Microscale Modeling of High-temperature Heat Transfer in Anisotropic Porous Materials

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Tuesday 7th May 2018

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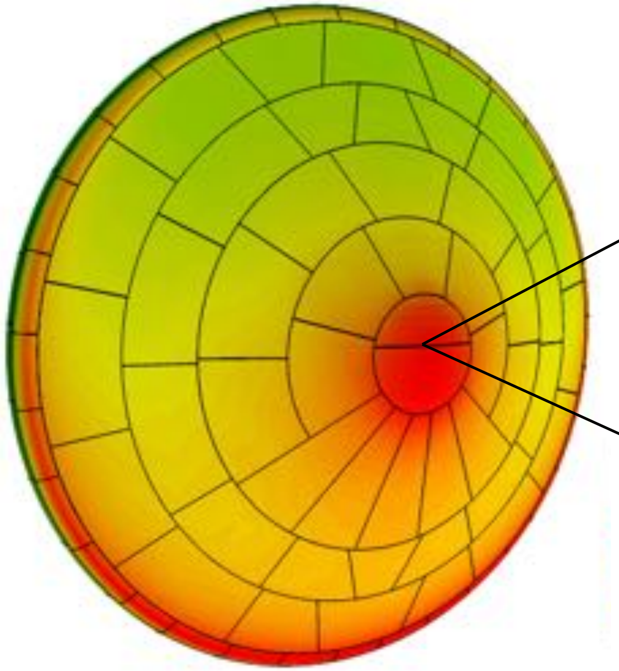
MOTIVATION & OBJECTIVES

Modeling Thermal Protection Systems (TPS)

Macroscale Modeling

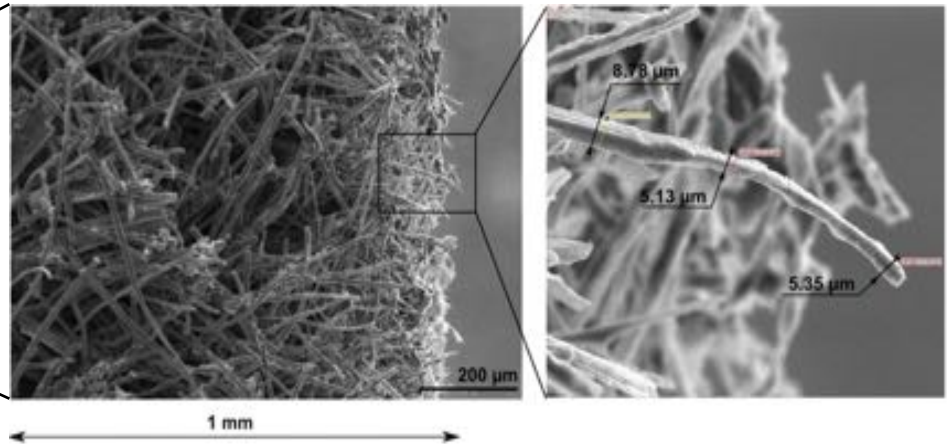
Full scale material response solvers, using volume-averaged techniques to solve conservation equations for ablation



Simulation of surface temperature for MSL heatshield

Microscale Modeling

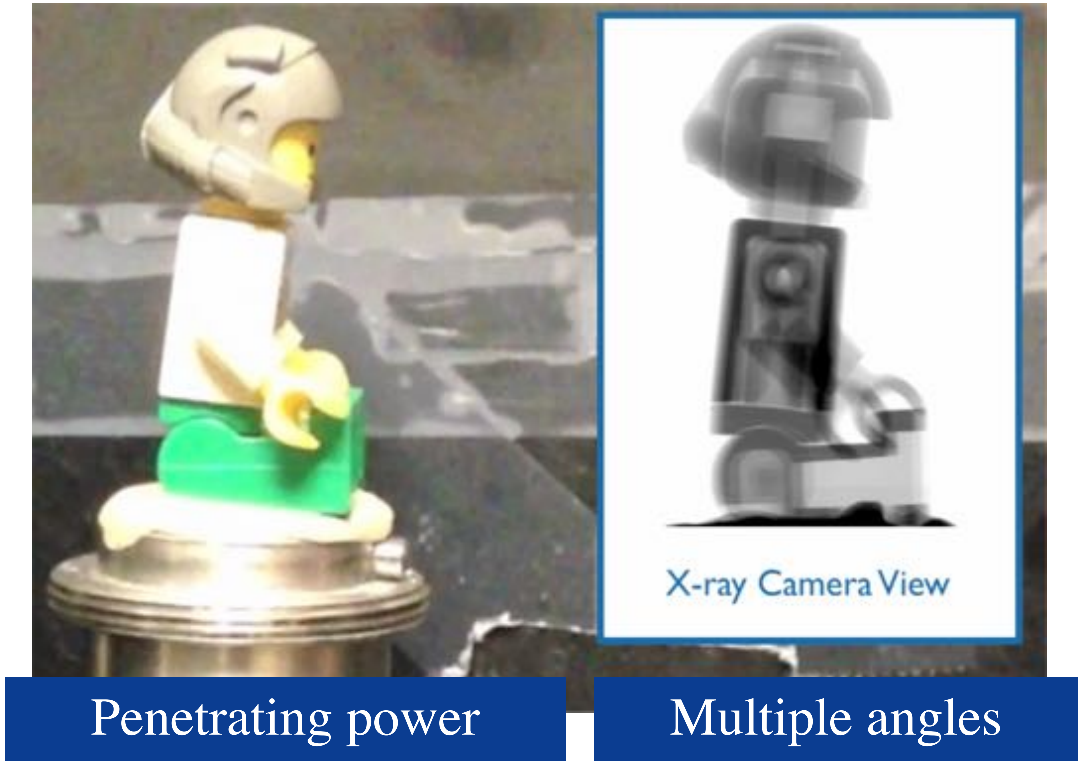
Used to inform material properties and material response parameters used in macro-scale modeling



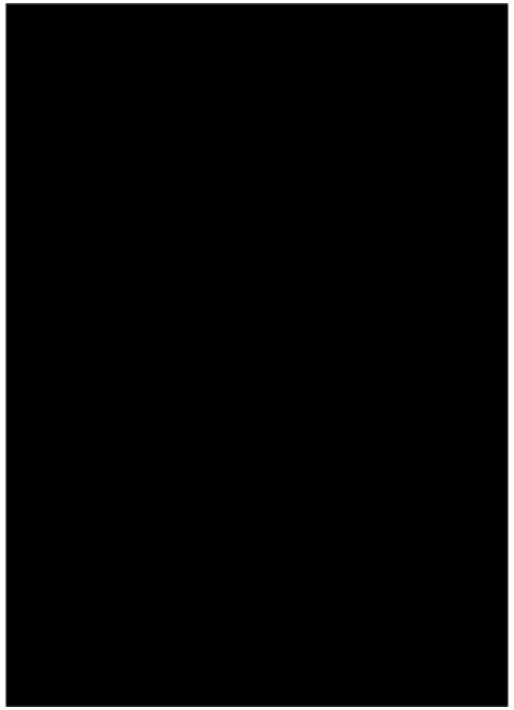
Lachaud and Mansour, *JTHT* 2013

X- Ray Microtomography

Collect X-ray images of the sample as you rotate it through 180°

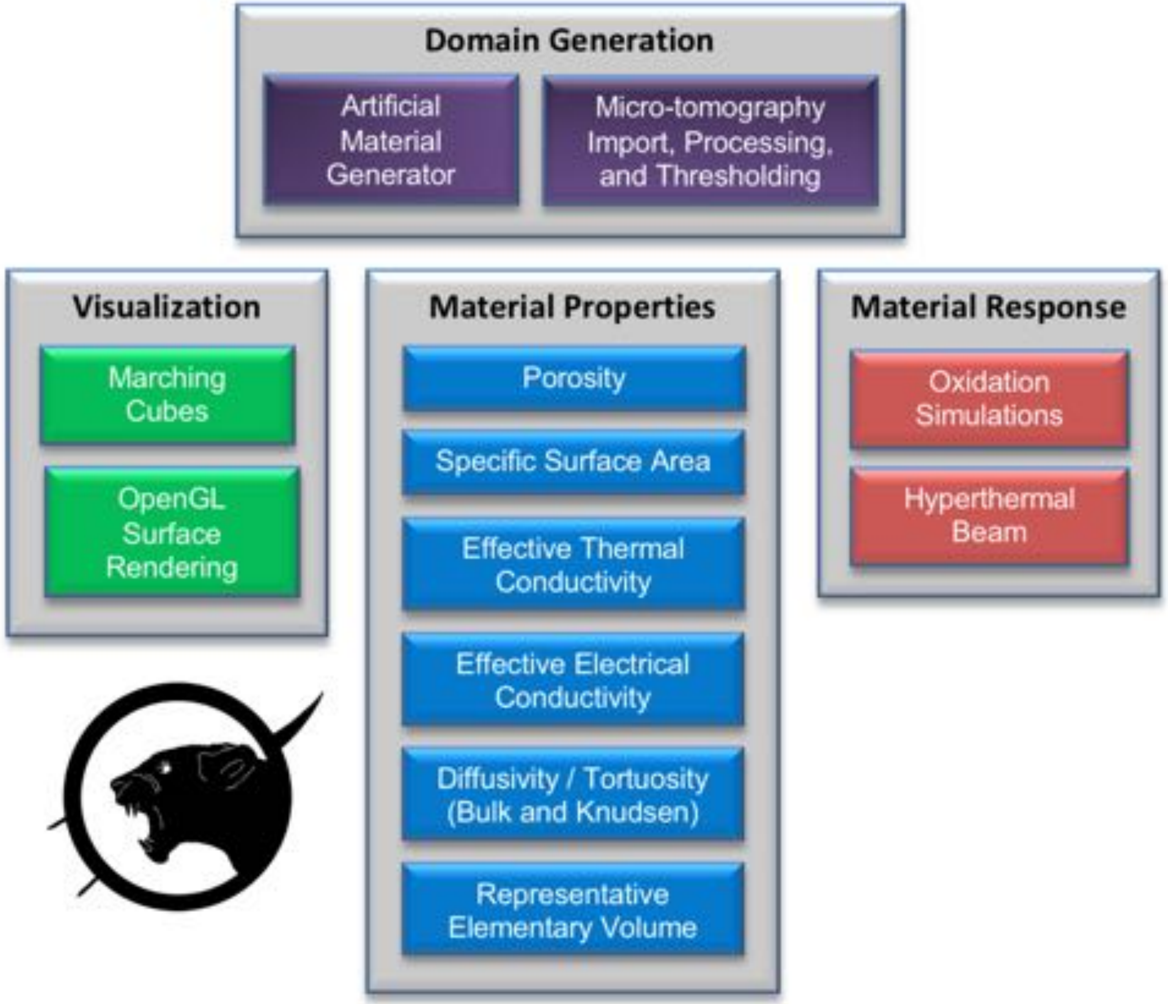


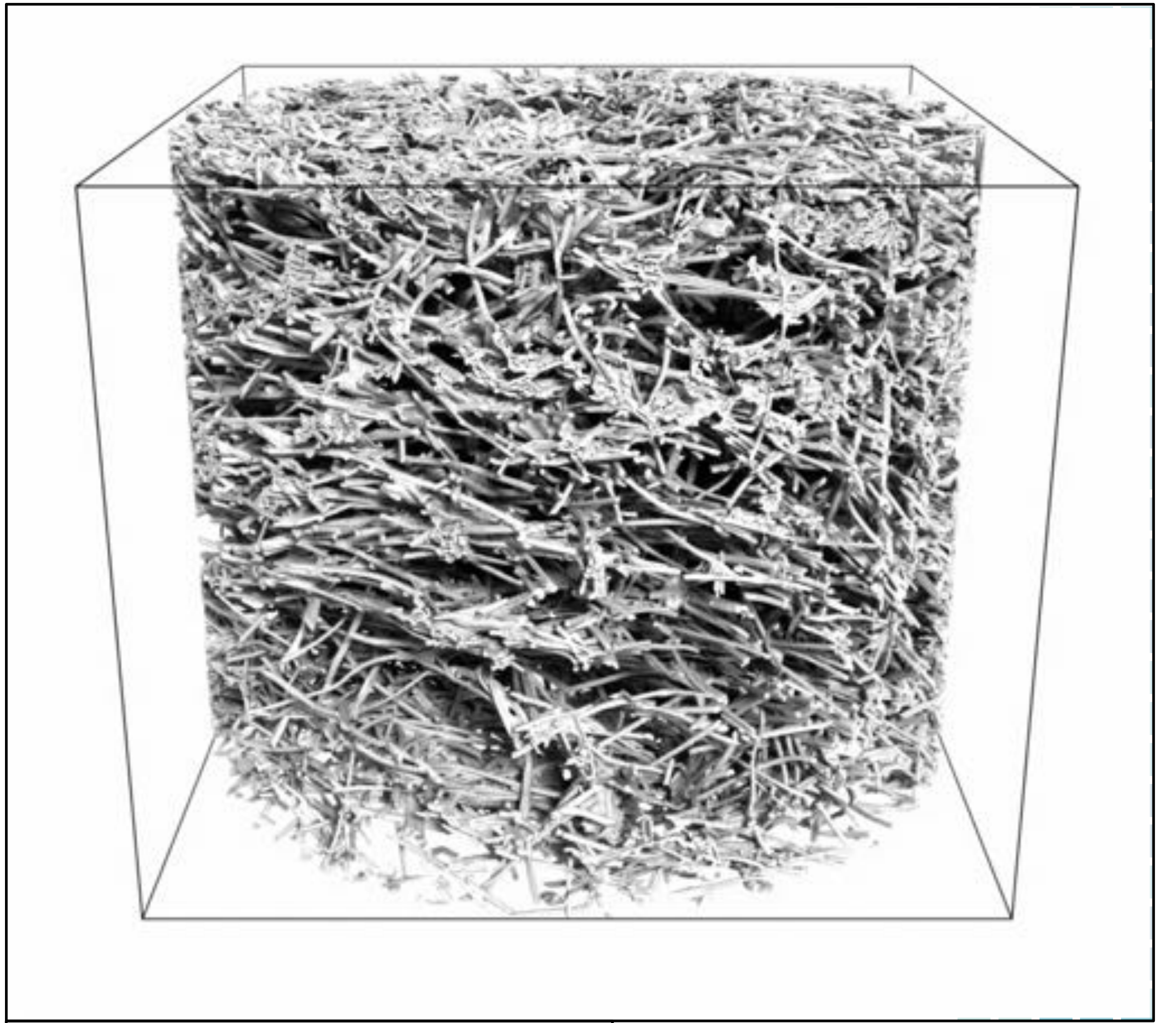
Use this series of images to reconstruct the 3D object



Courtesy of D. Parkinson (ALS)

Porous Microstructure Analysis (PuMA) software

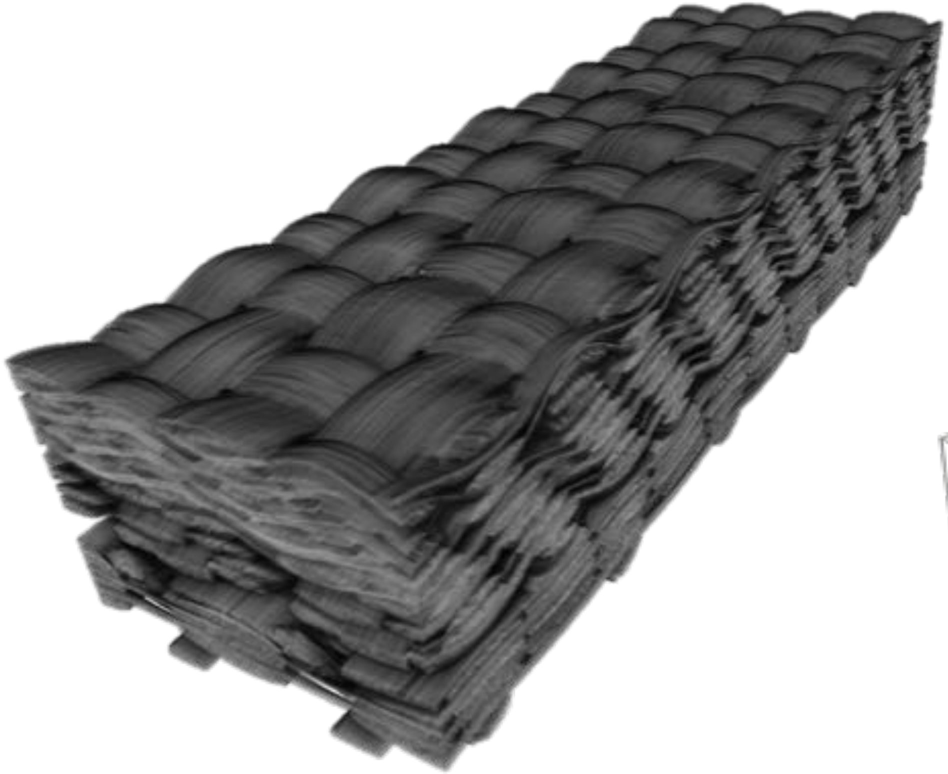



CT Reconstruction of FiberForm

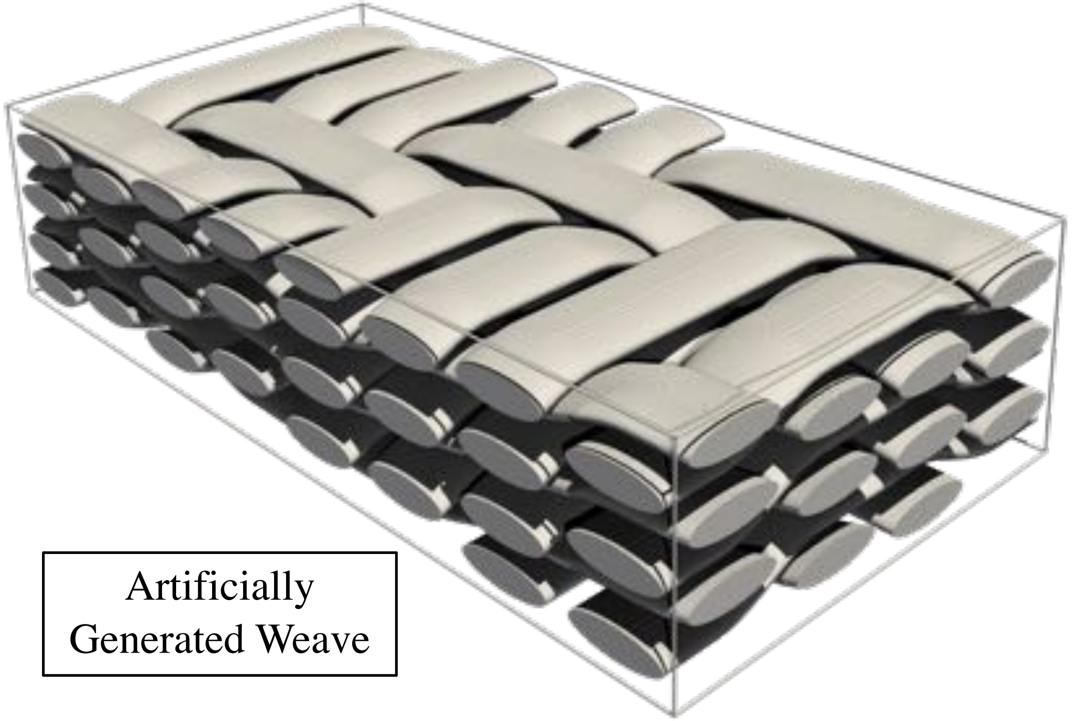
Ferguson, J. C., Panerai, F., Borner, A., & Mansour, N. N. (2018). PuMA: the Porous Microstructure Analysis software. *SoftwareX*, *7*, 81-87. https://software.nasa.gov/software/ARC-17920-1

Challenges in Micro-scale modeling

As NASA moves towards woven TPS materials, our modeling must adapt



12- ply real TPS weave

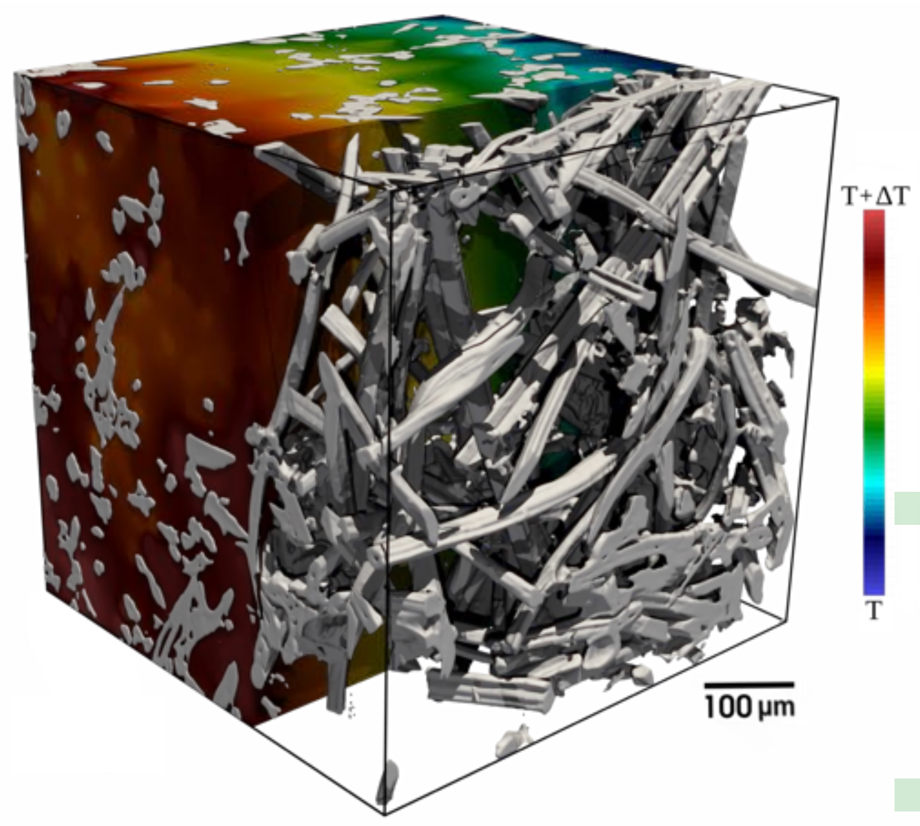


Artificially Generated Weave

Objectives

Formulate, implement and validate:

1. Finite Volume (FV) method to find the effective thermal conductivity due to anisotropic solid heat conduction
2. Ray Casting method for estimating the fiber orientation in CT reconstructions
3. Collision based Monte-Carlo method to find the View Factors (VF) to compute the effective radiative coefficient

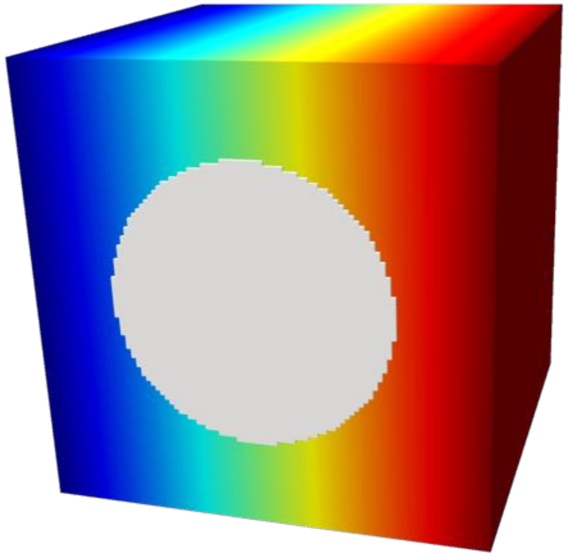


$$q = (k_c + k_r)\,\nabla T$$

SOLID HEAT CONDUCTION

Computing the effective thermal conductivity

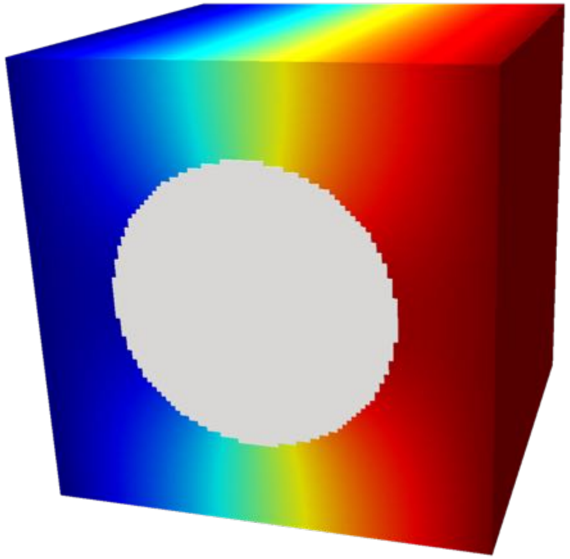
Impose initial linear
Temperature profile



$$T_{i,j,k} = \frac{i}{L_x}$$

CGLS →

Temperature converged
to Steady State



→

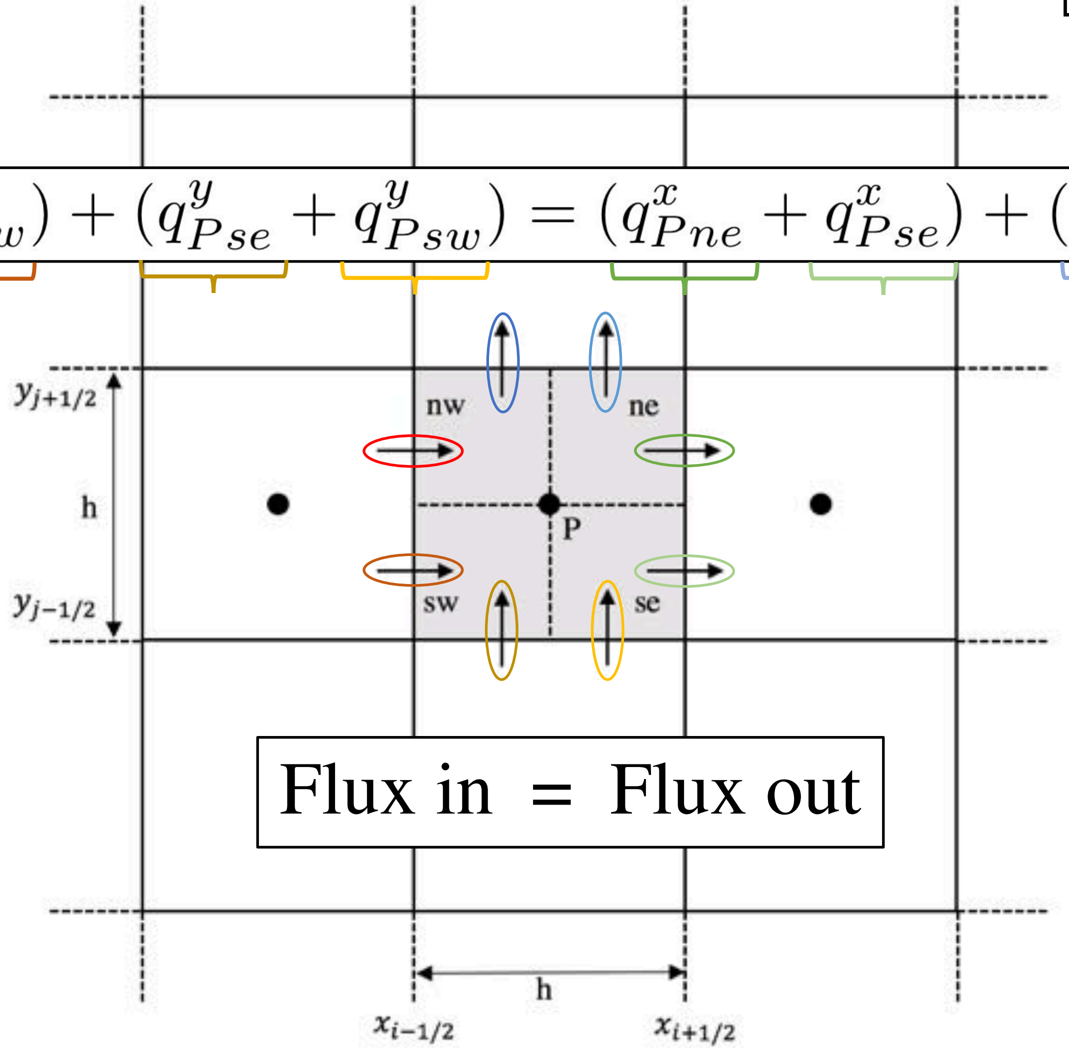
Compute Effective
Thermal Conductivity

$$k_{c,eff} = \boldsymbol{q}^x \cdot L_x$$

Finite Volume Method

$$\int_{z_{k-1/2}}^{z_{k+1/2}} \int_{y_{j-1/2}}^{y_{j+1/2}} \int_{x_{i-1/2}}^{x_{i+1/2}} \left(\frac{\partial q^x}{\partial x} + \frac{\partial q^y}{\partial y} + \frac{\partial q^z}{\partial z} \right) dxdydz = 0 \quad \text{where } \boldsymbol{q} = \begin{bmatrix} k^{xx} & k^{xy} & k^{xz} \\ k^{yx} & k^{yy} & k^{yz} \\ k^{zx} & k^{zy} & k^{zz} \end{bmatrix} \begin{bmatrix} \partial T/\partial x \\ \partial T/\partial y \\ \partial T/\partial z \end{bmatrix}$$

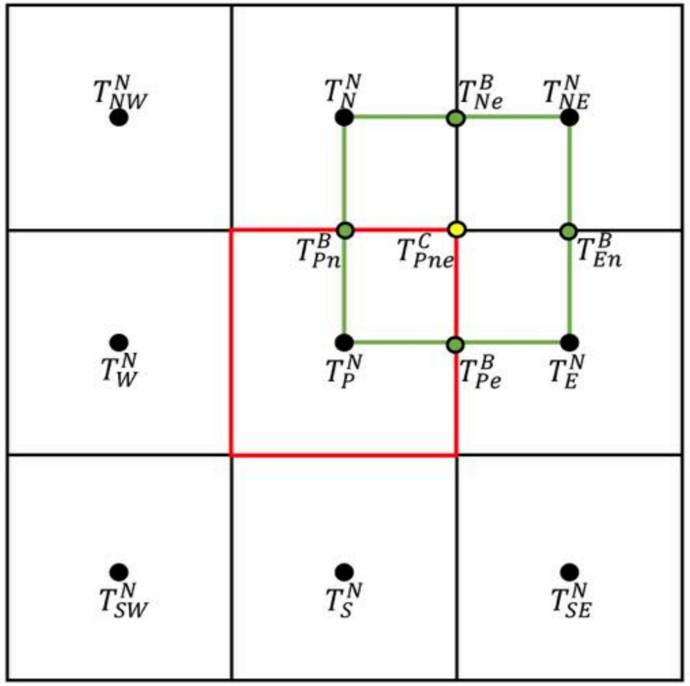
$$(q^x_{Pnw} + q^x_{Psw}) + (q^y_{Pse} + q^y_{Psw}) = (q^x_{Pne} + q^x_{Pse}) + (q^y_{Pne} + q^y_{Pnw})$$

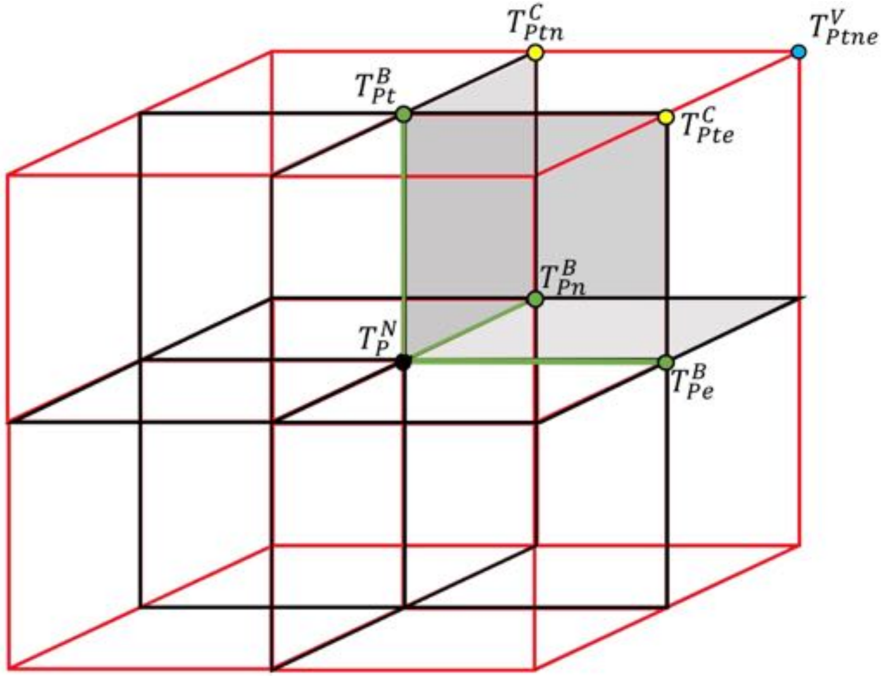


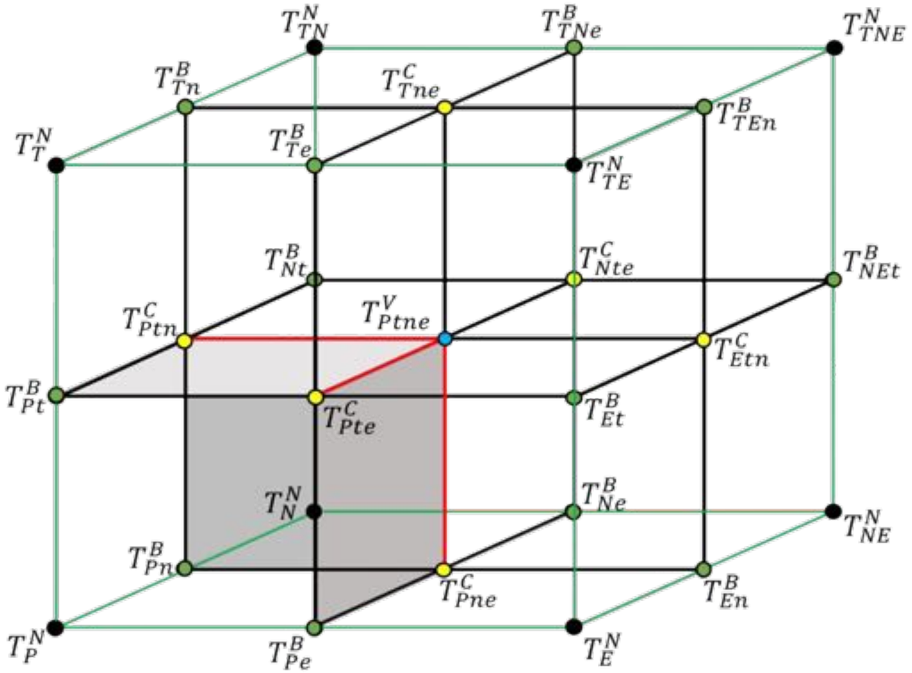
Flux in = Flux out

Multi-Point Flux Approximation (MPFA*)

- Integration carried out inside Control Volume (CV)
- Continuity of flux enforced inside Interaction Volume (IV)







*Ivar Aavatsmark. Multipoint flux approximation methods for quadrilateral grids. *9th International forum on reservoir simulation*, *Abu Dhabi*, pages 9–13, 2007.

Transmissibility Matrix

$$q^x_{Pne} = k^{xx}_P \overbrace{\frac{T^B_{Pe} - T^N_P}{h/2}} + k^{xy}_P \overbrace{\frac{T^B_{Pn} - T^N_P}{h/2}}$$

$$\boxed{q^x_{Pne}} = q^x_{Enw}$$

$$q^x_{Nse} = q^x_{NEsw}$$

$$q^y_{Pne} = q^y_{Nse}$$

$$q^y_{Enw} = q^y_{NEsw}$$

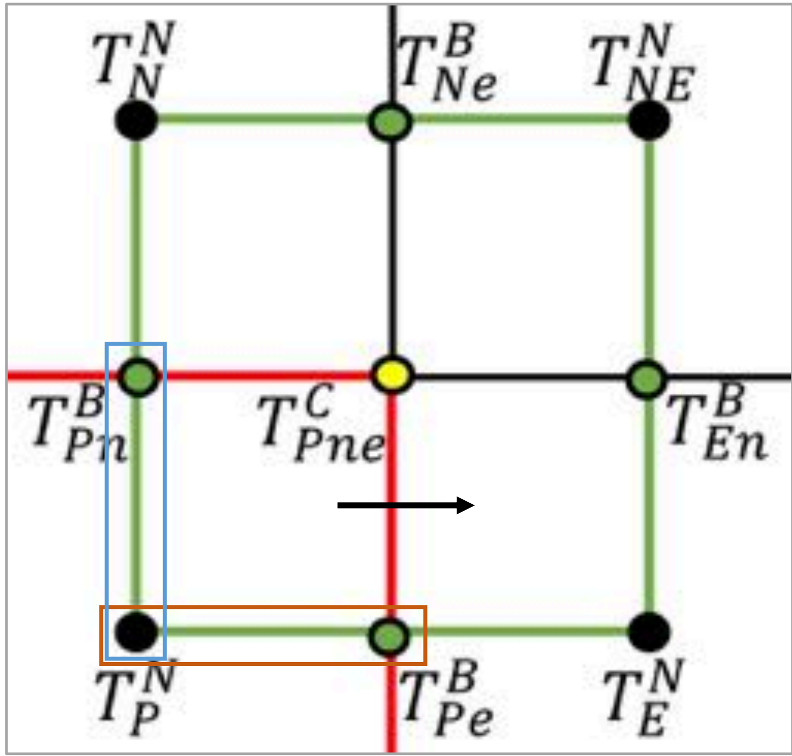
$$\boldsymbol{T}^N = \left[T^N_P, T^N_E, T^N_N, T^N_{NE}\right]^T \quad \boldsymbol{T}^B = \left[T^B_{Pe}, T^B_{Ne}, T^B_{Pn}, T^B_{En}\right]^T$$

$$\boldsymbol{q} = \boldsymbol{A}\,\boldsymbol{T}^B + \boldsymbol{B}\,\boldsymbol{T}^N$$

$$\boldsymbol{C}\,\boldsymbol{T}^B = \boldsymbol{D}\,\boldsymbol{T}^N \rightarrow \boldsymbol{T}^B = \boldsymbol{C}^{-1}\boldsymbol{D}\,\boldsymbol{T}^N$$

$$\boldsymbol{q} = \boldsymbol{E}\,\boldsymbol{T}^N \quad \text{where} \quad \boldsymbol{E} = \boldsymbol{B} + \boldsymbol{A}\boldsymbol{C}^{-1}\boldsymbol{D}$$

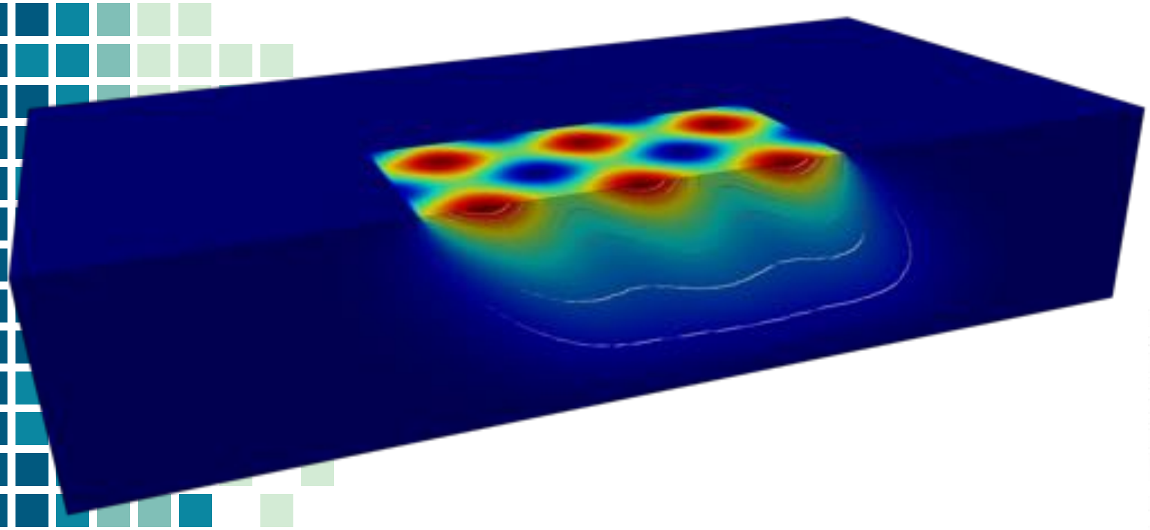
$$\boldsymbol{q}(x,t) = \boldsymbol{E}(x)\,\boldsymbol{T}^N(x,t)$$

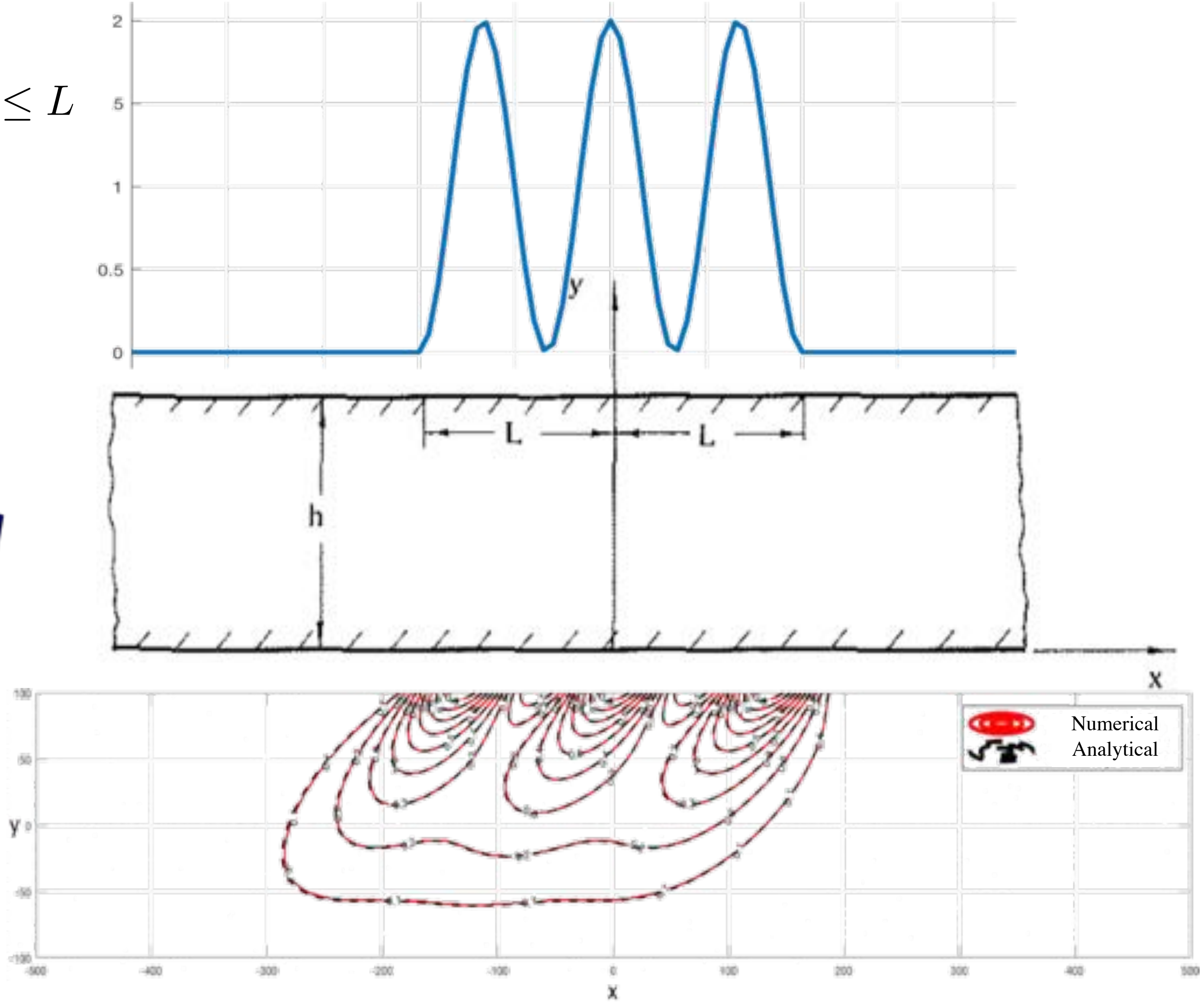


Analytical Case for Anisotropic sample

$$\boldsymbol{T}_{i,j,k} = \begin{cases} 0, & x < -L \\ \cos\left(3\pi/h\, x\right) + \cos\left(3\pi/h\, z\right), & -L \leq x \leq L \\ 0, & x > L \end{cases}$$

$$\boldsymbol{k}_{i,j,k} = \begin{bmatrix} 1 & 0.75 & 0.75 \\ 0.75 & 1 & 0.75 \\ 0.75 & 0.75 & 1 \end{bmatrix}$$

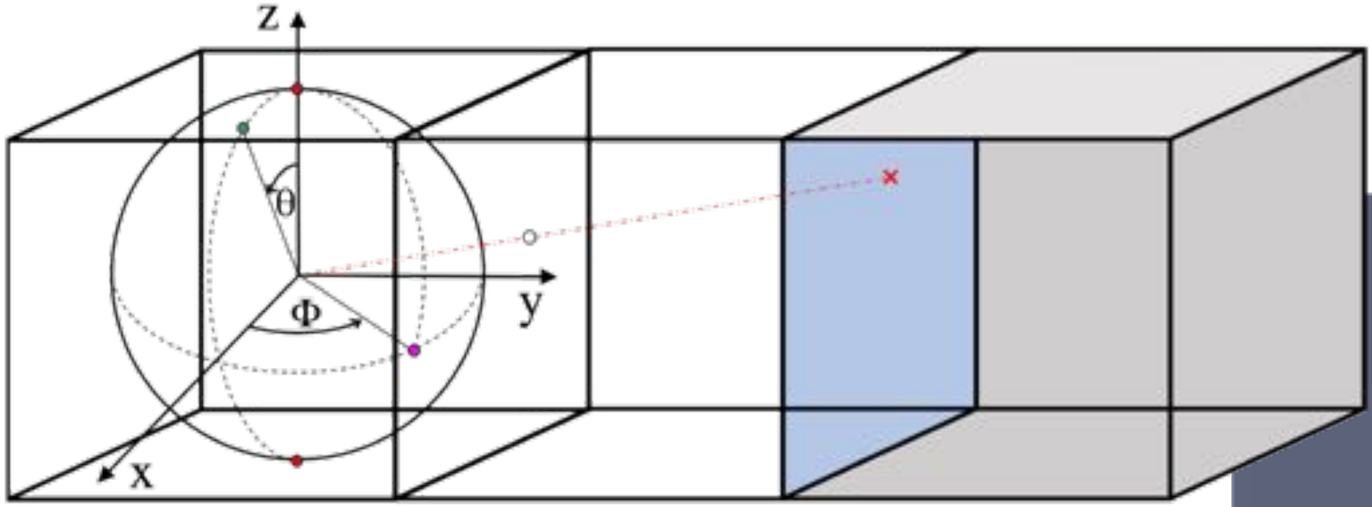


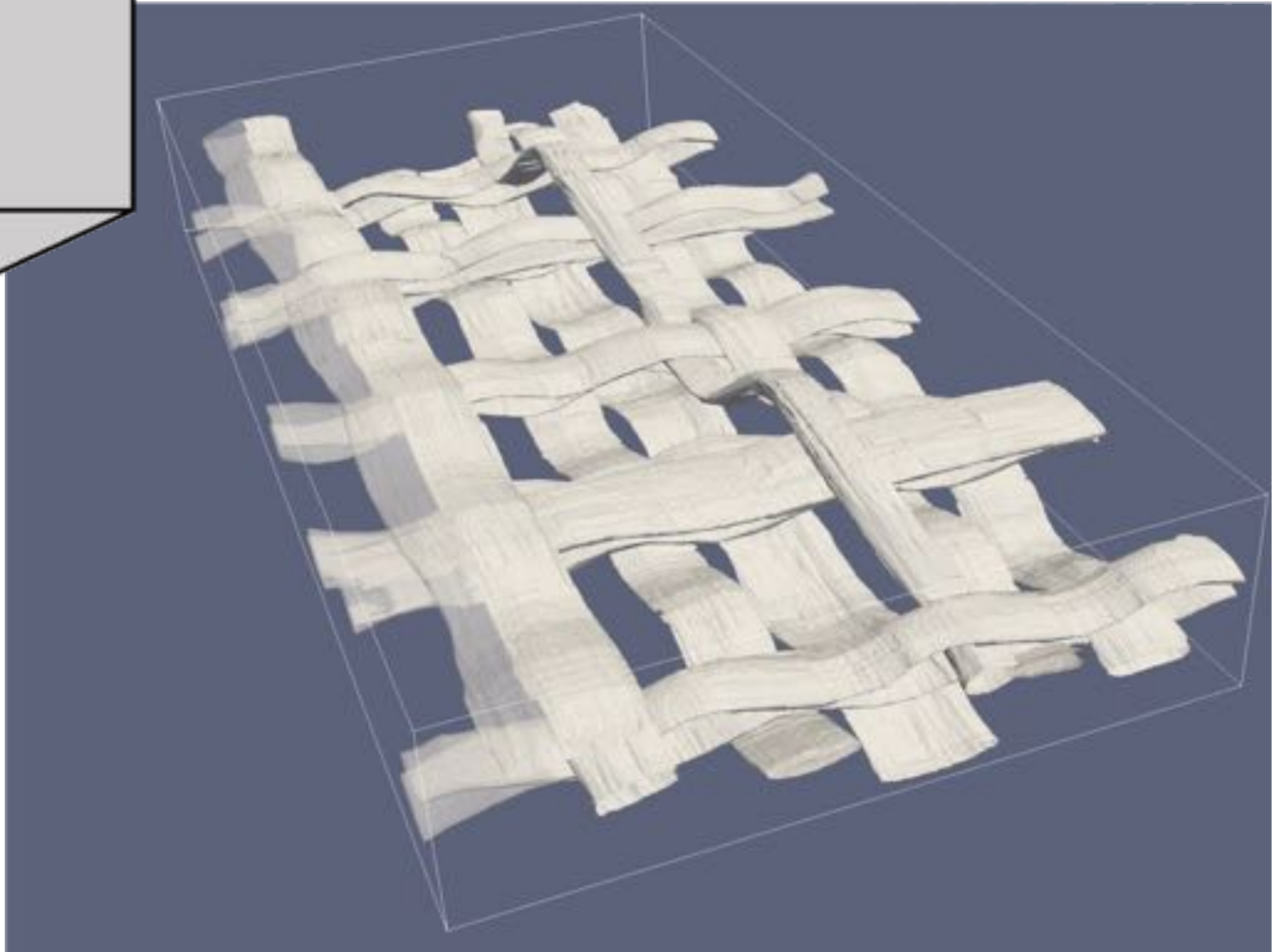


Zhang Xiangzhou, Steady-State Temperatures in an Anisotropic Strip, *ASME Journal of Heat Transfer,* 112(1), pp. 16-20.

FIBER ORIENTATION

Ray Casting Method



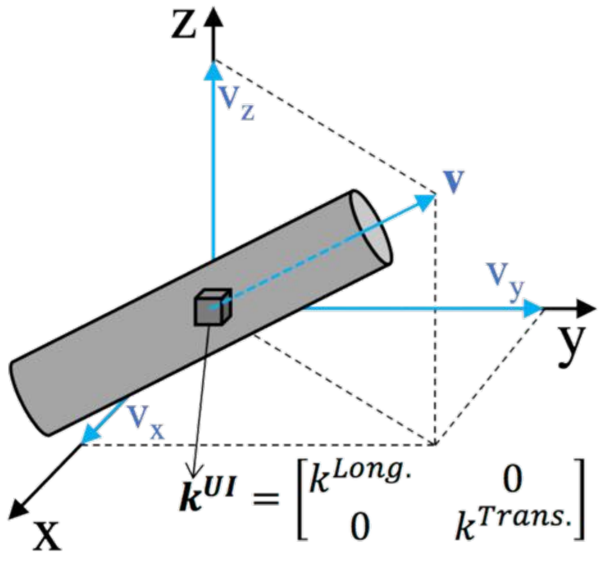


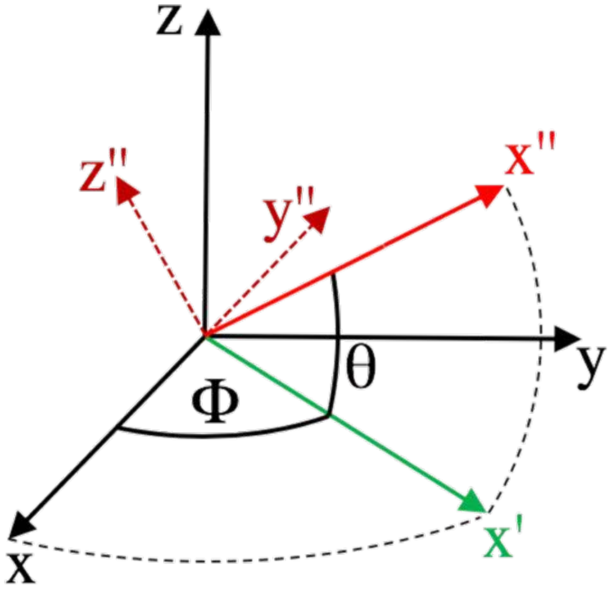
$$\theta \in [0, 180^\circ) \qquad \phi \in [0, 360^\circ)$$

$$N = \left(\frac{180^\circ}{d\psi} - 1\right)\left(\frac{360^\circ}{d\psi}\right) + 2$$

$$\boldsymbol{v} = v_x \boldsymbol{i} + v_y \boldsymbol{j} + v_z \boldsymbol{k}$$

Conductivity Tensor Rotation





$$\boldsymbol{v} = v_x \boldsymbol{i} + v_y \boldsymbol{j} + v_z \boldsymbol{k}$$

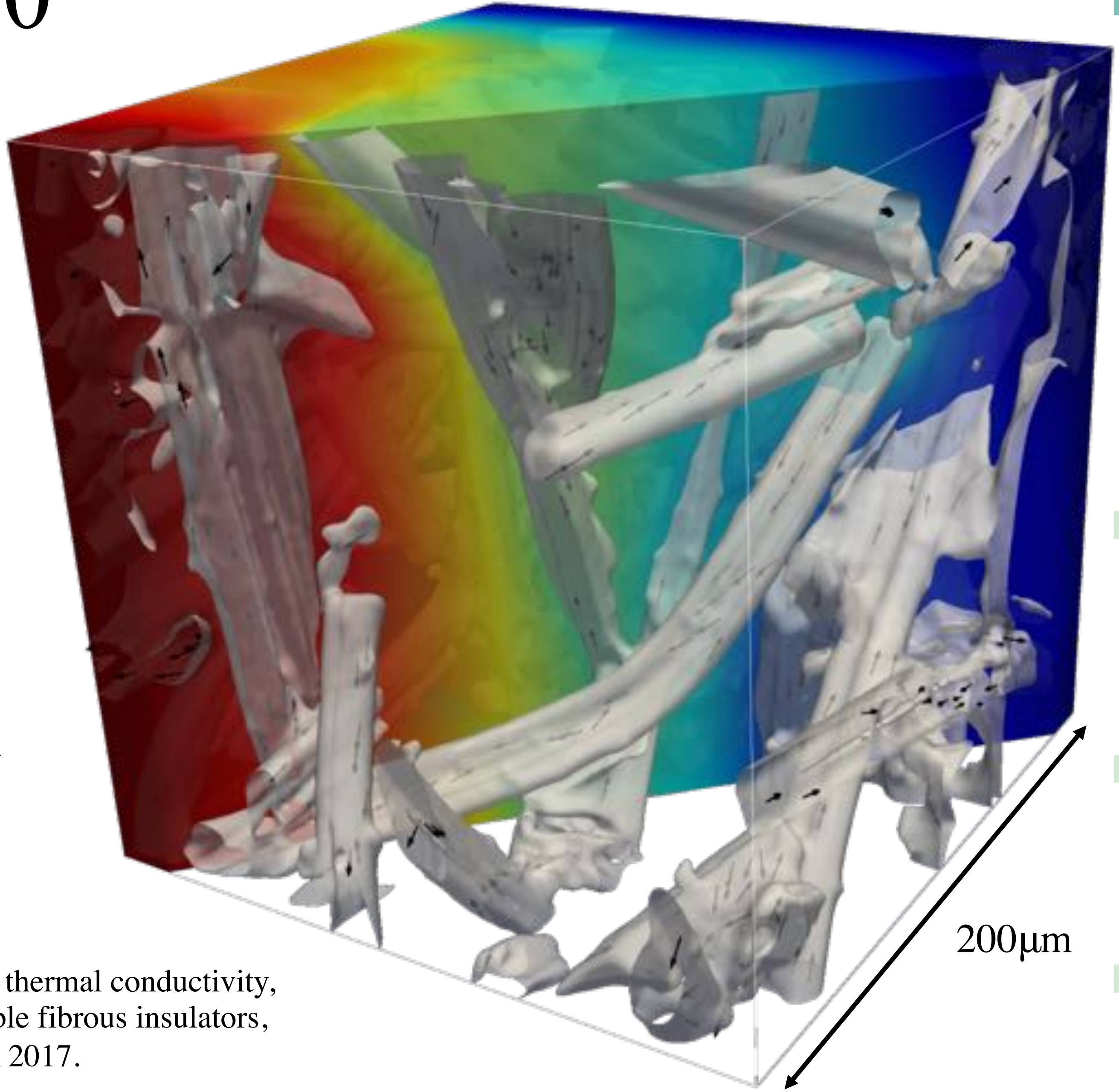
$$\boldsymbol{k}'' = \begin{bmatrix} k^{Long.} & 0 & 0 \\ 0 & k^{Trans.} & 0 \\ 0 & 0 & k^{Trans.} \end{bmatrix}$$

$$\theta = \arcsin v_z \quad \phi = \arctan \frac{v_y}{v_x}$$

$$\boldsymbol{q} = \underbrace{\left[\boldsymbol{R}^{-1} \boldsymbol{k}'' \boldsymbol{R}\right]}_{\boldsymbol{k}} \nabla T$$

$$\boldsymbol{R} = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

FiberForm 800^3

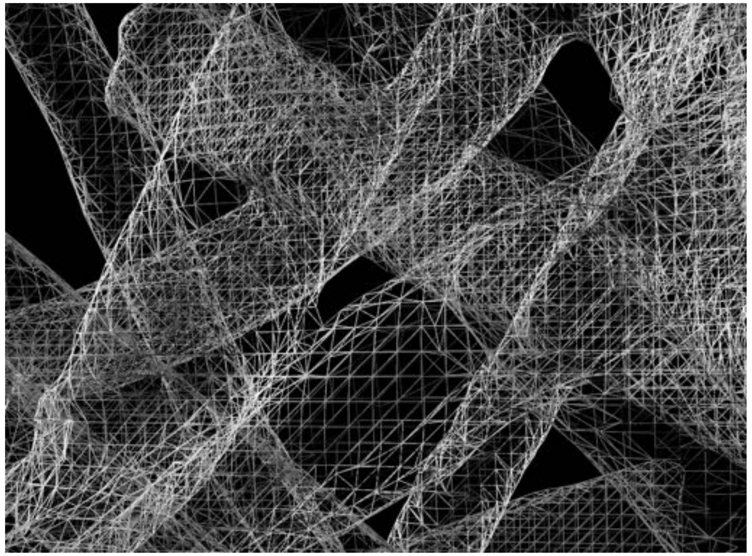


$$\boldsymbol{k}^{UI} = \begin{bmatrix} 12 & 0 \\ 0 & 1.2 \end{bmatrix} W/mK$$

F. Panerai et al., Micro-tomography based analysis of thermal conductivity, diffusivity and oxidation behavious of rigid and flexible fibrous insulators, *Int. Journal of Heat and Mass Transfer,* 108-801-811, 2017.

RADIATIVE HEAT TRANSFER

Radiation Model



- Marching Cubes for surface triangulation
- The total heat flux is computed by iteratively solving the sparse coupled linear system:

$$\underbrace{J_i}_{\text{Total radiation from surface } i} = \underbrace{\epsilon_i \sigma T_i^4}_{\text{Emitted radiation}} + \underbrace{(1-\epsilon_i)\sum_j \boxed{F_{ij}} J_j}_{\text{Reflected radiation}}$$

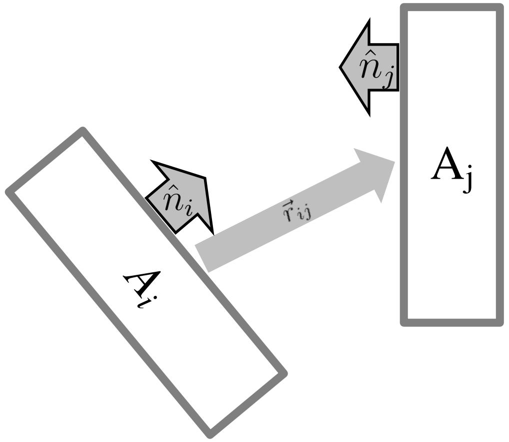
T_i: Temperature of surface i

ϵ_i: Emissivity of surface i

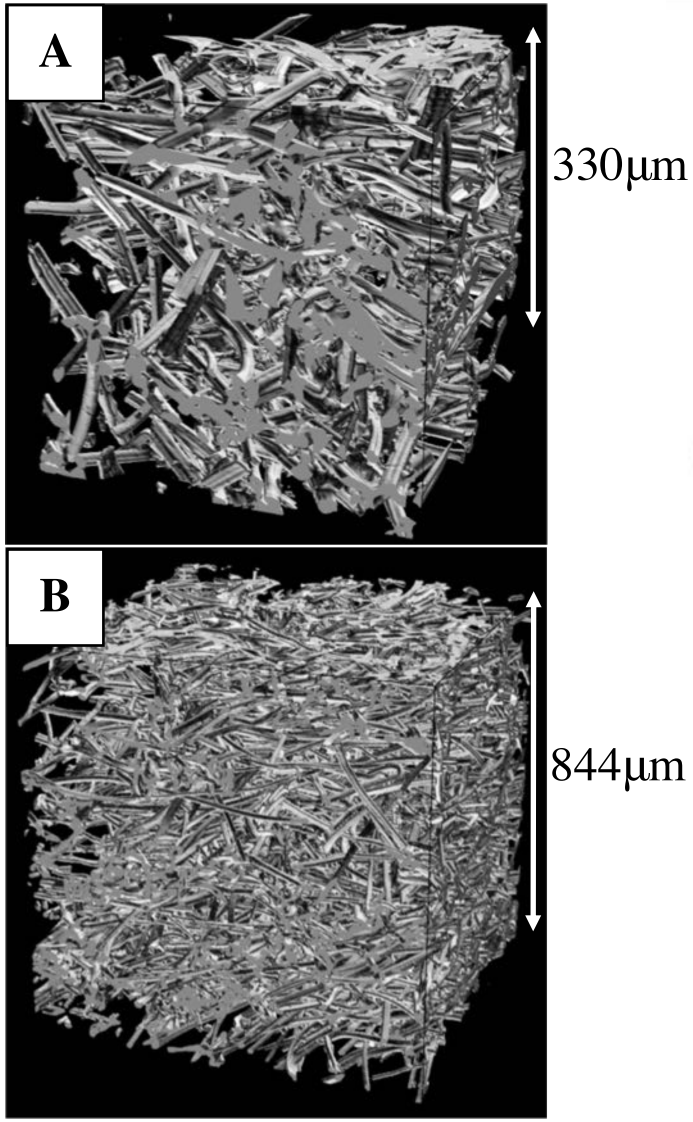
F_{ij}: View Factor, fraction of radiation from surface j reaching surface i)

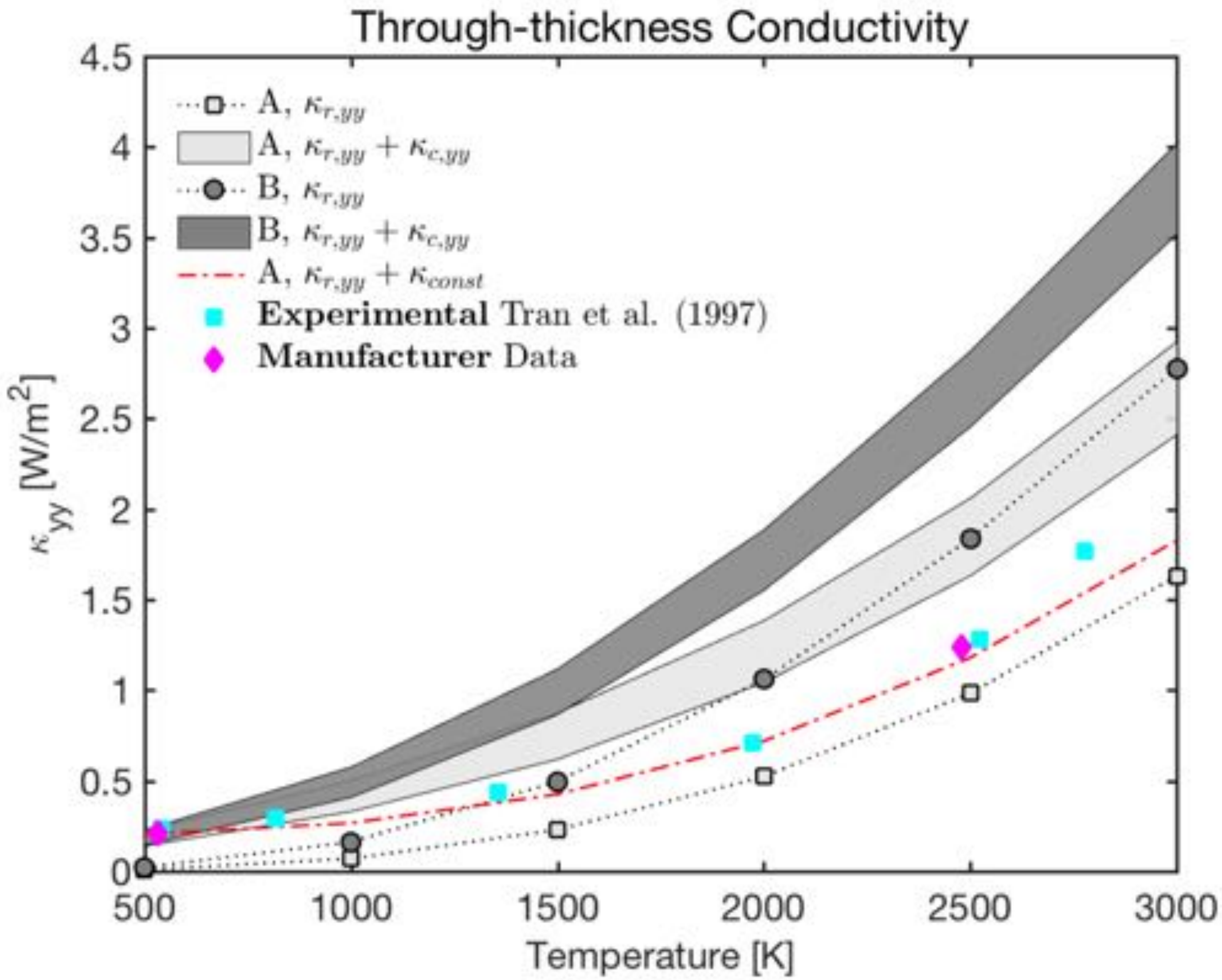
- View Factors are determined by projecting rays from each surface as:

$$F_{ij} = \frac{1}{A_i}\int_{A_i}\int_{A_j}\frac{|\hat{n}_i \cdot \hat{r}_{ij}||\hat{n}_j \cdot \hat{r}_{ij}|}{|\vec{r}_{ij}|^2} dA_j dA_i$$



Heat Transfer Calculation





Thank you