Modified Geometric Truncation of the Scattering Phase Function

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Abstract

Phase function of light scattering on large atmospheric particles has very strong peak in forward direction constituting a challenge for accurate numerical calculations of radiance required in remote sensing problems. Scaling transformation replaces original phase function with a sum of the delta function and a new regular smooth phase function. Geometric truncation is one of the ways to construct such a smooth function. The replacement phase function coincides with the original one outside the forward cone and preserves the asymmetry parameter. It has discontinuity at the cone.

Another simple functional form of the replacement phase function within the cone is suggested. It enables continuity and allows for a number of modifications. Three of them are considered in this study: preserving asymmetry parameter, providing continuity of the 1st derivative of the phase function, and preserving mean scattering angle.

Yet another problem addressed in this study is objective selection of the width of the forward cone. That angle affects truncation fraction and values of the phase function within the cone. A heuristic approach providing unambiguous criterion of selection of the truncation angle is proposed. The approach has easy numerical implementation.

Suggested modifications were tested on cloud phase function using discrete ordinates and Monte Carlo methods. It was shown that the modifications provide better accuracy of the radiance computation compare to the original geometric truncation with discrete ordinates while continuous derivative approach provides significant gain in computer time with Monte Carlo simulations.

Highlights

- Criterion for selection of the angle of truncation is proposed;
- New form of the replacement phase function within the forward cone;
- Preservation of integral characteristics of the phase function is considered;
- Continuity of the phase function at the angel of truncation is considered;
- Comparison of radiance computed various truncation methods is performed.

Keywords: phase function, truncation, similarity transformation, angle of truncation, delta-M

1. INTRODUCTION

Accurate numerical calculations of radiance transmitted through and, especially, reflected by the atmosphere are required in numerous remote sensing applications. Size distribution of the major scattering media in the atmosphere is such that effective size of scattering particles is much greater than wavelength of visible light. Consequently, scattering of light by those particles is highly anisotropic with a distinct forward scattering peak. The presence of that peak constitutes one the major problems of atmospheric radiative transfer (RT).

The problem of the forward scattering peak is handled with various methods that are collectively referred to as truncation of the scattering phase function. The essence of the approach is to replace the actual phase function with a weighted sum of the delta function and yet another (truncated) phase function that is relatively smoother than the actual one. The difference between the truncation methods is in the choice of the truncated phase function. Two groups can be distinguished among those methods. The first group preserves functional behavior of the phase function outside of the forward scattering cone [1, 2, 3]. The other group deals with Legendre moments of the phase function [4, 5, 6]. While representation of the phase function with Legendre moments bears clear theoretical advantages, its practical use may not be easy in cases when the phase function is given by a set of scattering angle – value pairs. The forward peak may be poorly resolved making computation of the moments a very challenging problem. Methods based on this representation will not be considered in this study.

New methods belonging to the first group will be presented. Term “geometric truncation” (GT) was suggested by Iwabuchi and Suzuki [3]. This term will be used to refer to their study. However, the ideas from other methods preserving phase function outside certain forward cone were used in this study. For this reason it is important give a brief recap of those studies. First, we note that works of Potter and Arking [1, 7, 8] used truncation technique without providing explicit functional form of the dependence of the truncated phase function on the scattering angle Θ withing the forward cone. Figures 1 of all of the above mentioned papers suggest that it is either $\sim\exp(-\Theta/\Theta_t)$ or $\sim\exp(-\Theta^2/\sigma^2)$. Those graphs also suggest that truncated phase function has continuous derivative at the angle of truncation. Mitrescu and Stephens [2] suggested replacement of the forward peak with a constant presenting phase function value at the angle of truncation. While they performed a study on the sensitivity to that angle, no clear recipe on a priori selection of such an angle was provided. Iwabuchi and Suzuki suggested similar technique but their truncated phase function was discontinuous at the angle of truncation while preserving asymmetry parameter of the original phase function.

All methods based on angle – value representation of the phase function require selection of the angle of truncation. Such selection is also needed in delta-fit technique [5]. However no objective recipe has been provided so far. Rozanov and Lyapustin [9] presented RMS error of radiance as a function of truncation angle for delta-fit and delta-S [2]. Their conclusion was “the optimal width <of the forward cone> cannot be evaluated in general case because the approximation error depends on the unknown intensity”. In this study another approach will be suggested based on how much information on single scattering can be retained from the original phase function and what part of the phase function range presents the diffraction peak. While there is no rigorous definition of the width of the forward cone and, therefore, there may be various methods of objective selection, the use of such objective criterion can be desirable in the situation of comparing accuracy of RT computations for different phase functions with different multiple scattering RT methods. This problem will be also addressed in this study.

2. STATEMENT OF THE PROBLEM AND NOTATION

If a plane parallel scattering medium is illuminated on its top by light coming in direction $\mu_0 = \cos\theta_0$, $\phi_0 = 0$, then the diffuse radiance inside the and its boundaries is a solution of the RT equation (RTE):

$$\mu\frac{\partial I}{\partial\tau}+I(\tau,\mathbf{\Omega})=\frac{\omega}{4\pi}\oint_{4\pi} d\mathbf{\Omega}'\, P(\mathbf{\Omega}'\to\mathbf{\Omega})I(\tau,\mathbf{\Omega}') \quad , \tag{1}$$

where τ is optical depth, ω – single scattering albedo (SSA), μ = cos θ, θ, ϕ are the polar and azimuth angles of the direction of propagation of light, *P*(μ, μ′, ϕ – ϕ′) is the scattering phase function normalized with condition:

$$\oint_{4\pi} d\mathbf{\Omega}'\, P(\mathbf{\Omega}'\to\mathbf{\Omega})=2\pi\int_0^{\pi} P(\Theta)\sin\Theta\, d\Theta=2\pi\int_{-1}^{1} P(x)\, dx=4\pi \quad . \tag{2}$$

We will also consider cumulative phase function widely used in Monte Carlo simulations of radiative transfer:

$$C(\Theta)=\frac{1}{2}\int_0^{\Theta} P(\Theta')\sin\Theta'\, d\Theta'=\frac{1}{2}\int_{\cos\Theta}^{1} P(x)\, dx \quad . \tag{3}$$

Assuming spherical symmetry of the scattering centers and thus phase function dependence on the scattering angle, phase function can be expanded in Legendre series

$$P(\Theta)=\sum_{l=0}^{\infty}(2l+1)\chi_l P_l(\cos\Theta);\quad \chi_l=\frac{1}{2}\int_{-1}^{1} P_l(x)P(x)\, dx;\quad x=\cos\Theta \quad . \tag{4}$$

Truncation of the phase function

$$P(\mathbf{\Omega}'\to\mathbf{\Omega})\to\tilde{P}(\mathbf{\Omega}'\to\mathbf{\Omega})=4\pi f\,\delta(\mathbf{\Omega}'\to\mathbf{\Omega})+(1-f)P'(\mathbf{\Omega}'\to\mathbf{\Omega}) \tag{5}$$

that also can be written as

$$P(x)\to\tilde{P}(x)=2f\,\delta(1-x)+(1-f)P'(x) \tag{6}$$

leads to the same RTE (similarity transformation [1 - 6, 9]

$$\mu\frac{\partial I}{\partial\tau'}+I(\tau',\mathbf{\Omega})=\frac{\omega'}{4\pi}\oint_{4\pi} d\mathbf{\Omega}'\, P'(\mathbf{\Omega}'\to\mathbf{\Omega})I(\tau',\mathbf{\Omega}') \tag{7}$$

with new single scattering albedo and optical depth:

$$\tau'=\tau(1-f\omega) \quad \omega'=\omega(1-f)/(1-f\omega) \tag{8}$$

If truncated phase function $P'(x)$ satisfies condition (2) then eqs. (5) and (6) ensure that replacement phase function $\tilde{P}(x)$ also does.

The original geometric truncation was suggested as [3]

$$P'(\Theta)=\frac{1}{1-f}\begin{cases} P_f, & 0\le\Theta\le\Theta_t \\ P(\cos\Theta), & \Theta>\Theta_t \end{cases} \quad , \tag{9}$$

so that the new phase function $\tilde{P}(x)$ remains unchanged for scattering angles greater than the angle of truncation Θ_t . Normality condition and preservation of the asymmetry parameter lead to the following system for the fraction of truncation *f* and constant P_f.

$$\begin{cases} 2f+P_f(1-x_t)=\int_{x_t}^{1} P(x)\, dx \\ 2f+\frac{1}{2}P_f(1-x_t^2)=\int_{x_t}^{1} P(x)\, x\, dx \end{cases} \tag{10}$$

where $x_t=\cos\Theta_t$. It is easy to see that equations (10) contain three variables: the fraction of truncation *f*, constant P_f, and the angle of truncation. The latter can be used as a variable there if either the fraction of truncation or constant P_f is defined. It was as $f=\chi_M$ (see delta-M truncation [4]) to determine P_f and Θ_t for a specific phase function and assumed a prerequisite of conservation of the first moment. They also noted that f and P_f can be determined if Θ_t is a prescribed parameter.

The strength of the method is that it retains scattering features everywhere outside the forward cone. It does not require knowledge of the Legendre coefficients χ_l. So, the approach based on delta-M determination of the fraction of truncation looks unnecessary complication: it requires accurate computation of the moments which is a challenging problem in some cases (especially of higher orders). Moreover, the use of $f=\chi_M$ is subjective since the choice of order M is also subjective. Therefore, some other way to prescribe Θ_t and determine *f* and P_f is needed. Section 3 of this paper suggests an objective determination of the angle of truncation.

It is also easy to see that the resulting phase function $\widetilde{P}(x)$ has discontinuity at the angle of truncation. Figure 1 shows phase function $\widetilde{P}(x)$ computed according to system (10) for different angles of truncation. The jump is of the order of magnitude of the original phase function at the angle of truncation. Section 3 suggests two modifications of the original geometric truncation providing continuous replacement of the phase function.

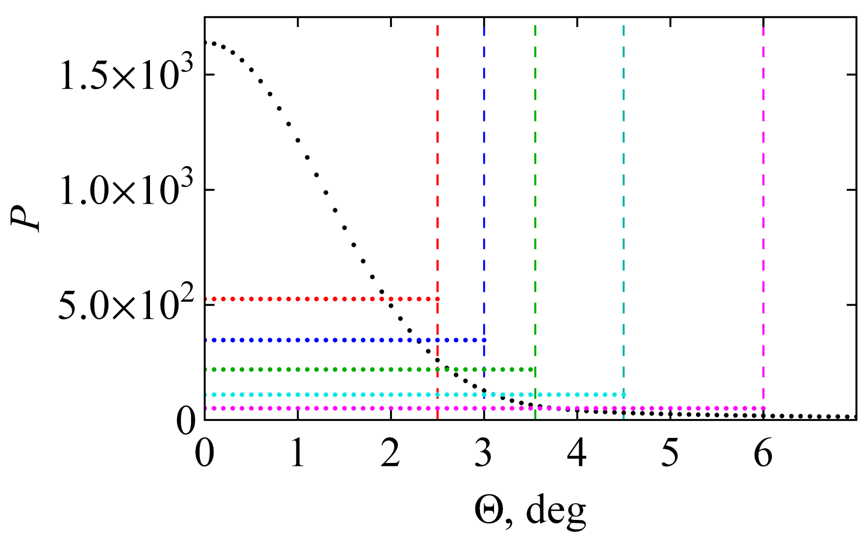


Figure 1. Phase function of cloud (micro-physical properties were taken from from OPAC)

3. OBJECTIVE CHOICE OF THE ANGLE OF TRUNCATION

Forward scattering peak of the phase function presents the major challenge in multiple scattering radiative transfer. It is desirable to replace the actual phase function as a sum of eq. (6). If the phase function is given as a function of the scattering angle then standard methods of calculus can be used to find characteristic points of the function such as extrema and points of infliction. In this study we will consider polydisperse scattering particles that are characterized with particle size distributions and their integral parameters. If effective diameter of such distribution D_{eff} is much greater than wavelength λ, so that Mie parameter $\pi D_{eff}/\lambda >>1$ then forward peak of the phase function is very strong. Plot of such a phase function itself is not evident. It is more fruitful to consider a plot of $\ln(P(\Theta))$ and its derivatives with the goal to find a point that can be considered as an objective limit of the forward peak.

Figs. 3(a) and 4(a) show $\ln(P(\Theta))$ for two different liquid clouds from OPAC database [10]. It is clear that the forward peak is limited by scattering angle of few degrees. Red ellipses show the transition range from the peak. Considering derivatives of $\ln(P(\Theta))$ one can see that there are two points that can be potentially used to determine the limit of the peak: minimum and infliction point of the 1st derivative. The former lies before the transition range, see Figs. 2(c) and 3(c). The infliction point provide more adequate estimate of the transition from the peak. It is also clear from Figs. 2(d) and 3(d) that cumulative phase function also has transition from one regime to another in the infliction point of log derivative of the phase function.

Phase functions from Figs. 2 and 3 were computed with Mie scattering code SPHER [11, 12]. These computations provide phase functions with any desirable angle resolution allowing differentiation with high accuracy. If the phase function is known from experiment or from numerical modeling then angular resolution may be poor. This and experimental/modeling errors may lead to derivative $d\ln(P(\Theta))/d\Theta$ to be a very noisy function so that further numerical differentiation becomes inadequate mean to find the infliction point. The example is shown in Fig. 4. The phase function from ice clouds data base [13, 14] look smooth but their 1st derivatives are noisy making further numerical differentiation troublesome. In this case the technique can be modified: actual values of $d\ln(P(\Theta))/d\Theta$ can be fitted with a polynomial of 4th or 5th degree and then the 1st positive infliction point of such polynomial can be found with standard means. The infliction points in Fig. 4 was found with 4th degree polynomial. Adding higher orders does not lead to significant change of the position of the infliction point.

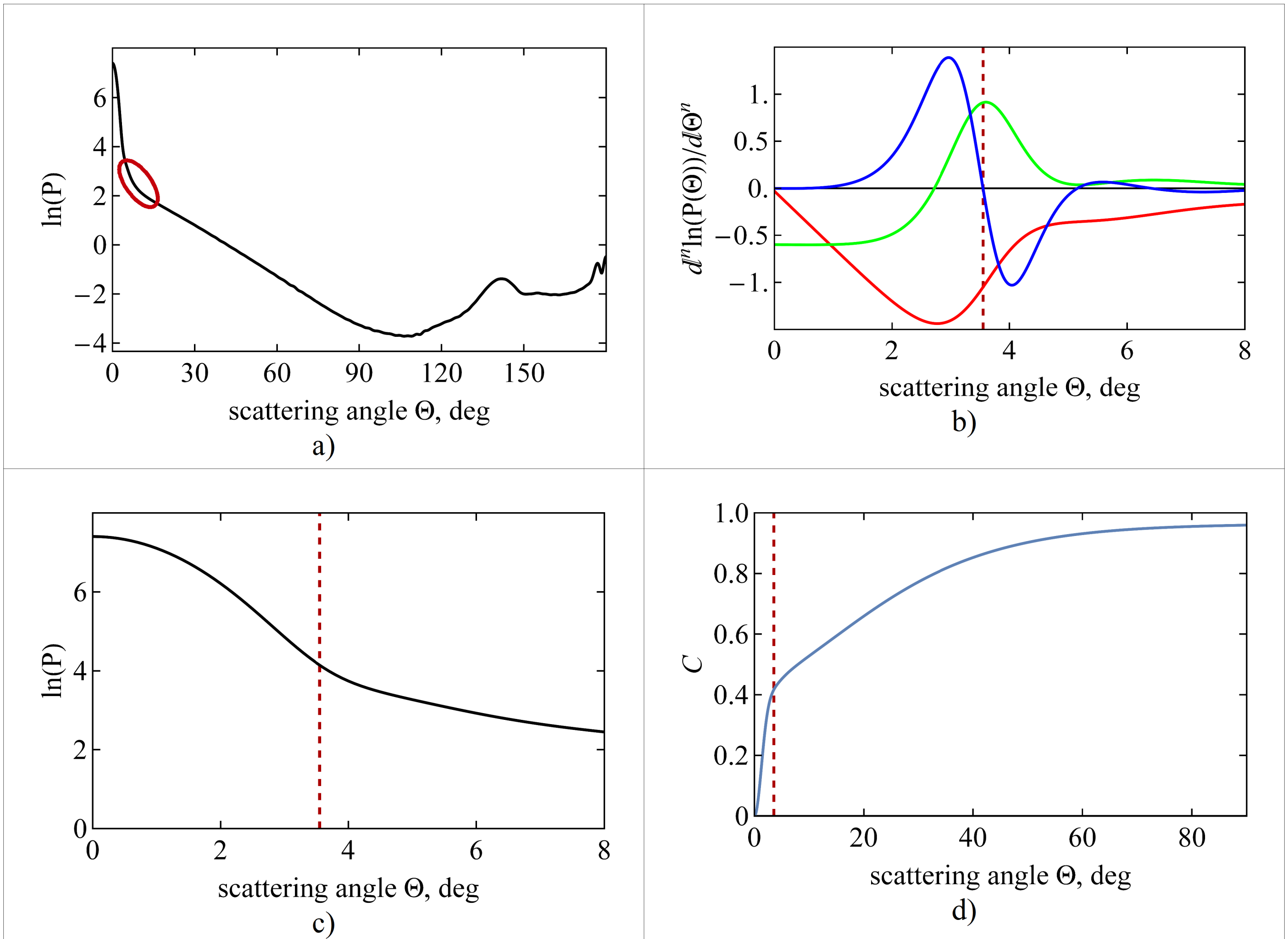


Figure 2. Log of phase function of maritime cumulus cloud, wavelength 1.5 mm (micro-physical properties were taken from from OPAC, CUMA). a – full range of scattering angle, b – first, second, and third derivatives of $\ln(P(\Theta))$, c – the same as a but for forward cone only, d – cumulative phase function. Dark red vertical line shows the first infliction point of $d\ln(P(\Theta))/d\Theta$.

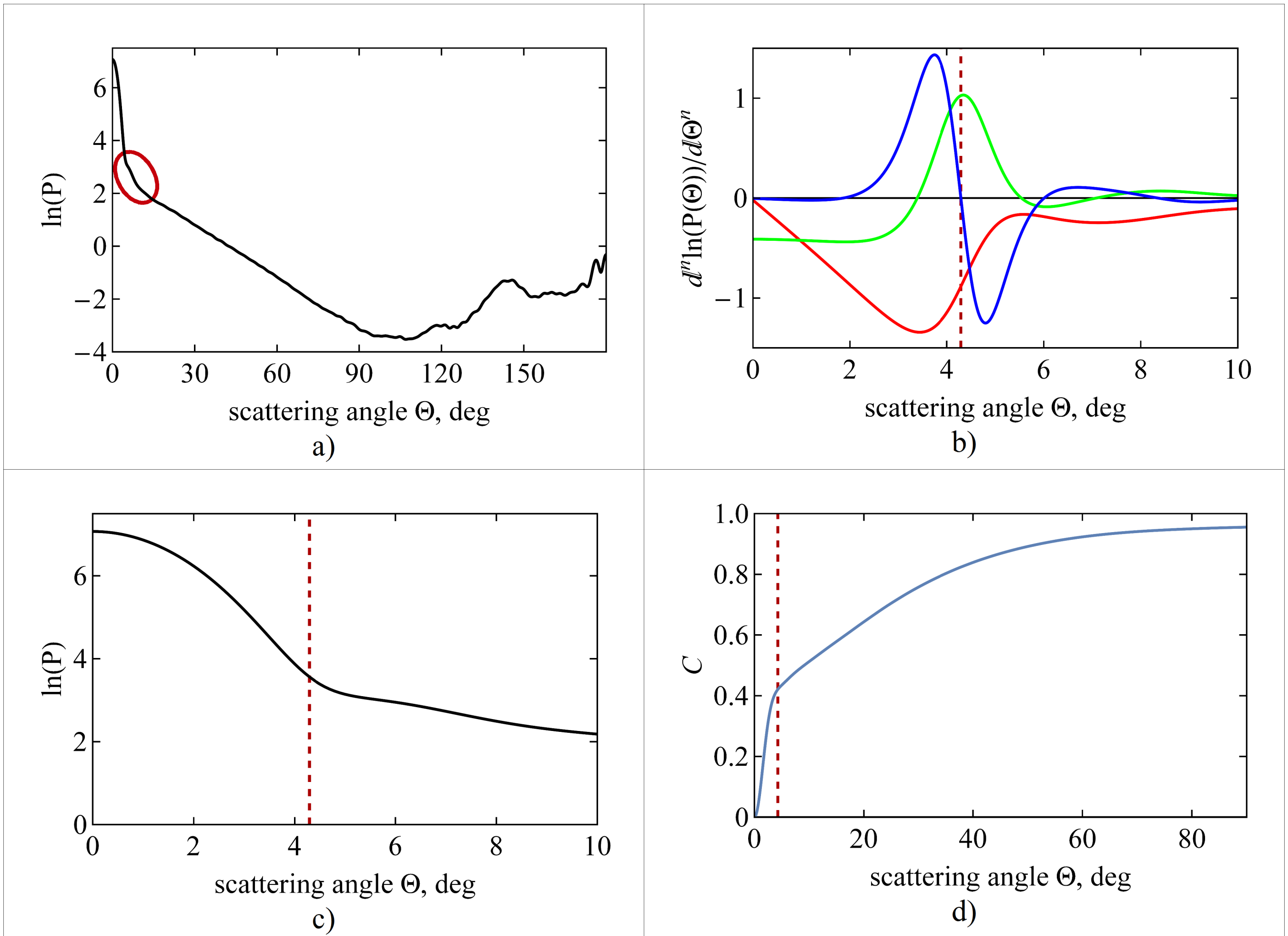


Figure 3. The same as Fig. 2 But for continental polluted cumulus cloud, wavelength 0.55 mm (microphysical properties were taken from from OPAC, CUCP).

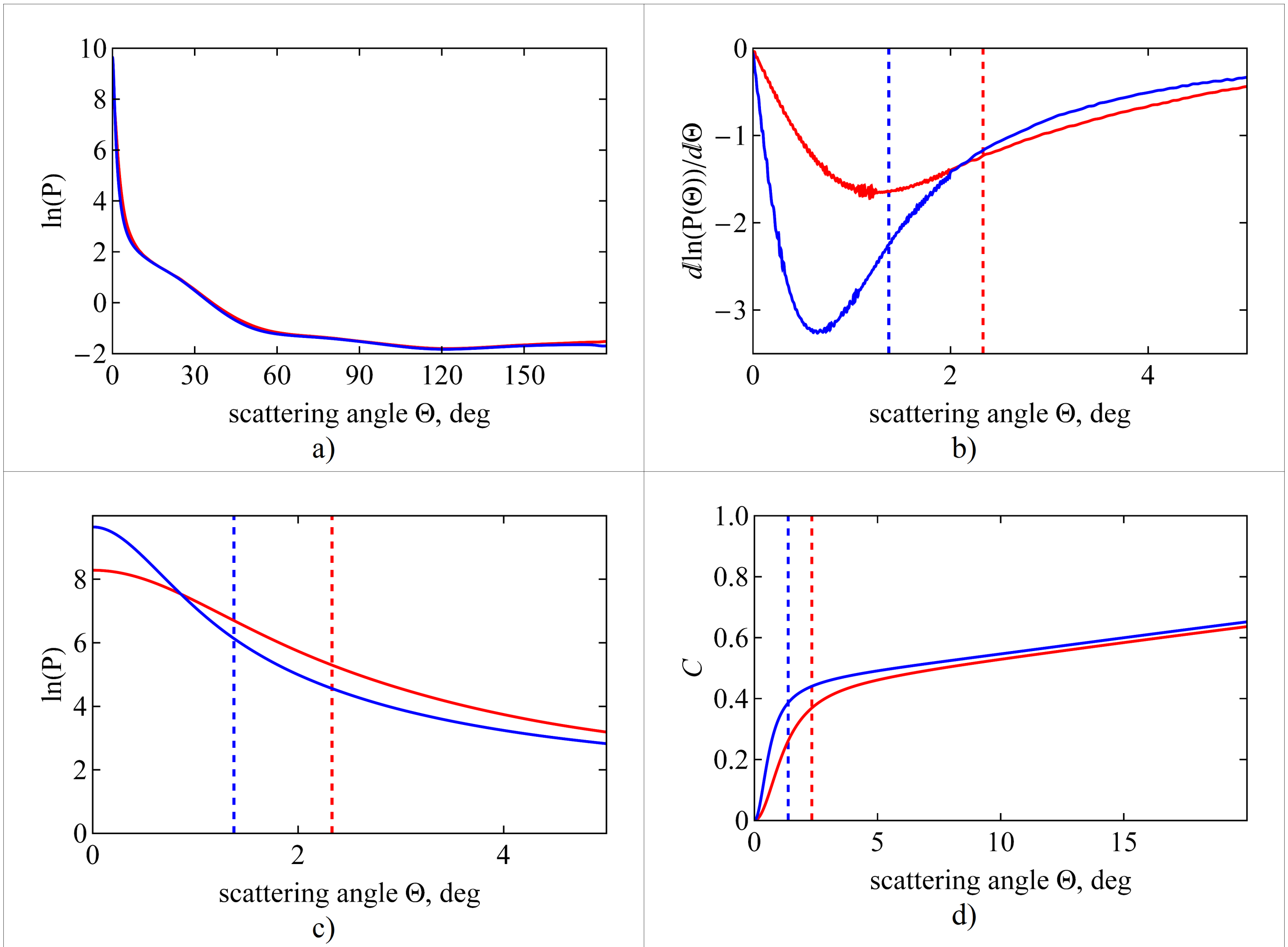


Figure 4. The same as Fig. 2 but for ice cloud, general habit mixture, wavelength 0.55 µm. B shows only 1^{st} derivative. Red - D_{eff} = 10 µm, blue D_{eff} = 20 µm. Vertical lines show the 1^{st} infliction points of the 1^{st} derivative found with polynomial fitting of the 1^{st} derivative. Phase function data from 13

4. CONTINUOUS MODIFICATION OF GEOMETRIC TRUNCATION

There are many ways to remove discontinuity of the replacement phase function $\widetilde{P}(x)$. If a constant value within the forward peak is desirable then the preservation of the asymmetry parameter (second equation in system (10)) needs to be replaced with continuity condition. However, preservation of the asymmetry parameter may be desirable, so it makes sense to replace actual phase function within the forward peak with a simple function allowing both conditions to be met. Let us consider the following

$$P'(\Theta)=\frac{1}{1-f}\begin{cases} a+b\cos\Theta, & 0\le\Theta\le\Theta_t \\ P(\cos\Theta), & \Theta>\Theta_t \end{cases}, \tag{11}$$

Then normality condition gives

$$2f+a(1-x_t)+b(1-x_t^2)/2=\int_{x_t}^{1} P(x)\,dx \quad . \tag{12}$$

Continuity condition gives

$$a+b\,x_t=P(x_t) \quad . \tag{13}$$

So far we have obtained 2 equations while one more equation is needed to derive parameters *a*, *b*, and *f*. Preservation of asymmetry parameter still can be used to close the system as it was used in the original study [3]. It leads to the following equation:

$$2f+a(1-x_t^2)/2+b(1-x_t^3)/3=\int_{x_t}^{1} P(x)\,x\,dx \quad . \tag{14}$$

Asymmetry parameter itself is a functional of the phase function. Therefore, preservation of its value can be replaced with preservation any other functional that may present physical sense in a particular application. Mean scattering angle (MSA) will be considered in this study as yet another example of this kind. MSA is defined as

$$\langle\Theta\rangle=\frac{1}{2}\int_0^{\pi}\Theta\,P(\Theta)\sin\Theta\,d\Theta=\frac{1}{2}\int_{-1}^{1}\arccos\mu\,P(\mu)\,d\mu \quad . \tag{15}$$

It is related to the mean backscattered fraction [15], see eqs. (15a), (15b) there*:

$$\frac{\langle\Theta\rangle}{\pi}=\frac{1}{4\pi}\int_0^1\int_{-1}^0\int_0^{2\pi} P(x_1,x_2,\phi)\,dx_1\,dx_2\,d\phi \quad . \tag{16}$$

Requiring preservation of MSA we obtain instead of (13)

$$a\,f_a(\Theta_t)+b\,f_b(\Theta_t)=\int_{x_t}^{1}\arccos\mu\,P(\mu)\,d\mu \quad , \tag{17}$$

where

$$f_a(\Theta_t)=\sin\Theta_t-\Theta_t\cos\Theta_t; \quad f_b(\Theta_t)=[\sin\Theta_t\cos\Theta_t-\Theta_t\cos 2\Theta_t]/4 \quad . \tag{18}$$

Conditions (14) and (17) present examples of preservation of a certain integral characteristics of the phase function. Along with (12) and (13) they do not provide smooth function behavior at the angle of truncation, i.e. derivative of the phase function is not continuous there. It is interesting to see consequences of dropping preservation of asymmetry parameter or MSA in favor of continuity of the 1st

* relationship (16) was well forgotten so it was independently rediscovered years later [16].

derivative of the phase function. For this reason yet another variant of modification will be considered with condition

$$-b\sin\Theta_t = \frac{dP(\Theta)}{d\Theta}\Big|_{\Theta=\Theta_t} \quad . \tag{19}$$

It is interesting to note that in this case matrix of the system is diagonal that makes the solution easier.

Figure 5 presents implementation of the original and new versions of geometric truncation. It is important to note that continuous derivative GT provides the smallest values of the replacement phase function $\tilde{P}(x)$ within the forward cone. This implies the greatest truncation fraction f_{GT} among other versions. Consequently effective single scattering albedo, see Table 1, will be the smallest for this version of truncation. A crucial effect of this on Monte Carlo modeling will be discussed in section 4.2.

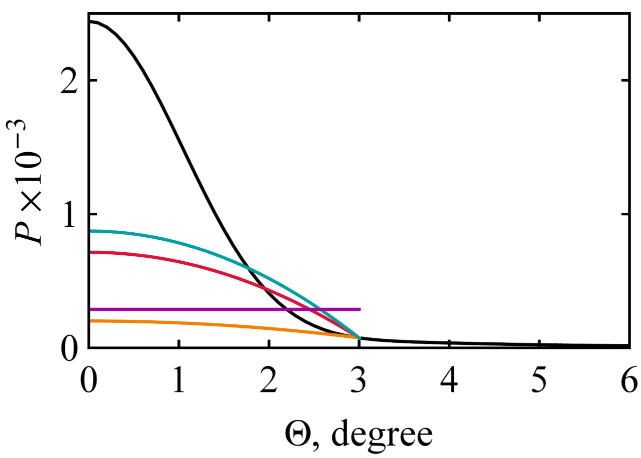


Figure 5. Original phase function $P(x)$ (black) and replacement phase functions $\tilde{P}(x)$ (without δ peak) for clean continental cumulus (OPAC CUCC, D_{eff} = 11.54 mm, λ = 0.55 mm), Θ_t = 3°: purple – original GT, red – continuous GT with preserved asymmetry parameter, cyan – continuous GT with preserved MSA, orange – continuous derivative GT.

5. NUMERICAL TESTS OF GEOMETRIC TRUNCATION WITH RADIANCE COMPUTATIONS

The original study [3] proposed geometric truncation for Monte Carlo modeling of radiance. It is very suitable in the cases when phase function is given as a set of angle – value pairs. However, as a general method of handling highly anisotropic forward scattering it also can be used with deterministic methods such as discrete ordinates and spherical harmonics. In this study both numerical tests were performed with DISORT [17], one of the most popular discrete ordinate RT solver, and the I3RC Monte Carlo code [18, 19].

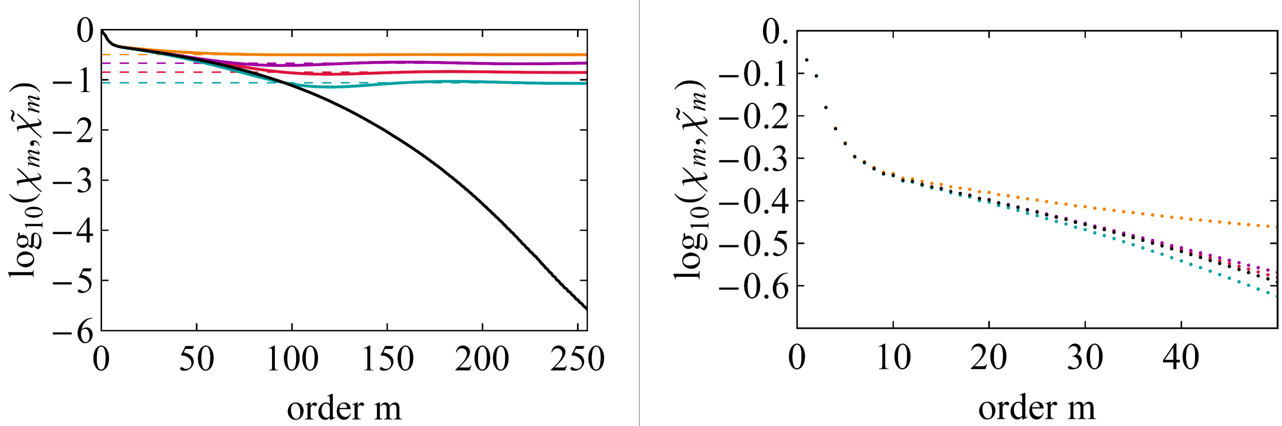
5.1. Test with DISORT

5.1.1. Behavior of the phase function moments.

Since DISORT heavily relies on the moment representation of the phase function, consequences of the original and new versions of geometric truncation on the moment decay with order need to be considered. Using definition of moments(4) and relationship (6) between the replacement phase function $\tilde{P}(x)$ and truncated phase function $P'(x)$ we obtain for the moments:

$$\tilde{\chi}_m = f + (1-f)\chi'_m \tag{20}$$

Figures 5 and 6 show decay of the moments $\widetilde{\chi}_m$ and χ'_m, respectively, in comparison with the original moments. It is clear from eq. (20) that moments $\widetilde{\chi}_m$ reach value of the truncation fraction f as an asymptote. Provided that the same angle of truncation was used, truncation fraction significantly depends on the version of geometric truncation. In the example considered in Figure 5 the angle of truncation $\Theta_t = 3°$ yields the following values of the fraction of truncation: original GT - $f = 0.215$, continuous GT with preserved asymmetry parameter - $f = 0.142$, continuous GT with preserved MSA - $f = 0.0875$, continuous derivative GT - $f = 0.318$. Figure 5,b shows that continuous derivative approach reveals the greatest deviation from the original moments while preservation of the asymmetry parameter the best one. However, up to order m = 10 moments $\widetilde{\chi}_m$ are very well numerically preserved regardless of the method of truncation. the moments of the truncated phase function are sign alternating. Figure 6 shows absolute values of them revealing the fastest decay occurs with continuous derivative approach and slowest with original approach.



a) b)

Figure 6. Legendre moments of the replacement phase function $\widetilde{P}(x)$ along with original moments (black). Original GT – purple, GT with preserved asymmetry parameter – red, GT with preserved MSA – cyan, continuous derivative GT – orange. a) shows truncation fraction in the same colors, see eq. (20); b) shows only orders up to 50, moments become visually distinguishable around order of 10.

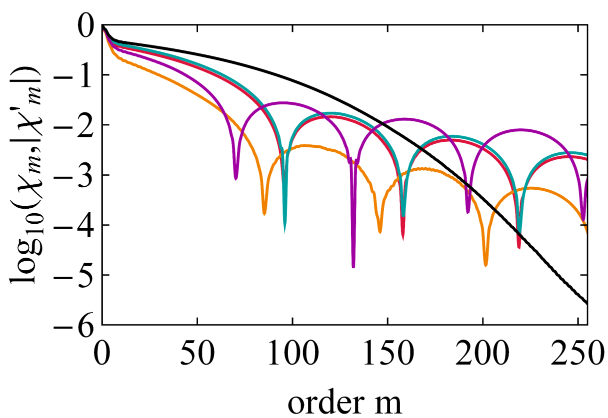


Figure 7. Legendre moments of the replacement phase function $\widetilde{P}(x)$ along with original moments (black). Original GT – purple, GT with preserved asymmetry parameter – red, GT with preserved MSA – cyan, continuous derivative GT – orange. a) shows truncation fraction in the same colors, see eq. (20); b) shows only orders up to 50, moments become visually distinguishable around order of 10.

5.1.2. radiance reflected by and transmitted through the cloud

Radiance calculations were performed for clean continental cumulus cloud (see OPAC [] for microphysical properties) at the wavelength of 0.55 μm. Single scattering albedo was 0.9999997. Two optical thicknesses ($\tau = 10$ and $\tau = 100$) and two Sun zenith angles ($\theta_0 = 36.9°$, $\mu_0 = 0.8$ and $\theta_0 = 49.5°$, $\mu_0 = 0.65$) were considered. Version 2 of DISORT was used as an RT solver. That version uses Delta-M truncation without an option to turn it off. For this reason performance of Delta-M was also tested as it affects test runs of with geometric truncation. Reference run used 200 streams (yielding truncation fraction $f_{DM} = \chi_{200} = 3.379 \times 10^{-4}$) with original moments up to the order of 255. Reference runs used 100 streams.

Table 1. Parameters of GT affecting RT calculations with DISORT.

Truncation method	truncation fraction, f_{GT}	Effective single scattering albedo, ω', see eq. (8)	Delta-M truncation fraction, $f_{DM} = \chi'_{100}$
Delta-M 100	0	0.99999969	7.714×10^{-2}
Original GT	0.2150	0.99999962	-2.674×10^{-2}
GT with preserved asymmetry parameter	0.1419	0.99999965	-5.218×10^{-3}
GT with preserved MSA	8.755×10^{-2}	0.99999967	-5.634×10^{-3}
Continuous derivative GT	0.3180	0.99999956	-3.413×10^{-3}

Table 1 presents truncation fractions due to geometric truncation and due to delta-M approach used in DISORT. The latter is always much smaller than the former meaning that the major portion of power concentrated in the forward peak is taken care of by geometric truncation while unavoidable influence of delta-M is significantly smaller. Effective single scattering albedo also presented in Table 1 is that after geometric truncation. It does not include further correction applied within DISORT. The code was supplied with parameters coming into RTE (7): ω', χ'_l , and effective optical thickness after geometric truncation.

It is known fact (see, e.g. [Rozanov and Lyapustin 2010], Figure 5 there) that delta-M truncation has to be supplemented with single scattering correction of radiance otherwise relative difference of radiance obtained with that method and reference solution oscillates as a function of solar and view zenith angles with magnitude of tens of per cent. Such correction is indeed implemented in DISORT. A user of the code should should supply as many as possible Legendre moments of the phase function for accurate reconstruction of single scattering. The algorithm employs first m moments for multiple scattering RT then corrects for single scattering using all supplied moments. In the context of this study, supplying all available moments of truncated phase function χ'_l is not enough – this enables correction to P'. Substantial change of the algorithm is needed to implement yet another correction for single scattering with the original phase function.

Such change of the algorithm is beyond of the scope of this study. But that correction is not needed for comparison the original GT with new versions proposed here. It can be done without single scattering correction, i.e. without the use of higher order moments of phase function. Comparison of relative difference of radiances computed with different truncation techniques without correction allows to estimate performance of the techniques themselves while using correction as described above gives delta-M an advantage. So, comparison with correction only allows relatively impartial comparison of GT techniques between each other. Figures 8 thorough 13 show reference radiance along with relative differences between the reference solution and those obtained with truncation techniques. It is important to notice the drastic difference in the performance of delta-M technique with and without correction. Among GT techniques continuous derivative approach shows the best performance for reflected radiance and better than the original GT for transmitted radiance with and without correction. It also shows the best performance without correction since delta-M approach generates very oscillating solution. Performance of the continuous derivative approach with correction to the truncated phase function *P'* is comparable with delta-M approach with correction to the original phase function *P*. Therefore, continuous derivative approach is the best option in the situation when Legendre moments of higher orders cannot be easily estimated, e.g. if the phase function is given by a table of angle and values with insufficient resolution of the forward peak.

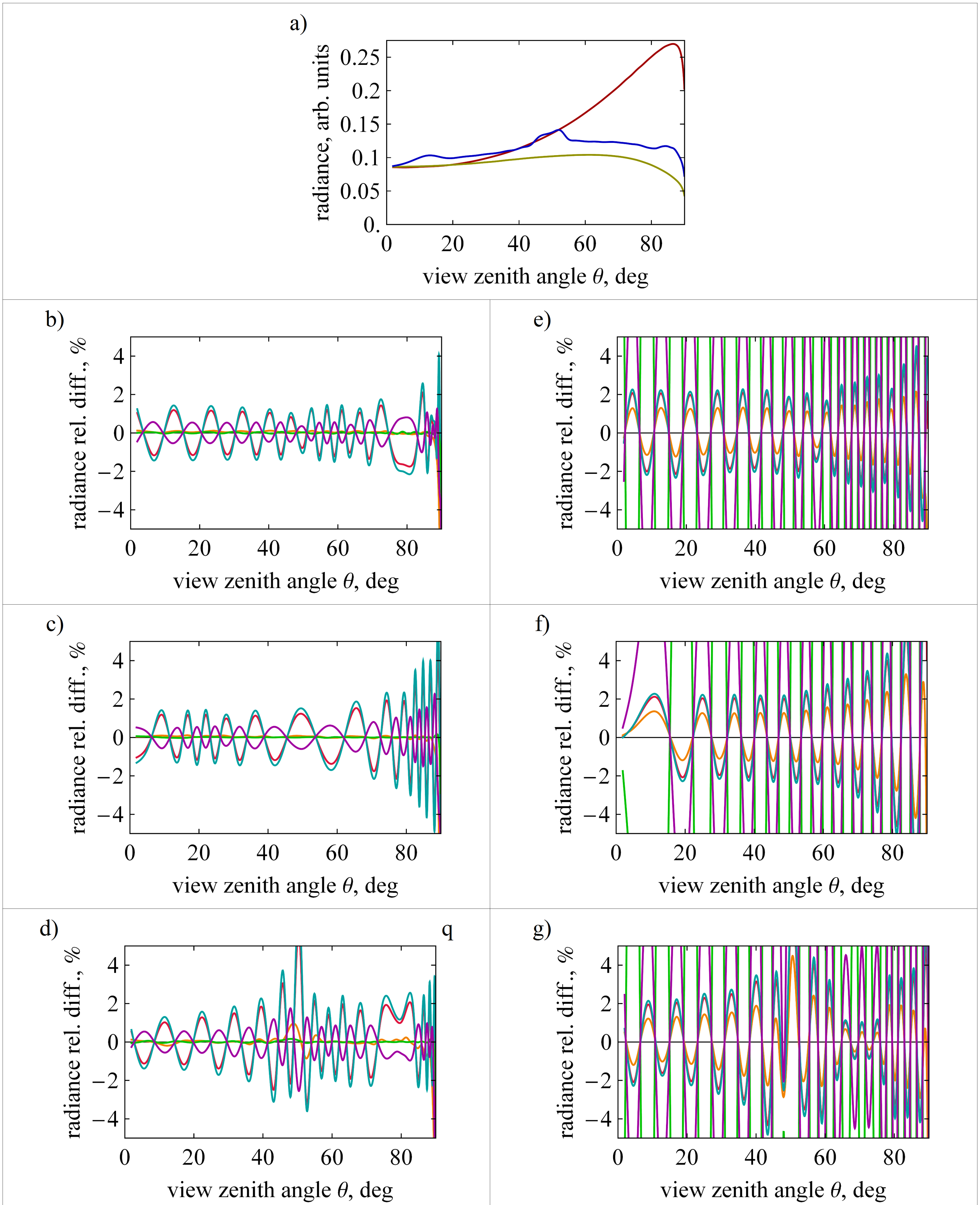


Figure 8. Reflected radiance in the case of OPAC CUCC cloud as a function of view zenith angle with $\tau_t = 10$ and $\mu_0 = 0.65$. a) reference radiance for different relative azimuth: dark red – $\phi = 0°$, olive – $\phi = 90°$, blue – $\phi = 180°$; b) through g) relative difference of radiance between various types of phase

function truncation and reference : green – delta-M, purple – original GT, red – GT with preserved asymmetry parameter, cyan – GT with preserved MSA, orange – continuous derivative GT; b), c), d) – with single scattering correction (see text), e), f) g) – without correction; b) and e) – $\phi = 0°$, c) and f) – $\phi = 90°$, d) and g) – $\phi = 180°$.

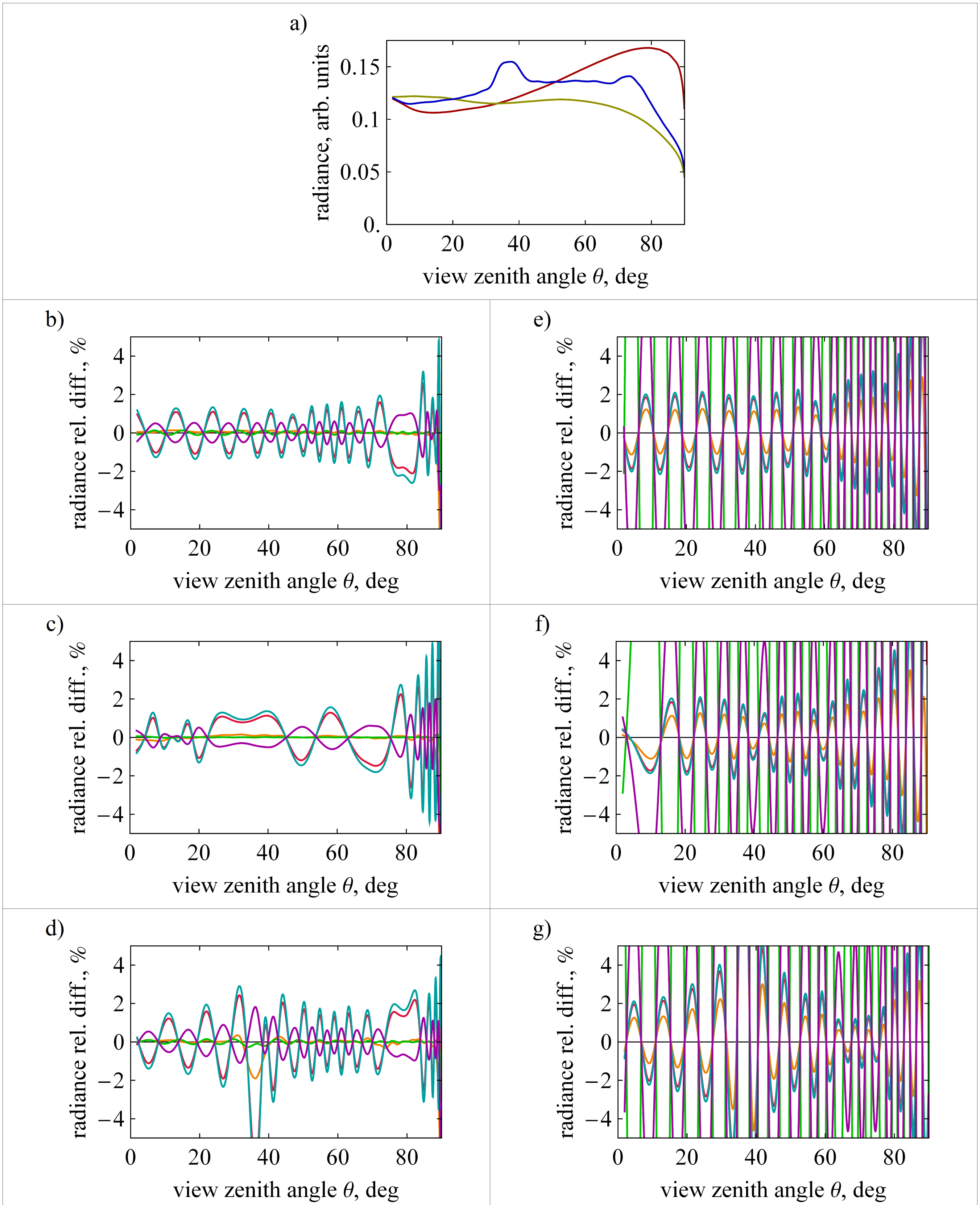


Figure 9. The same as 8 but for $\mu_0 = 0.80$.

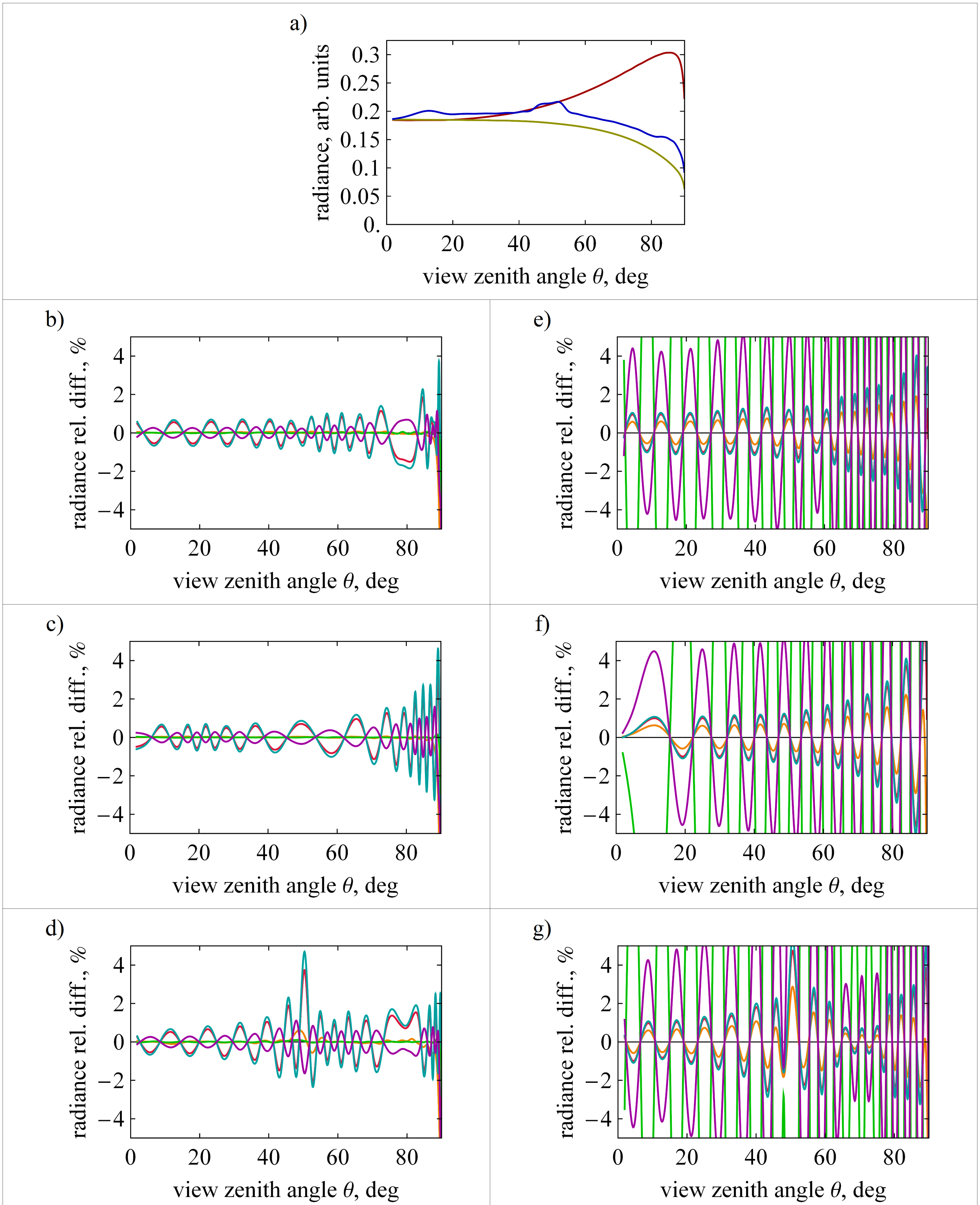


Figure 10. The same as Fig. 8 but for $\tau_t = 100$.

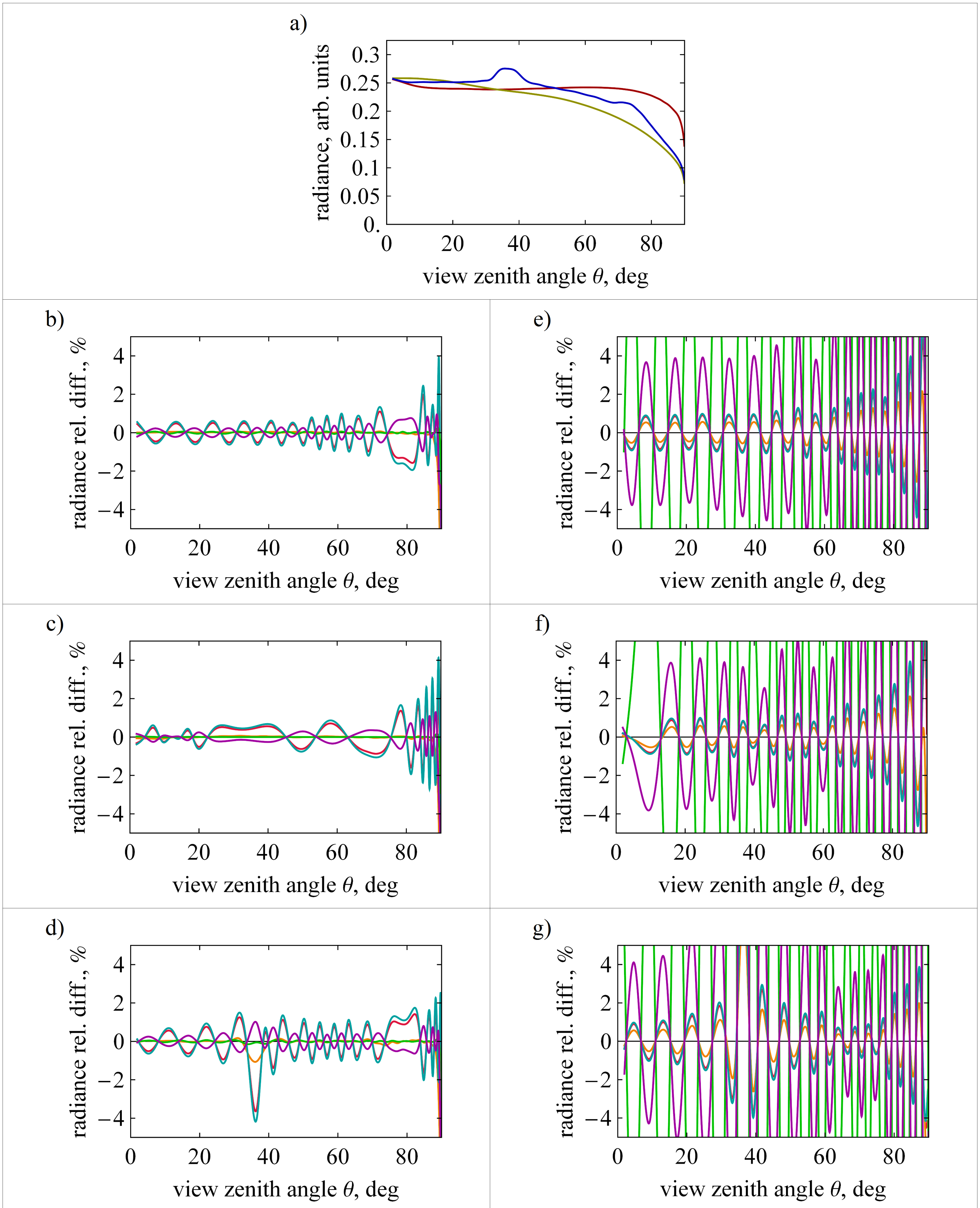


Figure 11. The same as Fig. 10 but for $\mu_0 = 0.80$.

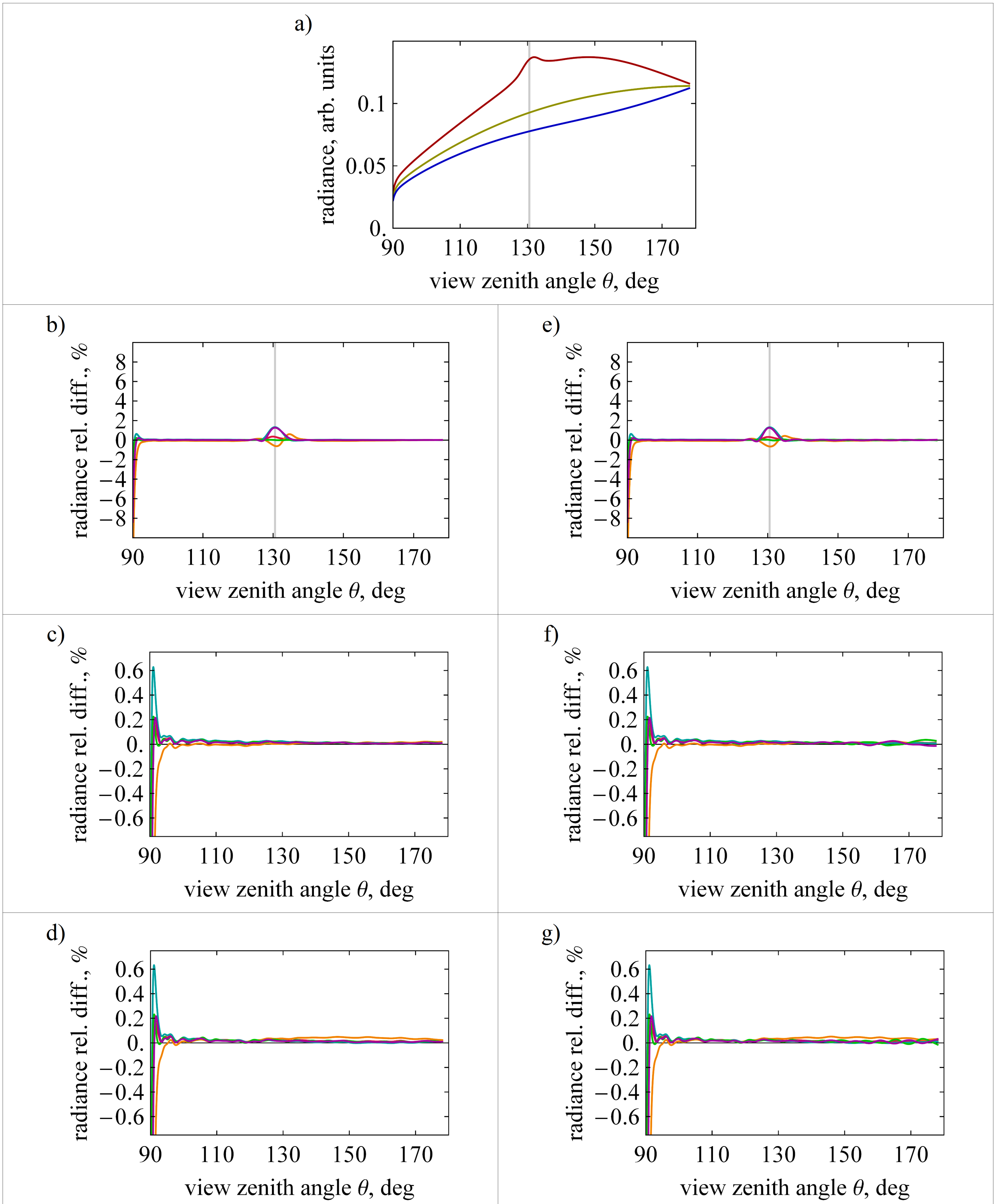


Figure 12. The same as Fig. 8 but for reflected radiance.

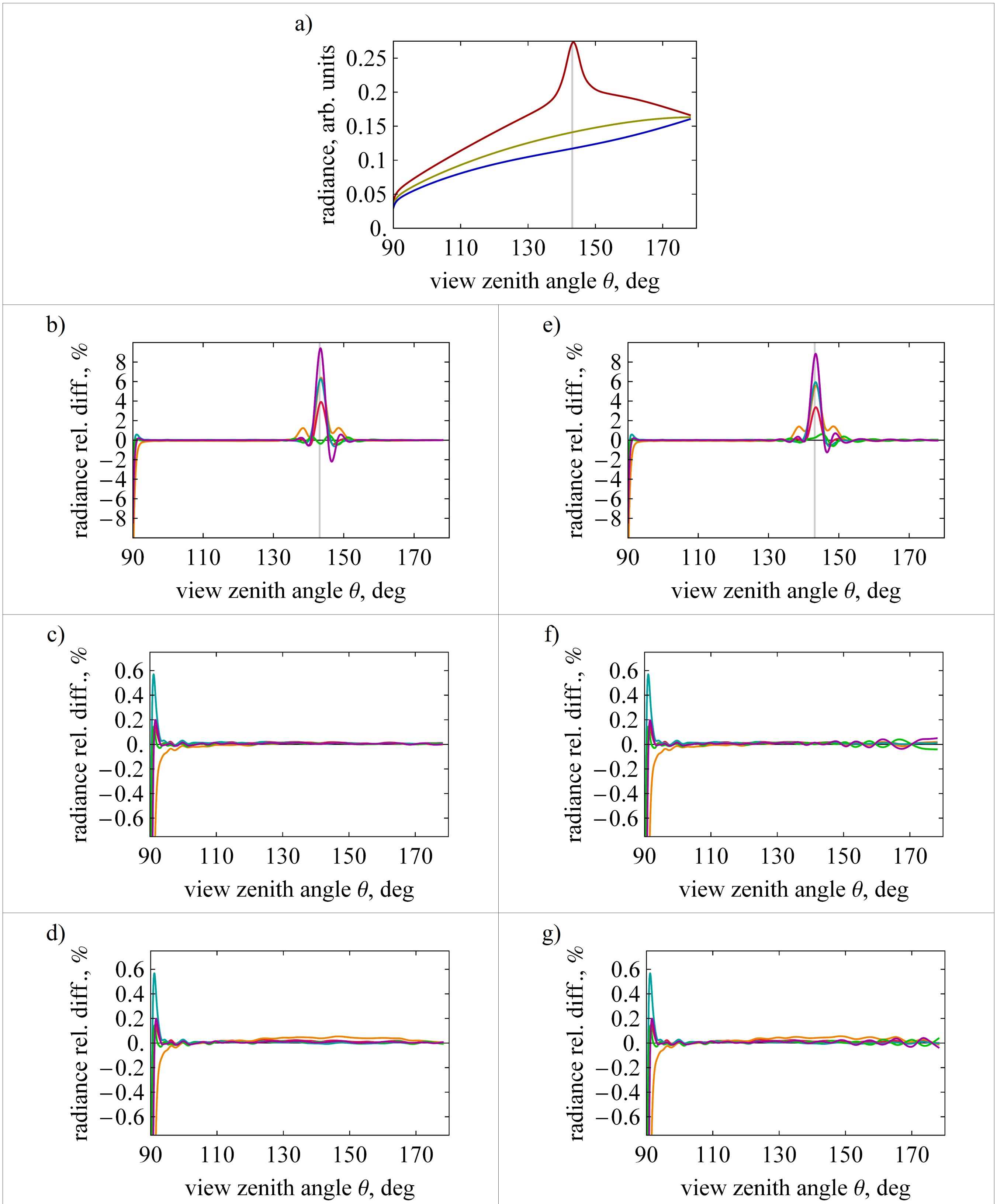


Figure 13. The same as Fig. 12 but for $\mu_0 = 0.80$.

5.2. Test with Monte Carlo modeling

Monte Carlo modeling was performed with I3RC Monte Carlo code with the following set up: 1D model, the same medium as in previous section, and 1.6×10^{7} photon stories were played in batches by 10^{4} photons. Phase function in all cases was represented by Legendre moments up to order of 255. Table 2 gives values of the last moments along with the difference $1 - \omega(')$ for all variants of GT. Figures 14 and 15 present reflected radiance along with relative difference between the reference solutions and those obtained with various versions of GT. While the graphs of the relative difference show quickly changing functions of the view zenith angle, the absolute value of the relative error is within 4% for vast range of angles. It is also easy to see that the solution with continuous derivative is the least variable. It is also clear that there is certain systematic positive bias in all graph. The reason for this bias is in specific design of the RT code. It requires all input variable to be in machine single precision. In the case of cloud with single scattering albedo very close to unity, machine representation of the values listed in Table 1 may shifted them to the nearest available machine number. Such a shift can cause bias in albedo of ~ 1% between original phase function and SSA and truncated ones. In this situation continuous derivative GT appears to be the best option as providing the least variable deviation from the solution with original parameters.

Monte Carlo simulations of radiative transfer are known to be very time consuming. Therefore, it is important to check the impact of the truncation techniques on computer time. Table 3 presents computer times per one batch of 10^{4} photons for different optical thicknesses of the cloud and Sun zenith angles. It is not surprising that the greater SSA supplement is the shorter computer time is. Greater SSA supplement means greater effective absorption and shorter photon lifetime leading to faster simulation. For this reason continuous derivative GT is the best option providing significant advantage in computer time comparing to the original GT (about 10% depending on geometry) and comparing with modeling with the original optical parameters (>30%).

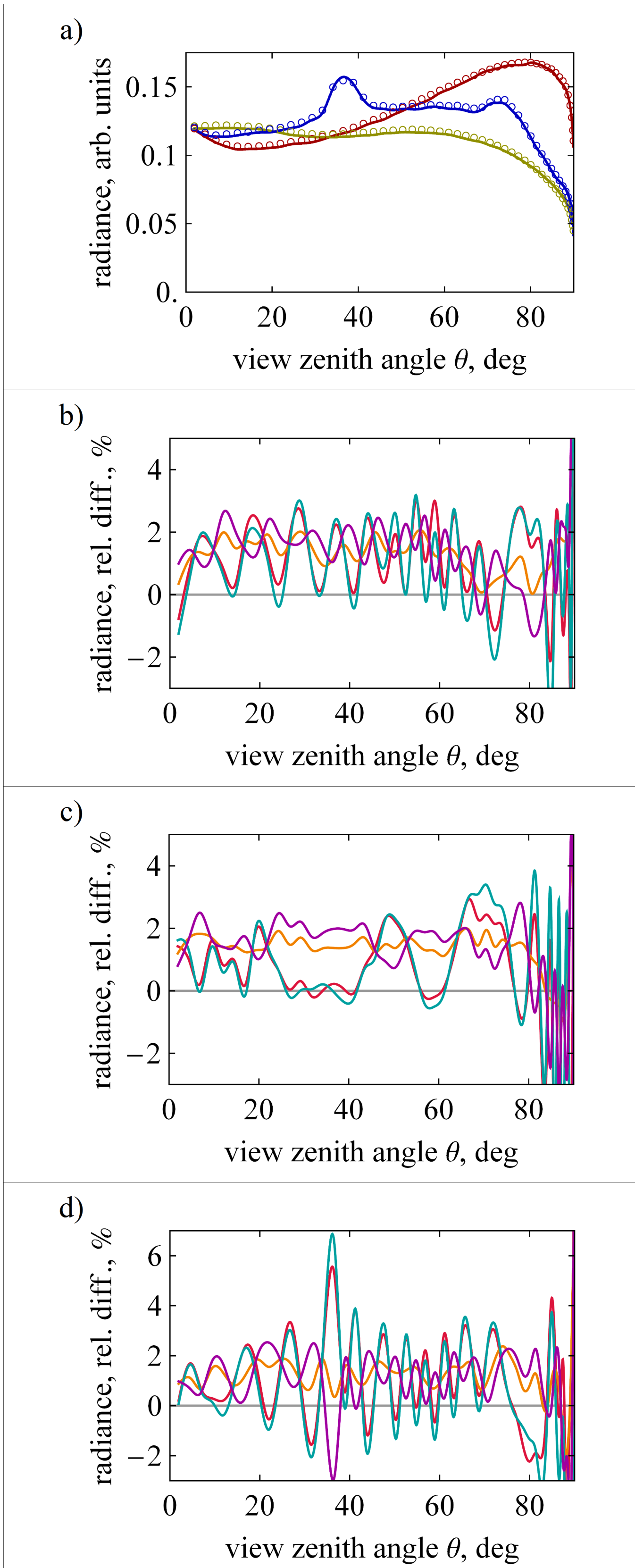


Figure 14. reflected radiance (a) and relative difference (b - d) in the case of clean cumulus continental cloud for $\tau = 10$, $\mu_0 = 0.80$. a) solid line

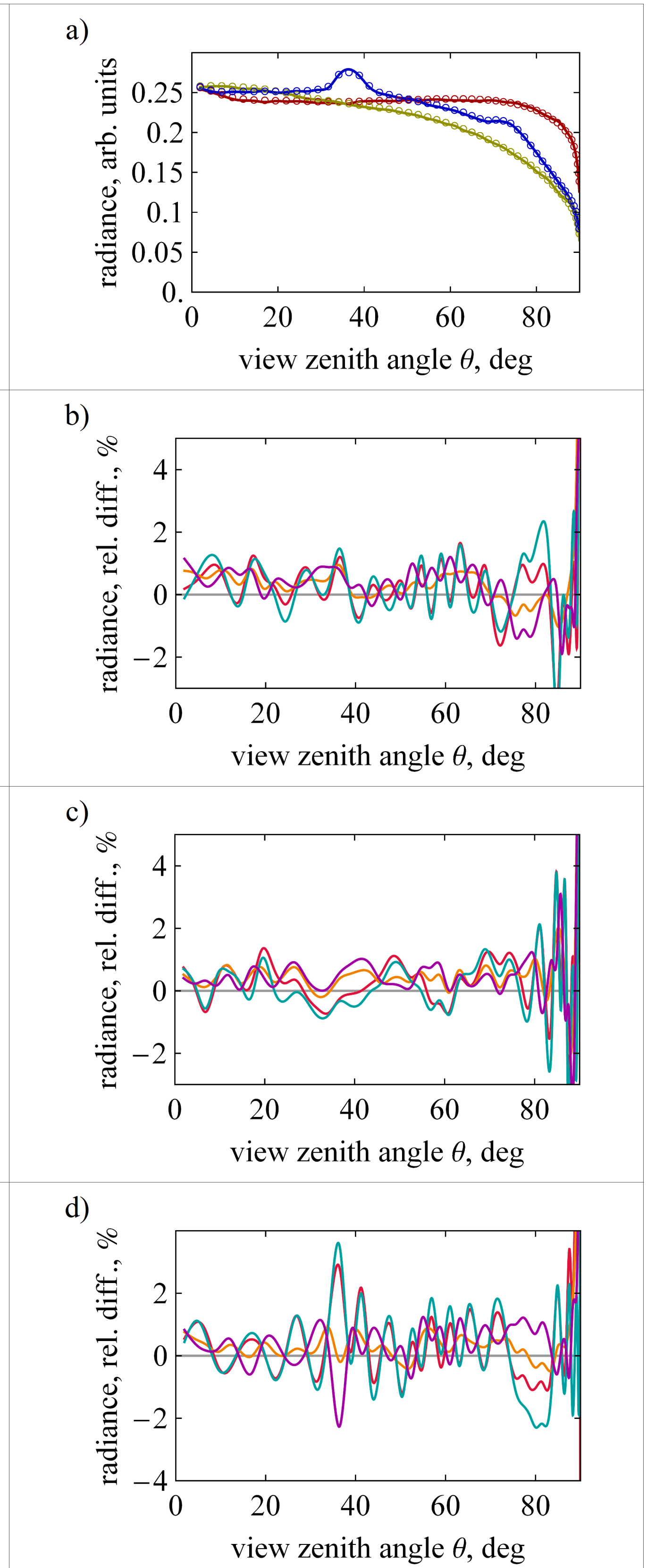


Figure 15. The same as Fig. 14 but for $\tau = 100$

Monte Carlo, open circles - DISORT	

CONCLUSION

The study considered variations of the original geometric truncation of the scattering phase function. It addressed problems of objective selection of the angle of truncation and discontinuity of the replacement phase function at the angle of truncation. A robust criterion of selection of the forward cone limit was suggested. The criterion is based on numerical evaluation of the phase function derivative and search for its first infliction point. The search for that infliction point can be done with either numerically differentiation of the phase function or with fitting the first derivative with a polynomial of 4^{th} or 5^{th} degree and finding first positive infliction point of that polynomial.

Three variations of the replacement phase function were suggested. Unlike the original approach, all new versions provide continuous replacement of the phase function. The normality condition for the replacement phase function provides one equation relating parameters of such replacement. Imposed continuity condition ultimately requires either to use more complex functional form of the phase function within the forward cone if some other conditions (such as preservation of asymmetry parameter) need to be met. A simple weighted sum of a constant and cosine of scattering angle was considered. This form allows to preserve the asymmetry parameter. However, it also provide flexibility in the choice of another condition instead of preservation of asymmetry parameter. Those other conditions can be requirements to preserve a certain functional of the original phase function or a certain property of the phase function. Among the conditions of the former type, preservation of the mean scattering angle was considered. The latter type of conditions was presented by the requirement for the replacement phase function to have continuous derivative at the angle of truncation.

It was found that the method preserving mean scattering angle returns the smallest fraction of truncation while derivative continuity approach returns the greatest one given the same angle of truncation. This makes continuous derivative geometric truncation the fastest option for Monte Carlo modeling. It was also shown that this version provides the least variable relative difference between simulations with and without truncation.

Calculations of reflected and transmitted radiance showed that continuous derivative GT provides the best match with the reference solution among considered geometric truncation methods regardless of the use of single scattering correction. It also provides accuracy comparable with delta-M under the unfavorable use of that correction. Overall, this version of GT fixes deficiencies of the previous versions of the truncation approaches based on preservation of the phase function outside the forward cone: simple functional form, easy implementation, and smooth continuity at the angle of truncation.

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Table 2. single scattering albedo supplement and the highest order moment of the phase function for original medium and different truncation methods

absorption	Original phase function	Original GT	GT, asymmetry preserved	GT, MSA preserved	GT, continuous derivative
$1 - \omega(')$, 10^{-7}	3.1	3.8	3.5	3.3	4.4
$\chi_{255}^{(')}$	2.626×10^{-6}	7.994×10-4	-2.040×10-3	-2.494×10^{-3}	-7.126×10^{-5}

Table 3. Average times for 10^4 photons simulations with different geometries and truncation methods; computations performed on the Intel Xeon E5620 2.4GHz.

Geometry	Original phase function time, s	Original GT		GT, asymmetry preserved		GT, MSA preserved		GT, continuous derivative	
		time, s	% of original	time, s	% of original	time, s	% of original	time, s	% of original
$\mu_0 = 0.65$, $\tau = 10$	58.75	47.4	80.7	50.27	85.6	55.82	95.0	41.66	70.9
$\mu_0 = 0.65$, $\tau = 100$	560.3	443.9	79.2	485.8	86.7	517.6	92.4	387.3	69.1
$\mu_0 = 0.80$, $\tau = 10$	57.38	44.93	78.3	48.97	85.3	53.22	92.8	39.02	68.0
$\mu_0 = 0.80$, $\tau = 100$	617.5	456.5	73.9	531.2	86.0	586.4	95.0	419.0	67.9