Implementation Recommendations and Usage Boundaries for the Two-Dimensional Probability of Collision Calculation

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Agenda and Overview

- **Introduction**
  - **Motivation and objectives**
  - **Review latest 2D-Pc vs *Brute Force Monte Carlo* comparison for archived conjunctions**
  - **Review previous discussions of 2D-Pc validity criteria**
- **Analysis**
  - **Investigate “offset-from-TCA” variations as an indicator of 2D-Pc inaccuracy**
  - **Develop a diagnostic boundary test to indicate potential large-amplitude 2D-Pc method underestimations**
- **Conclusions and Recommendations**

Motivation and Objectives

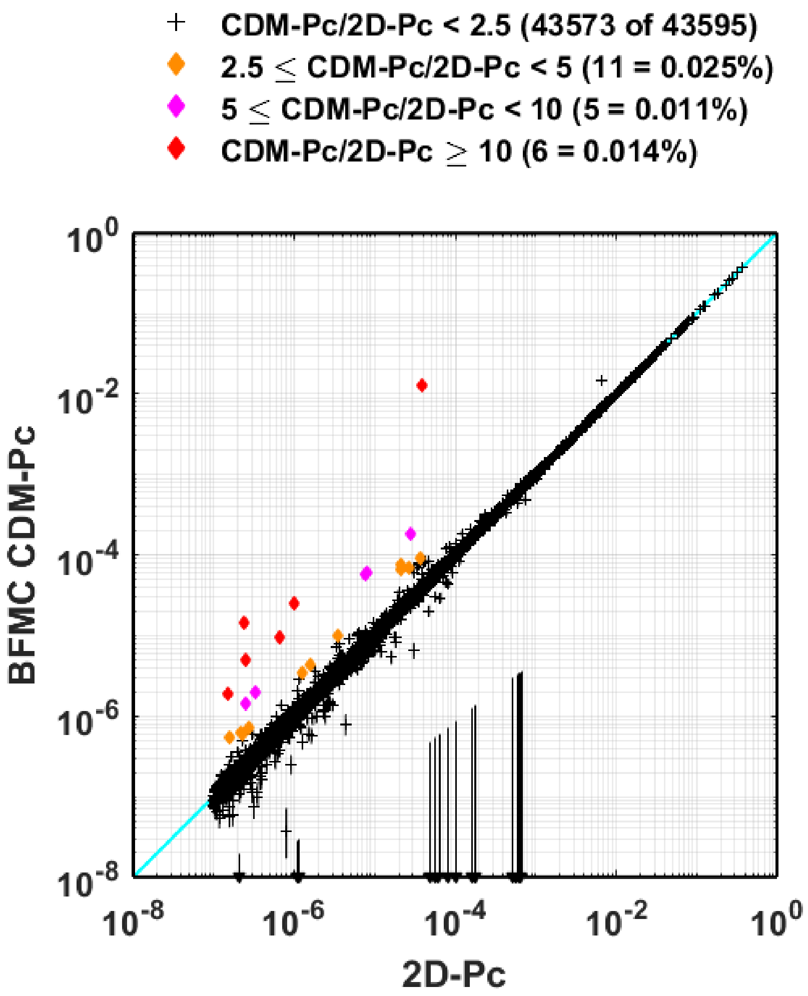
- **Motivation** The probability of collision (Pc) between two Earth-orbiting satellites can often but not always be approximated adequately using the semi-analytical "2D-Pc" formulation[1,2,3]

- **Objective** Develop a test to indicate when the 2D-Pc method reliably provides sufficiently accurate Pc estimates, so that high-fidelity *Brute Force Monte Carlo*[3] (BFMC) Pc simulations do not have to be executed for all conjunctions

[1]J.L. Foster and H.S. Estes, "A Parametric Analysis of Orbital Debris Collision Probability and Maneuver Rate for Space Vehicles," NASA/JSC-25898, Aug. 1992.
[2]M.R. Akella and K.T. Alfriend, "The Probability of Collision Between Space Objects," *Journal of Guidance, Control, and Dynamics*, Vol. 23, No. 5, pp. 769-772, 2000.
[3]D.Hall *et al* "High-Fidelity Collision Probabilities Estimated Using Brute Force Monte Carlo Simulations" AAS 18-244, Aug. 2018

BFMC-Pc vs 2D-Pc Comparison for Archived Satellite Conjunctions

- **43,595 CARA archive conjunctions**
  - **2017-05-01 to 2018-11-15 with 2D-Pc ≥ 10^{-7}**
- **2D-Pc works well for the vast majority[1]**
  - **But there are many more differences between 2D-Pc and BFMC-Pc than expected from random variations**
- **2D-Pc significantly <u>underestimates</u> BFMC-Pc for an extremely small (but measurable) fraction of events**
  - **BFMC-Pc/2D-Pc ≥ 2.5 for 22 of the analyzed conjunctions (0.05%)**
  - **Diamonds show such "large-amplitude" 2D-Pc underestimation failures**
  - **Most concerning type of 2D-Pc approximation inaccuracies**



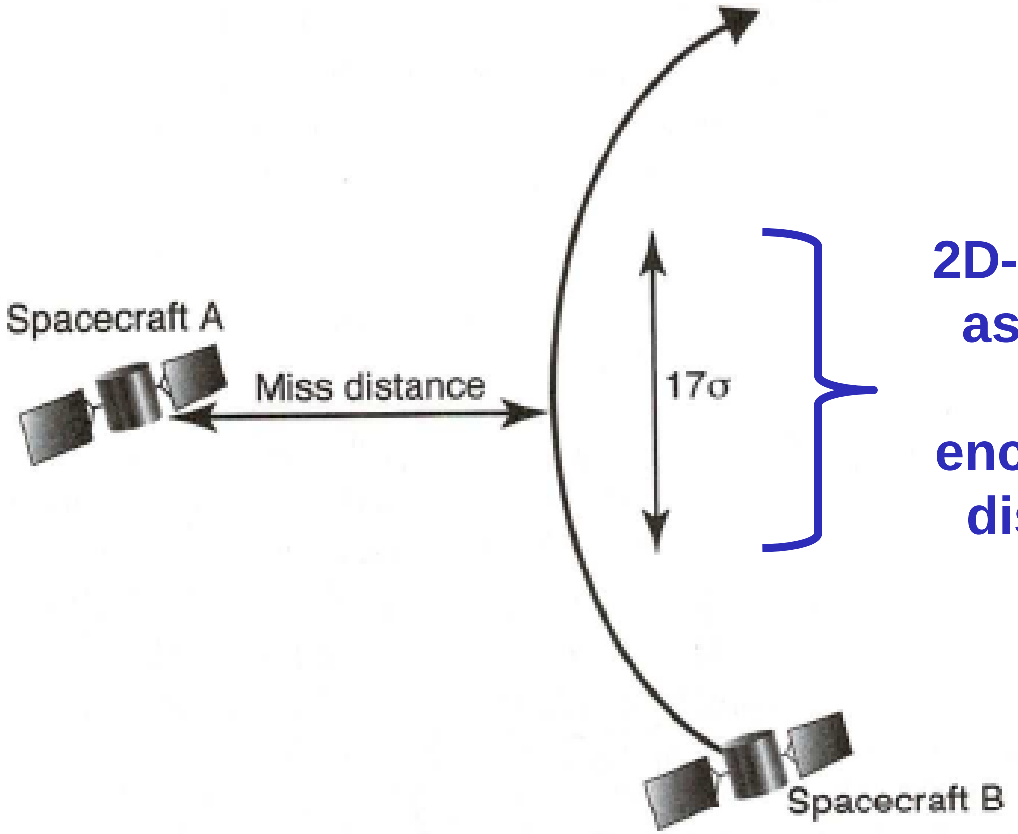
[1]D.Hall *et al* (2018) "High-Fidelity Collision Probabilities Estimated Using Brute Force Monte Carlo Simulations" AAS 18-244

Why Does 2D-Pc Occasionally Fail?

- The formulation of the 2D-Pc approximation assumes the following[1,2]
  1. At TCA the primary-to-secondary relative position uncertainty distribution can be approximated as Gaussian
  2. During the conjunction, the relative satellite trajectories can be approximated as linear
  3. During the conjunction, the relative position covariance can be approximated as constant
- 2D-Pc estimates can be inaccurate if any of these three assumptions are violated sufficiently

[1] J.L. Foster and H.S. Estes, "A Parametric Analysis of Orbital Debris Collision Probability and Maneuver Rate for Space Vehicles," NASA/JSC-25898, Aug. 1992.
[2] M.R. Akella and K.T. Alfriend, "The Probability of Collision Between Space Objects," *Journal of Guidance, Control, and Dynamics*, Vol. 23, No. 5, pp. 769-772, 2000.

Chan (2008)[1] Analysis of the Encounter Region for Valid 2D-Pc Estimation

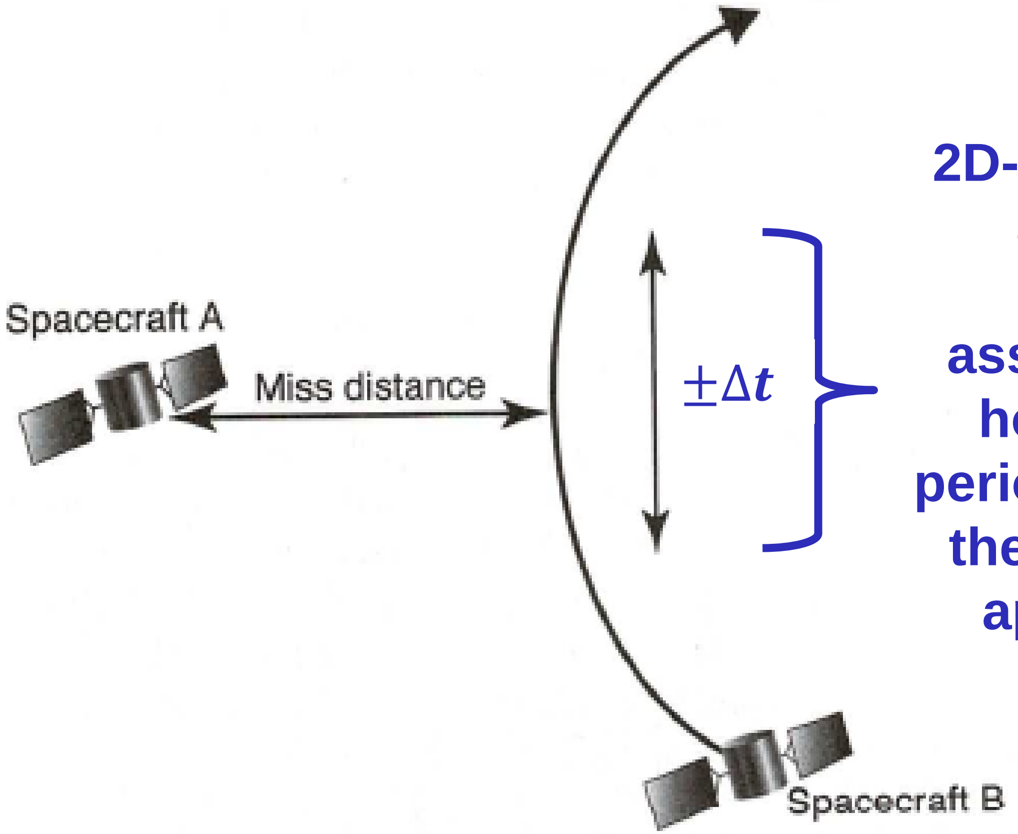


2D-Pc linear motion assumption must hold over an encounter traversal distance of ±8.5σ

Figure 3.1. Requisite path length for integration. The length must be 17σ for the rectilinear approximation to be valid.

[1]K.Chan (2008) "Spacecraft Collision Probability" The Aerospace Press, El Segundo CA

Coppola (2012)[1] Analysis of the Time Interval for Valid 2D-Pc Estimation



2D-Pc linear motion and constant covariance assumptions must hold over a time period $\pm\Delta t$ relative to the time of closest approach (TCA)

Short-term encounter validity interval: $\Delta t = \max(|\tau_0|, |\tau_1|, \tau_1 - \tau_0)$

[1]V.Coppola (2012) "Evaluating the Short Encounter Assumption of the Probability of Collision Formula" AAS 12-248, Feb 2012

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Offset-from-TCA 2D-Pc Estimates

- Normally, 2D-Pc estimates are calculated using states and covariances at TCA

$$P_{c,0} = P_c(t = 0)$$

- However, 2D-Pc estimates can also be calculated using states and covariances propagated to times offset from TCA

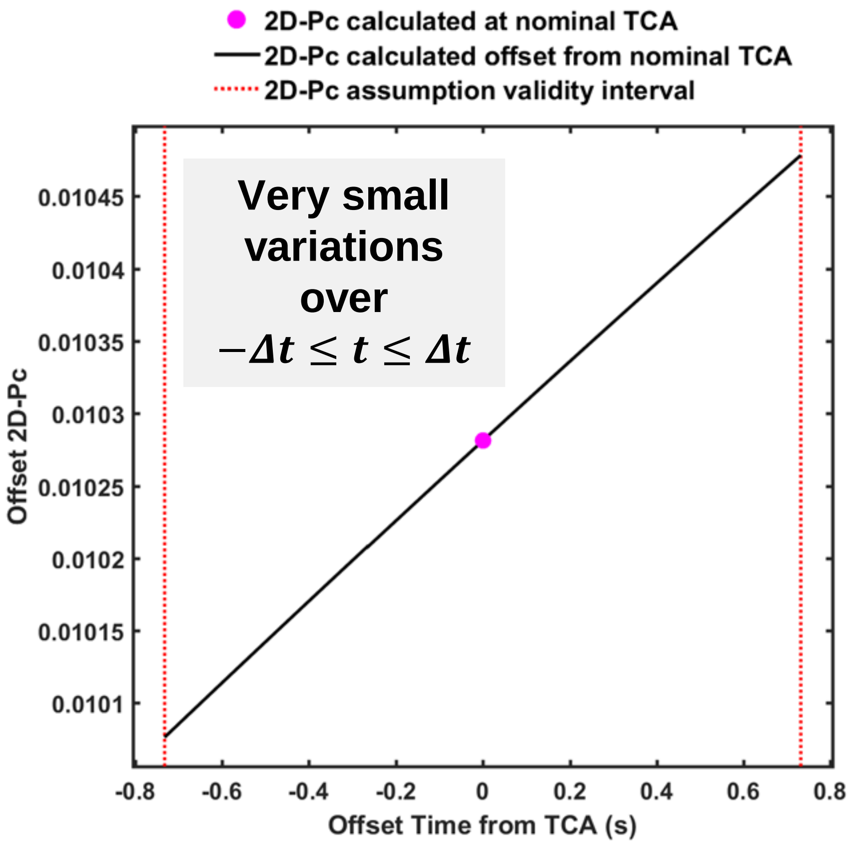
$$P_{c,t} = P_c(t \neq 0)$$

- If the 2D-Pc assumptions are satisfied, then this yields about the same Pc estimates within the validity interval

$$P_{c,t} \approx P_{c,0} \quad \text{for} \quad |t| \leq \Delta t$$

IDEA: Variations in offset-from-TCA estimates could provide an indicator of 2D-Pc inaccuracy (S. Alfano, Nov 2018)

Known AQUA conjunction[1] with BFMC-Pc = 2D-Pc



[1]D.Hall *et al* (2018) "High-Fidelity Collision Probabilities Estimated Using Brute Force Monte Carlo Simulations" AAS 18-244

Using Offset-from-TCA Variations as an Indicator of Overall 2D-Pc Accuracy

- **Offset-from-TCA 2D-Pc variations can be measured using the extrema found during the validity time interval**

$$P_c^{max} = \max_{|t| \leq \Delta t} [P_{c,t}] \qquad P_c^{min} \text{ similar}$$

- **A "variation metric" can be defined as**

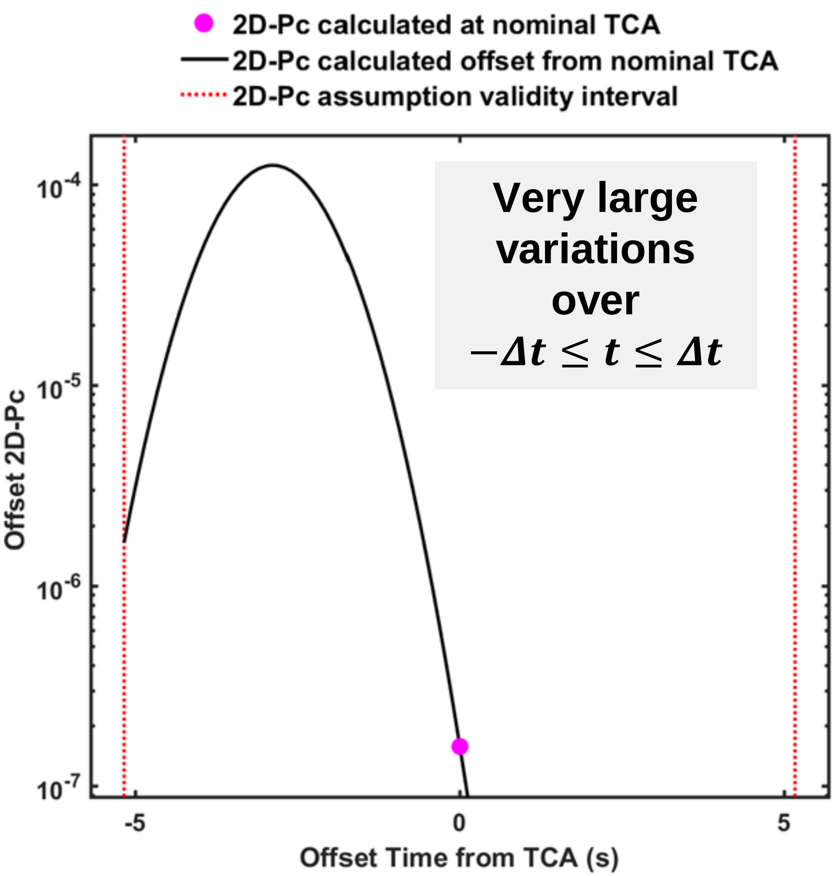
$$V = \log_{10}[P_c^{max} / P_c^{min}]$$

so that

$$V \rightarrow 0 \text{ for small variations}$$
$$V \rightarrow \infty \text{ for large variations}$$

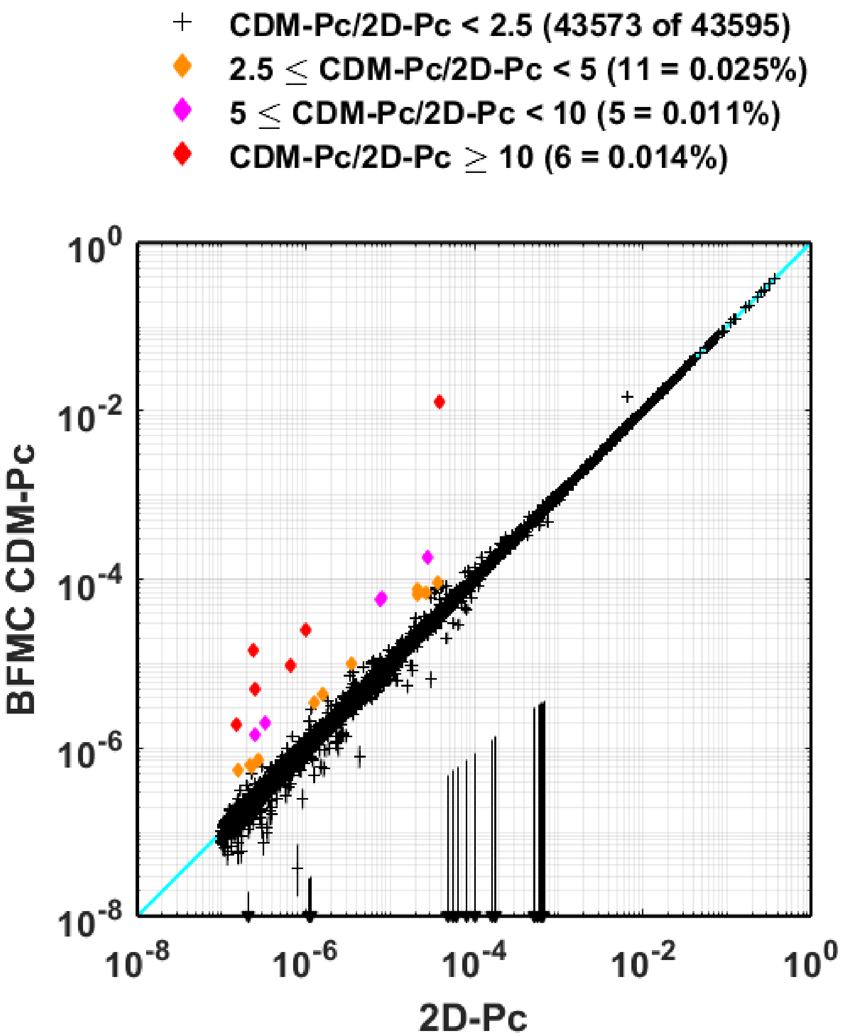
OBSERVATION: All conjunctions found to have large 2D-Pc underestimation inaccuracies also have large variation metrics, V

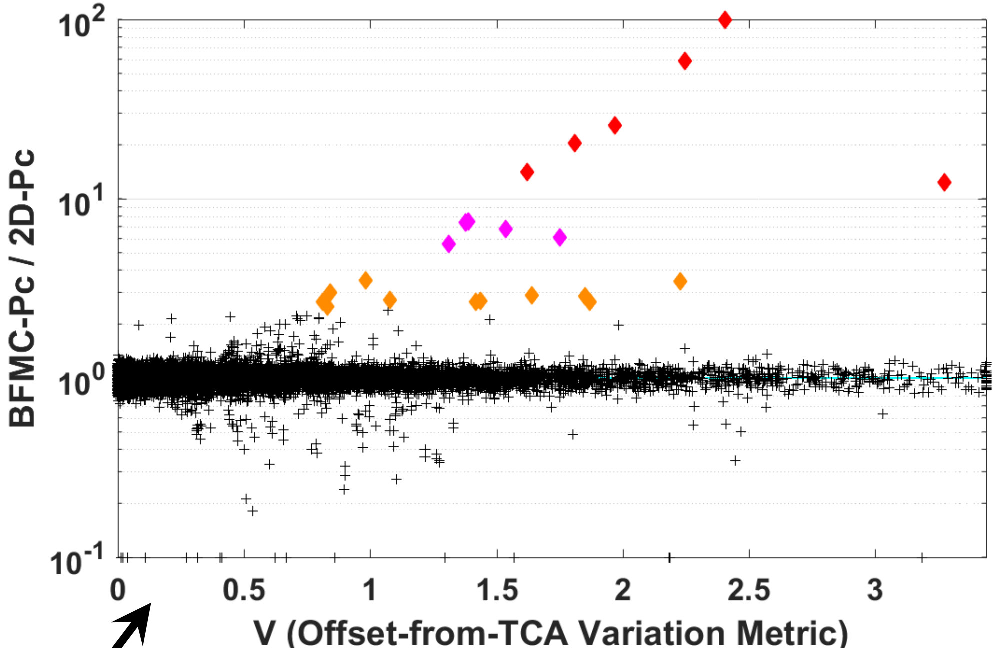
VAN ALLEN conjunction[1] with BFMC-Pc = 320 × 2D-Pc



[1]D.Hall *et al* (2018) "High-Fidelity Collision Probabilities Estimated Using Brute Force Monte Carlo Simulations" AAS 18-244

Distribution of 2D-Pc Underestimates as a Function Offset-from-TCA Variations





OBSERVATION: All conjunctions found to have large 2D-Pc underestimation inaccuracies also have large variation metrics, V, but not vice versa

Boundary Condition that Isolates 2D-Pc Method Underestimation Failures

- **All 2D-Pc underestimations exceeding ×2.5 occur above a boundary variation metric of**

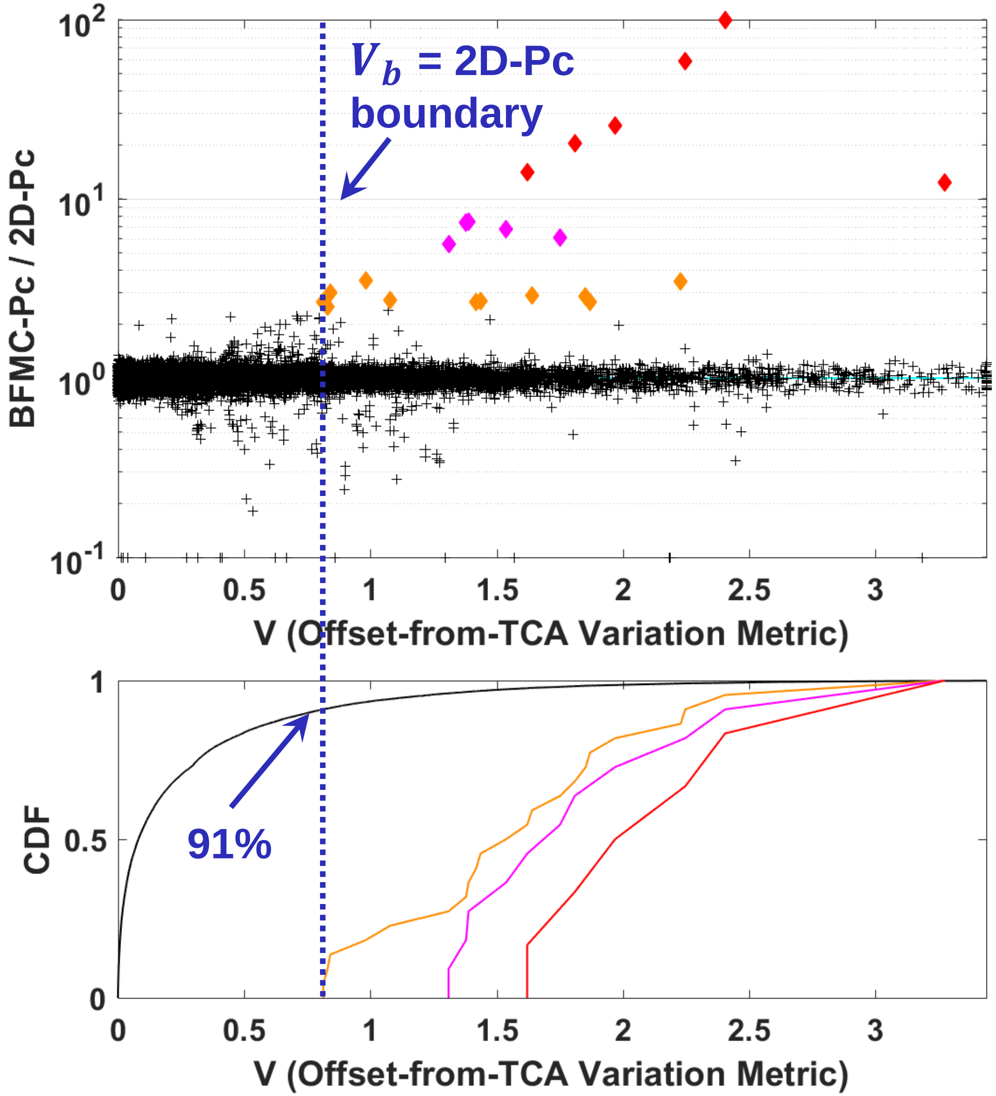
$$V_b = 0.8$$

- **The majority of events have variation metrics smaller than this boundary value**

$$V \leq V_b \text{ for 91\% of events}$$

$$V > V_b \text{ for 9\% of events}$$

2D-Pc Boundary Test:
If $V > V_b$ then 2D-Pc *could* underestimate the true Pc by a factor of 2.5 or more



2D-Pc Boundary Test
Missed Detection Frequencies

Includes all analyzed data

Number of 2D-Pc Underestimate Missed Detections
(among 43,595 conjunctions with 2D-Pc >= 1e-7
occuring 2016-10-01 to 2019-01-22)

Fraction Calculated with BFMC (%)	2D-Pc Underestimation Factor 2.0	2.5	3.0	5.0	10.0
0.0	34	22	14	11	6
1.0	30	18	10	8	3
2.0	26	14	8	6	1
3.0	22	10	5	3	0
5.0	16	5	1	0	0
10.0	7	0	0	0	0
15.0	2	0	0	0	0
20.0	1	0	0	0	0
30.0	1	0	0	0	0
40.0	0	0	0	0	0
50.0	0	0	0	0	0

No missed detections in this data set
Missed detection fraction <= 1e-4
Missed detection fraction > 1e-4

Includes only BFMC-Pc $\geq 10^{-5}$

Number of 2D-Pc Underestimate Missed Detections
(among 43,595 conjunctions with 2D-Pc >= 1e-7
occuring 2016-10-01 to 2019-01-22)

Fraction Calculated with BFMC (%)	2D-Pc Underestimation Factor 2.0	2.5	3.0	5.0	10.0
0.0	16	10	8	6	3
1.0	14	8	6	4	1
2.0	13	7	5	3	0
3.0	12	6	4	2	0
5.0	10	4	2	0	0
10.0	6	0	0	0	0
15.0	1	0	0	0	0
20.0	0	0	0	0	0
30.0	0	0	0	0	0
40.0	0	0	0	0	0
50.0	0	0	0	0	0

No missed detections in this data set
Missed detection fraction <= 1e-4
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Conclusions and Recommendations

- **A diagnostic boundary test can be used to identify potential 2D-Pc method underestimation failures**
  - **Analysis based on a set of ~44,000 CARA conjunctions analyzed using *Brute Force Monte Carlo* simulations to establish truth Pc values**
  - **Boundary test uses the amplitude of "offset-from-TCA" variations as an indicator of accuracy in the 2D-Pc approximation**
  - **The test identifies all large-amplitude 2D-Pc underestimations detected to date, with a high false-alarm rate**
- **RECOMMENDATIONS: For future conjunctions with 2D-Pc $\geq 10^{-7}$**
  - **HIGH PRIORITY: Estimate BFMC-fidelity Pc values for the 10% of events with the largest offset-from-TCA variation metrics**
  - **MEDIUM PRIORITY: Estimate BFMC-fidelity Pc values for the 20% of events with the largest offset-from-TCA variation metrics**
  - **LOW PRIORITY: Estimate BFMC-fidelity Pc values for all events**

Backup Slides

Using Offset-from-TCA Variations as an Indicator of 2D-Pc Underestimation

- The variation metric introduced previously

$$V = \log_{10}\left[P_c^{max}/P_c^{min}\right]$$

was developed to indicate any 2D-Pc inaccuracies, both underestimates and overestimates

- A slightly different metric works somewhat better for indicating underestimates alone

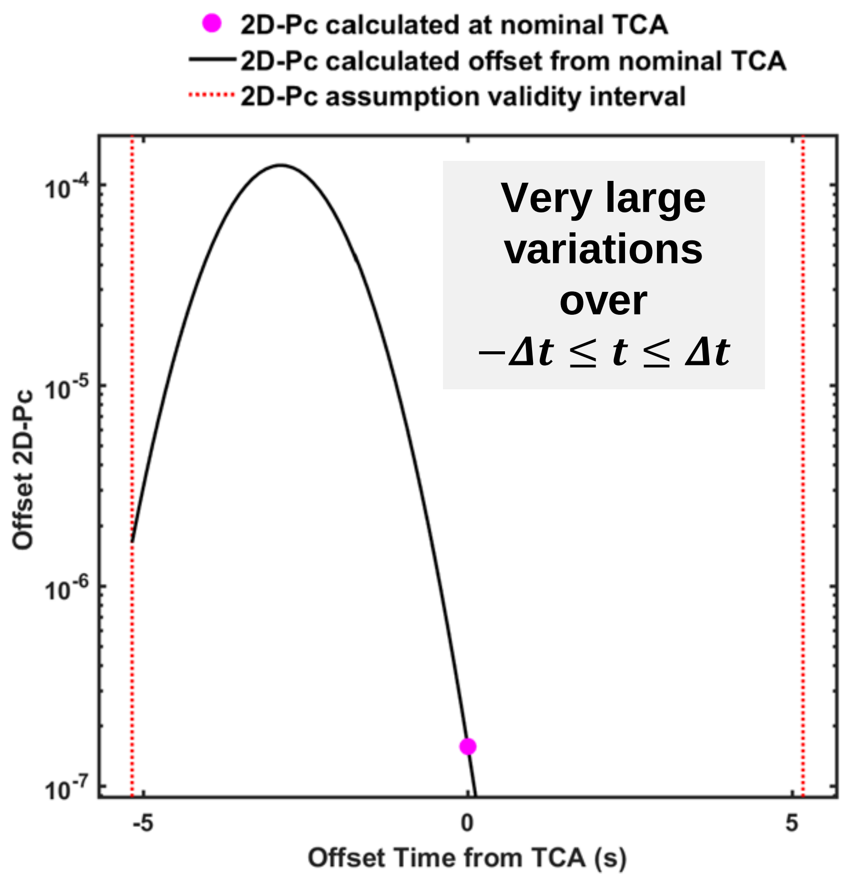
$$V = \log_{10}\left[P_c^{max}/P_c^{mid}\right]$$

with

$$P_c^{mid} = (P_{c,0} + P_c^{min})/2$$

This analysis focuses on using this metric to establish a "2D-Pc validity boundary" within which no large-amplitude 2D-Pc underestimates exist

VAN ALLEN conjunction[1] with BFMC-Pc = 320 × 2D-Pc



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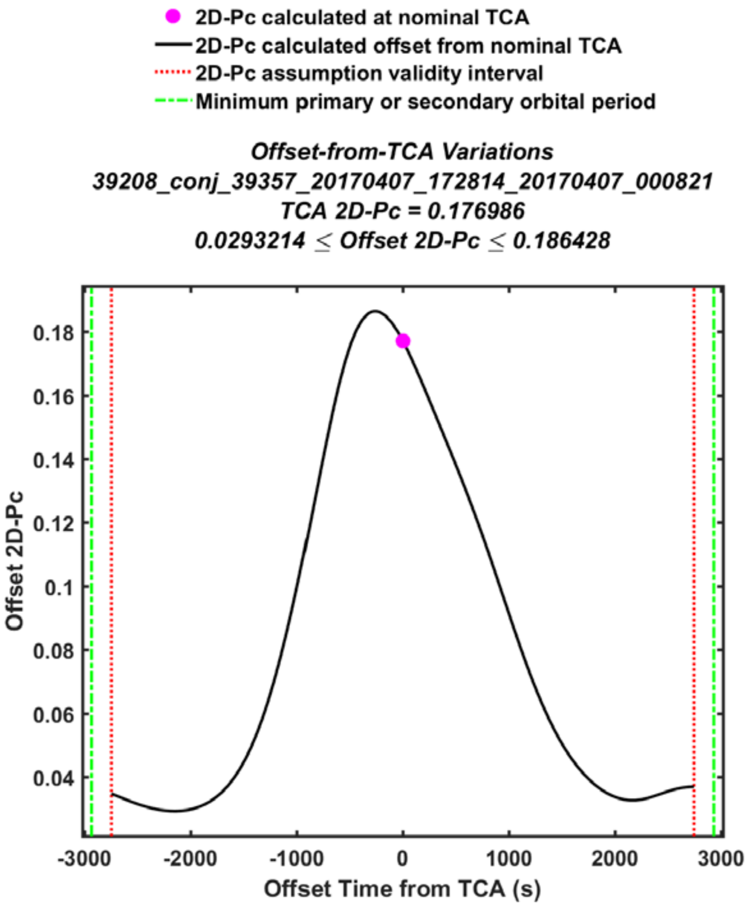
Offset Variations for Other Likely 2D-Pc Failures
(from SSPAT Low Relative Velocity Project Data Set)

Long Duration Conjunction:

$N_{min} = 2 \qquad \Delta t/T_{orb} = 0.94$



Repeating Conjunction

$N_{min} = 4 \qquad \Delta t/T_{orb} = 2.4$