) was assigned a very generous I_{sp} for this trade (a 10 or 20 sec increase over Kerosene might be more attainable without increasing the expansion ratio), but the LOx/LCH4 option would not win the trade based on mass or volume at any of the three payload sizes.

MARS ASCENT VEHICLE EXAMPLES

The Mars Ascent Vehicle (MAV) is designed as part of a Mars Sample Return campaign.^{xi,xii} A payload, including samples, of 16 kg suggests small stages that are highly sensitive to propellant mass, akin to the smaller stages in the above example. The MAV project has focused on a two-stage solid rocket motor option and a single-stage hybrid rocket option. Revisit the previously-considered traditional bi-propellant liquids to see that liquids' stage mass performance doesn't measure up for this mission.

During MAV planning, the engineering team used the equations in this paper for initial solids sizing.^{xiii} These studies on the solids two-stage vehicle identified the importance of non-propulsion inert masses: that is, masses that cannot be non-dimensionalized to propellant mass by the use of the inert mass fraction. Table 6 enumerates these for the two-stage solids design and estimates how much of that still applies to a liquid rocket option. For this case, the constants for the low inert mass fraction technology level are used for both liquids cases, but the non-propulsion-driven inerts are omitted for the "best" option. The "best" set of assumptions is like saying, "assume the stage factors include all the avionics, reaction control, and structure functions."

Table 6. Current Values of the 2-Stage Solids Design.

	SOLIDS	LIQUIDS, EXPECTED	LIQUIDS, "BEST"
MISSION PAYLOAD	16 kg	16 kg	16 kg
STAGE 2 INERTS: AVIONICS, RCS, STRUCTURE	34	22	0
STAGE 1 INERTS: INTERSTAGE AND AERODYNAMIC TAIL	14	7	0
"STAGE PAYLOAD" STAGE 2	50	38	16
"STAGE PAYLOAD" STAGE 1	128	<sizing-dependent>	

For this MAV trade, the reference trajectory, based on the 2-stage solids design, assumed a total of 4000 m/s of ideal ΔV . Then ideal rocket equation rearrangements of equation (8) were used. For two-stage options, the ΔV share between stages was varied to find the minimum Gross Liftoff Mass (GLOM). While it is likely that stage assumptions could affect the required ΔV , this analysis assumes that to be of secondary effect, and leaves it to another study. Suffice it to choose stage thrust-to-weight ratios that could deliver a reasonable trajectory with limited losses: Stage 1 FW_p of 1.5, and Stage 2 FW_p of 0.8, if there's a second stage.

Figure 9 shows the performance of multiple options: two propellant combinations, two technology levels, and one or two stages. As is often the case, the GLOM should be minimized or at least controlled

for an optimum vehicle, but especially so for MAV: because this launch vehicle is a payload of a payload (return launch component on a lander), mass increases here would magnify back through most of the sample return campaign. Therefore, compare each liquid design's performance to the two-stage solids GLOM at the bottom of each plot. The highly mature storable propellant combination, nitrogen tetroxide and monomethyl hydrazine is shown in plots (a) and (c), at expected and best case mass assumptions, respectively. Note that, because of the increasing penalty with decreasing stage masses, the two-stage option is only slightly less massive than the single-stage option.

Another thought for liquids is to consider the denser, though less mature, propellant combination of 90% hydrogen peroxide with kerosene, Figure 9 (b) and (d). The higher bulk density with similar I_{sp} allows these stages to perform better, but still falls short of the solids option.

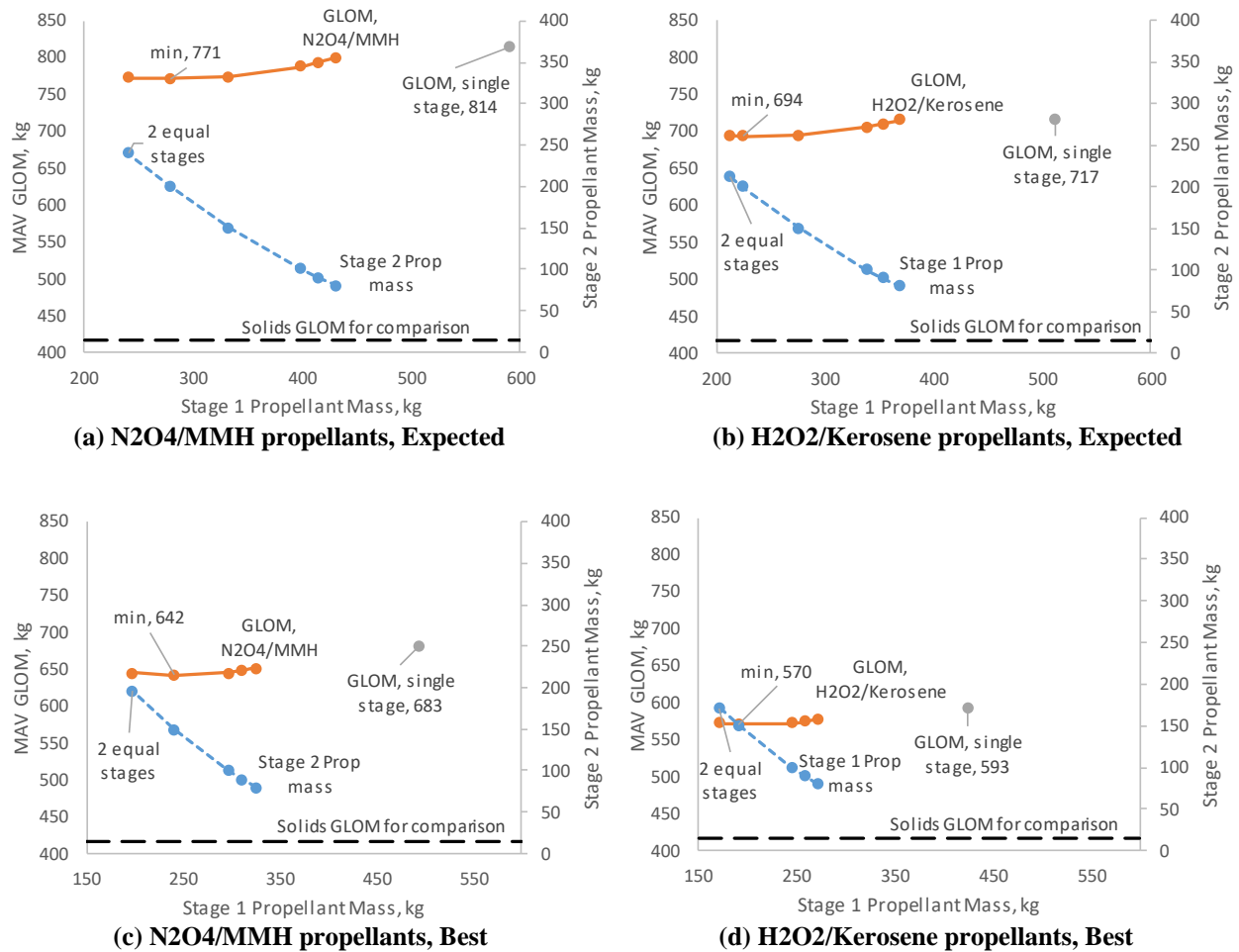


Figure 9. MAV Single- and Two-stage Liquid Propulsion Estimates Heavier than Baseline 2-stage Solid.

CONCLUSION

The “technology factor” sizing correlation explored herein can successfully analyze the high-level relative performance differences between stage design options. This correlation semi-empirically shows the effects of:

- Propellant choice due to bulk density – including novel propellant combinations or OFs,
- Stage thrust-to-weight selection, and
- Overall scale – including where each propellant type is “large enough” for scale not to matter.

This analysis is most relevant in the following instances:

- Mission concept generation for launch vehicles and propulsion stages.
- Early screening to get down to a couple of options.
- Evaluation of novel technology/system proposals (e.g., as a sanity check on the proposer’s numbers – a) “how aggressive must they be to solve the scale problem?” or b) “can this new development be a drop-in replacement to some existing stage?”)

The equation for single-stage performance showed how propellant mass fraction can be traded against exit velocity or Specific Impulse. This correlation performs across propellant types, but when the project moves to the next level of fidelity, additional mathematical definition will be required to trade mass for I_{sp} within the chosen stage (e.g. as the nozzle grows, specific impulse increases, but so does inert mass fraction).

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