Satellite Conjunction "Probability," "Plausibility," and "Possibility": A Categorization of Competing Satellite Conjunction Assessment Risk Analysis Paradigms

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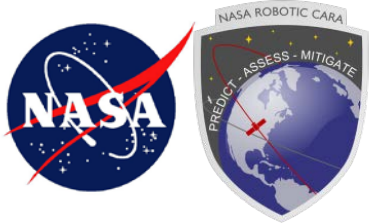
Agenda

- **Introduction and definition of categorization terms**
  - Probability, Plausibility, and Possibility (imperative, subjunctive, optative)
- **Categorization supporting/amplifying considerations**
  - Fundamental question, null hypothesis, and required supporting data
- **Probabilistic techniques**
  - Vanilla Pc calculation, Wald sequential probability ratio, Pc Uncertainty
- **Plausibilistic techniques**
  - Pc Sensitivity, Maximum Pc
- **Possibilistic techniques**
  - Ellipsoid “overlap”
- **Categorization summary and observations**
- **Conclusions and future work**



Introduction

- **Number of different conjunction assessment risk analysis methodologies/parameters proposed in critical literature**
  - Proposed over a long period (decades)
  - Proposed episodically and often in isolation
- **Approaches have different foundational CA philosophies**
  - CA practitioners must choose methods that agree with philosophical alignment
- **What is actually needed is development of full CA philosophy**
  - Large project for another day
- **However, can substantially assist development of philosophy by providing categorization of existing major risk analysis methods**
  - Tease out different features
  - Identify conceptual points of difference
  - Develop vocabulary to discuss different approaches (and potential future approaches) meaningfully

Linguistic Mood and Conjunction Assessment

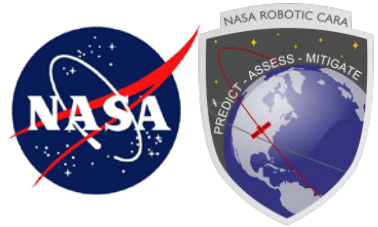
- **Indo-European languages developed three linguistic moods in order to specify the level of certainty of described event/circumstance**
  - Indicative mood: modality of certainty; indicates that something will occur
  - Subjunctive mood: modality of less certainty; indicates that something may occur
  - Optative mood: modality of potentiality; indicates that something could occur
- **Examining risk analysis methods suggests applying same categories**
  - Indicative: (ostensible) precise calculation of collision likelihood
    - "Probability"
  - Subjunctive: considers additional errors and "likelihood of likelihoods"
    - "Plausibility"
  - Optative: establishes or negates mere possibility of collision event
    - "Possibility"
- **Use of Probability/Plausibility/Possibility nomenclature may not be an improvement**
- **Actual situation more a spectrum from Probability to Possibility**

Probability/Plausibility/Possibility: Notional Definitions

- **Probability**
  - Determination of (ostensible) actual probability that a serious close approach event will take place
  - Dispute in literature regarding whether these constitute actual probabilities
    - But intent is that they be probabilities, so reasonable category to apply
- **Possibility**
  - Determination whether a collision is simply possible
  - Implication is that mitigation action would be pursued even for mere possibility
  - Statistics involved here, as some threshold for possibility required
- **Plausibility**
  - Term for approaches that seem to stand between probability and possibility
  - Often a probabilistic calculation of sorts that attempts to take account of additional uncertainties in input data
    - Can be more rigorously probabilistic or more speculative and notional

Risk Analysis Fundamental Question

- **Decision processes should be guided by a fundamental question**
- **Should get at the core issue**
  - "Will the two satellites collide" is God's eye truth and cannot be known, so one has to proceed statistically; fundamental question should be statistical as well
- **Must not be too general or too specific**
  - Otherwise does not illuminate the decision process
- **Needs to focus on decision nexus**
  - For CA risk assessment, question is whether to pursue a mitigation action
- **Needs to make evident the "default" position with regard to the basic decision**
  - When the data are ambiguous or not definitive, should the decision favor or refrain from a mitigation action



The Null Hypothesis

- **CA risk analysis bears outward similarity to hypothesis testing**
  - Employs a conjunction risk test statistic
  - Test statistic compared to a threshold, which defines a critical region
  - Strength of test greater the further test statistic pushes into critical region
  - Can define confidence interval on test statistic, to determine strength of test with more precision
- **Degree of similarity between CA risk analysis and classic hypothesis testing presently an open question**
  - Subject of dispute in current critical literature
    - But many tenets of hypothesis testing presently disputed within statistical sciences
  - CA operators often do not conceive of their activities explicitly in these terms
- **However, analogy strong enough that is useful as point of description to aid risk assessment technique characterization**

Individual RA Techniques: Vanilla Pc Calculation (1 of 2)

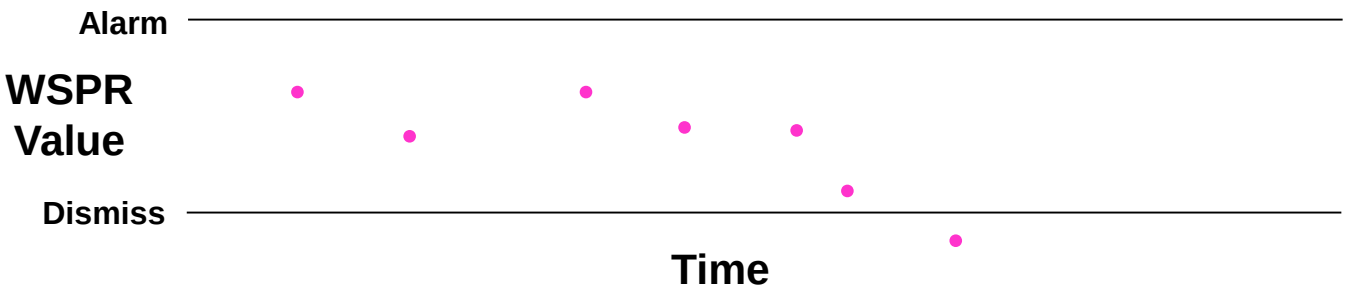
- **Calculated by one of a number of techniques**
  - 2- or 3-D analytical; Monte Carlo; even some geometric proposals
- **Pc represents the likelihood that the actual miss distance will fall within the hard-body radius circle**
- **Pc value compared to a mitigation threshold**
  - If Pc value greater than threshold, typically mitigation action pursued
- **Problem: "dilution region"**
  - Two ways Pc can be low
    - Satellite positions known with precision, covariances small, very little likelihood that actual miss distance will fall within HBR
    - Satellite positions known only poorly, covariances large, range of possible miss distances so large that little likelihood that actual miss distance will fall within HBR
  - Latter case is called "dilution region"
    - In such a situation, low Pc does not provide evidence that situation is safe

Individual RA Techniques: Vanilla Pc Calculation (2 of 2)

- **If Pc is only risk analysis parameter, then particular, endemic default conditions emerge**
  - If outside of dilution region, calculation is robust; compare Pc to threshold
  - If in dilution region and Pc high, situation is high-risk; compare Pc to threshold
  - If in dilution region and Pc low, cannot definitively conclude situation is safe, but can justify refraining from mitigation action
    - According to modeling, most conjunctions that satellites encounter are with untracked debris; no way to mitigate, so just accept as background risk
    - Dilution region events are very similar to this (a little bit known about positions, but too little to conclude definitively that an unsafe situation exists)
    - These events therefore also treated as a portion of background risk that is simply accepted as a condition of operating satellites
- **Given the above, propose following fundamental question**
  - Do the presented data justify a decision to mitigate the conjunction?
- **Associated null hypothesis**
  - The actual miss distance is greater than the HBR

Individual RA Techniques: WSPRT (1 of 2)

- **WSPRT = Wald Sequential Probability Ratio Test**
- **Computes ratio between the current collision likelihood and that which the two objects typically experience, apart from this event**
  - Ratio calculable from Pc values and $P_{C|0}$, the two-object background risk
- **User-defined probabilities of false alarm and missed detection allow definition of alarm (mitigate) and dismiss (ignore) conditions**
  - Also includes third possibility between these, which is to wait for more data
- **Ratio evaluated sequentially with each information update**
  - Time-series situation shown below

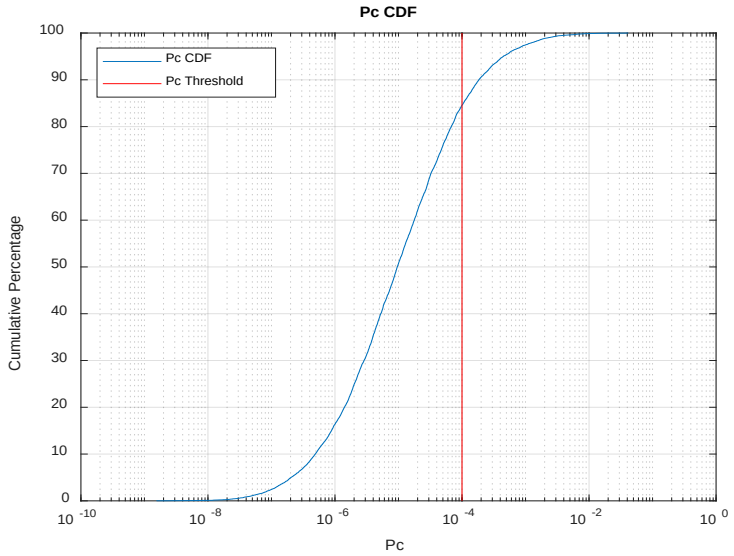


Individual RA Techniques: WSPRT (2 of 2)

- **In some ways, fundamental question and null hypothesis not relevant to WSPRT because these concepts baked into construct**
  - By nature treats alarm, dismissal, and ambiguous situation innately
  - Thus intrinsically directed to CA decision, and no null hypothesis needed
- **However, actual implementation in CA construct forces modification**
  - Mitigation decision must be made by satellite maneuver commitment point
  - If WSPRT between thresholds at this point, cannot wait longer and must decide
- **Could potentially accommodate opposite forms of default position**
  - Intra-constellation use of WSPRT embraced mitigation as default
  - Satellite-catalogue-based use refrained from mitigation as default
  - This particular technique thus more fungible than some others
- **For now, will assign fundamental question used for full catalogue**
  - Do present data and background risk analysis justify a mitigation action?
- **Null hypothesis**
  - The actual miss distance is greater than the HBR

Individual RA Techniques: CARA Pc Uncertainty (1 of 2)

- **18 SPCS covariances known not always to be fully representative**
- **Historical state and covariance data can be analyzed against reference orbits for every satellite to develop covariance corrections**
- **PDFs of these corrections can be used to generate corrected family of secondary covariances, which can then be used to calculate Pcs**
- **Output produces PDF of Pc values**
  - Mitigation action sought if enough of Pc probability density over threshold



Individual RA Techniques: CARA Pc Uncertainty (2 of 2)

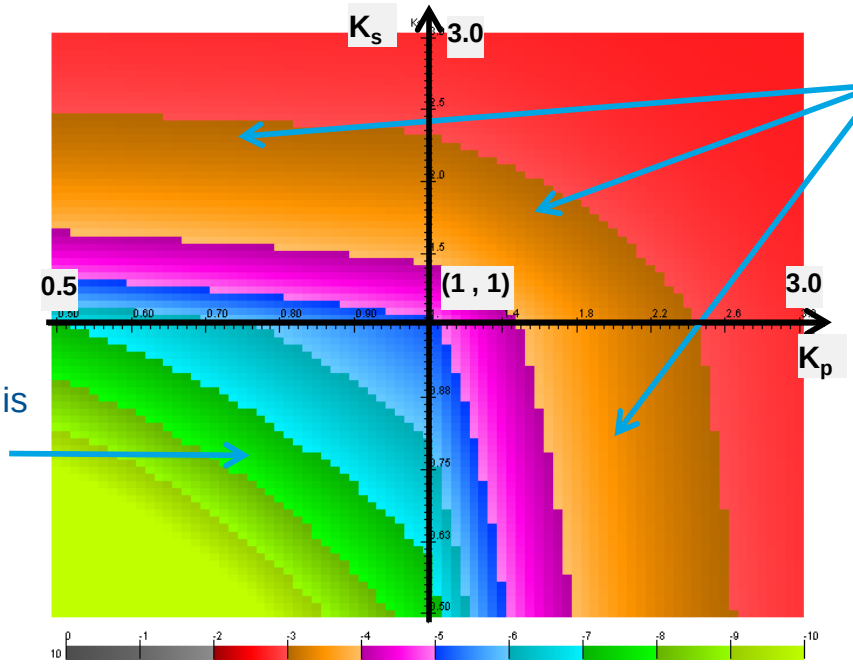
- **Takes some steps toward a plausibilistic construct**
  - Considers what range of Pc values would be if current secondary's covariance suffered from same levels and frequencies of irrealism as in past
- **However, level of speculation kept to a minimum**
  - Actual frequency of each covariance modification known, so rigorous PDF of covariance alternations constructed
  - This allows actual PDF/CDF of expected Pc values
- **Fundamental question should recognize this pedigree, but only slight broadening of that for vanilla Pc calculation needed**
  - Given the current data and historical covariance realism information, does the Pc range of values justify a decision to mitigate?
- **Null hypothesis is the same as for vanilla Pc**
  - The actual miss distance is greater than the HBR

Individual RA Techniques: Pc Sensitivity (1 of 3)

- **Same foundational concept as Pc Uncertainty**
  - Attempt to account for covariance irrealism
  - However, no access to broad set of covariance realism data to construct PDF
- **Instead, uses interval analysis**
  - Analytical/heuristic effort to establish scale factor boundaries for covariances
    - E.g., covariance can be expected to be between 0.25 to 4 times representative size
  - Presume uniform distribution between these boundaries
- **Apply range of scale factors to both primary and secondary covariances in gridded fashion, and calculate Pcs**
- **If any Pc exceeds set threshold, then take as alert condition and consider mitigation action**

Individual RA Techniques: Pc Sensitivity (2 of 3)

Example of display: Kp in [0.5 ; 3.] and Ks in [0.5 ; 3.]



If Primary's and Secondary's covariance are pessimistic the risk is over-estimated

If Primary's or Secondary's covariance is optimistic the risk is under-estimated

Figure from Laporte (2018)

- **In above plot, primary scaling on x-axis; secondary on y-axis**
- **If any color above threshold (perhaps brown or red), potentially risky situation**

Individual RA Techniques: Pc Sensitivity (3 of 3)

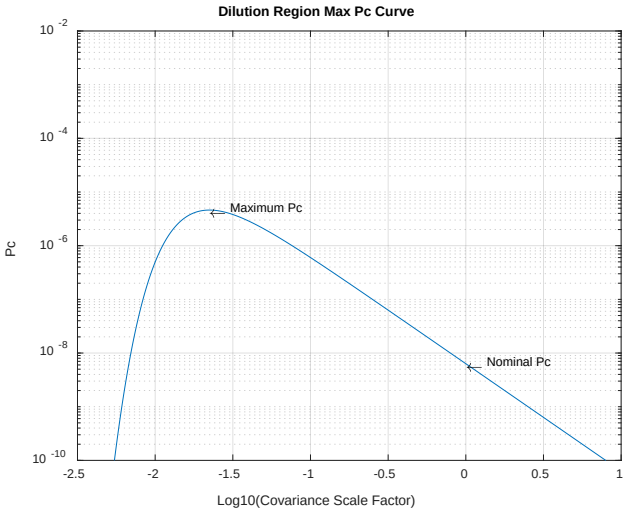
- **Pc Sensitivity journeys further into plausibility**
  - Considers range of Pc values, like Pc Uncertainty
  - However, (somewhat) loosely establishes scale factor range and merely presumes a PDF (uniform distribution), so overall more speculative
- **Technique does not suggest particular default position**
  - CNES implementation, however, seems to favor similarity with vanilla Pc
    - Overall threshold made more lenient from vanilla Pc situation, but even single Pc above modified threshold will trigger mitigation action consideration
    - Leans away from pushing ambiguous situation to mitigation because only one Pc violation needed to require it—reasonably easy to get null hypothesis rejection
- **Proposed fundamental question**
  - Given the current data and covariance realism assumptions, does the Pc range of values justify a decision to mitigate?
- **Proposed null hypothesis**
  - The actual miss distance is greater than the HBR

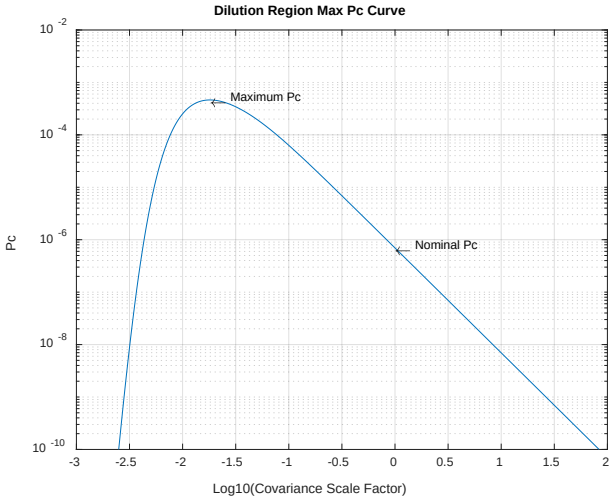
Individual RA Techniques: Maximum Pc (1 of 3)

- **Technique originally constructed to address issue of dilution region**
  - If in non-dilution region, use covariances as submitted
  - If in dilution region
    - Freeze miss distance
    - Contract secondary covariance incrementally, maintaining aspect ratio, and calculate Pc at each contraction step
    - Find maximum Pc value and if above threshold, pursue mitigation action
- **Essentially moves along P_c curve by varying covariance for given miss distance until maximum P_c found for covariance smaller than or equal to given value**
  - Grounding claim is that more data would produce a smaller covariance, and could to the max value of P_c
- **Similar to Pc Sensitivity approach, but with scaling factors limited to range of 0 to 1**

Individual RA Techniques: Maximum Pc (2 of 3)

- **Top graph example event**
  - Nominal Pc of 6.3E-09; maximum Pc (peak of curve) 4.6E-06
  - "True" Pc (which could have been obtained with more adequate data) could be as high as 4.6E-06
  - Below threshold of 1E-04, so can conclude that this dilution region event not dangerous
- **Bottom graph example event**
  - Nominal Pc of 6.9E-07; maximum Pc (peak of curve) 4.6E-04
  - Here, maximum Pc above threshold, so situation remains inconclusive





Individual RA Techniques: Maximum Pc (3 of 3)

- **Approach operates even more strongly in realm of plausibility**
  - Constraining assumptions about (secondary or joint) covariance relaxed to aspect ratio only
- **Technique promotes a different default response**
  - Constructed to demonstrate when a safe situation exists; ambiguous otherwise
  - Thus naturally fits a null hypothesis that favors mitigation
    - Can reject this null hypothesis when safety criteria established
- **Proposed fundamental question**
  - Given the data and assumptions regarding possible values of the covariance, does the maximum Pc value justify dismissal of the event?
- **Proposed null hypothesis**
  - The miss distance is less than the HBR

Individual RA Techniques: "Overlapping" Ellipsoids (1 of 2)

- **Again, motivated by dilution region phenomenon for vanilla Pc**
- **To ensure a safe situation, defines a maximum "overlap" between primary and secondary covariance ellipsoids**
  - First approach: requires that a certain sigma level of joint covariance remain smaller than nominal miss distance by HBR length
  - Second approach: requires that a certain sigma-level of primary and secondary covariance ellipsoids be separated by HBR length
  - Finally, requires mitigation actions to enforce these overlap distances
- **Still a statistical method certainly, but notably more conservative than Pc approaches**
  - With dilution region, increased uncertainty leads to lower likelihood and less frequency of mitigation action
  - With ellipse overlap approach, increased uncertainty leads to larger required separation distances and greater frequency of mitigation action

Individual RA Techniques: "Overlapping" Ellipsoids (2 of 2)

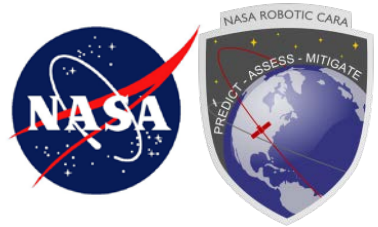
- **Technique focuses on limiting mere possibility of conjunction**
  - Much more demanding safety standard
- **Default action is to mitigate**
  - To guarantee safety, must be prepared to mitigate unless conservative safety standard met
- **Fundamental question simply restatement of this requirement**
  - Do the presented data rule out the possibility of a collision?
- **Different formulations of null hypothesis possible**
  - Miss distance less than HBR—standard form used here so far
  - Covariance ellipsoids overlap to a non-discountable degree—more specific to particular technique used here

RA Techniques Categorization: Summary Table

	Technique	Fundamental Question	Null Hypothesis	Data Required
Probability ↑	Vanilla Pc Calculation	Do the presented data justify a decision to mitigate the conjunction?	The actual miss distance is greater than the hard-body radius	Immediate CDM
	Wald Sequential Probability Ratio Test	*Do the presented data and background risk analysis justify a decision to mitigate the conjunction?*	*The actual miss distance is greater than the hard-body radius*	Immediate CDM + background risk between primary and secondary
	CARA Pc Uncertainty	Given the current data and historical covariance realism information, does the Pc range of values justify a decision to mitigate?	The actual miss distance is greater than the hard-body radius	Immediate CDM + large historical archive of by-object covariance realism data
	Pc Sensitivity	*Given the current data and covariance realism assumptions, do the Pc range of values justify a decision to mitigate?*	*The actual miss distance is greater than the hard-body radius*	Immediate CDM + scale-factor end-points for primary and secondary covariance
	Maximum Pc	Given the data and assumptions regarding possible values of the covariance, does the maximum Pc value justify dismissal of the event?	The actual miss distance is less than the hard-body radius	Immediate CDM + expected covariance aspect ratio
Possibility ↓	Ellipse Overlap	Do the data rule out the possibility of a collision?	The covariance ellipses overlap to a non-discountable degree	Immediate CDM

Discussion/Observations

- **Different major risk analysis techniques span full spectrum**
  - Three probabilistic, two plausibilistic, one possibilistic, although more of a continuum than discrete binning, and bin titles not necessarily all that helpful
- **Some techniques favor particular fundamental question and null hypothesis; others less so**
  - Vanilla Pc and ellipse overlap strongly directed to particular operational approach and use
  - WSPRT and Pc Sensitivity seem most flexible
- **In midst of spectrum (middle of plausibility bin), null hypothesis flips from refraining from mitigation to presuming it**
  - Not a surprise, given general trend from more permissive to more conservative
- **Techniques at extremes of spectrum require least amount of conjunction information**
  - Middle-of-spectrum approaches, which try to expand safety without becoming overly conservative, require additional information for such an expansion



Conclusions and Future Work

- **Wide range of risk analysis techniques extant in literature, with full swath of risk-tolerant to risk-adverse orientations**
- **Categorization provides vocabulary to allow fruitful comparative discussion of approaches**
- **Categories and associated features also can help current CA practitioners choose risk analysis approaches that suit their particular operational philosophy**
- **Categorization results can provide building blocks for overarching CA philosophy**
  - To be developed by CARA as part of CA Handbook initiative