National Aeronautics and Space Administration

Small Launch Vehicle Sizing Analysis With Solid Rocket Examples

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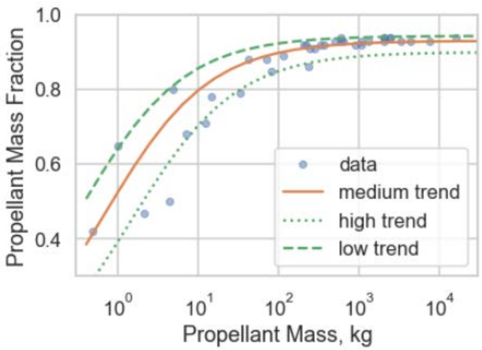
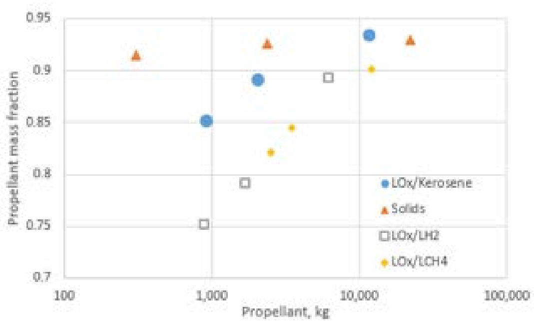
Good modeling and a "trade space mentality" generates much launch vehicle information for little cost

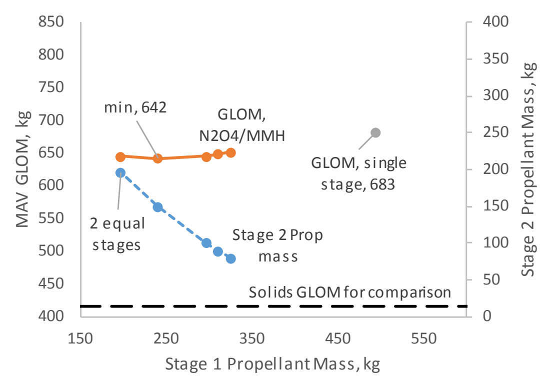


a semi-empirical inert mass fraction model that is explicit and transparently fits a broad base of historical stage data



Single-stage performance comparisons

Mars Ascent Vehicle examples



A trade space mentality might seek to answer one of the following questions:

Relative to a reference case –

- Which propellant choice accomplishes the mission in a smaller gross mass, volume, or length?
- Is this different propellant combination able to deliver more performance than a reference propellant combination, given similar technology level and construction standards?
- Is this technology improvement more impactful for propellant A or propellant B?
- Is this technology improvement more impactful for mission A or mission B?
- Can Stage B be swapped in for Stage A for equal or greater performance?

This mass model is empirically adjustable and easy to implement, but not too simple.

- Stage inert mass (fraction) is a sum of the effects of 3 factors:
  - Mass due to **volume**
    - Propellant bulk density – including novel propellant combinations
  - Mass due to **thrust**
    - estimates engine mass & other thrust- and loads-driven structure
  - Mass due to **size** (diameter proxy for small stage effects)
    - estimates how mass efficiency suffers as stages get smaller
- Correlation coefficients become "technology factors" (material, technique, safety factor, etc.)

$$\frac{inert\ mass}{propellant\ mass} = \boldsymbol{f_i} = \boldsymbol{f_{i,vp,ref}}\left(\frac{\rho_p}{\rho_{p,ref}}\right)^{-1} + \boldsymbol{C_F} FW_p + \boldsymbol{C_{mpref}}\left(\frac{m_p}{m_{p,ref}}\right)^{-\frac{2}{3}}$$

Useful Values for the Technology Parameters

MODEL PARAMETER	HIGH ESTIMATE	MEDIUM	LOW ESTIMATE
$f_{i,vp,ref}$ (ref is at typical LOx/Kerosene density, 2.7 OF)	0.030	0.022	0.015
C_{FW}	0.040	0.031	0.025
A_{mpref} (ref is at 10,000 lbm)	0.186	0.127	0.077

	DENSITY RATIO $\frac{\rho_p}{\rho_{p,ref}}$	C_{mpref} LOW	C_{mpref} MEDIUM	C_{mpref} HIGH
STORABLES	**1.17**	0.165	0.113	0.068
LOX/KEROSENE	**1**	0.186	0.127	0.077
LOX/LH2	**0.35**	0.407	0.278	0.168

– C_{mpref} can be related to propellant bulk density to extend to other propellant combinations

$$C_{mpref} = A_{mpref}\left(\frac{\rho_p}{\rho_{p,ref}}\right)^{-\frac{3}{4}}$$

Mass due to volume assesses across propellants combinations, common & novel

- Reference: LOX and Kerosene at a 2.7 oxidizer-to-fuel (OF) ratio
- $\boldsymbol{f_{i,vp,ref}}/\rho_{p,ref}$ constitute reference tanks' combined specific volume
  - How much less new propellant can fit in a stage of the same volume as the reference stage?
  - How much more volume must envelope the same mass of new propellant as the reference stage?

Mass fraction due to volume = $\boldsymbol{f_{i,vp,ref}} \left(\frac{\rho_p}{\rho_{p,ref}} \right)^{-1}$

$$\rho_p = \frac{OF + 1}{\frac{OF}{\rho_{oxidizer}} + \frac{1}{\rho_{fuel}}}$$

Mass due to thrust corrects for thrust changes induced by propellant choice & mission needs

Add in the factor for thrust-to-weight (propellant) ratio (FW_p)

$$\boldsymbol{f}_{\boldsymbol{i,large}} = \boldsymbol{f}_{\boldsymbol{i,vp,ref}} \left(\frac{\rho_p}{\rho_{p,ref}}\right)^{-1} + \boldsymbol{C}_{\boldsymbol{F}} FW_p$$

Rewrite formula: f_i new as function of reference f_i

Example: Delta IV core as function of Atlas V core

$$f_{i,\Delta IV} = f_{i,AV} \frac{\rho_{p,ref}}{\rho_{p,\Delta IV}} + C_F \left(FW_{p,\Delta IV} - \frac{\rho_{p,ref}}{\rho_{p,\Delta IV}} FW_{p,AV} \right)$$

$$0.134 = 0.079 \cdot 2.84 + C_F \left(1.69 - 2.84 \cdot 1.49 \right)$$

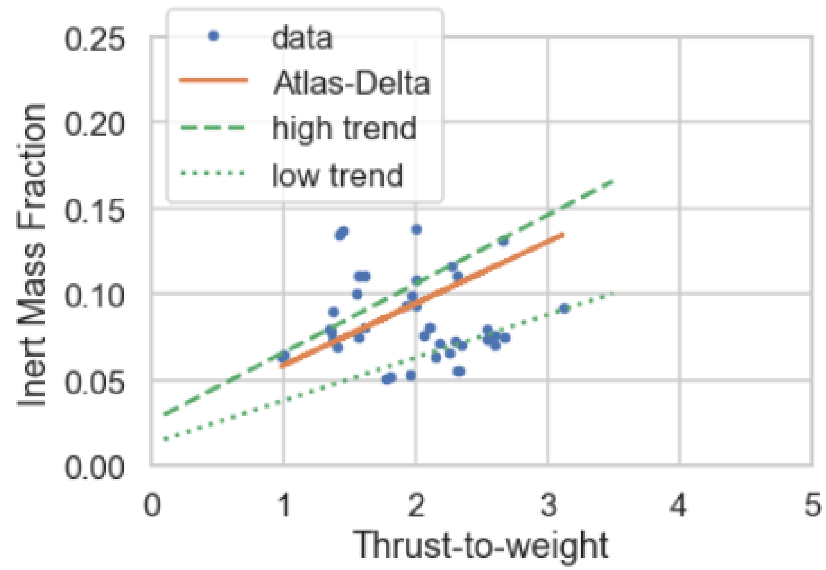
$$0.224$$

The term C_F multiplies is like cutting off thrust-associated mass

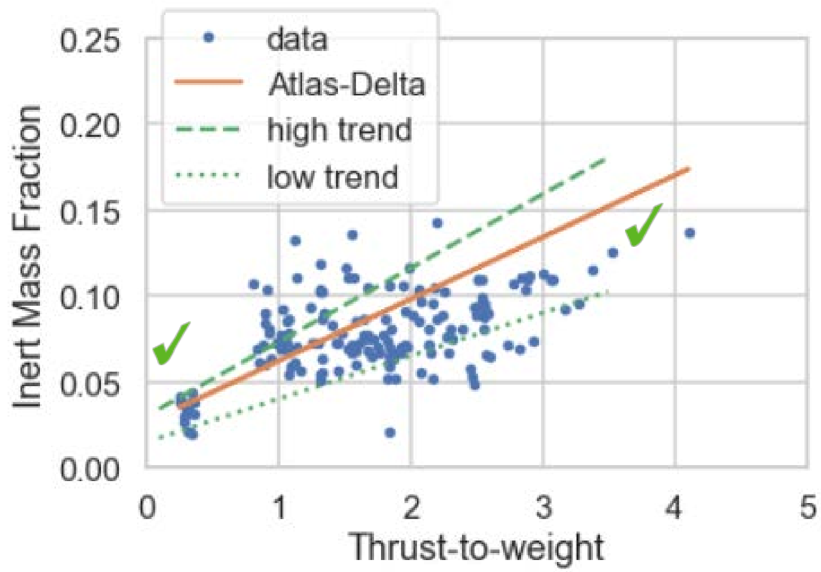
Solve for $\boldsymbol{C}_{\boldsymbol{F}}$ *= 0.036* → $f_{i,vp,ref}$ *= 0.026*

Mass due to thrust trends are supported by database of designed stages

Storables: two families

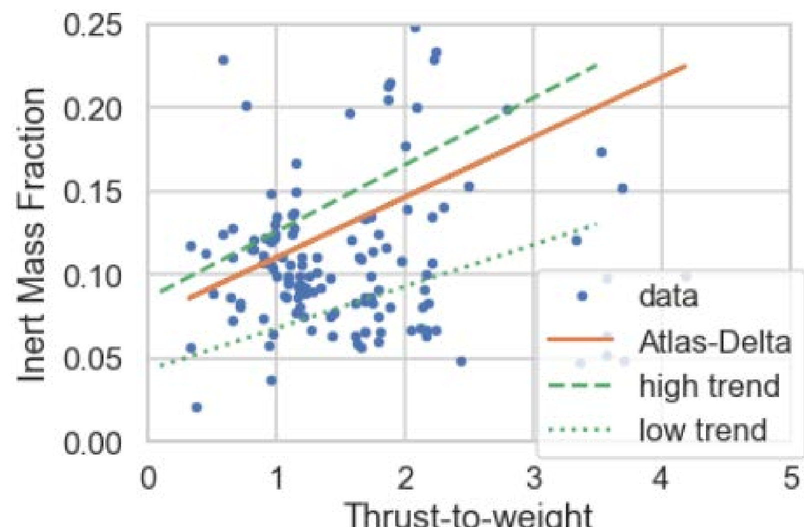


LOx/Kerosene: anchored at extremes



$$\boldsymbol{f_{i,large}} = \boldsymbol{f_{i,vp,ref}} \left(\frac{\rho_p}{\rho_{p,ref}}\right)^{-1} + \boldsymbol{C_F} F W_p$$

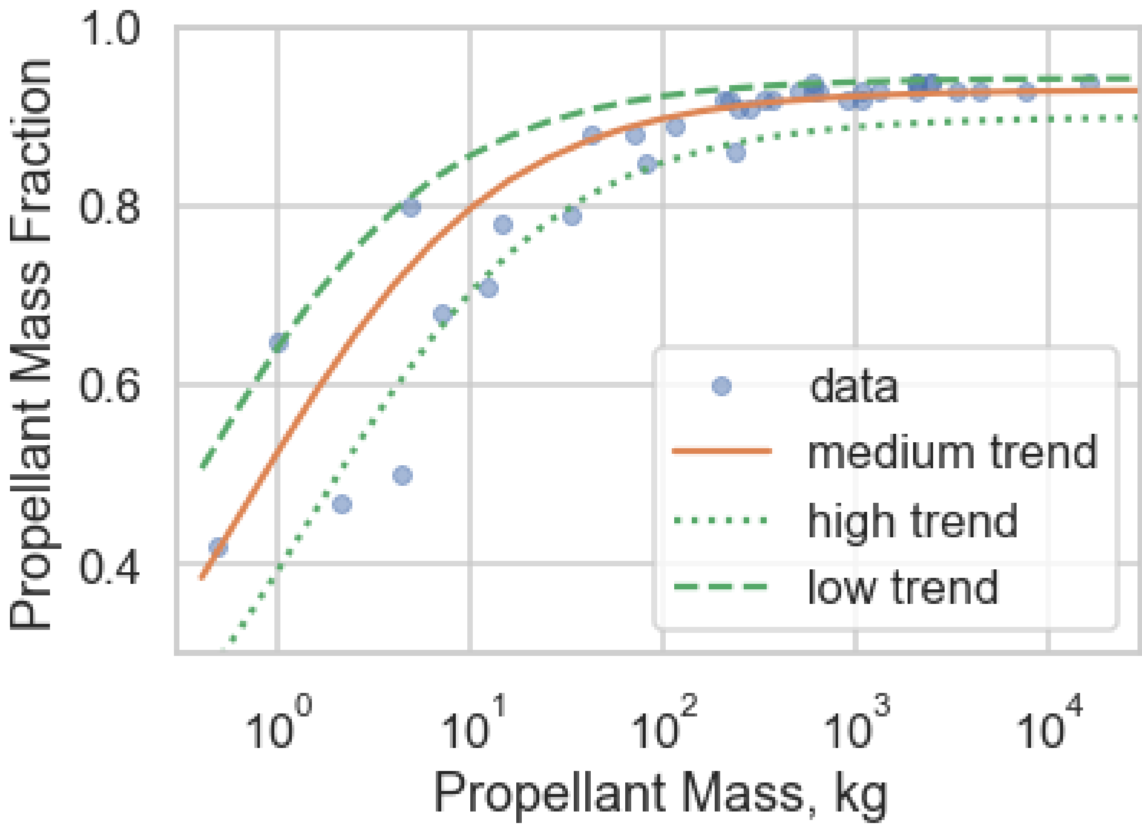
LOx/Hydrogen



Mass Due to Size is exemplified by solids analogous to a diameter effect

$$f_i m_p = f_{i,min} m_p + m_{min}(D) = f_{i,min} m_p + C \cdot m_p^{1/3}$$

$$f_i = f_{i,min} + \boldsymbol{C_{mpref}} \left(\frac{m_p}{m_p, ref} \right)^{-2/3}$$

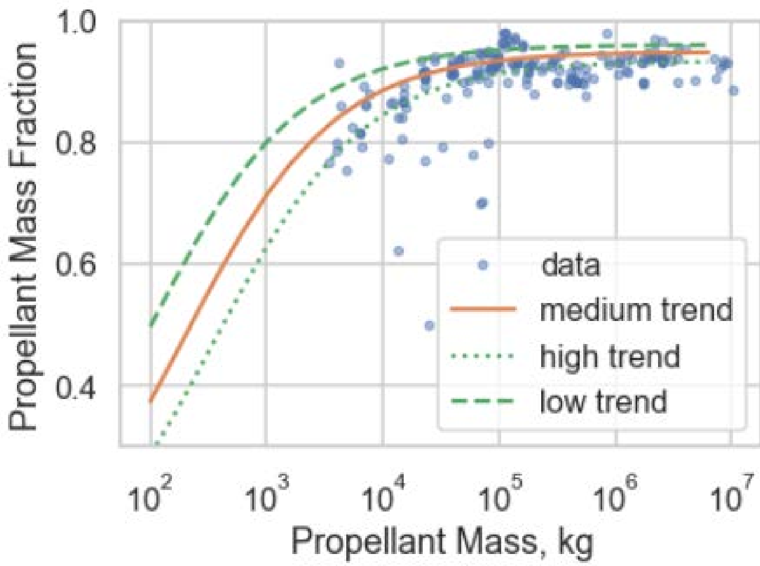


$$\lambda = \frac{m_p}{m_p + m_i} = \frac{1}{1 + f_i}$$

$$f_i = \frac{m_i}{m_p} = \frac{1}{\lambda} - 1$$

Mass due to size trends are supported by database of designed stages

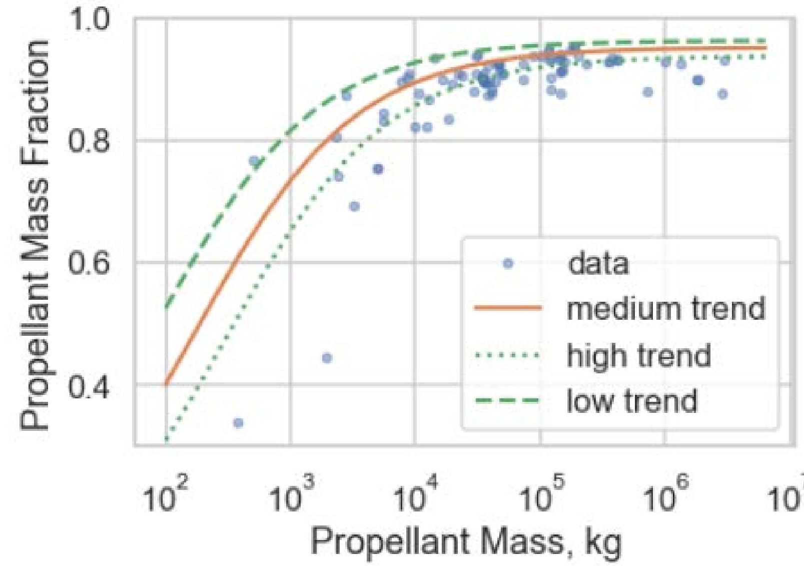
LOx/Kerosene



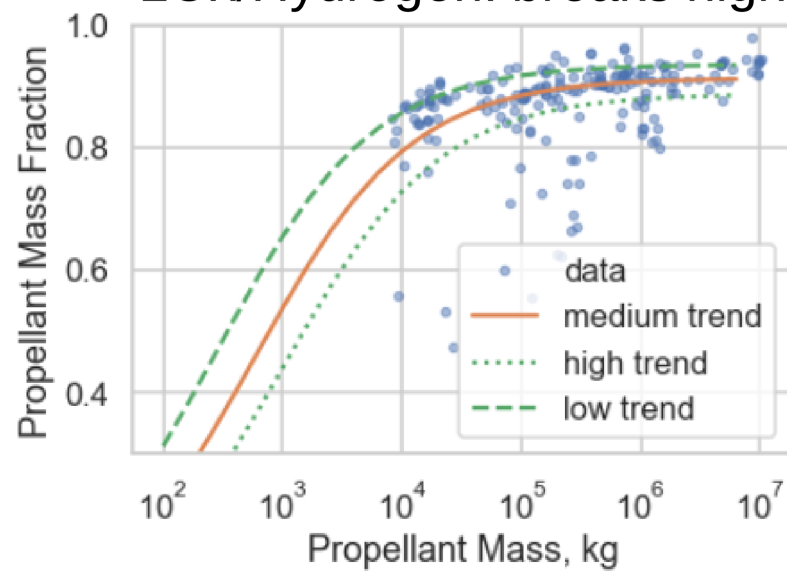
$$C_{mpref}\left(\frac{m_p}{m_{p,ref}}\right)^{-2/3}$$ where,

$$C_{mpref} = A_{mpref}\left(\frac{\rho_p}{\rho_{p,ref}}\right)^{-\frac{3}{4}}$$

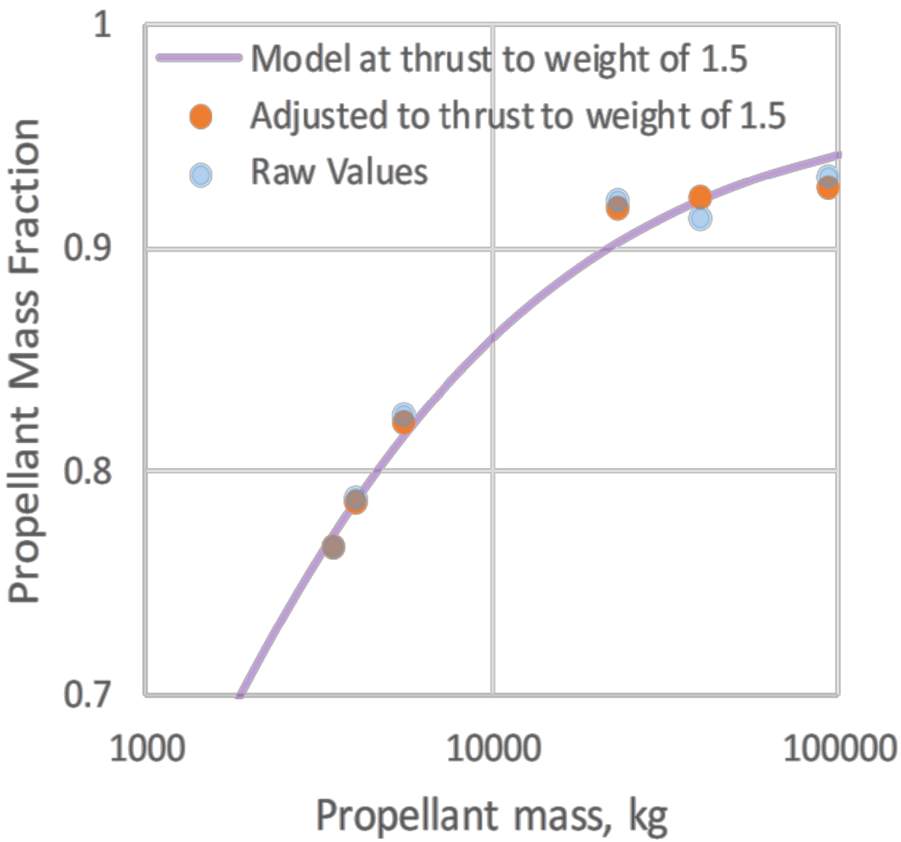
Storables: largest range

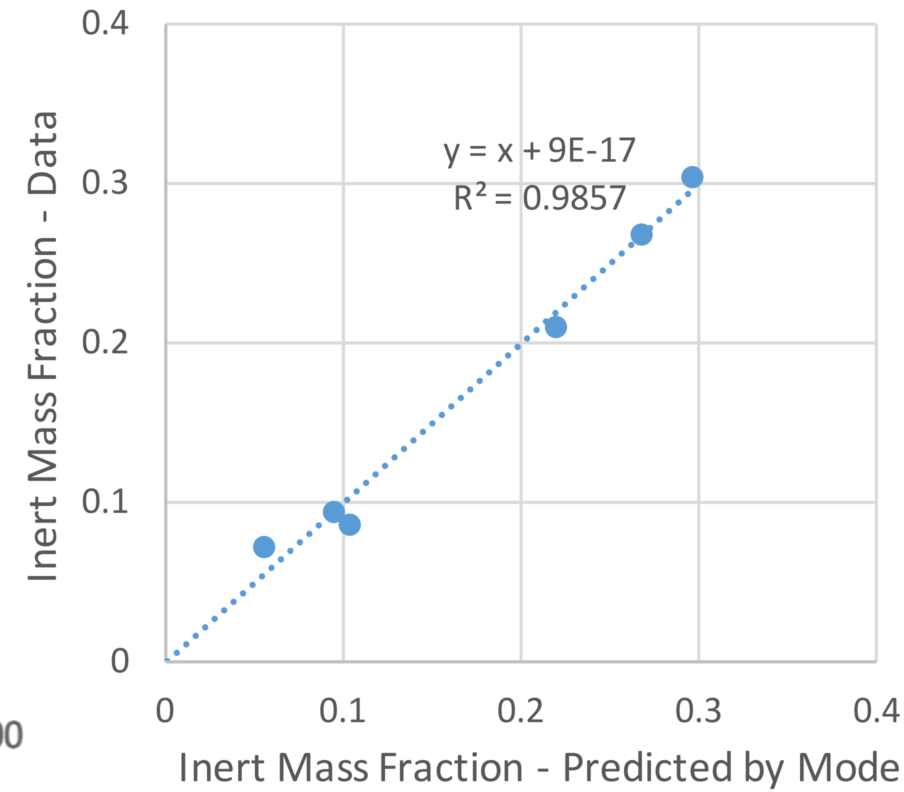


LOx/Hydrogen: breaks higher



This model captures Molniya family first-order changes over a propellant mass range of 30 X





Model and Parameters Summary:

Liquids –

$$f_i = f_{i,vp,ref}\left(\frac{\rho_p}{\rho_{p,ref}}\right)^{-1} + C_F FW_p + C_{mpref}\left(\frac{m_p}{m_{p,ref}}\right)^{-\frac{2}{3}}$$

$$C_{mpref} = A_{mpref}\left(\frac{\rho_p}{\rho_{p,ref}}\right)^{-\frac{3}{4}}$$

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$f_{i,vp,ref}$ (ref is at typical LOx/Kerosene density, 2.7 OF)	0.030	0.022	0.015
C_{FW}	0.040	0.031	0.025
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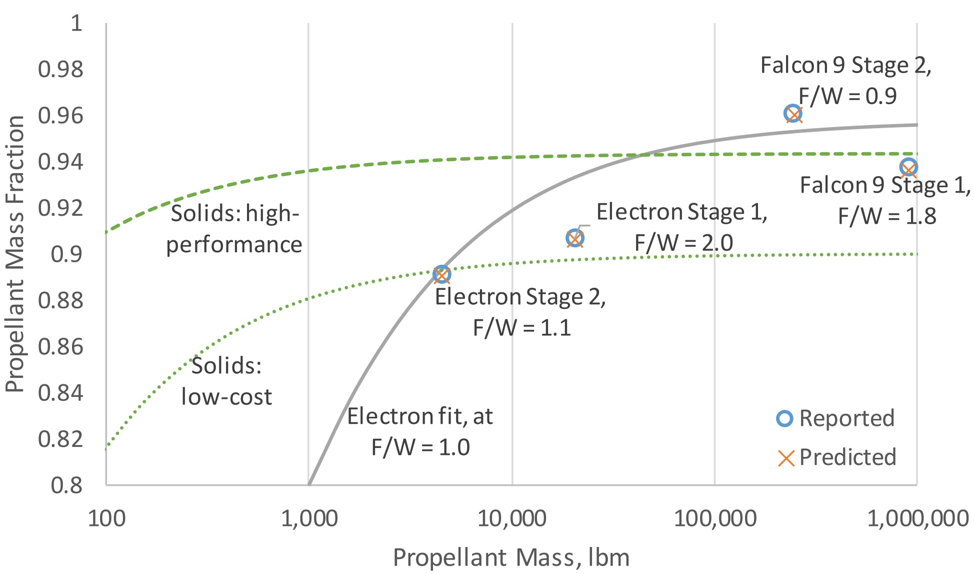
Solids –

$$f_i = f_{i,min} + C_{mpref}\left(\frac{m_p}{m_p, ref}\right)^{-2/3}$$

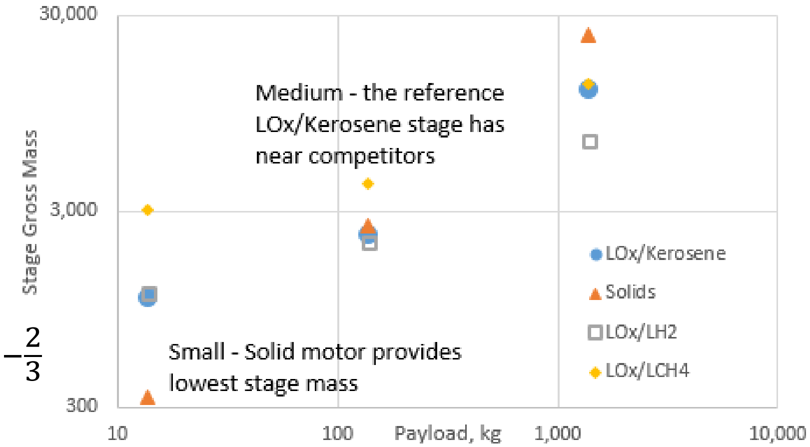
	HIGH $f_{i,min}$	MEDIUM	LOW $f_{i,min}$
λ_{max}	0.9	0.93	0.943
$f_{i,min}$	0.111	0.075	0.06
C_{mpref}	0.0052	0.003	0.0018

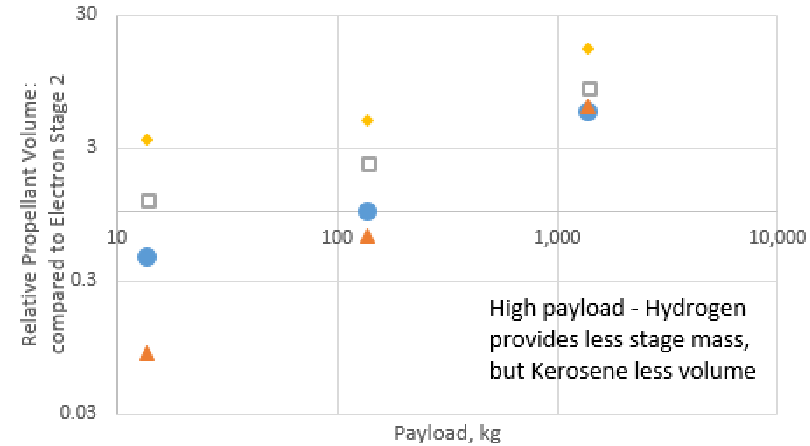
Single Stage comparison shows mass fraction outweighing Isp as scale decreases

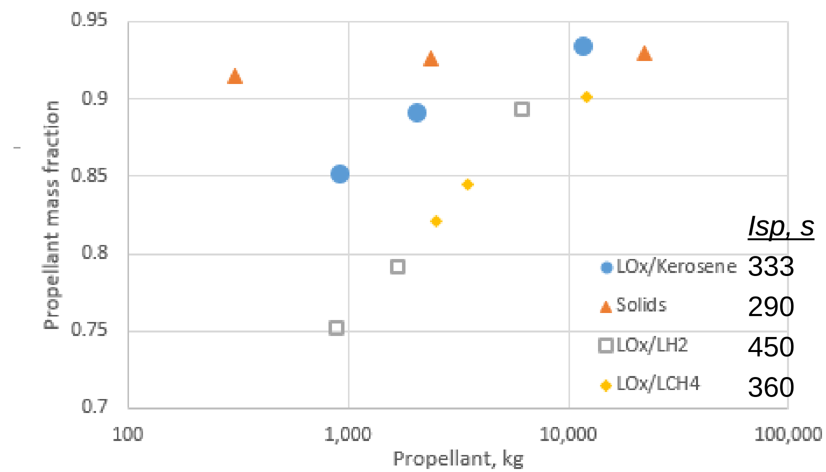
$$f_i = 0.0142\left(\frac{\rho_p}{\rho_{p,ref}}\right)^{-1} + 0.03FW_p + 0.044\left(\frac{m_p}{m_{p,ref}}\right)^{-\frac{2}{3}}$$



$$e^{-\frac{\Delta V}{V_e}} = 1-\lambda\left(1-\frac{m_{pay}}{m_{gross}+m_{pay}}\right)$$







Example: Mars Ascent Vehicle studies identified the importance of non-propulsion inert masses

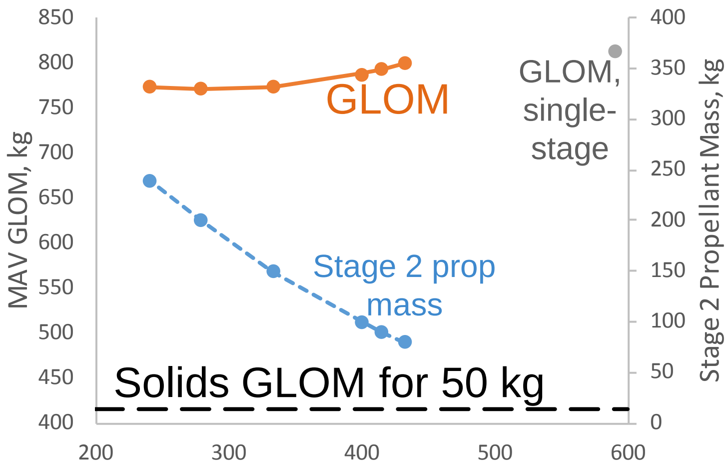
- Mission
  - Sample return from Mars surface to low Mars orbit, $\Delta V \approx 4$ km/s
  - Transit & surface mission duration: months
  - Baselines: two-stage solid & single-stage hybrid in competition
  - Compare: single or two-stage liquids (cryogenic not an option)

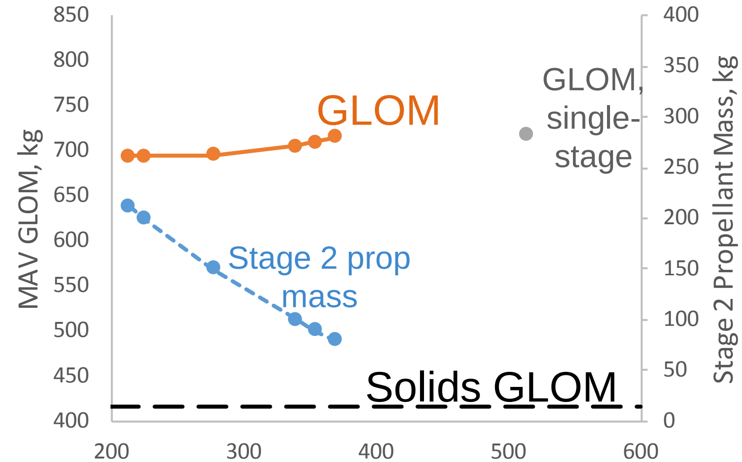
	SOLIDS	LIQUIDS, EXPECTED	LIQUIDS, "BEST"
MISSION PAYLOAD		**16 kg**	
STAGE 2 INERTS: AVIONICS, RCS, STRUCTURE	34	22	0
STAGE 1 INERTS: INTERSTAGE AND AERODYNAMIC TAIL	14	7	0
"STAGE PAYLOAD" STAGE 2	**50**	**38**	**16**
"STAGE PAYLOAD" STAGE 1	128	*<sizing-dependent>*	

MAV mission drives solution away from liquid propulsion despite 1/3 the "stage payload"

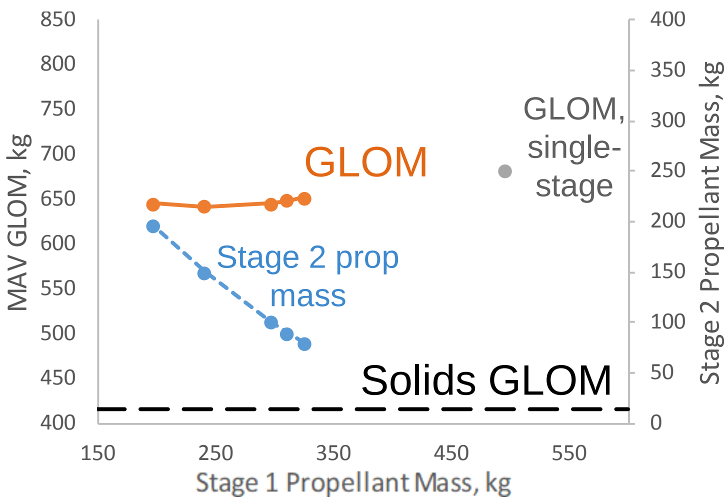
N_2O_4/MMH | 90% H_2O_2/Kerosene

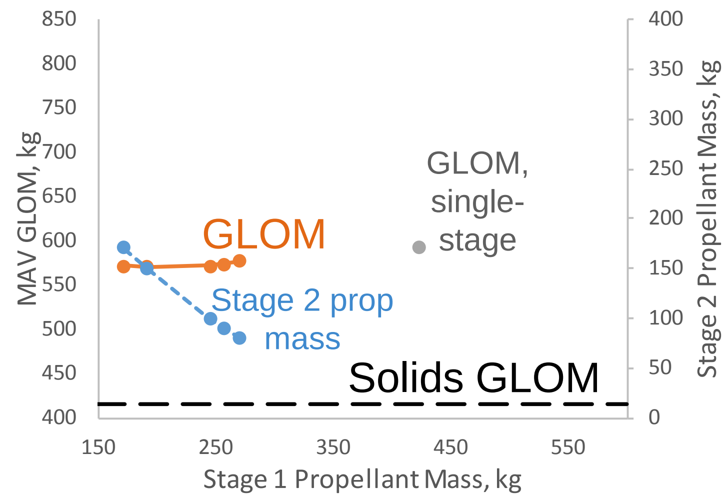
Expected, 38 kg stage payload





Best, 16 kg stage payload

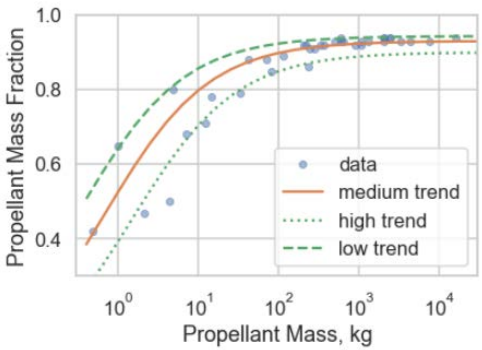




N_2O_4 = nitrogen tetroxide
MMH = monomethyl hydrazine

H_2O_2 = hydrogen peroxide

This sizing correlation is a powerful tool for comparing stage design performance differences

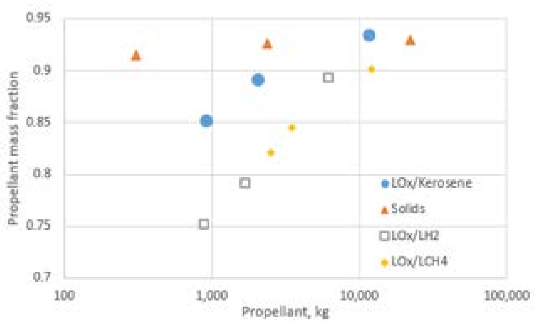


semi-empirically shows the effects of

Propellants choice & *OF* ratio

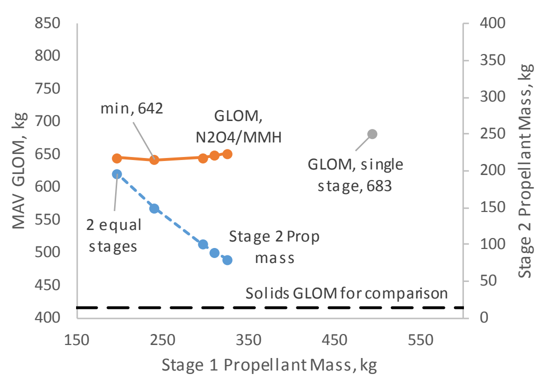
Stage FW_p

Overall scale (propellant mass)



demonstrated where mass fraction outweighs *Isp* with single-stage performance comparisons

Demonstrated Mars Ascent Vehicle trade outcome: liquids don't measure up



Questions?

Nomenclature

A_{mpref}	global coefficient of mass scaling
AV	pertaining to Atlas V
C_F	coefficient of thrust-to-weight
C_{mpref}	coefficient of mass scaling
CAD	computer-aided design
DAC	design-analysis cycle
ΔIV	pertaining to Delta IV
f_i	inert mass fraction
FW_p	thrust-to-weight ratio (using propellant weight)
GLOM	gross liftoff mass
H2O2	hydrogen peroxide
I_{sp}	specific impulse
LCH4	liquid methane
LH2	liquid hydrogen
LOx	liquid oxygen
MAV	Mars ascent vehicle
MMH	monomethyl hydrazine
N2O4	nitrogen tetroxide
m	mass
m_i	inert mass
m_{gross}	gross mass, stage
m_p	propellant mass
m_{pay}	payload mass
OF	oxidizer-to-fuel ratio
RCS	reaction control system
ref	reference
SRM	solid rocket motors
V_e	exit velocity
vp	propellant volume
ΔV	change in velocity
λ	propellant mass fraction
ρ_p	propellant bulk density