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MIL-STD-461G And You: Requirements Tailoring for Space Applications – Sponsored by TC-8

MIL-STD-461G And You

Monday AM #2

Common Mode Conducted Emissions

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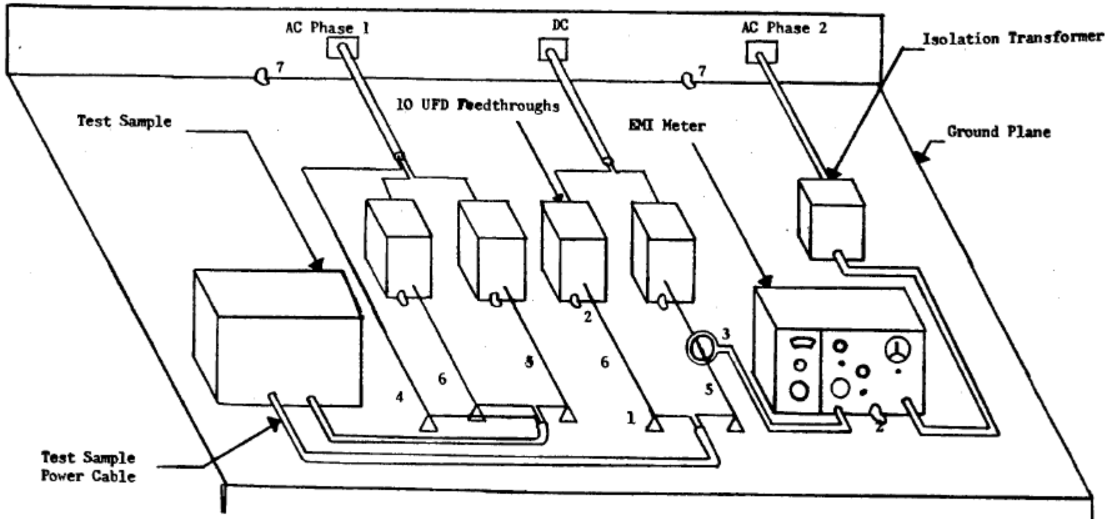
Acronym List

BCE	Bulk Current Emissions
CE	Conducted Emissions
CISPR	Comité International Spécial des Perturbations Radioélectriques
CM	Common Mode
CMCE	Common Mode Conducted Emissions
CS	Conducted Susceptibility
CUT	Cable Under Test
EMC	Electromagnetic Compatibility
EMI	Electromagnetic Interference
EUT	Equipment Under Test
GEVS	General Environmental Verification Specification
GSFC	Goddard Space Flight Center
MIL-STD	Military Standard
NASA	National Aeronautics and Space Administration
RE	Radiated Emissions
RS	Radiated Susceptibility

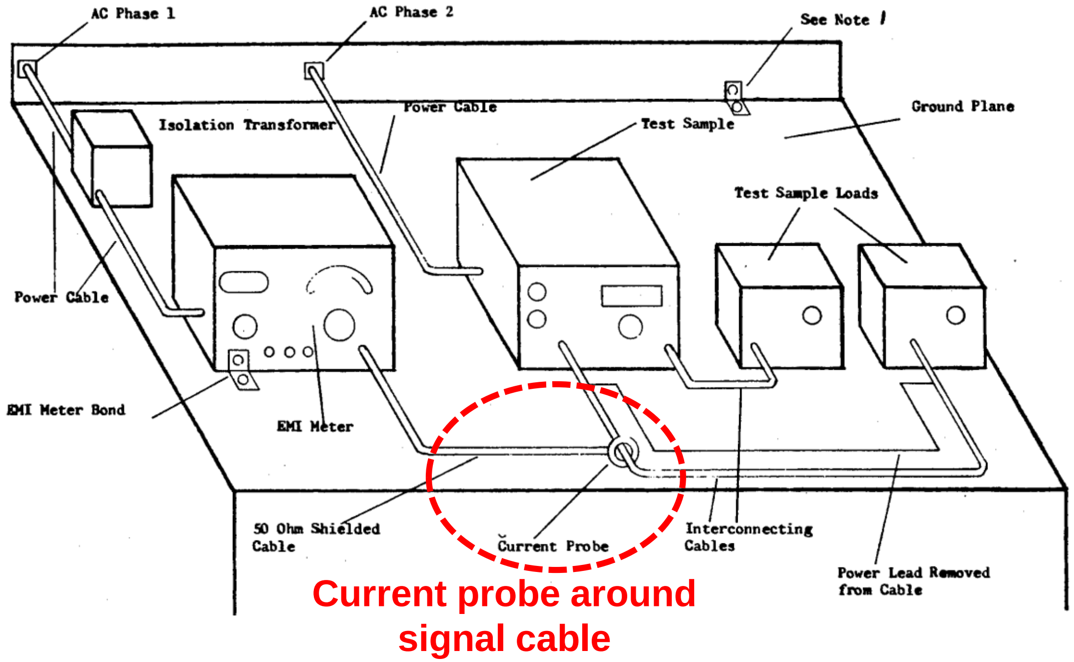
Overview

- Brief history of common mode conducted emissions (CMCE) measurements
- CMCE, radiated emissions, crosstalk, and net integrated average current
- Cable above ground, transmission line theory, current distributions, standing waves, peaks and nulls, etc.
- Damping resistance and the absorbing clamp
- Summary

MIL-STD-462 (1967)



CE01/CE03 on Power Lines



CE02/CE04 on Signal Lines

MIL-STD-461C (1987)

2.2 CE01 limits.

2.2.1 AC and DC leads. Electromagnetic emissions shall not appear on AC and DC leads in excess of the values as shown on Figure 3-1. The limits shall be met when measured with an effective bandwidth not exceeding the primary power frequency plus 20% of the power frequency for AC power leads or 75 Hz for DC power leads.

2.2.2 Interconnecting leads. If compliance with this requirement is required for interconnecting leads, limits shall be developed on a case-by-case basis considering the intentional transmission, its specified power level, necessary information bandwidth, and pulse rise time. Such limits must be approved by the Command or agency concerned.

3. CE03

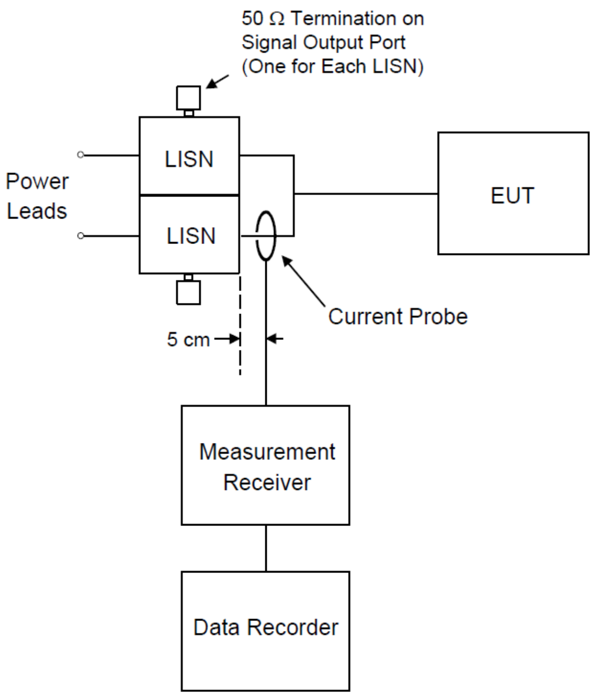
3.1 CE03 applicability. This requirement is applicable for AC and DC leads, which obtain power from other sources or provide power to other equipment and subsystems. The requirement is not applicable for interconnecting leads, unless otherwise specified by the Command or agency concerned.

3.2 CE03 limits.

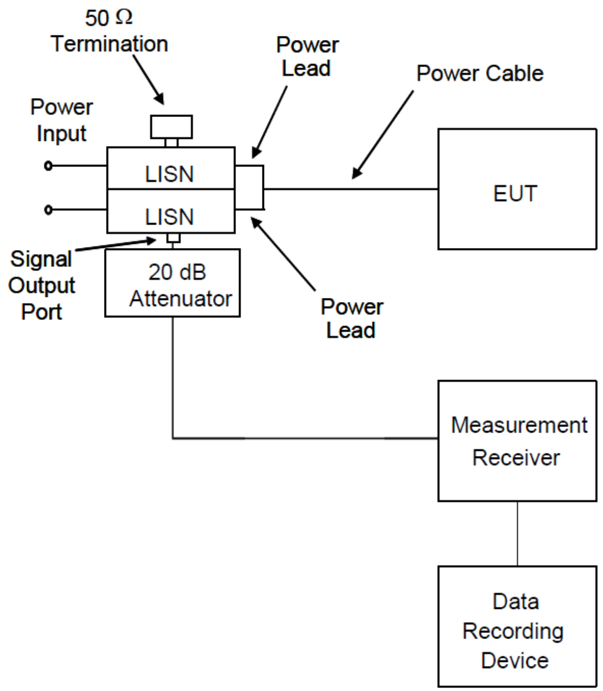
3.2.1 AC and DC leads. Electromagnetic emissions shall not appear on AC and DC leads in excess of the values as shown on Figures 3-2 and 3-3 for narrowband and broadband emissions, respectively. Conducted switching spike emissions (including ON/OFF switching) on AC and DC power leads shall meet the requirements of CE07.

3.2.2 Interconnecting leads. If compliance with this requirement is required for interconnecting leads, limits shall be developed on a case-by-case basis considering the intentional transmission, its specified power level, necessary information bandwidth, and pulse rise time. Such limits must be approved by the Command or agency concerned.

MIL-STD-461G (2015)



CE101 on Power Lines
30 Hz – 10 kHz
Current Measurement



CE102 on Power Lines
10 kHz – 10 MHz
Voltage Measurement across LISN

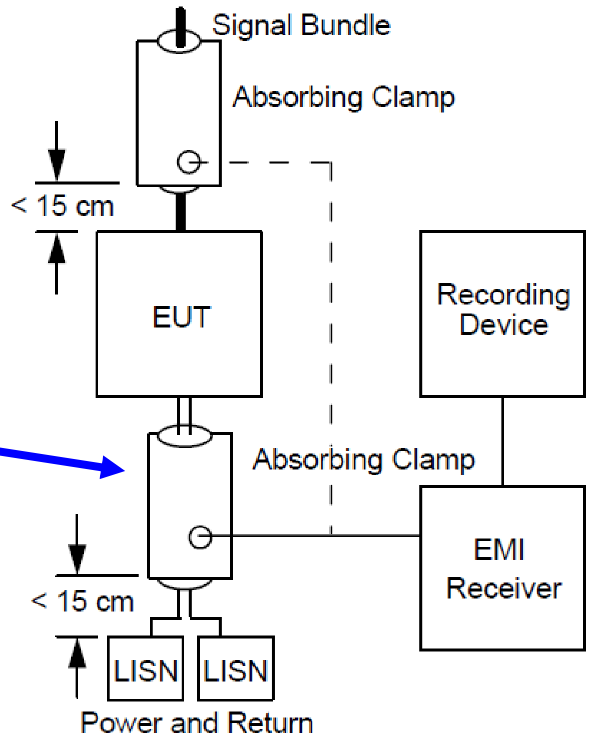
NO COMMON MODE MEASUREMENT ON SIGNAL LINES

SL-E-0002, Book 3 – Space Shuttle (2001)

Bulk Current Emission (BCE) technique defined as Radiated Emissions (RE) test method below 200 MHz

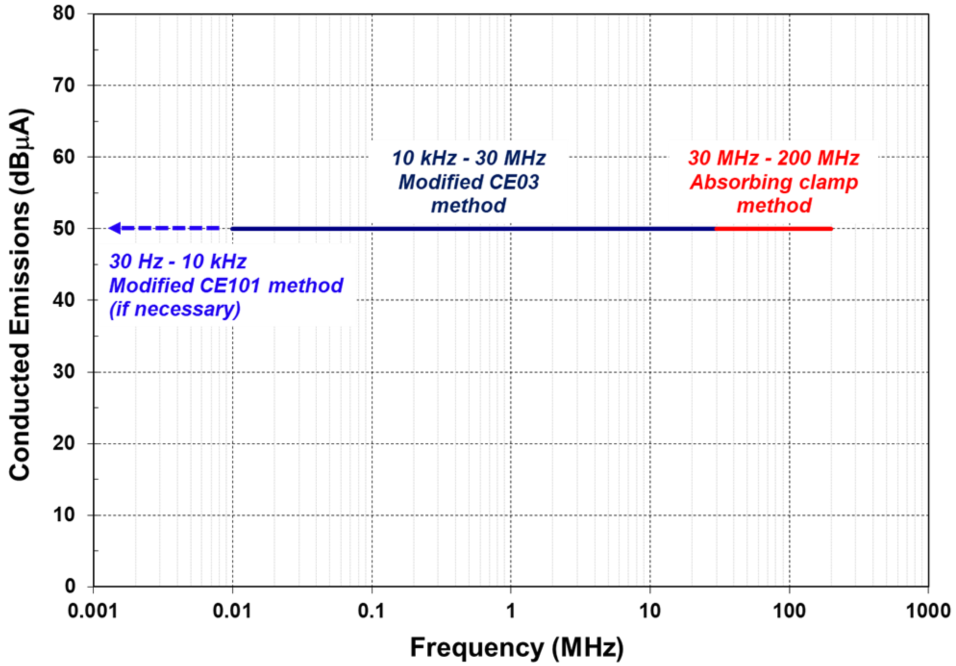
FIGURE 5-29

(RE102-2B) BCE TEST SETUP

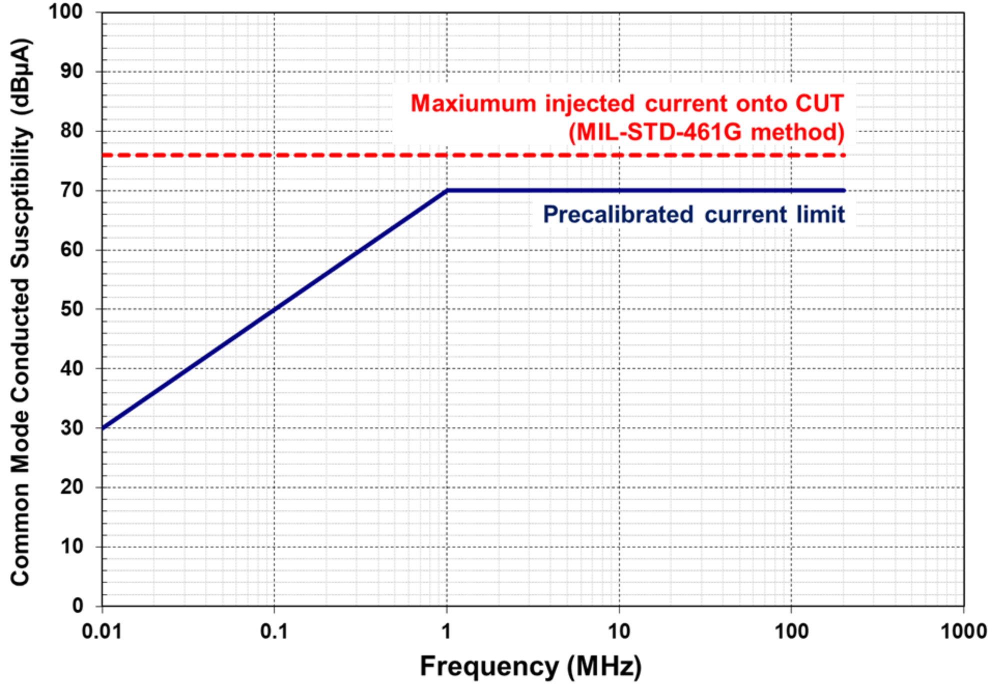


Absorbing clamp (more later)

NASA/GSFC's General Environmental Verification Standard (GEVS)

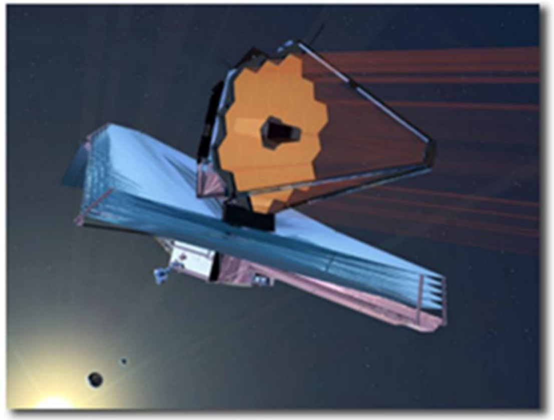


Equipment must not emit more than 50 dBµA



Approximately equivalent to tailored RS103 limit of 2 V/m

Space Applications

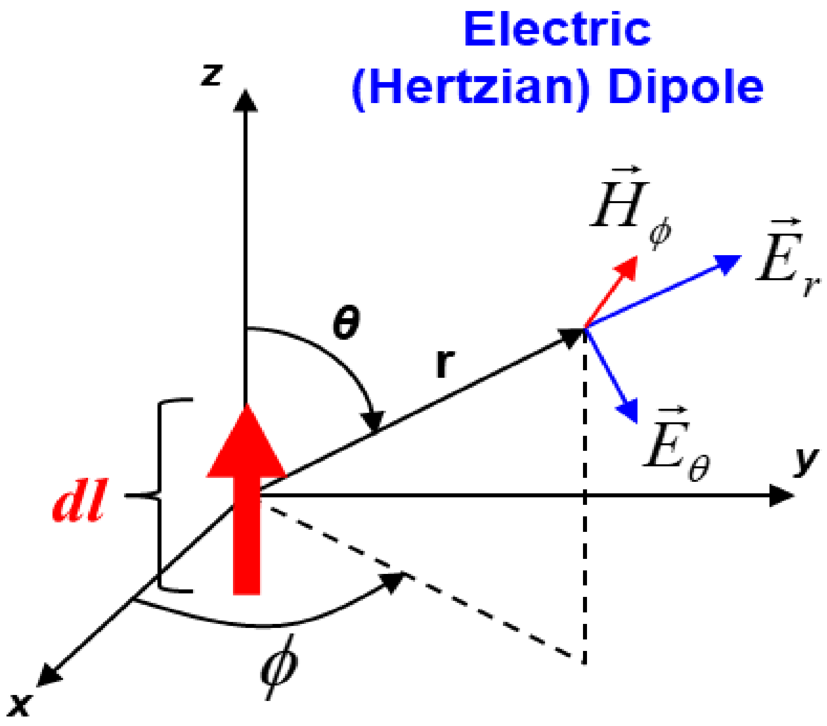


- Highly sensitive science instrumentation
- Not much use of electromagnetic spectrum below 200 MHz

Below 200 MHz, dominant concern is

CROSSTALK

CMCE and Radiated Emissions (RE)



Electric field components:

$$E_r = \frac{2Idl}{4\pi}\eta_0\beta_0^2 \cos\theta \left(\frac{1}{\beta_0^2 r^2} - j\frac{1}{\beta_0^3 r^3}\right) e^{-j\beta_0 r}$$

$$E_\theta = \frac{Idl}{4\pi}\eta_0\beta_0 \sin\theta \left(j\frac{1}{\beta_0 r} + \frac{1}{\beta_0^2 r^2} - j\frac{1}{\beta_0^3 r^3}\right) e^{-j\beta_0 r}$$

$$E_\phi = 0$$

Far field:

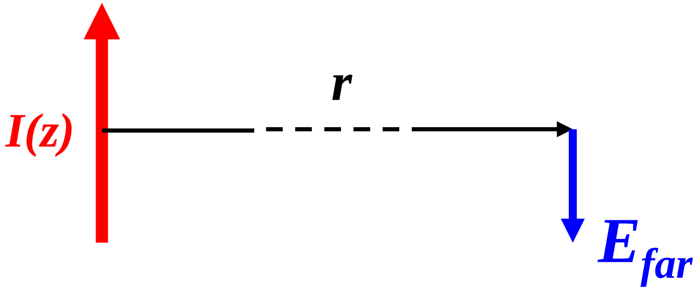
$$E_{far} = j\frac{Idl}{4\pi}\eta_0\beta_0 \sin\theta \frac{e^{-j\beta_0 r}}{r}\overrightarrow{a_\theta}$$

$$\left|E_{far}\right| = CONSTANT \cdot Idl$$

Total Electric Field (Far Field)

At a distance r from the center of a wire of length l:

$$|E_{far}| \approx CONSTANT \cdot \int_0^l I(z)dz = CONSTANT \cdot l \cdot \underbrace{\frac{1}{l}\int_0^l I(z)dz}$$



Far field emissions are determined by ***net integrated average current****,* ***NOT*** *by peak current*

Inductive Crosstalk

Coupled potential (Faraday's Law):

$$|V_{emf}| = \left|\frac{d\Phi}{dt}\right|$$

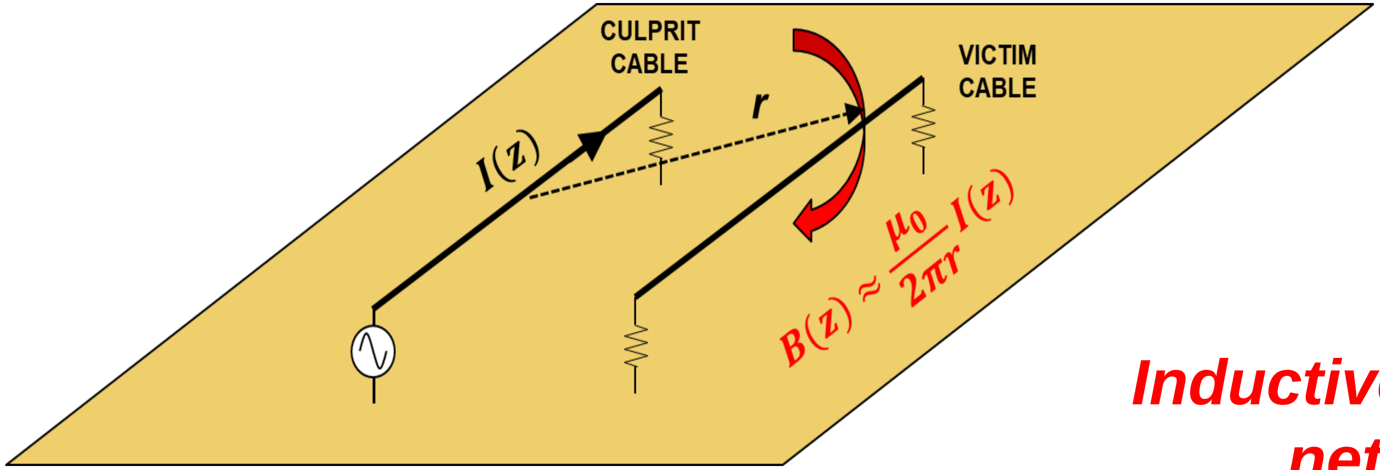
Electrically short cable:

$$\Phi = BA$$

Electrically long cable:

$$\Phi = h\int_0^l B(z)dz = \frac{\mu_0 h}{2\pi r}\int_0^l I(z)dz$$

$$\Phi = h\int_0^l B(z)dz = \frac{\mu_0 hl}{2\pi r}\cdot\frac{1}{l}\int_0^l I(z)dz$$

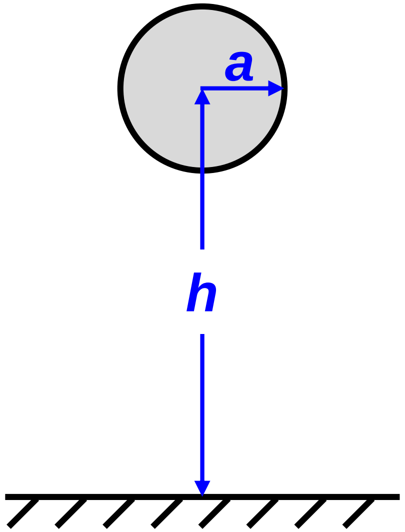


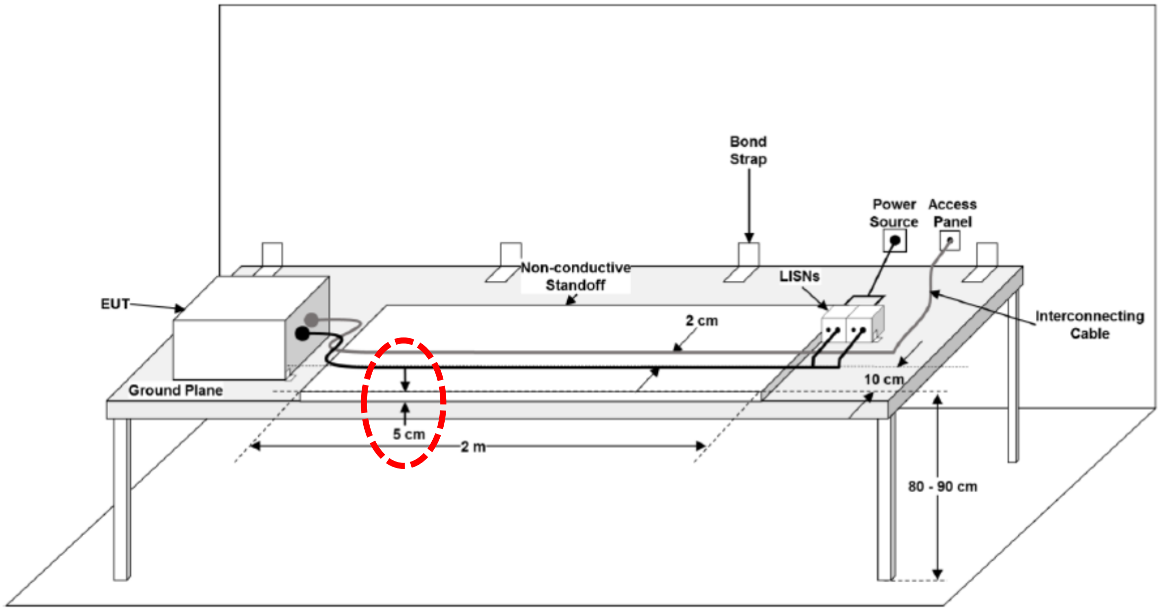
Inductive crosstalk is also determined by net integrated average current, NOT by peak current

Wire-Above-Ground Model

Any cable from which we want to measure CMCE must be modeled as a ***wire-above-ground*** *with:*

- *h = height above ground plane (5 cm per MIL-STD-461G)*
- *a = cable/wire radius*

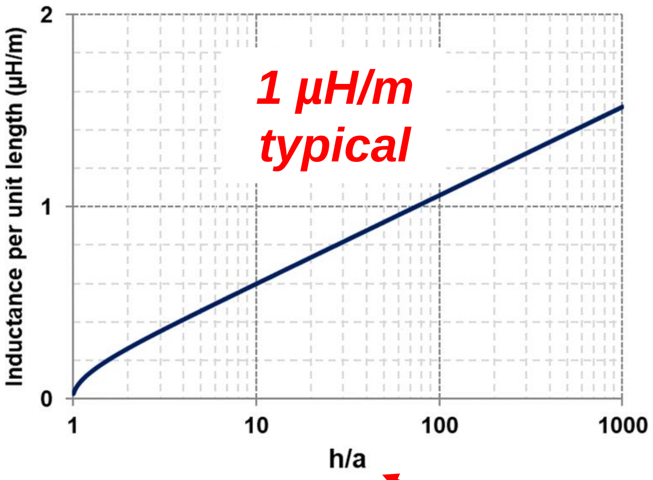




Wire-Above-Ground Model (cont.)

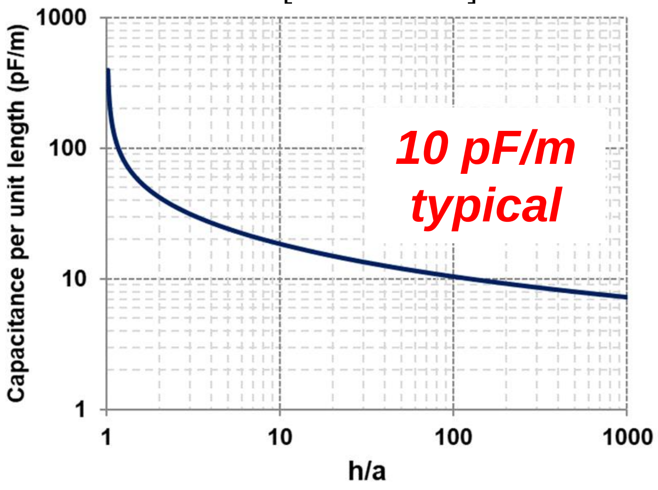
Inductance:

$$L = \frac{\mu_0}{2\pi} ln\left[\frac{h}{a} + \sqrt{\left(\frac{h}{a}\right)^2 - 1}\right]$$



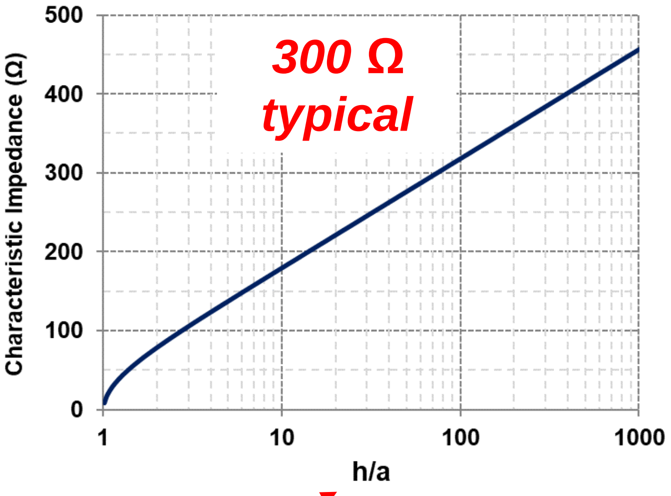
Capacitance:

$$C = \frac{2\pi\varepsilon_0}{ln\left[\frac{h}{a} + \sqrt{\left(\frac{h}{a}\right)^2 - 1}\right]}$$



Characteristic Impedance:

$$Z_0 = 60 \cdot ln\left[\frac{h}{a} + \sqrt{\left(\frac{h}{a}\right)^2 - 1}\right]$$



All logarithmic functions of h/a

Transmission Lines 101

Typical case wire-above-ground transmission line represents shielded cable with shield ***terminated to chassis*** *at both ends…*

Z_S, Z_L → 0

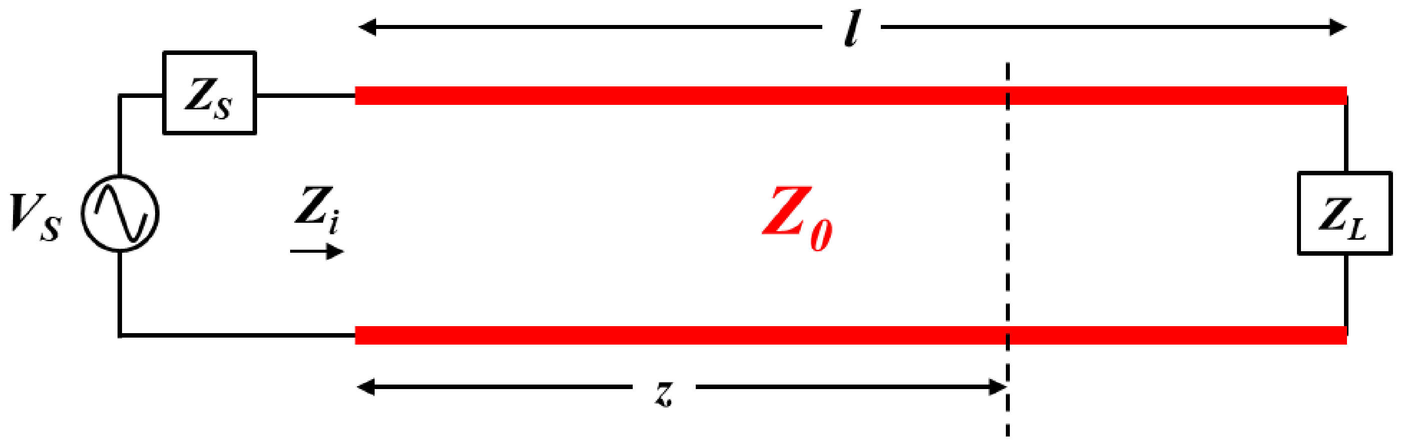
Mismatched impedance → reflections → ***STANDING WAVES***

Reflection coefficient (source end):

$$\Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0}$$

Standing wave ratio (source end):

$$SWR_S = \frac{1 + |\Gamma_S|}{1 - |\Gamma_S|}$$



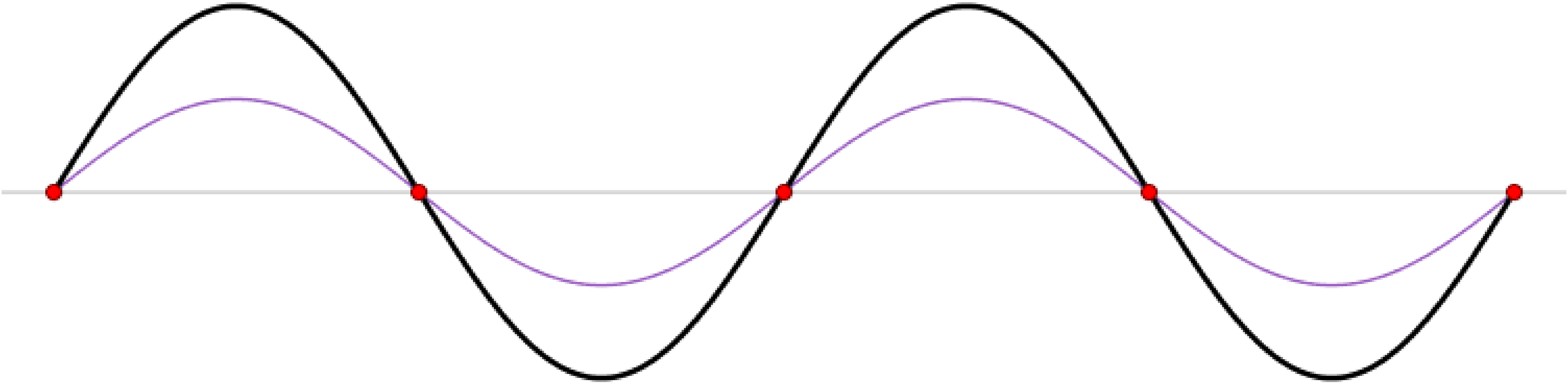
Reflection coefficient (load end):

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Standing wave ratio (load end):

$$SWR_L = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$$

Standing Waves (Animation)



Transmission Line Current Distribution and Input Impedance

Details in backup slides…

$$I(z) = \frac{V_S}{Z_0 + Z_S} \cdot \frac{1 - \Gamma_L e^{-j2\beta l} e^{j2\beta z}}{1 - \Gamma_S \Gamma_L e^{-j2\beta l}}$$

Matched line, $Z_L = Z_0$, $\Gamma_L = 0$:

$$I(z) = \frac{V_S}{Z_0 + Z_S}$$

Constant current amplitude determined by Z_0

Input impedance of lossless transmission line:

$$Z_i = Z_0 \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)} \qquad \beta = \frac{2\pi}{\lambda} \qquad \lambda = \frac{c}{f} = \frac{300}{f_{MHz}}$$

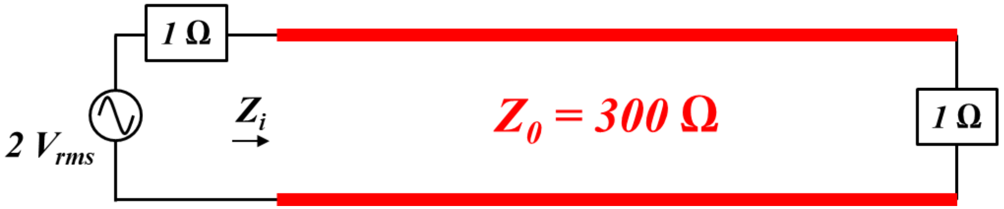
$l = (2n-1)\lambda/4$: $Z_i = \frac{Z_0^2}{Z_L}$

Shorted termination looks like open circuit (Current nulls)

$l = n\lambda/2$: $Z_i = Z_L$

Shorted termination looks like short circuit (Current peaks)

Source End Current vs. Frequency

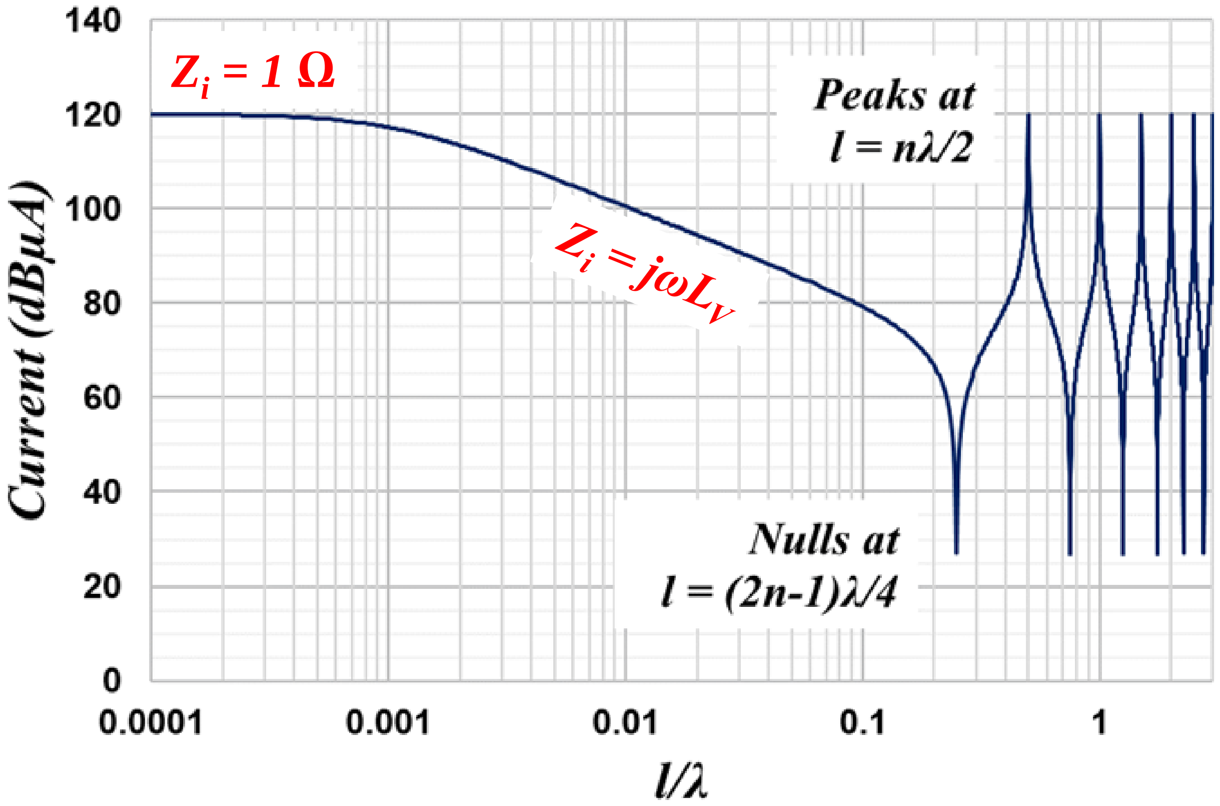


- At low frequencies (l << λ), source end current = DC current normalized to 1 A (120 dBμA)
- At mid frequencies, loop inductance dominates
- Source end current minimum (null):

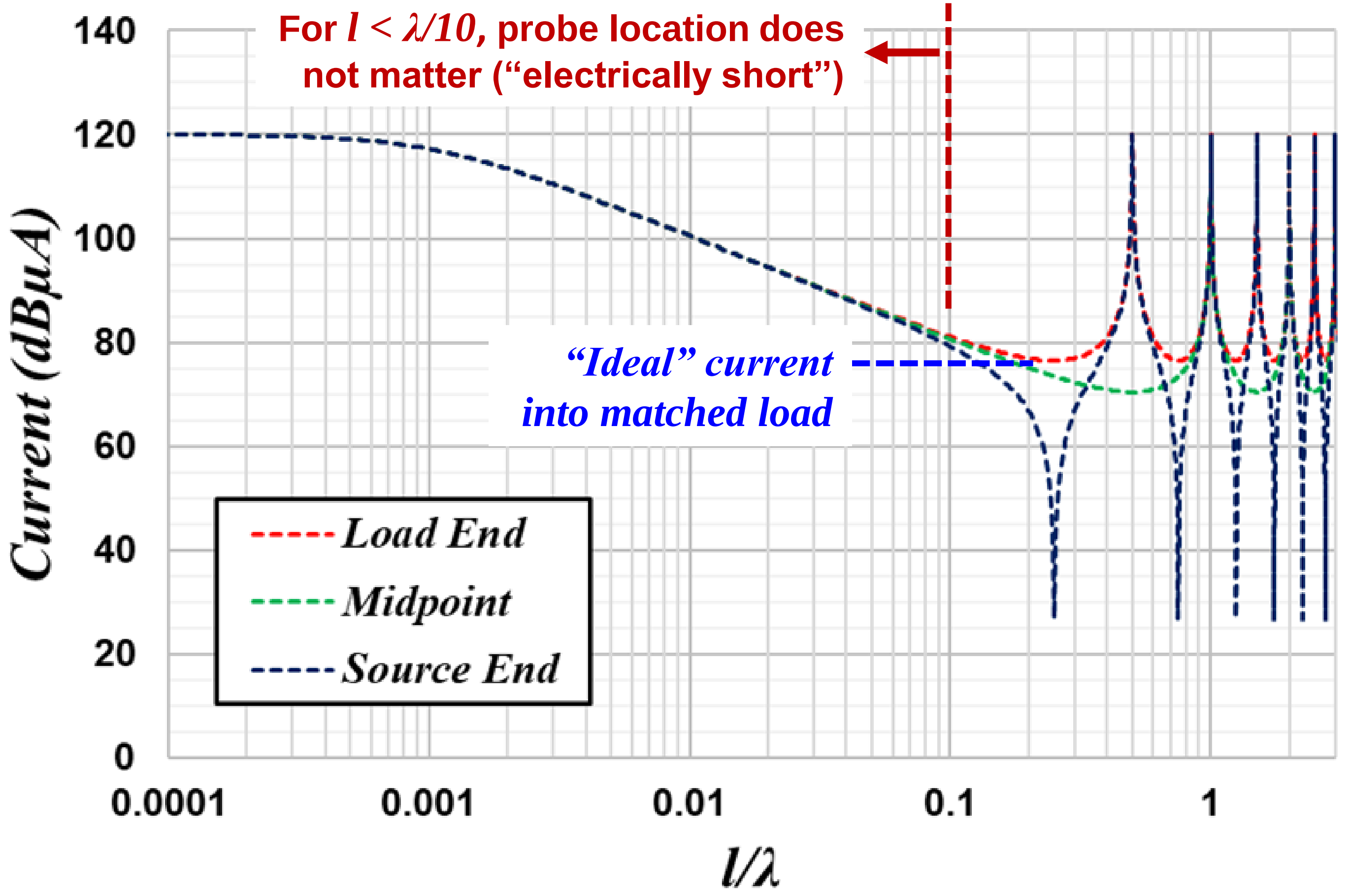
  @ *l = (2n-1)λ/4*
- Source end current maximum (peak):

  @ *l = nλ/2*

- ***For 4 meter cable (2 m in front of ground plane + 2 meter to wall):***
  - ***Nulls at odd multiples of 18.75 MHz***
  - ***Peaks at multiples of 37.5 MHz***



Current Probe Locations: When Does It Matter?



For *l* > *λ/10*, choosing a single probe location adjacent to EUT could cause:

- *false positive (test failure due to exaggerated emission level)*
- *false negative (test passes because method masks a real emission that could pose a problem)*

Placing probe at load end raises nulls to equivalent levels for matched load

→ Still leaves the peaks…

CMCE, Electrically Long Cables (f $\geq$ 30 MHz)

- Ideally, we want a normalized measurement of emissions that is independent of cable configuration *(e.g. matched transmission line)*
- This will provide an assessment of frequency content of emissions from EUT that may be more effectively used to assess compatibility with rest of platform in the flight configuration
- ***Remember:***

$l = (2n-1)\lambda/4$: $Z_i = \frac{Z_0^2}{Z_L}$

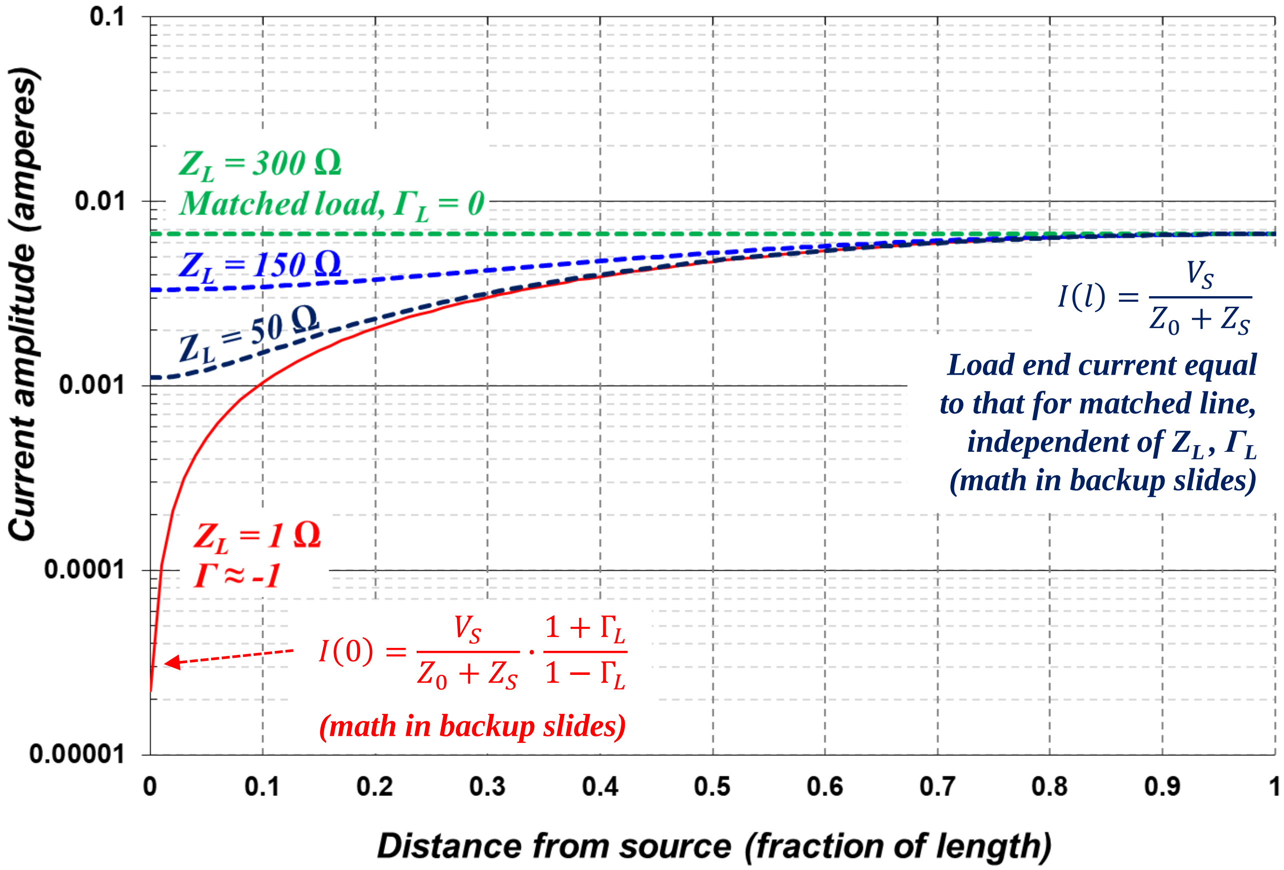
Increasing Z_L reduces Z_i
→ Increases current at nulls

$l = n\lambda/2$: $Z_i = Z_L$

Increasing Z_L increases Z_i
→ Decreases current at peaks

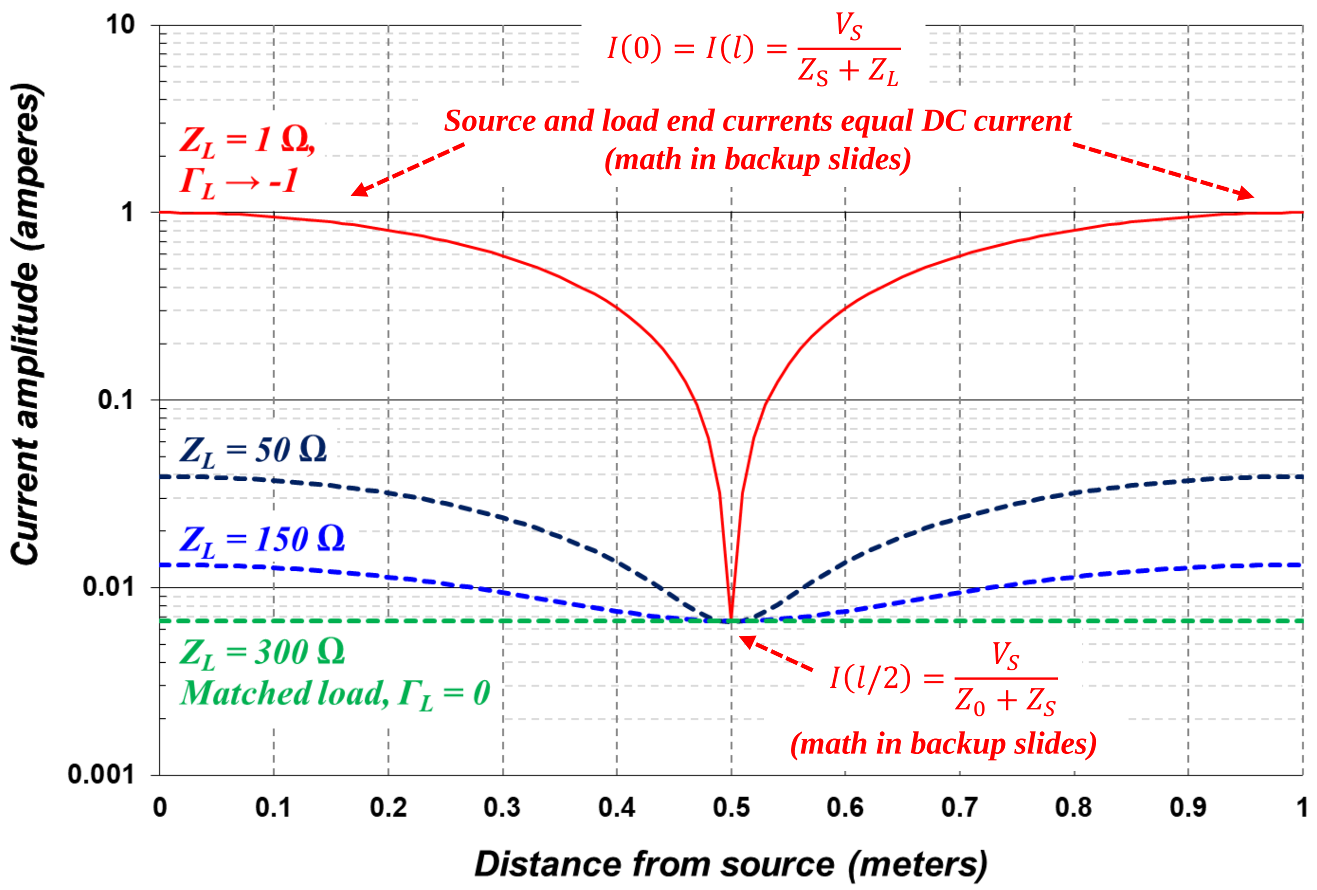
Damping at Nulls ($l = \lambda/4$ example)

- Added damping resistance increases minimum current at source end while leaving maximum current at load unchanged
- Load end current equals that for matched line, independent of Z_L and Γ_L
- When $Z_L = Z_0$, current amplitude is constant across the length of the cable
- At null frequencies, damping resistance has no effect on maximum current, and it makes current more uniform along its length
- Specific position of current probe for CMCE measurement is no longer crucial



Damping at Peaks ($l = \lambda/2$ example)

- Added damping resistance decreases maximum current at source and load ends, bringing it to "ideal" current for matched load when $Z_L = Z_0$
- Current at midpoint (λ/4 from load) equals "ideal" current for matched load, independent of Z_L *and* Γ_L
- Damping provides more uniform current along length
- Again, specific position of current probe for CMCE measurement is no longer crucial



A Closer Look at Resonant Peaks for $l = n\lambda/2$

Up to this point, we have considered only the envelope of the current distribution

- *Only spatial dependence considered*
- *Time dependence ignored*

For the resonant peaks for which $l = n\lambda/2$, it is instructive to consider the full time domain representation:

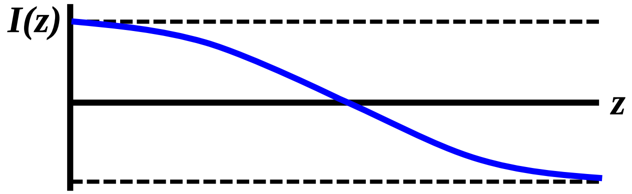
Details in backup slides…

$$I(z,t) = |I(z)|e^{j\theta(z)} \cdot e^{j(\omega t - \beta z)}$$

$$RE[I(z,t)] = |I(z)| \cdot \cos[\omega t - \beta z + \theta(z)]$$

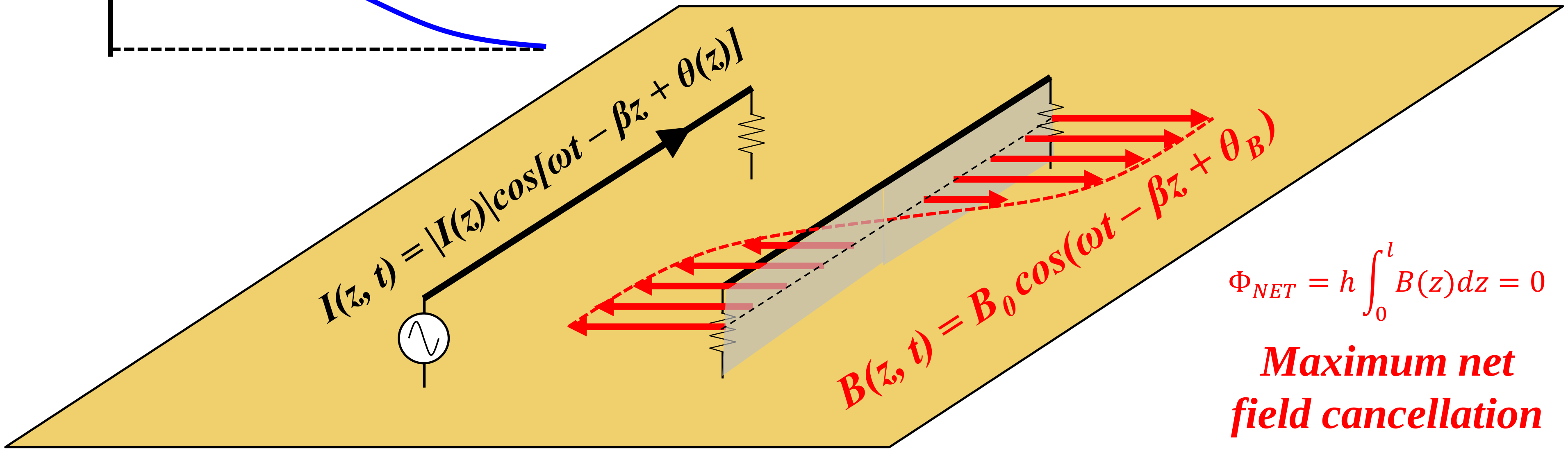
A Closer Look at Resonant Peaks for $l = n\lambda/2$ (cont.)

"Snapshot in time":



$$I_{AV} = \frac{1}{l}\int_0^l I(z)dz = 0$$

Average current = 0

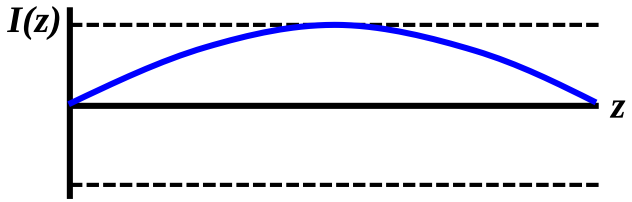


$$\Phi_{NET} = h\int_0^l B(z)dz = 0$$

Maximum net field cancellation

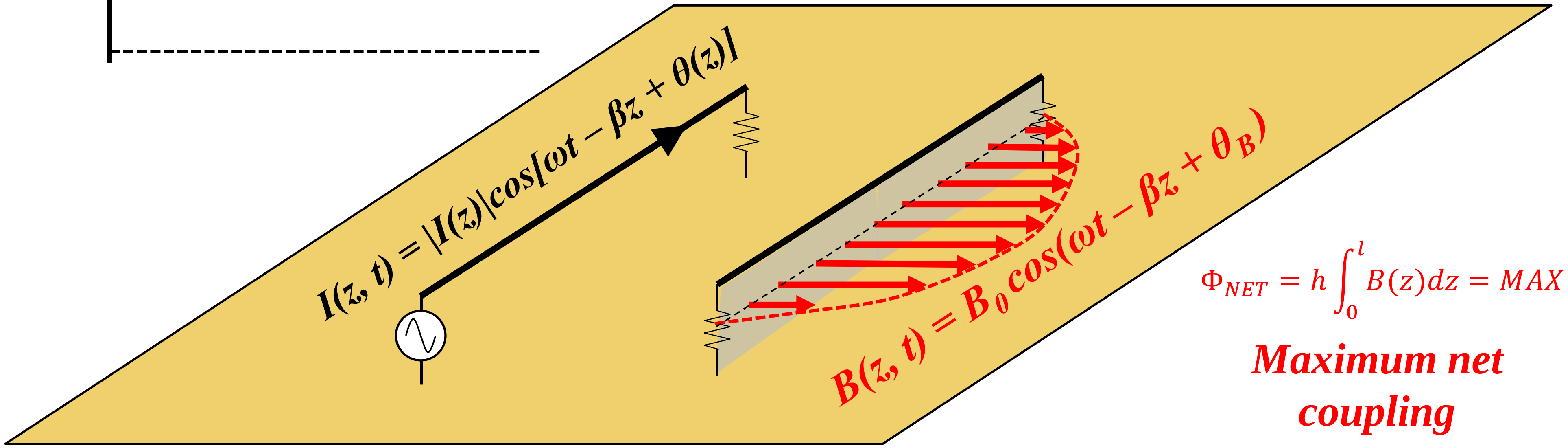
A Closer Look at Resonant Peaks for $l = n\lambda/2$ (cont.)

"Snapshot in time":



$$I_{AV} = \frac{1}{l}\int_0^l I(z)dz = \frac{2}{\pi}\cdot I_{peak}$$

Average current = maximum



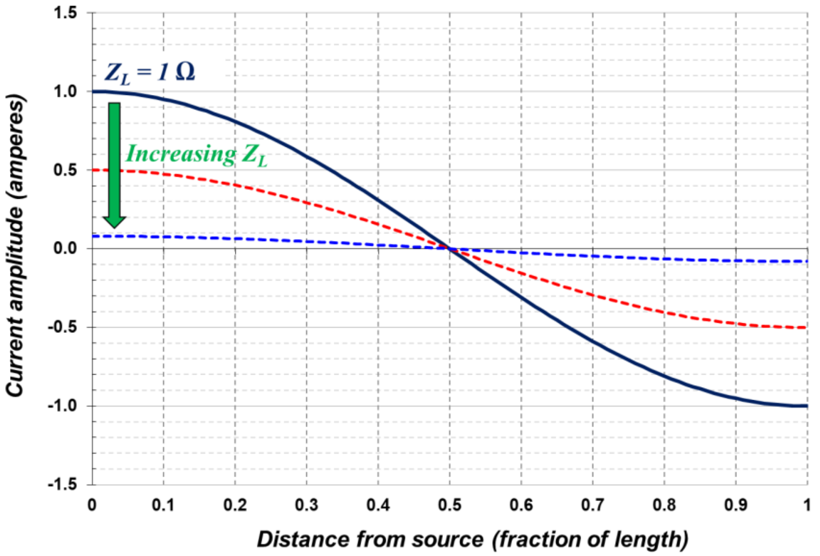
$$\Phi_{NET} = h\int_0^l B(z)dz = MAX$$

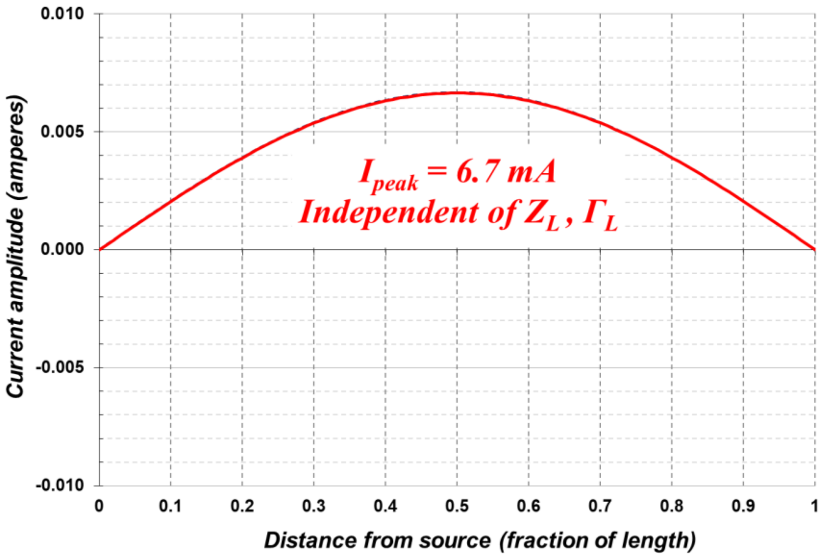
Maximum net coupling

A Closer Look at Resonant Peaks for $l = n\lambda/2$ (cont.)

$$I_{AV}(t) = \frac{V_S}{Z_0 + Z_S} \cdot \frac{2}{n\pi} \cdot \sin \omega t$$

Average current independent of Z_L, Γ_L
Same as average current into matched load
Inversely proportional to n
(Derivation in backup slides)

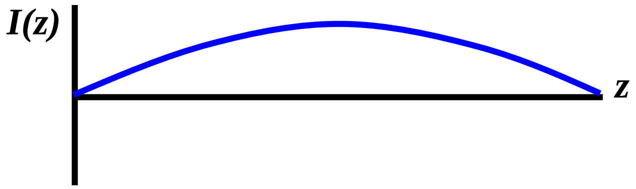




Increasing damping resistance reduces peak amplitude at "snapshots in time" corresponding to maximum field cancellation (no effect on coupling)

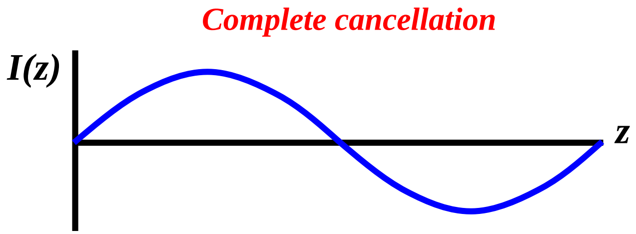
Increasing damping resistance has no effect on peak amplitude at "snapshots in time" corresponding to maximum coupling

A Closer Look at Resonant Peaks for $l = n\lambda/2$ (cont.)



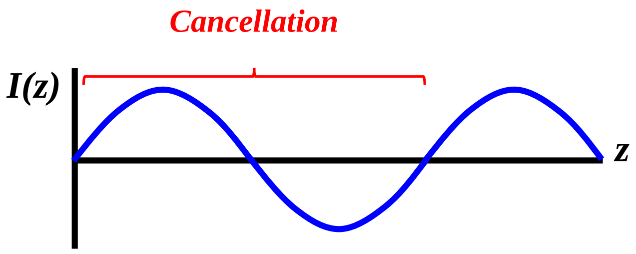
$n = 1$

$$(I_{AV})_{MAX} = \frac{V_S}{Z_0 + Z_S} \cdot \frac{2}{\pi}$$



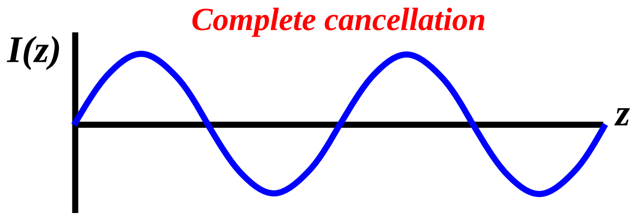
$n = 2$

$$(I_{AV})_{MAX} = 0$$



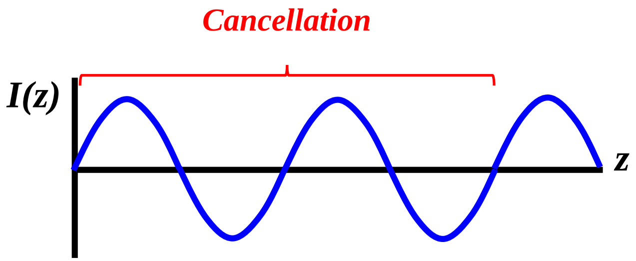
$n = 3$

$$(I_{AV})_{MAX} = \frac{V_S}{Z_0 + Z_S} \cdot \frac{2}{3\pi}$$



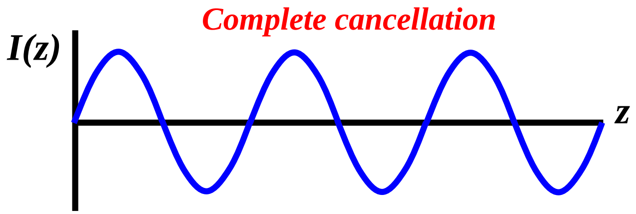
$n = 4$

$$(I_{AV})_{MAX} = 0$$



$n = 5$

$$(I_{AV})_{MAX} = \frac{V_S}{Z_0 + Z_S} \cdot \frac{2}{5\pi}$$



$n = 6$

$$(I_{AV})_{MAX} = 0$$

A Closer Look at Resonant Peaks for $l = n\lambda/2$ (cont.)

$$\Phi_{NET}(t) \propto I_{AV}(t) \Rightarrow \Phi_{NET}(t) = \frac{1}{n}\Phi_0 \cdot \sin\omega t$$

Φ_0 = peak amplitude of coupled flux for n = 1

Net coupled flux decreases with frequency along with average current

Coupled potential into victim loop:

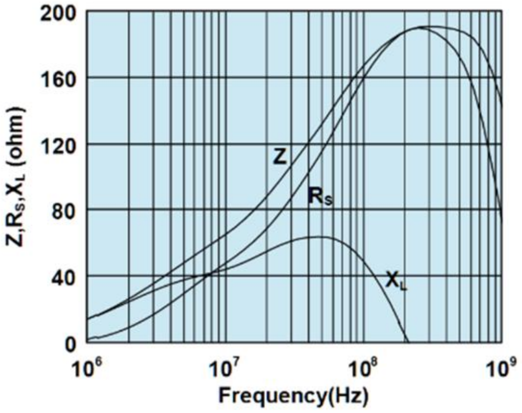
$$V_V = -\frac{d\Phi_{NET}(t)}{dt} = -\frac{\omega}{n}\Phi_0 \cdot \cos\omega t$$

$$V_V = -\frac{2\pi f}{n}\Phi_0 \cdot \cos\omega t$$

Frequency dependence cancels; Coupled potential has constant peak amplitude with frequency

Absorbing Clamp

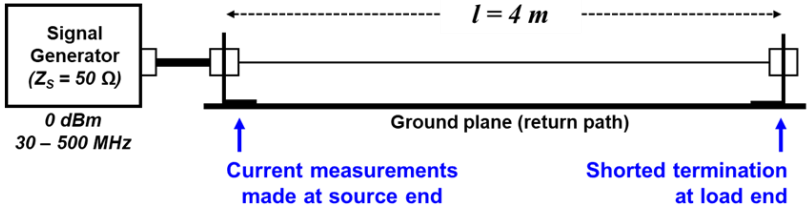
- A matched termination at all frequencies would reduce the current emissions at frequencies for which $l < \lambda/10$, which is not desirable
- Inserting such a connection would require breaking the shield termination and inserting a 300 Ω resistor, which is neither desirable nor practical
- Enter the absorbing clamp…
  - Specified in CISPR 16
  - Current probe followed by ferrite ring absorber elements
  - Adds resistive impedance above 30 MHz and acts to isolate the rest of the cable, minimizing the standing waves associated with signals on an electrically long mismatched transmission line
  - Specified for Space Shuttle program to address radiated emissions below 200 MHz

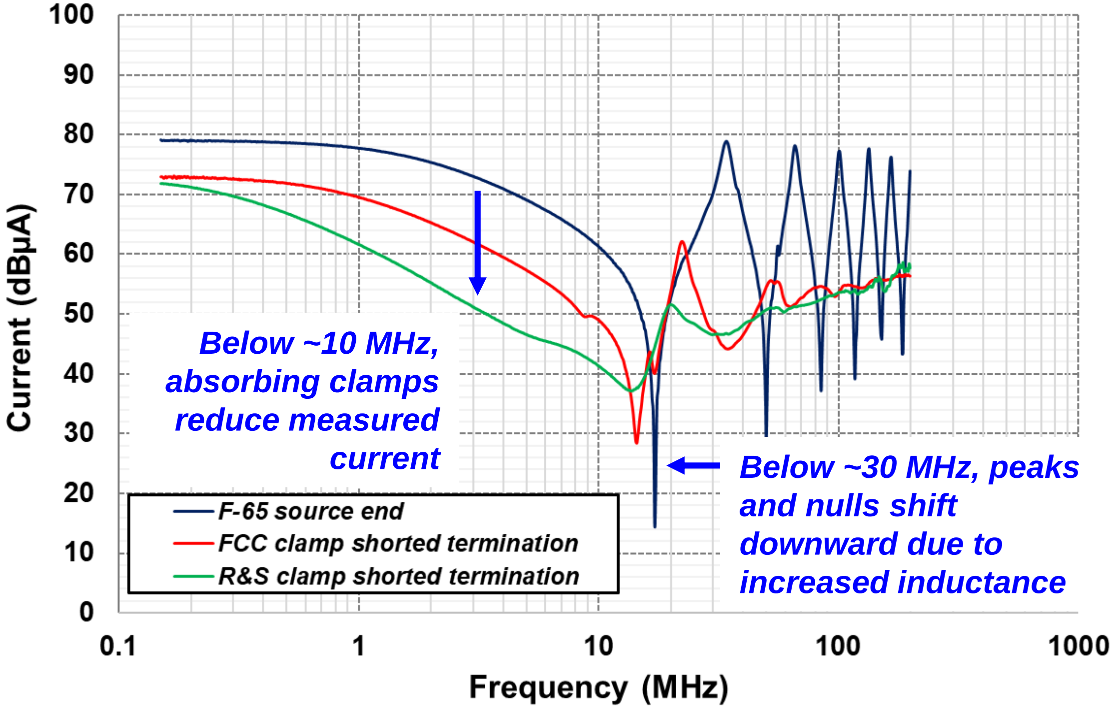


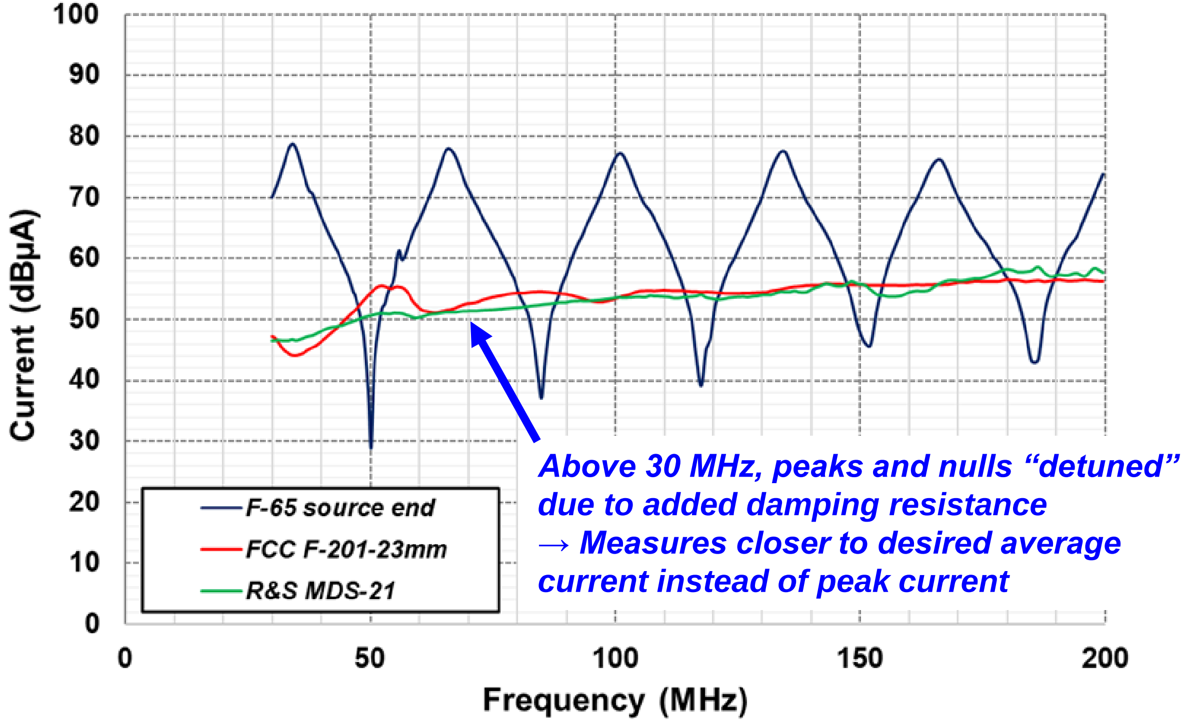
Impedance, reactance, and resistance vs. frequency.

(Representative)

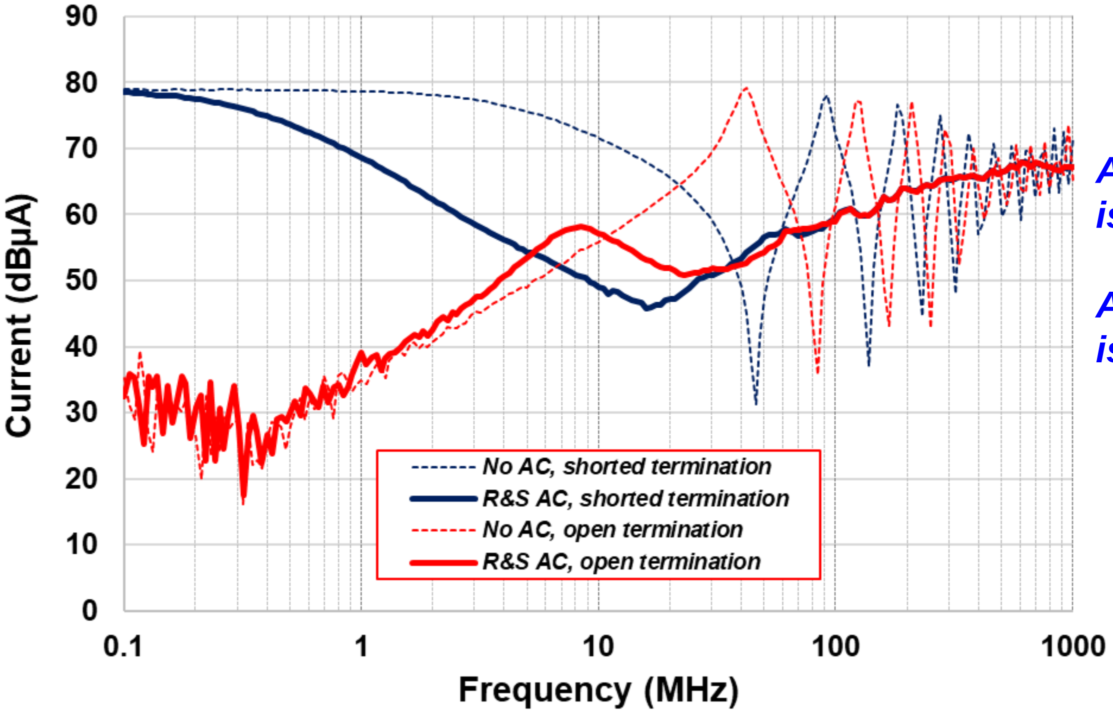
Measured Results: Comparison of Standard Current Probe to Absorbing Clamps







Absorbing Clamp on 2 m Wire, Shorted vs. Open Terminations



Above 30 MHz, termination is insignificant

Absorbing clamp provides isolation from rest of cable

Summary

- CMCE measurements on cables provides excellent tool for assessing risk of radiated emissions and crosstalk at system level
- For typical case of shielded cable with shield terminated to chassis at both ends, cables must be considered as wire-above-ground transmission line with shorted termination at each end
- Current distribution will exhibit predictable pattern of peaks and nulls
    - Nulls at odd multiples of $\lambda/4$
    - Peaks at multiples of $\lambda/2$
- For frequencies for which $l < \lambda/10$, current is constant over length
    - CMCE measurements may be performed with current probe at any location
- For f < 30 MHz, current probe should be placed at “load end” to ensure that peak current is captured
- For f > 30 MHz, absorbing clamp should be used to measure approximate average current and to get closer to ideal normalized measurement of EUT emissions independent of cable configuration

QUESTIONS?

BACKUP SLIDES

(for the mathochists)

Inductive Crosstalk Revisited (Electrically Short Cables)

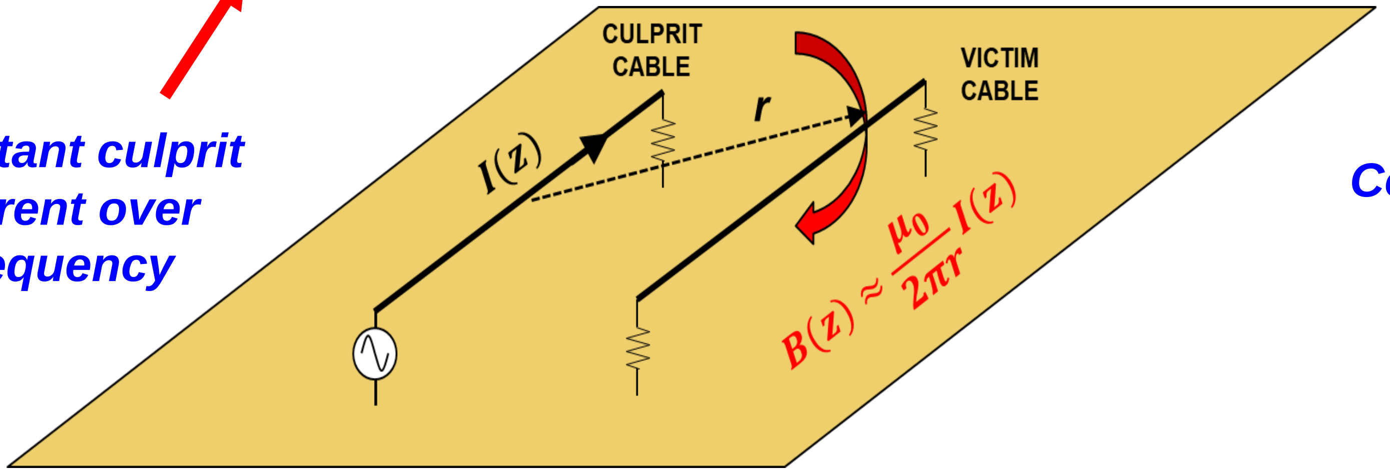
Coupled potential increases with frequency:

Faraday's Law: $\left|V_{emf}\right| = \left|\frac{d\Phi}{dt}\right| = j\omega BA \propto f \cdot I$

Victim cable impedance mostly inductive – also increases with frequency:

$$Z_V = j\omega L_V \propto f$$

Constant culprit current over frequency

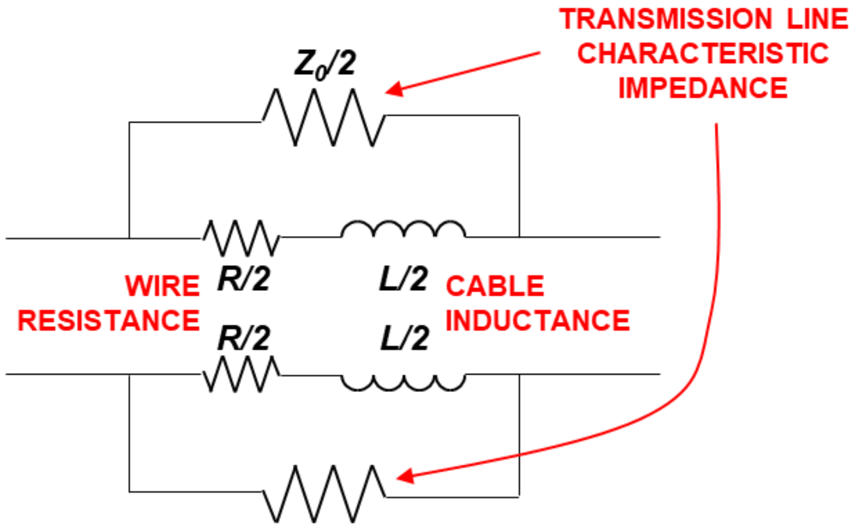


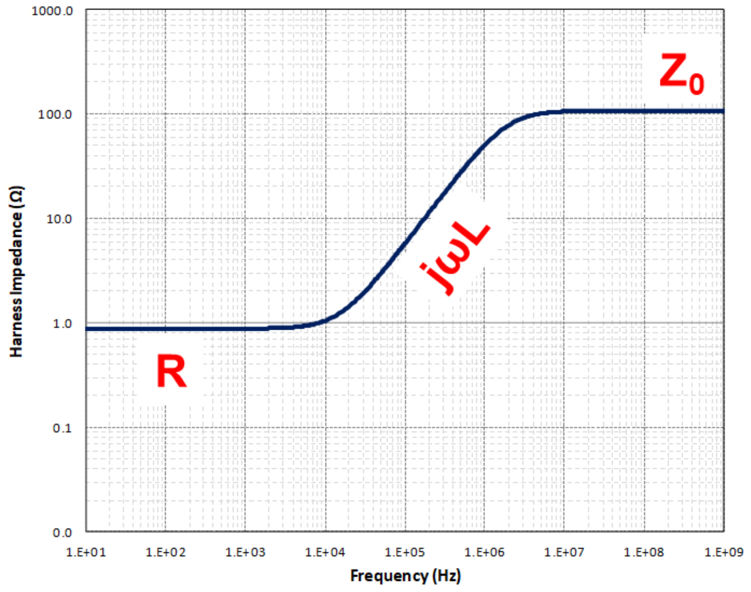
Constant victim current over frequency

Wire-Above-Ground – Shorted at Both Ends

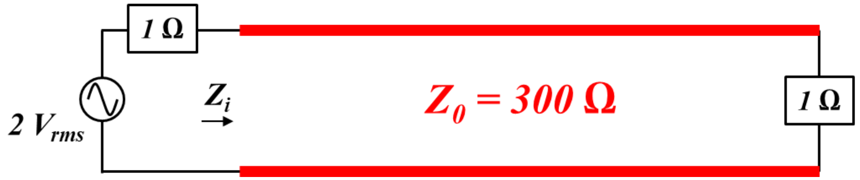
Typical case is shielded cable with shield ***terminated to chassis*** *at both ends*

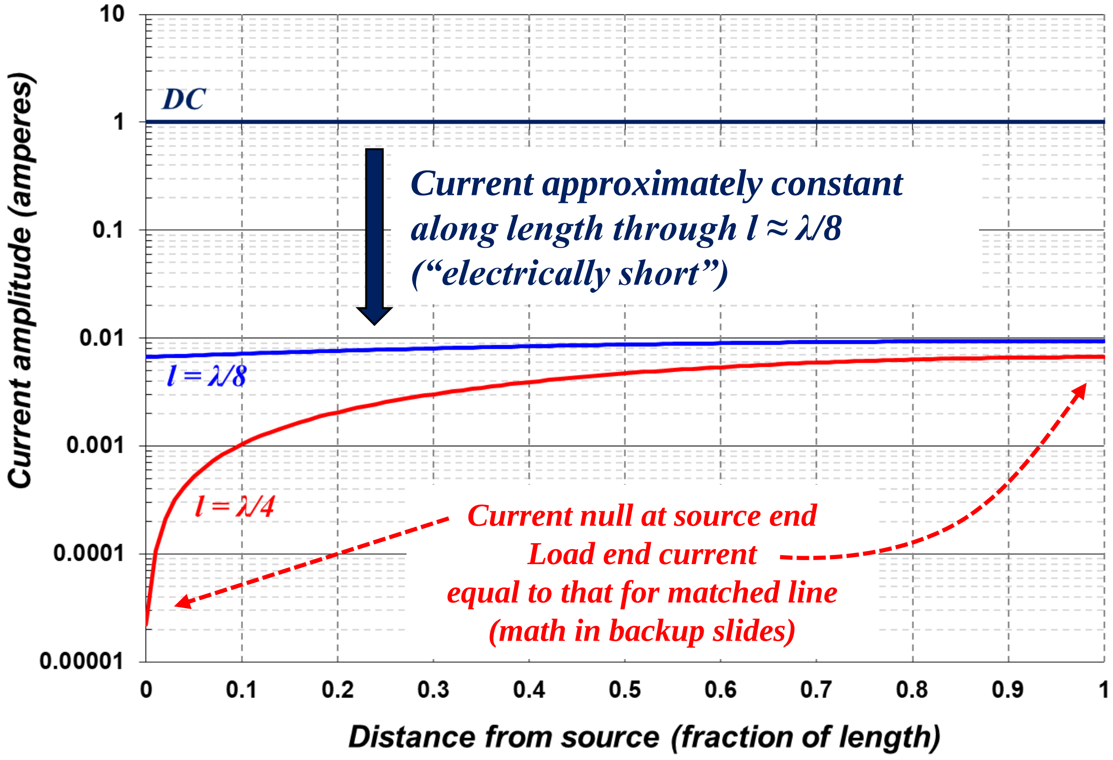
- *At very low frequencies, wire/shield resistance dominates*
- *At "midrange" frequencies for which cable is "electrically short" (l ≤ λ/10), inductance dominates*
- *When cable is "electrically long" (l > λ/10), characteristic impedance dominates*

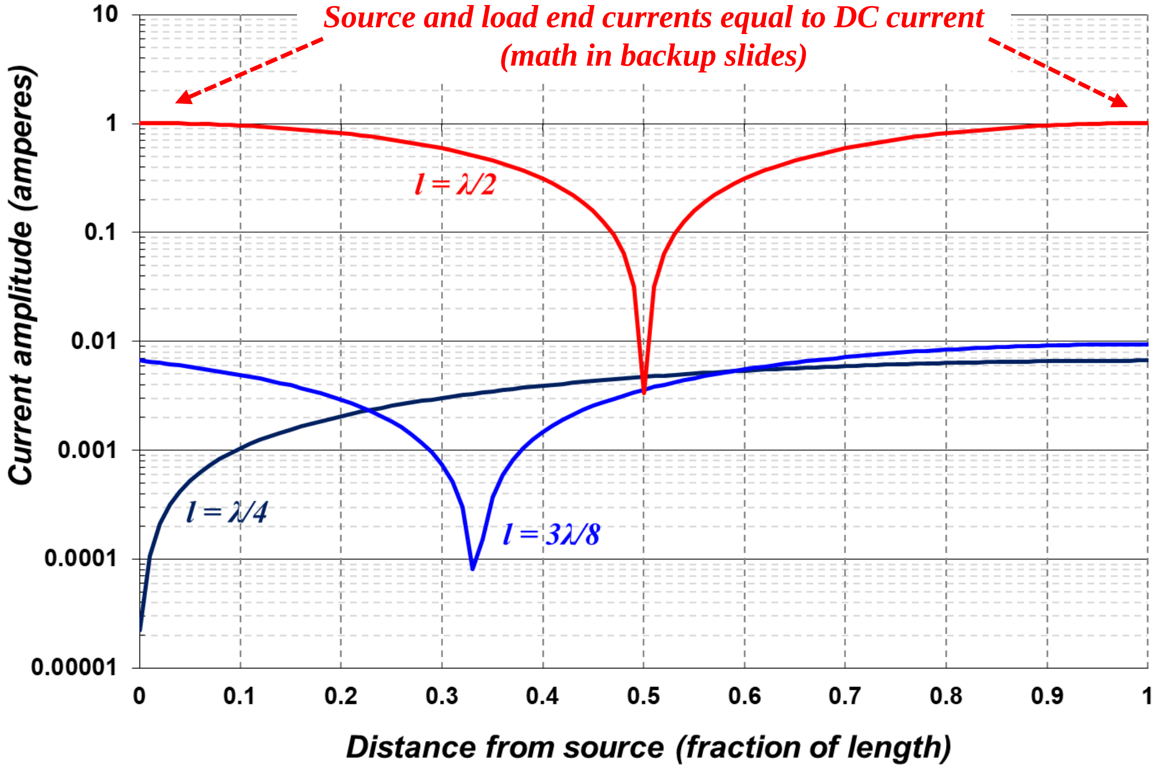




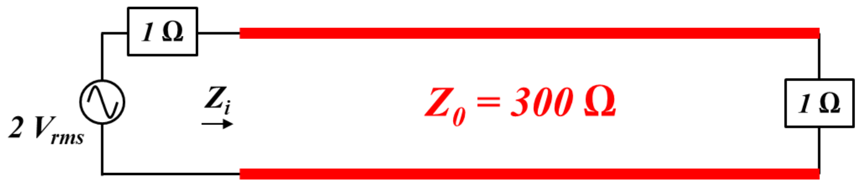
Current Distribution, DC to $l = \lambda/2$

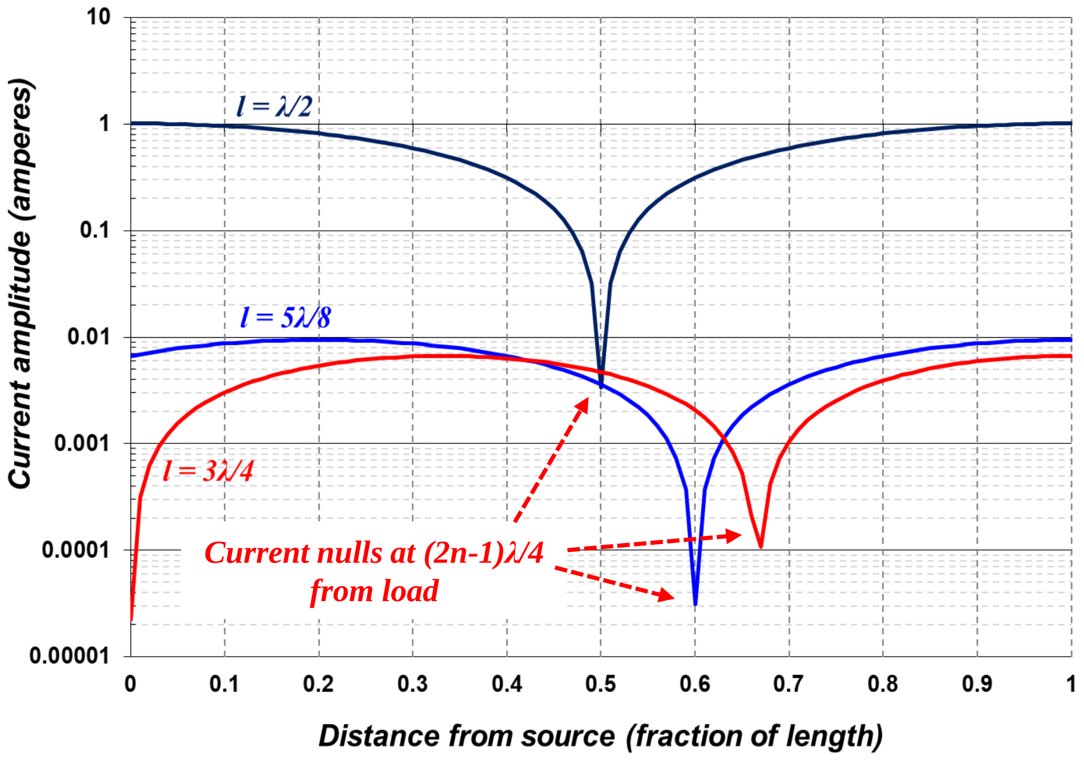


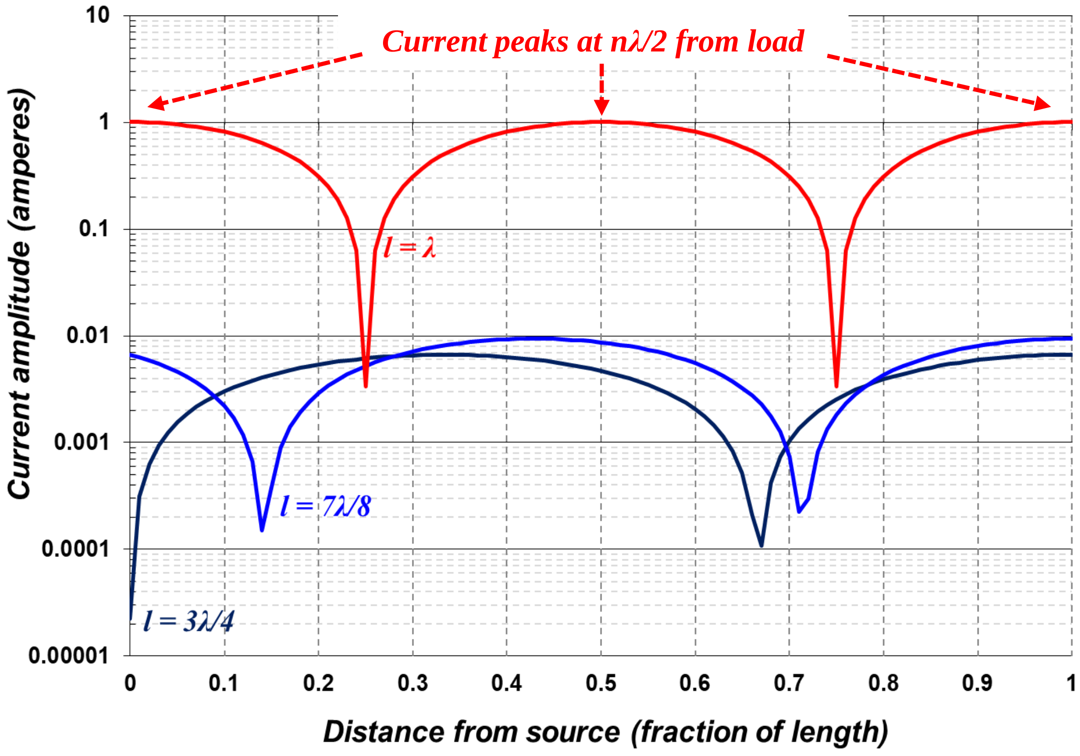




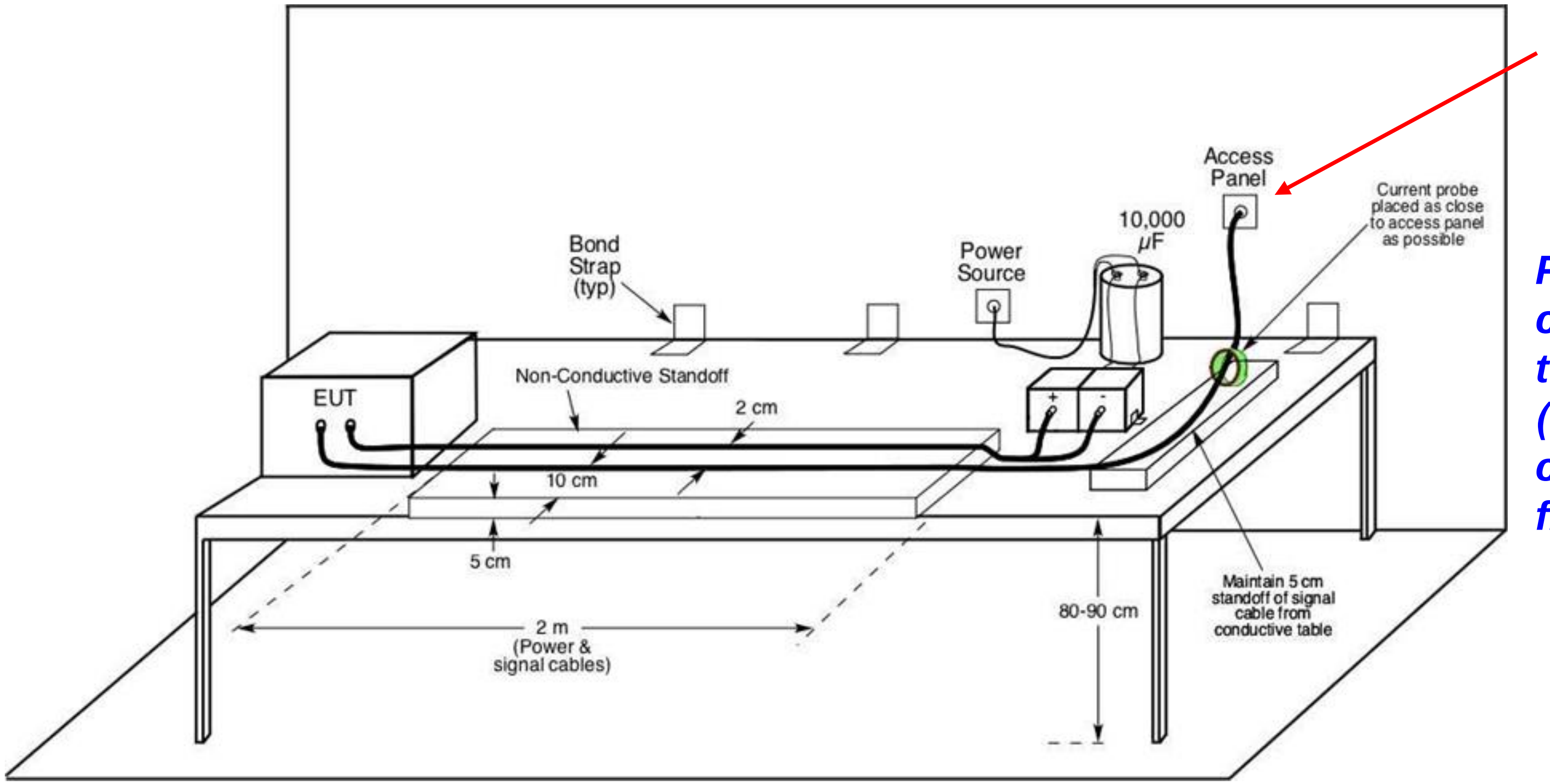
Current Distribution, *l* = *λ/2 to l = λ*







CMCE, *f* ≤ *30 MHz*



Cable shields terminated at access panel

For f < 30 MHz, use standard current probe placed as close to access panel as possible (“load end”) in order to capture peak current at all frequencies

Current Distribution on Mismatched Transmission Line, *l = (2n-1)λ/4*

$$I(z) = \frac{V_S}{Z_0 + Z_S} \cdot \frac{1 - \Gamma_L e^{-j(2n-1)\pi} e^{j2\beta z}}{1 - \Gamma_S \Gamma_L e^{-j(2n-1)\pi}} \Rightarrow e^{-j(2n-1)\pi} = -1$$

$$I(z) = \frac{V_S}{Z_0 + Z_S} \cdot \frac{1 + \Gamma_L \cos(2\beta z)}{1 + \Gamma_S \Gamma_L}$$

For $Z_S \rightarrow 0$,
$\Gamma_S \rightarrow -1$

$$I(0) \approx \frac{V_S}{Z_0 + Z_S} \cdot \frac{1 + \Gamma_L}{1 - \Gamma_L}$$

For $Z_L \rightarrow 0$, $\Gamma_L \rightarrow -1$:
$I(0) \rightarrow 0$
Current null at source end

$$I(l) \approx \frac{V_S}{Z_0 + Z_S} \cdot \frac{1 - \Gamma_L}{1 - \Gamma_L} = \frac{V_S}{Z_0 + Z_S}$$

Load end current same as current into matched load (Independent of Γ_L)

Current Distribution on Mismatched Transmission Line, *l* = *nλ/2*

$$I(z) = \frac{V_S}{Z_0 + Z_S} \cdot \frac{1 - \Gamma_L e^{-j2\pi} e^{j2\beta z}}{1 - \Gamma_S \Gamma_L e^{-j2\pi}} \Rightarrow e^{-j2\pi} = 1 \Rightarrow I(z) = \frac{V_S}{Z_0 + Z_S} \cdot \frac{1 - \Gamma_L \cos(2\beta z)}{1 - \Gamma_S \Gamma_L}$$

For z = 0 and z = l (endpoints) :

$$I(0) = I(l) = \frac{V_S}{Z_0 + Z_S} \cdot \frac{1 - \Gamma_L}{1 - \Gamma_S \Gamma_L}$$

$$= \frac{V_S}{Z_0 + Z_S} \cdot \frac{1 - \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right)}{1 - \left(\frac{Z_S - Z_0}{Z_S + Z_0}\right) \cdot \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right)}$$

$$= V_S \cdot \frac{(Z_L + Z_0) - (Z_L - Z_0)}{(Z_0 + Z_S)(Z_L + Z_0) - (Z_s - Z_0)(Z_L - Z_0)}$$

$$= V_S \cdot \frac{2Z_0}{Z_0 Z_L + Z_0^2 + Z_S Z_L + Z_S Z_0 - (Z_s Z_L - Z_s Z_0 - Z_L Z_0 + Z_0^2)}$$

$$= V_S \cdot \frac{2Z_0}{2Z_0 Z_L + 2Z_S Z_0}$$

$$I(0) = I(l) = \frac{V_S}{Z_S + Z_L}$$

Same as DC current (Resonant peak)

For z = l/2 (midpoint):

$$I(l/2) = \frac{V_S}{Z_0 + Z_S} \cdot \frac{1 + \Gamma_L}{1 - \Gamma_S \Gamma_L}$$

For $Z_S \rightarrow 0$, $\Gamma_S \rightarrow -1$, $|\Gamma_L| < |\Gamma_S|$:

$$I(l/2) \approx \frac{V_S}{Z_0 + Z_S} \cdot \frac{1 + \Gamma_L}{1 + \Gamma_L} \approx \frac{V_S}{Z_0 + Z_S}$$

$$I(l/2) \approx \frac{V_S}{Z_0 + Z_S}$$

Same as current into matched load

For $\Gamma_L = \Gamma_S \approx -0.99$:

$$I(l/2) \approx \frac{V_S}{Z_0 + Z_S} \cdot 0.5$$

Half of current into matched load

For $\Gamma_L = \Gamma_S = -1$:

$$I(l/2) \approx \frac{V_S}{Z_0 + Z_S}$$

Same as current into matched load

Current Distribution on Mismatched Transmission Line

Envelope of current amplitude on mismatched transmission line of length l as function of distance z from source:

$$I(z) = \frac{V_S}{Z_0 + Z_S} \cdot \frac{1 - \Gamma_L e^{-j2\beta l} e^{j2\beta z}}{1 - \Gamma_S \Gamma_L e^{-j2\beta l}}$$

$$I(z) = \frac{V_S}{Z_0 + Z_S} \cdot \frac{1 - \Gamma_L e^{-j2\beta l} e^{j2\beta z}}{1 - \Gamma_S \Gamma_L e^{-j2\beta l}} \cdot \frac{1 - \Gamma_S \Gamma_L e^{j2\beta l}}{1 - \Gamma_S \Gamma_L e^{j2\beta l}}$$

$$I(z) = \frac{V_S}{Z_0 + Z_S} \cdot \frac{1 + \Gamma_S \Gamma_L \Gamma_L e^{j2\beta z} - \Gamma_L e^{-j2\beta l} e^{j2\beta z} - \Gamma_S \Gamma_L e^{j2\beta l}}{(\Gamma_S \Gamma_L)^2 - 2\Gamma_S \Gamma_L \cos(2\beta l) + 1}$$

$$NUM_{Re} = 1 + \Gamma_S {\Gamma_L}^2 \cos(2\beta z) - \Gamma_L \cos(2\beta z - 2\beta l) - \Gamma_S \Gamma_L \cos(2\beta l)$$

$$NUM_{Im} = \Gamma_S {\Gamma_L}^2 \sin(2\beta z) - \Gamma_L \sin(2\beta z - 2\beta l) - \Gamma_S \Gamma_L \sin(2\beta l)$$

Current Distribution on Mismatched Transmission Line (cont.)

Magnitude:

$$|I(z)| = \frac{V_S}{Z_0 + Z_S} \cdot \frac{\sqrt{(NUM_{Re})^2 + (NUM_{Im})^2}}{(\Gamma_S \Gamma_L)^2 - 2\Gamma_S \Gamma_L \cos(2\beta l) + 1}$$

Phase:

$$\theta(z) = \tan^{-1}\left(\frac{NUM_{Im}}{NUM_{Re}}\right)$$

Full time domain representation:

$$I(z,t) = |I(z)| e^{j\theta(z)} \cdot e^{j(\omega t - \beta z)}$$

$$RE[I(z,t)] = |I(z)| \cdot \cos[\omega t - \beta z + \theta(z)]$$

Average Current as Function of Time, *l = nλ/2*

$$I(z,t) = \frac{V_S}{Z_0+Z_S}\cdot\frac{1-\Gamma_L e^{j2\beta z}}{1-\Gamma_S\Gamma_L}e^{j(\omega t-\beta z)} = \frac{V_S}{Z_0+Z_S}\cdot\frac{e^{j\omega t}}{1-\Gamma_S\Gamma_L}\left(e^{-j\beta z}-\Gamma_L e^{j\beta z}\right)$$

$Z_S \to 0, \Gamma_S \to -1$:

$$I(z,t) = \frac{V_S}{Z_0+Z_S}\cdot\frac{e^{j\omega t}}{1+\Gamma_L}\left(e^{-j\beta z}-\Gamma_L e^{j\beta z}\right)$$

Average current as function of time:

$$I_{AV}(t) = \frac{V_S}{Z_0+Z_S}\cdot\frac{e^{j\omega t}}{1+\Gamma_L}\cdot\frac{1}{l}\int_0^l\left(e^{-j\beta z}-\Gamma_L e^{j\beta z}\right)dz = \frac{V_S}{Z_0+Z_S}\cdot\frac{e^{j\omega t}}{1+\Gamma_L}\cdot\frac{1}{j\beta l}\cdot\left[-e^{-j\beta z}-\Gamma_L e^{j\beta z}\right]_0^l$$

$$= \frac{V_S}{Z_0+Z_S}\cdot\frac{e^{j\omega t}}{1+\Gamma_L}\cdot\frac{1}{j\beta l}\cdot\left[-e^{-j\beta l}-\Gamma_L e^{j\beta l}+1+\Gamma_L\right]$$

➡ $\beta = \frac{2\pi}{\lambda}$ $\quad l = \frac{n\lambda}{2}$ ➡ $\beta l = n\pi$

n odd: $e^{-jn\pi} = e^{jn\pi} = -1$

n even: $e^{-jn\pi} = e^{jn\pi} = 1$

n odd:

$$I_{AV}(t) = \frac{V_S}{Z_0+Z_S}\cdot\frac{e^{j\omega t}}{1+\Gamma_L}\cdot\frac{1}{jn\pi}\cdot\left[1+\Gamma_L+1+\Gamma_L\right]$$

$$= \frac{V_S}{Z_0+Z_S}\cdot\frac{e^{j\omega t}}{1+\Gamma_L}\cdot\frac{2}{jn\pi}\cdot\left[1+\Gamma_L\right]$$

$$= \frac{V_S}{Z_0+Z_S}\cdot\frac{2}{n\pi}\cdot\frac{1+\Gamma_L}{1+\Gamma_L}\cdot\left[-je^{j\omega t}\right]$$

$$= \frac{V_S}{Z_0+Z_S}\cdot\frac{2}{n\pi}\cdot e^{j\left(\omega t-\frac{\pi}{2}\right)}$$

n even:

$$I_{AV}(t) = \frac{V_S}{Z_0+Z_S}\cdot\frac{e^{j\omega t}}{1+\Gamma_L}\cdot\frac{1}{jn\pi}\cdot\underbrace{\left[-1-\Gamma_L+1+\Gamma_L\right]}$$

Average current = 0 when l is integral multiple of full wavelength (full cancellation)

$$Re\left[I_{AV}(t)\right] = \frac{V_S}{Z_0+Z_S}\cdot\frac{2}{n\pi}\cdot\sin\omega t$$

Average current independent of Z_L, Γ_L
Same as average current into matched load
Inversely proportional to n

Experimental Results – Standard Current Probe (F-65)