Determining Appropriate Risk Remediation Thresholds from Empirical Conjunction Data Using Survival Probability Methods

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Agenda and Overview

- Introduction
- Semi-Empirical Formulation
  - Cumulative collision probabilities
  - Risk remediation thresholds
  - Nominal vs conservative estimates
- Variations with Mission Parameters
  - Mission lifetime (remaining on-orbit duration)
  - Satellite size (hard-body radius)
  - Maneuver responsiveness (commit time)
- Discussion and Conclusions

Motivation and Objectives

- **Motivation** **Requirements for collision risk remediation need to be analyzed and quantified for CARA, and the wider the conjunction risk assessment community**

- **Objective** **Demonstrate how archived conjunction data can be used to estimate risk remediation collision probability thresholds and associated maneuver rates**

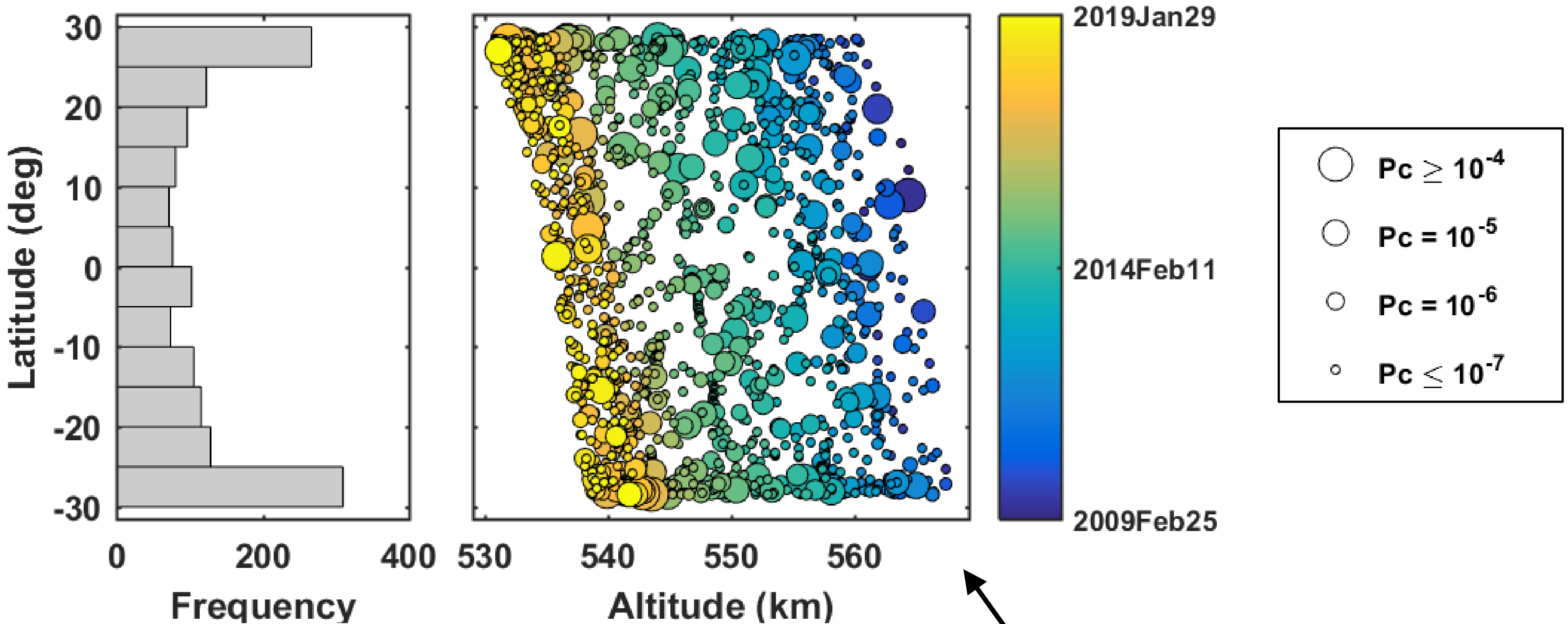
Problem Statement

Problem: **Determine risk remediation parameters that ensure a satellite will survive its remaining on-orbit lifetime without colliding with another cataloged object at a specified confidence level (e.g., 99.9%)**

Methodology: **Semi-empirically estimate the following**

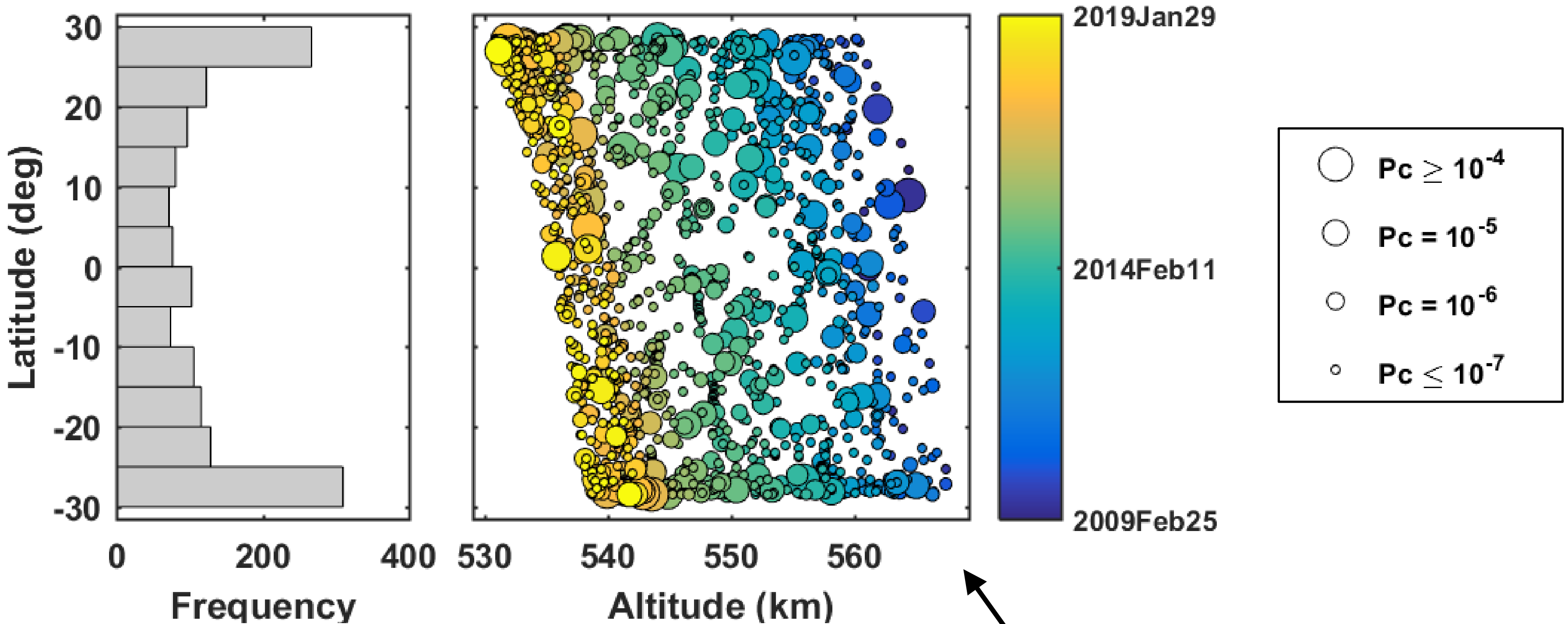
1. **Collision probability that accumulates during a mission's remaining lifetime**
2. **Collision probability threshold required for performing a risk mitigation maneuver (RMM)**
3. **Required maneuver rates (RMMs per year)**

Long-Duration Conjunction Sequence Observed for HST (SCN 20580 – 9.92 years)



Each circle represents a conjunction with archived states and covariances that allow collision probability (Pc) estimation

Resampling to Create Semi-Empirical Conjunction Sequences for Model Missions

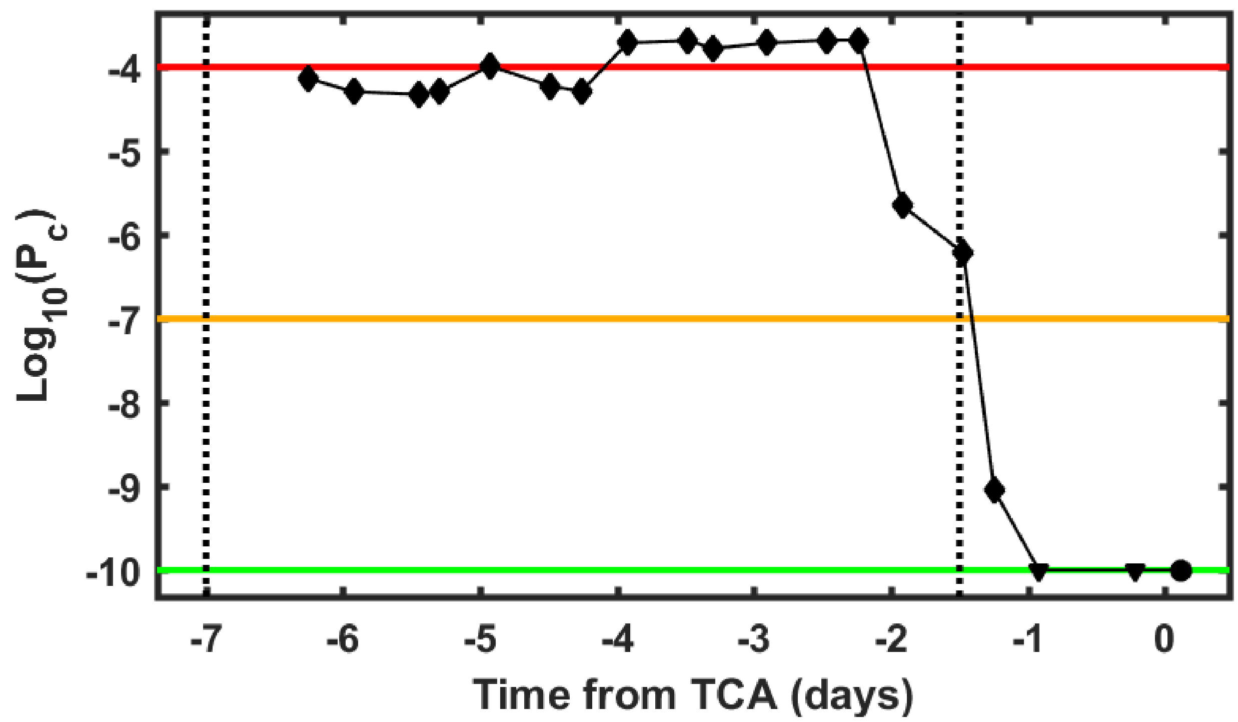


This sequence can be re-sampled to create model sequences for hypothetical missions that occupy HST-like orbits

Pc Update Series for a Conjunction

HST Conjunction Collision Probability Updates
Secondary: SCN 12586 @ 2016-02-11 02:40:52

Pc values estimated using HBR = 10 m
Last update 1.5 to 7 days before TCA: Pc = 2.23e-6



Risk Mitigation Maneuver (RMM) Commit Time Limits and Last-Update Pc Values

HST Conjunction Collision Probability Updates
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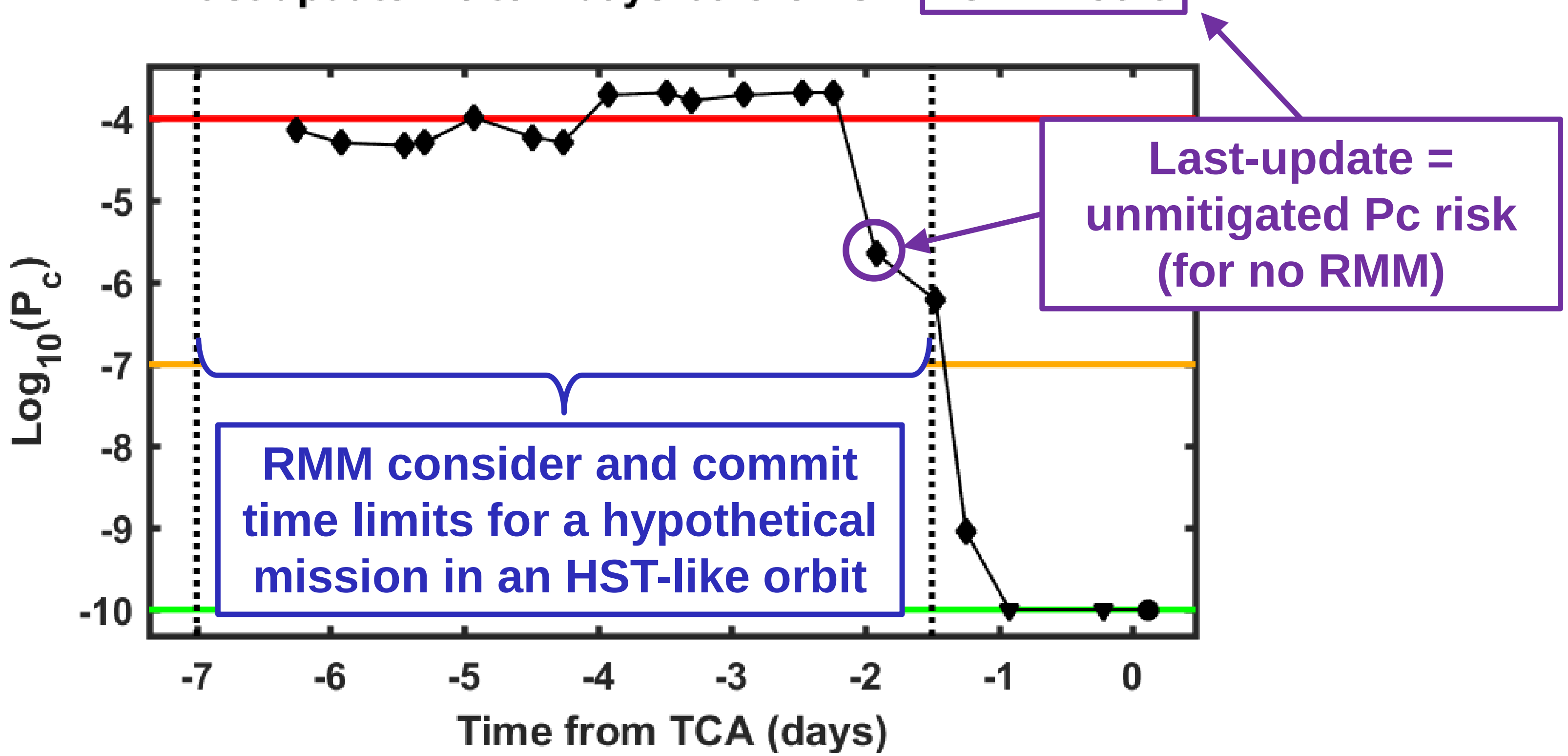

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Cumulative Survival and Collision Probabilities

- Single conjunction survival probability

$$S_c = 1 - P_c$$

- Cumulative survival probability for a sequence of statistically independent conjunctions

$$S_{cum} = \prod_i S_{c,i}$$

- Cumulative collision probability

$$P_{cum} = 1 - S_{cum} = 1 - \prod_i (1 - P_{c,i})$$

Cumulative collision probabilities can be estimated by combining a sequence of conjunction collision probabilities

Cumulative Collision Probability as a Function of Time

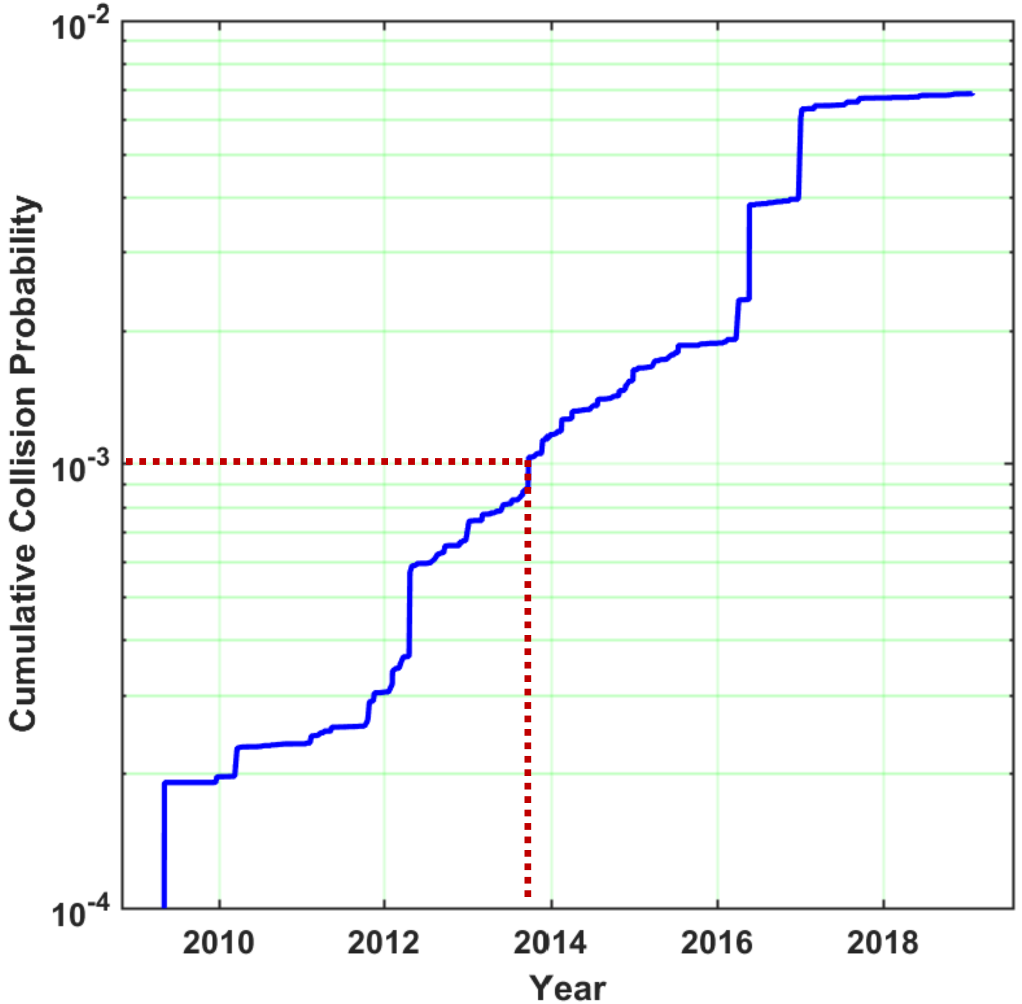
A hypothetical HST-like mission orbiting during 2009-2019 would have reached a cumulative Pc limit of

$$P_{cum} = 10^{-3}$$

after an on-orbit period of

$$\text{Duration} \approx 4.4 \text{ years}$$

Cumulative collision probabilities over extended on-orbit durations can be estimated semi-empirically



Cumulative Collision Probability as a Function Conjunction Pc Value

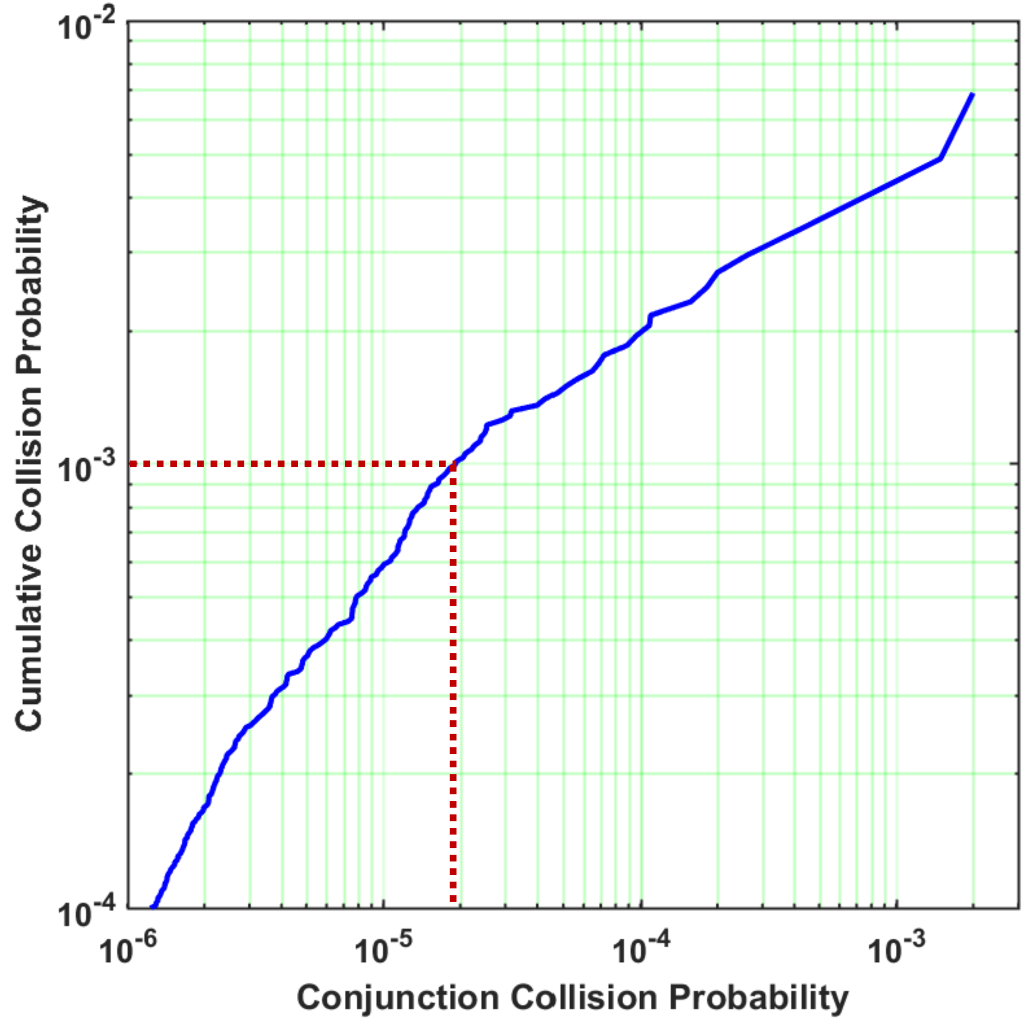
The HST-like mission could have reduced its cumulative Pc to a goal value of

$$P_{cum} \leq 10^{-3}$$

by executing perfectly effective risk mitigation maneuvers at a threshold of

$$P_c > P_{RMM} \approx 2 \times 10^{-5}$$

Risk mitigation maneuver Pc thresholds can be estimated semi-empirically



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Regular vs Conservative Estimation Methods

- For a single conjunction, the worst-case scenario* is that the collision probability equals the remediation threshold – the highest Pc that can go unmitigated

$$P_c = P_{RMM}$$

- The cumulative probability accounting for this worst-case includes one additional event with the threshold probability

$$P_{cum}^{cons} = 1 - (1 - P_{RMM})\left[\prod_i \left(1 - P_{c,i}\right)\right]$$

Conservative **risk remediation estimates add one threshold-level event to model conjunction sequences**

Regular **estimates do not add such an event**

*Assuming the satellite's maneuver execution system is perfectly safe and reliable

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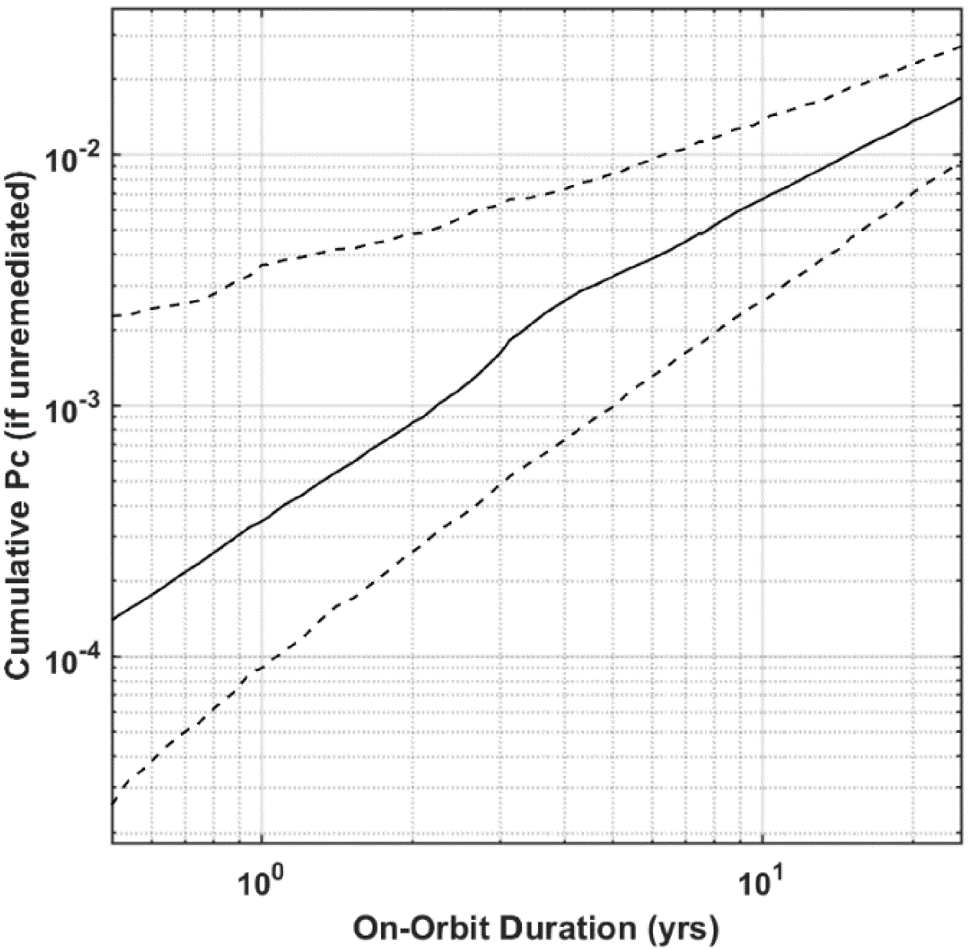
Semi-Empirical Cumulative Pc Estimates Increase with On-orbit Duration

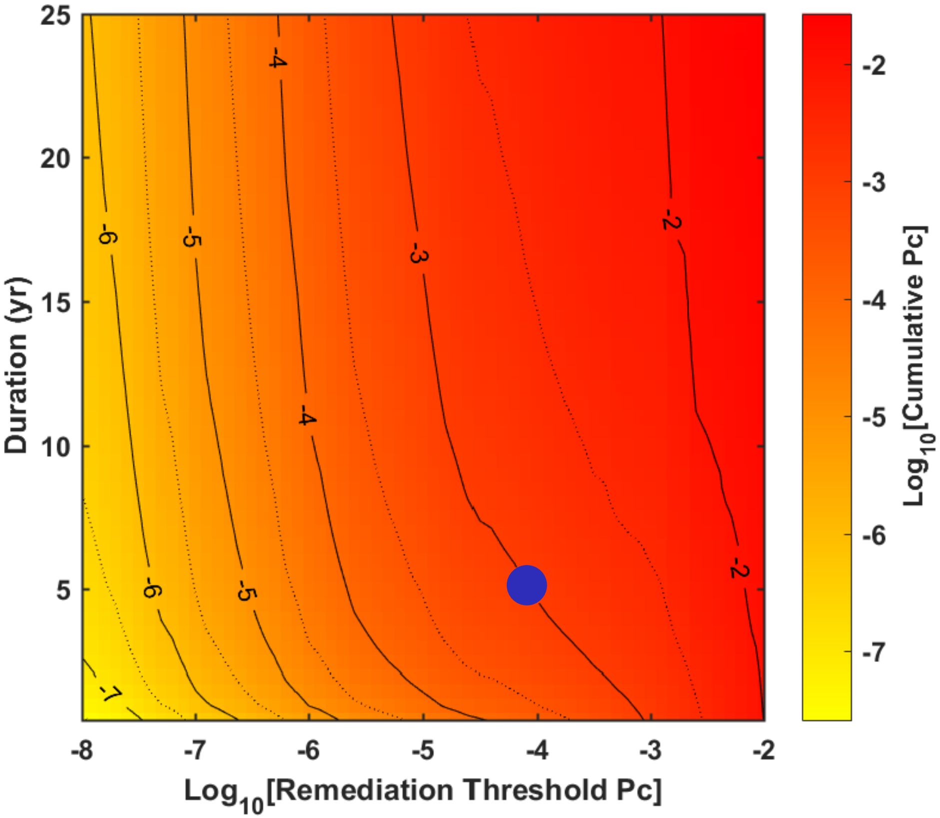
- **Cumulative Pc estimated for HST's orbital regime**
  - Hard-body radius = 10 m
  - Solid line: median over all state/covariance samples
  - Dashed lines: 95% range of sampling variations
- **For 10 years on-orbit**
  - Median estimate: $P_{cum} \approx 0.007$
  - 95%: $0.003 \leq P_{cum} \leq 0.013$

The semi-empirical method provides rough estimates



Remediation Maneuver Thresholds and Rates

- **Hypothetical mission parameters:**
  - Orbital regime: HST (CARA LEO-2)
  - Remaining duration = 5 years
  - Cumulative Pc goal ≤ 10^{-3}
  - HBR = 10 m
  - RMM type = Translational
  - RMM commit time = 1.5 days
  - RMM consider time = 7.0 days
- **Conservative mode estimates:**
  - Pc threshold: $P_{RMM} \approx 8.5 \times 10^{-5}$
  - RMM rate: $\dot{N}_{RMM} \approx 1.1/\text{year}$



Remediation thresholds change as a function of a mission's remaining on-orbit duration

Remediation Requirements Vary Significantly with a Mission's Remaining On-Orbit Duration

Mission parameters: HST-like orbit, HBR = 10 m
Cumulative collision risk requirement: $P_{cum} \leq 10^{-3}$
Conservative mode estimates

	Duration = 2 years	Duration = 5 years	Duration = 15 years
P_{RMM}	4.1×10^{-4}	8.5×10^{-5}	1.1×10^{-5}
$\dot{N}_{RMM}$ (yr^{-1})	0.3	1.1	5.7

Flying shorter missions significantly reduces the remediation activities required to achieve a lifetime cumulative risk goal

Remediation Requirements Vary Significantly with a Mission's Hard Body Radius

Mission parameters: HST-like orbit, 5 year duration
Cumulative collision risk requirement: $P_{cum} \leq 10^{-3}$
Conservative mode estimates

	HBR = 5 m	HBR = 10 m	HBR = 20 m
P_{RMM}	4.6×10^{-4}	8.5×10^{-5}	2.9×10^{-5}
$\dot{N}_{RMM}$ (yr^{-1})	0.1	1.1	7.7

Flying smaller satellites significantly reduces the remediation activities required to achieve a lifetime cumulative risk goal

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Remediation Requirements Vary Significantly with a Mission's Hard Body Radius

Mission parameters: HST-like orbit, 5 year duration
Cumulative collision risk requirement: $P_{cum} \leq 10^{-3}$
Conservative mode estimates

	HBR = 5 m	HBR = 10 m	HBR = 20 m
P_{RMM}	4.6×10^{-4}	8.5×10^{-5}	2.9×10^{-5}
$\dot{N}_{RMM}$ (yr^{-1})	0.1	1.1	7.7

Corollary: Artificially inflating mission hard-body radii can significantly increase risk remediation workloads and activities

Remediation Requirements Vary Significantly with a Mission's Maneuver Commit Time

Mission parameters: HST-like orbit, 5 year duration, HBR = 10 m
Cumulative collision risk requirement: $P_{cum} \leq 10^{-3}$
Conservative mode estimates

	Commit at TCA-6 hours	Commit at TCA-24 hours	Commit at TCA-36 hours
P_{RMM}	1.2×10^{-4}	9.6×10^{-5}	8.5×10^{-5}
$\dot{N}_{RMM}$ (yr^{-1})	0.3	0.7	1.1

Flying more responsive mission maneuver systems significantly reduces the remediation activities required to achieve a lifetime cumulative risk goal

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Discussion

- **Advantages of the semi-empirical method:**
  1. **All required states and covariances can be extracted from the CARA conjunction archive (no simulations required)**
  2. **Different mission and satellite parameters can be studied by resampling the archived states and covariances**
- **Disadvantages of the semi-empirical method:**
  1. **Provides rough estimates, limited by sampling variations**
  2. **Results only apply to orbital regimes occupied by primary satellites contained in the conjunction archive**
  3. **Assumes archived state/covariance distributions can be applied to model missions**

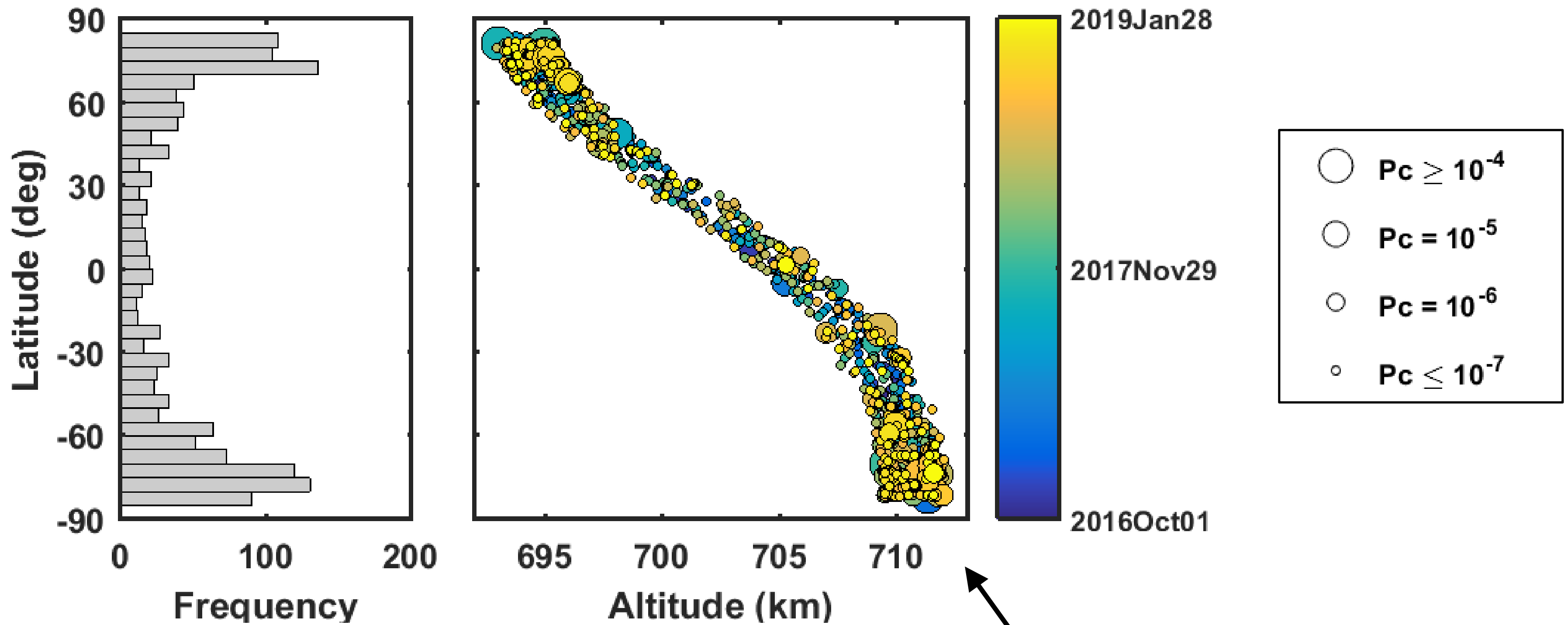
The semi-empirical method has limitations, but can be used effectively for testing/calibrating simulation software, as well as for some pre-mission planning

Conclusions

- Archived conjunction data can be used to estimate collision risk remediation Pc thresholds and maneuver rates
  - Allows varying mission parameters such as HBR, on-orbit duration, cumulative Pc goal, and maneuver commit times
  - Provides rough estimates, limited by sampling variations
  - Enables simulation model testing and pre-mission planning
  - Can also incorporate "collision consequences" (see paper)
- Semi-empirical analysis quantifies the reduction in collision risk mitigation activities achieved by
  - Flying shorter missions
  - Flying smaller satellites
  - Flying more responsive satellites

Back-Up Slides

Long-Duration Conjunction Series Observed for AQUA (SCN 27424 – 2.32yrs)



Each circle represents a conjunction with archived states and covariances that allow collision probability (Pc) estimation

Conjunction Sequences for Model Missions

- Approximate number of conjunctions expected for a model satellite deployed near an observed primary's orbit

$$N_{mod} \approx N_{obs} \times \left[{T_{mod}}/{T_{obs}} \right]$$

- States and covariances sampled from the archive*

$$\left\{ \mathrm{X}_{i,k}^{pri}, \mathrm{C}_{i,k}^{pri}, \mathrm{X}_{i,k}^{sec}, \mathrm{C}_{i,k}^{sec} \right\} \qquad i = 1 \ldots N_{mod} \qquad k = 1 \ldots N_{sample}$$

- Collision probability sequence experienced during the mission

$$P_{c,i,k} = \mathrm{PcFunction}\left(H, \mathrm{X}_{i,k}^{pri}, \mathrm{C}_{i,k}^{pri}, \mathrm{X}_{i,k}^{sec}, \mathrm{C}_{i,k}^{sec} \right) \qquad H = \mathrm{HBR}$$

Method samples archived states and covariances to create long-term conjunction sequences for model missions

*Assumes statistical independence, and that the model and observed missions have similar state and covariance distributions

Cumulative Pc Estimation for Model Missions

- **Remediated cumulative collision probability for a model mission that performs risk mitigation maneuvers (RMMs)**

$$P_{cum}^{rem} = 1 - \prod_i \left(1 - P_{c,i}^{rem}\right)$$

- **Remediated collision probabilities**

$$P_{c,i}^{rem} = \begin{cases} P_{c,i} & \text{For } P_{c,i} \leq P_{RMM} \text{ (no RMM required)} \\ \rho_t \, P_{RMM} & \text{Translational RMM} \\ \rho_r \, P_{c,i} & \text{Rotational RMM} \end{cases}$$

Method estimates P_c thresholds for executing risk mitigation maneuvers, P_{RMM}, and associated maneuver rates, $\dot{N}_{RMM}$

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***Suppressing the sampling realization index k for brevity**

Mission parameters: HST-like orbit, 5 year duration, *HBR* = 10 m

Cumulative collision risk requirement: $P_{cum} \leq 10^{-3}$

Regular Mode Estimates	Conservative Mode Estimates
Pc Threshold: $P_{RMM} \approx 1.1 \times 10^{-4}$	Pc Threshold: $P_{RMM} \approx 8.5 \times 10^{-5}$
Maneuver rate: $\dot{N}_{RMM} \approx 0.9$ per year	Maneuver rate: $\dot{N}_{RMM} \approx 1.1$ per year