Turbulent Axial Odometer Model

Michael E. Olsen[1] Randolph P. Lillard[2]

[1]NASA Ames Research Center, Moffett Field, CA 94035

[2]NASA Johnson Space Center, Houston, TX 77058

AIAA Aviation Forum

Introduction

- High Reynolds number experiments expanded understanding
- Flat plate flowfields
  1. c_f vs. Re_θ .... (6% revision)
  2. $[\kappa, B^+]$ .... $([.41, 5] \mapsto [.385, 4.1])$
  3. R^+_{ij} .... (R^+_{11} increases with Re_θ)
- First two items easy, third — not so much
- Reynolds-stress models should benefit from better match of <u>all</u> R_{ij}
- T_{ijk} depend directly on R_{ij} fields (and their derivatives)
- Attached flow: R^+_{11} and R^+_{33} generally *don't matter*
- Separated flow: $\partial_k T_{ijk}$ and full R_{ij} tensor *do matter*

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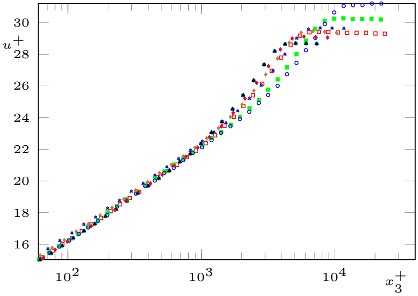
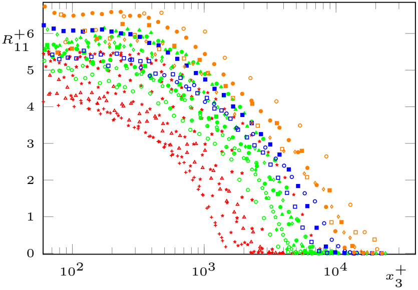


$u_1^+(x_3^+)$

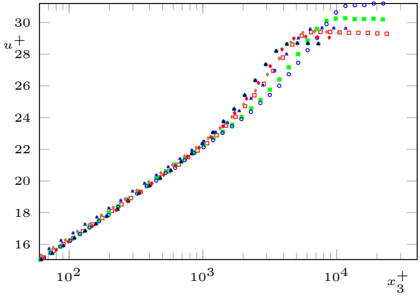


$R_{11}^+(x_3^+, Re_\theta)$

1. Recent Experiments (1990's on) : R_{ij} are not pure functions of u_τ
   R_{11}^+ grows with Re_θ, arguably R_{13}^+ and R_{33}^+ *are* pure F(u_τ)
2. Challenge: reproduce this behavior in a RANS model

Motivation: Why do we care about R_{11}^+ , R_{33}^+ ?

1. Evidence for this behavior in R_{ij} for canonical separated flows
2. Separated flows: all R_{ij} important (no longer a TSL — Mohr's circle)
3. Modeling $\partial_k(T_{ijk})$: accurate R_{ij} predictions necessary



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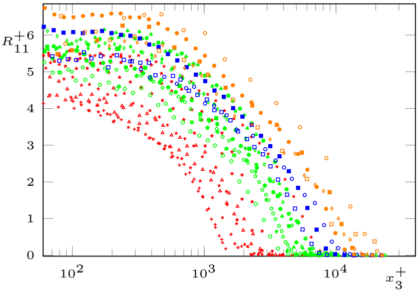

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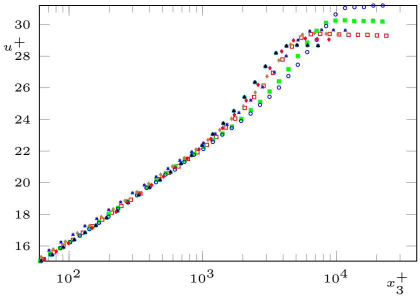

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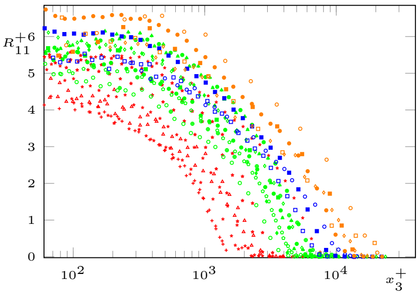

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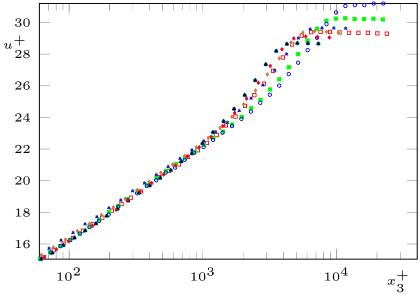

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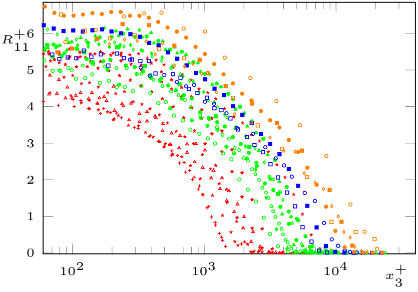

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Motivation: An Outer Scale in a Field Equation

- If wishes were horses

  Cebeci-Smith would still be among us (Re_θ unavailable)
- Physical phenomenon responsible — very long structures seen in high Re
- Concept: How long has this streamline been in turbulent flow?

An Equation for streamline length l_p (An odometer):

$$\rho\partial_t(l_p) + \rho u_i \partial_i(l_p) = \rho(u_i u_i)^{\frac{1}{2}}; (l_p|_0 = 0)$$

Turn this length into a Reynolds number, $R_o = k^{\frac{1}{2}} l_p/\nu$:

$$\partial_t(\rho R_o) + \partial_i(\rho u_i R_o) = \rho\sqrt{u_i u_i}\frac{\sqrt{k}}{\nu}$$

Add boundary layer sync and laminar reset:

$$\partial_t(\rho R_o) + \partial_i(\rho u_i R_o) = \rho\frac{\sqrt{u_i u_i}\sqrt{k}}{\nu} + \partial_i\left((\mu + \sigma_t\mu_t)\partial_i R_o\right) - \frac{\rho\omega R_o}{(1 + R_T)}$$

(Simple BC! Inflow: $R_o|_0 = 0$, Wall: $\partial R_o = 0$)

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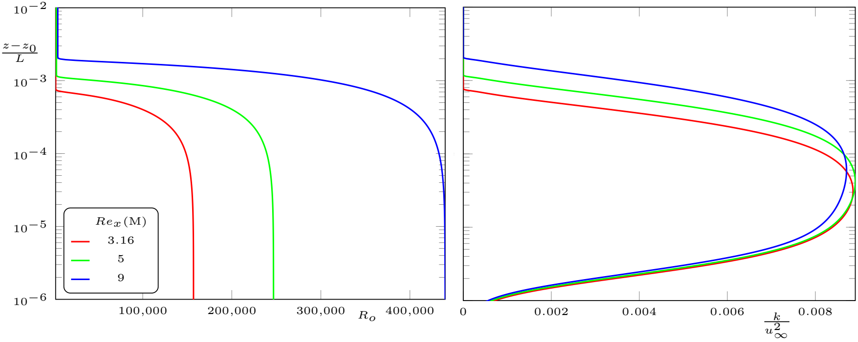
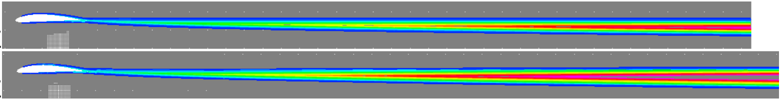


R_o k

Simplest flowfield: (and it works as designed)

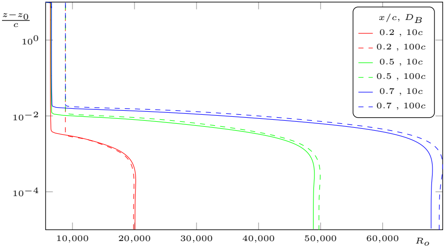
- Extremely small away from the turbulent flow
- $\partial_3 R_o \approx 0$ in log layer (linearly proportional to x_1)
- So far - So good, but for a vehicle?

10C
100C

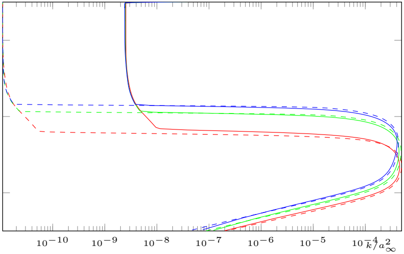


A more representative case, and it works as designed

- Two different solutions with Farfield Boundary 10C or 100C away
- Boundary layer solutions insensitive to farfield boundary distance—



R_o



k

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$$\begin{aligned}
\partial_t\left(R_{ij}\right)+\partial_k\left(u_k R_{ij}\right)=&-R_{jk}\partial_k\overline{U_i}-R_{ik}\partial_k\overline{U_j}-\partial_k T_{ijk}+\nu\partial_k\partial_k R_{ij}\\
&+\Pi_{ij}-2\nu\overline{\partial_k(u_i')\partial_k(u_j')}\\
\partial_t\left(T_{ijk}\right)+\partial_l\left(u_l T_{ijk}\right)=&-T_{ijl}\partial_l\overline{U_k}-T_{jkl}\partial_l\overline{U_i}-T_{kil}\partial_l\overline{U_j}\\
&+R_{ij}\partial_l R_{kl}+R_{jk}\partial_l R_{il}+R_{ki}\partial_l R_{jl}\\
&+\nu\partial_l\partial_l T_{ijk}\\
&+\Pi_{ijk}-\partial_l(Q_{ijkl})-\varepsilon_{ijk}
\end{aligned}$$

$$\begin{aligned}
\Pi_{ij}=&\frac{1}{\rho}\left[\overline{u_j'\partial_i(p')}+\overline{u_i'\partial_j(p')}\right]\\
\Pi_{ijk}=&\frac{1}{\rho}\left[\overline{u_i'u_j'\partial_k(p')}+\overline{u_j'u_k'\partial_i(p')}+\overline{u_k'u_i'\partial_j(p')}\right]\\
Q_{ijkl}=&\overline{u_i'u_j'u_k'u_l'}\\
\varepsilon_{ijk}=&2\nu\left(\overline{u_i'\partial_l(u_j')\partial_l(u_k')}+\overline{u_j'\partial_l(u_k')\partial_l(u_i')}+\overline{u_k'\partial_l(u_i')\partial_l(u_j')}\right)
\end{aligned}$$

$$\partial_t (\rho k) + \partial_l (\rho u_l k) = \rho \left[R_{ij} S_{ij} - \beta^* k \omega \right] + \partial_l \left((\mu + \sigma_k \mu_T) \partial_l k \right) - A_4 \partial_l (\rho$$

$$\partial_t (\rho \omega) + \partial_l (\rho u_l \omega) = \alpha \rho S^2 - \beta \rho \omega^2 + \partial_l \left((\mu + \sigma_\omega \mu_T) \partial_l \omega \right)$$

$$\partial_t (\rho R_{ij}) + \partial_l (\rho u_l R_{ij}) = A_0 \rho \omega \left(R_{ij}^{(eq)} - R_{ij} \right)$$

$$\partial_t (\rho T_{ijk}) + \partial_l (\rho u_l T_{ijk}) = A_0 \rho \omega \left(T_{ijk}^{(eq)} - T_{ijk} \right)$$

$$\partial_t (\rho R_o) + \partial_i (\rho u_i R_o) = \rho \frac{\sqrt{u_i u_i} \sqrt{k}}{\nu} + \partial_i \left((\mu + \sigma_t \mu_t) \partial_i R_o \right) - \frac{\rho \omega R_o}{(1 + R_T)}$$

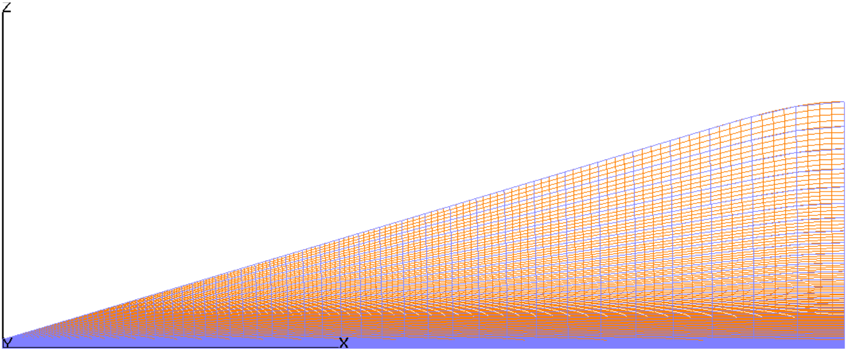
where:

$$R_{ij}{}^{(eq)} = \frac{2}{3} k \delta_{ij} - \frac{A_1}{\omega} (\mathcal{P}_{ij} - \frac{1}{3} \bar{\mathcal{P}} \delta_{ij}) + \ldots$$

$$\psi = \max(\Psi_L, \Psi_R \ln(1 + R_o / R_m)) \qquad A_6 = \frac{2}{3} \frac{1 + \psi^2}{R_{NN} + \psi} \qquad A_1 = \psi / A_6$$

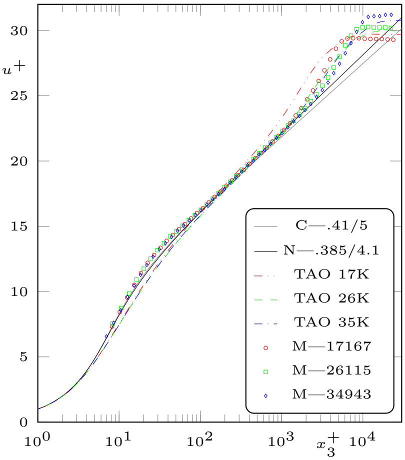
$$\mathcal{K} = \frac{1 + \psi^2}{A_6} \qquad \beta^* = \psi / A_6 \qquad \beta = \beta^* / n_D \qquad \sigma_\omega = \frac{\beta / A_6^2 - \alpha}{\mathcal{K} A_8^2}$$

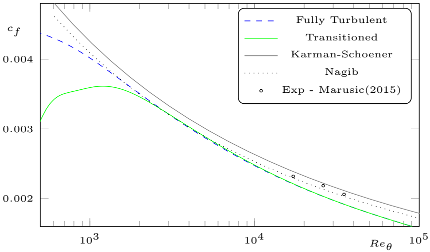
Flat Plate Solution



- $M_\infty = 0.2$
- $Re_L = 100^6$
- "Flight" freestream turbulence
- 513×513 grid (Grid convergence checked)

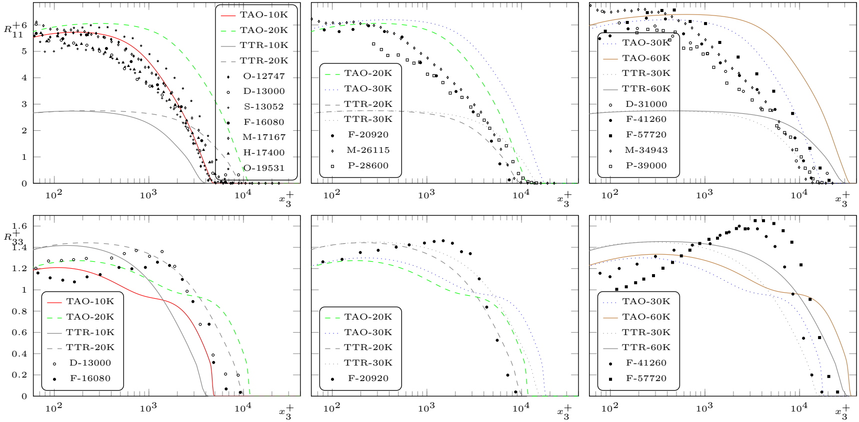
Flat Plate Velocity and Skin Friction





- Retained law of the wall axial velocity distribution
- Able to get good $c_f(Re_\theta)$ predictions
- –Did no harm–

Flat Plate Reynolds-stresses



- Much improved R^+_{11} predictions
- Better R^+_{33} predictions
- Overall much improved R_{ij} prediction behavior (Mohr's circle)

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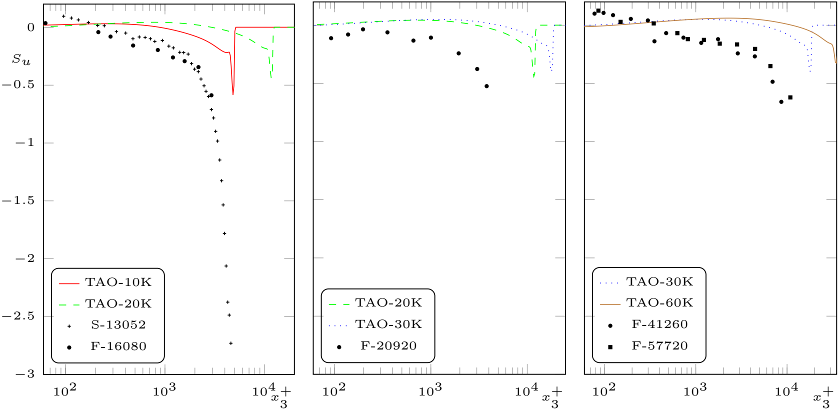
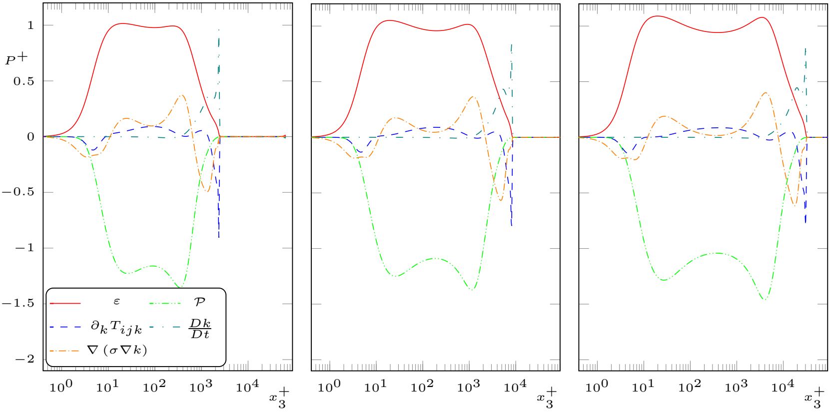


- $Su = T_{111}/R_{11}^{1.5}$
- Not the most important transport term, but checkable
- Overall prediction encouraging (low in log region, high at edge)
- B.L. Edge position not identical in CFD/experiment

Given $\partial_k(T_{ijk})$ plausible, what does the tke balance look like?



$Re_x = 2 \times 10^6$ $Re_x = 10 \times 10^6$ $Re_x = 50 \times 10^6$

- Transport Small, except at BL edge
- $\mathcal{P} = \varepsilon$ dominant balance in log layer

Conclusions/Future Directions

Conclusions

- R_o works as a turbulent odometer/outer scale
- Much improved R_{ij} predictions obtained
- T_{ijk} consistent with experiment (depends on R_{ij} predictions)

Future directions

- Matching/tuning more experiments (esp those with T_{ijk})
  - Junction Flow Experiment
  - Driver CS0, Spinning Cylinder
- Separated flows (promising results with earlier versions)
  - Junction Flow Experiment
  - Johnson Bump

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