

Does orbital angular momentum have effect on laser's scattering by molecular atmosphere?

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Highlights

1. Scattering of laser with orbital angular momentum (OAM) by small particles is studied.
2. The 3D CPML FDTD technique is applied in the solution of OAM laser beam scattering.
3. The OAM has no effect on laser's scattering by molecular atmosphere.

Abstract: Lasers with orbital angular momentum (OAM) have potential applications in communication technology, manipulation of particles, and remote sensing. Because of its unusual light-scattering properties, the OAM laser's interaction with a molecular atmosphere must be studied to ensure that it is not lossy for communication or remote-sensing applications that involve its transmission through an atmospheric environment. In this study, the finite-difference time-domain (FDTD) method [21] is applied to calculate the light scattering of the purely azimuthal (the radial mode number is assumed to be zero) Laguerre-Gaussian (LG) beams with OAM by very small dielectric particles. Not like Lorentz-Mie solutions, the FDTD method can calculate for particles off the central axis of the LG beam. It is found that when the particles are very small, and the topological charge number of the OAM of a laser is not extremely large, the laser's OAM has little effect on the scattering phase function. This suggests that Rayleigh theory can be applied directly to calculate the light scattering by atmospheric molecules. The transmission of a laser beam with OAM in a molecular atmosphere is not different from that of a regular Gaussian beam.

Keywords: Laser; orbital angular momentum; scattering; molecular atmosphere.

An electromagnetic (EM) wave can carry two forms of angular momentum around the propagation direction: Light spin angular momentum (SAM) related to polarized electromagnetic field vectors spinning around the EM-beam axis and the orbital angular momentum (OAM) related to the spatial field distribution (wave-front helical shape) of light [1-9]. Light's OAM can be generated by propagating a beam through a spiral phase plate [3], diffracting on a fork-like or pitchfork hologram [4-7], applying a q-plate with a SAM sign-change [8, 9], or converting a Hermite-Gaussian beam into a Laguerre-Gaussian beam by using two cylindrical lenses [2]. Due to the nature of the spatial phase distribution, OAM beams have potential applications in manipulation of particles in optical tweezers [10], high-bandwidth information encoding in optical communications [11], high-dimensional quantum information encoding [12-14], and optical detection [15, 16]. The interaction of OAM laser beams with particles has been studied both experimentally [17] and theoretically [17-21]. Several previous studies have used a Lorenz-Mie solution to calculate the light scattering of an OAM laser beam [17-20]. With this type of analytical solution, it is difficult to incorporate an off-axis beam. Indeed, these preliminary theoretical studies only consider the on-axis case. For the OAM laser beam, the on-axis is a special situation in which we would expect the OAM effect on light scattering would be maximized. It is impossible to extrapolate meaning to the off-axis case. For instance, Rury and Freeling [18] report that angular momentum induces transparency through particles; however, they consider a small particle positioned at the central axis of an OAM laser where the intensity is at a minimum. In this case, because its size is much smaller than the laser doughnut's radius, the tiny particle has nearly no interaction with the laser. As soon as the particle is moved off axis, the situation is changed drastically [21]. In Sun et al. [21], the finite-difference time domain (FDTD) method [22-24] with a convolutional perfectly matched layer (CPML) [25] is applied to calculate the scattering of the

purely azimuthal (the radial mode number p is assumed to be zero) Laguerre-Gaussian (LG) beams [21] with the OAM (i.e. topological charge number $L > 0$) by small dielectric particles. It's found that for OAM beam's interaction with dielectric particles, the forward peak in the conventional phase function (P_{11}) is reduced, and the light-scattering peak occurs at side-scattering angles, depending on particle sizes. The reduction of forward-scattering peak means that, in laser communications most of the particle-scattered noise cannot enter the receiver, thus the received light is optimally the original OAM-encoded signal. This feature of the OAM beam also implies that in lidar/radar remote sensing of atmospheric particles, most of the multiple-scattering energy will be away from the lidar/radar sensors, and this may result in an accurate profiling of particle layers in the atmosphere or in the oceans, or even in the ground when a ground penetrating radar (GPR) with OAM is applied [21]. With such unusual light-scattering properties, it is imperative to consider the effect of molecular scattering in the atmosphere. If such scattering using an OAM proves to be excessively lossy, it reduces the usefulness of the OAM laser in optical communication and remote-sensing applications. In this study, we used the same method as reported in [21] to study the scattering of the OAM laser by dielectric particles much smaller than the incident wavelength to investigate whether an OAM laser's scattering by a molecular atmosphere is different from Rayleigh scattering.

Figure 1 illustrates a particle positioned at the center of the FDTD computational domain and a laser beam with its central axis at a distance of d from the center of the computational domain. The FDTD computational domain is bounded by the CPML [21, 24, 25], and the spatial cell size is set at $\Delta s = \lambda/200$, where λ is the incident wavelength [21-25]. The CPML is set to be $6 \Delta s$ thick and a free space of $6 \Delta s$ is set between the particle surface and the CPML inner surface. The time step for the FDTD simulation is set to be $\Delta t = \Delta s / (2c)$, where c is light speed in free space [21-24].

Figure 2 shows phase function P_{11} of spherical particles with a refractive index of 1.33 and size parameter $x = \pi D/\lambda = 0.48, 0.24, 0.12,$ and 0.06 , where D denotes the diameter of the particle, under the incidence of Laguerre-Gaussian beams [21] with $p = 0$ and topological charge numbers $L = 0, 6, 48$, corresponding to Gaussian and Laguerre-Gaussian beams, respectively. The laser waist $w_0 = 400\Delta s$. The distance of the central axis of the beam from the particle center is $d = 200\Delta s$. We can see that the light-scattering phase function of the OAM Laguerre-Gaussian beam ($L = 6$) is nearly coincident with that from the Gaussian beam ($L = 0$) for $x < 0.48$. When $x = 0.48$, the OAM ($L = 6$) scattering is slightly weaker in the forward direction and stronger in the backward direction than the Gaussian beam ($L = 0$). When $L = 48$, the effect of the OAM on the light scattering becomes more significant for larger particles. However, when $x = 0.06$, this effect also becomes insignificant.

As illustrated in Fig. 3, the effect of the OAM on light scattering by particles is due to the phase change incurred by the OAM inside the particle, which is determined by a function $f(LD/R)$, where L is the OAM topological charge number, D is the particle size, and d is the distance between the particle center and the central axis of the OAM laser beam. LD/d is approximately the phase change in the range of the particle due to OAM. However f is also a function of particle's refractive index and shape. For beams of the same L , when d increases or D decreases, LD/d decreases and effect of the OAM on light scattering becomes smaller. This means that when particles are very small with respect to the wavelength, thus within the particle the OAM-incurred phase difference is also very small, the light scattering of the OAM laser beam is not different from that of a Gaussian beam or plane-wave incidence. As illustrated in the final panel of Fig. 2, when a particle is smaller than about 1/50 of the wavelength ($x = \sim 0.06$), its light scattering is very close to Rayleigh scattering. For a particle of $x = 0.06$, we found that when OAM $L < 480$, the scattering is not

significantly different from Rayleigh scattering. When OAM $L > 480$, there is very small in the forward scattering of P_{11} . Thus we can conclude that an OAM of $L < 1000$ should have a negligible effect on the light scattering of atmospheric molecules ($x \ll 0.06$).

In this study, the FDTD method is used to calculate the scattering of a purely azimuthal Laguerre-Gaussian beams having OAM by small dielectric particles. It is found that when the particles are very small, and the order of the OAM of a laser is not extremely high, the laser's OAM has little effect on the scattering phase function. This implies that Rayleigh theory can be directly applied to calculate the light scattering of an OAM laser beam by atmospheric molecules, and the transmission of an OAM laser beam through the atmosphere is not different from that of a Gaussian beam. Note here that, although we set a specific distance of the central axis of the beam from the particle center ($d = 200\Delta s$) in this study, this does not affect the conclusion.

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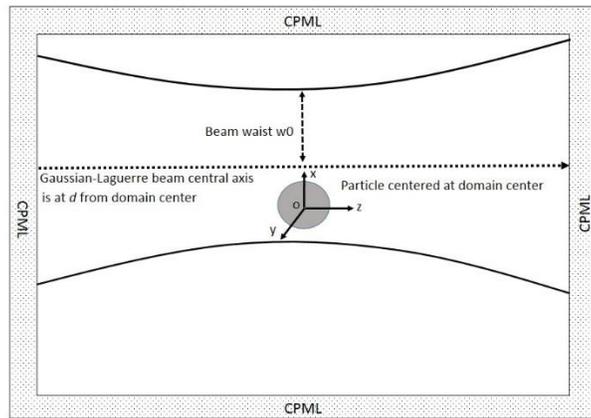


Figure 1. Illustration of a particle positioned at the center of the FDTD computational domain and a laser beam with its central axis at a distance d from the center of the computational domain. The FDTD computational domain is bounded by the CPML [21].

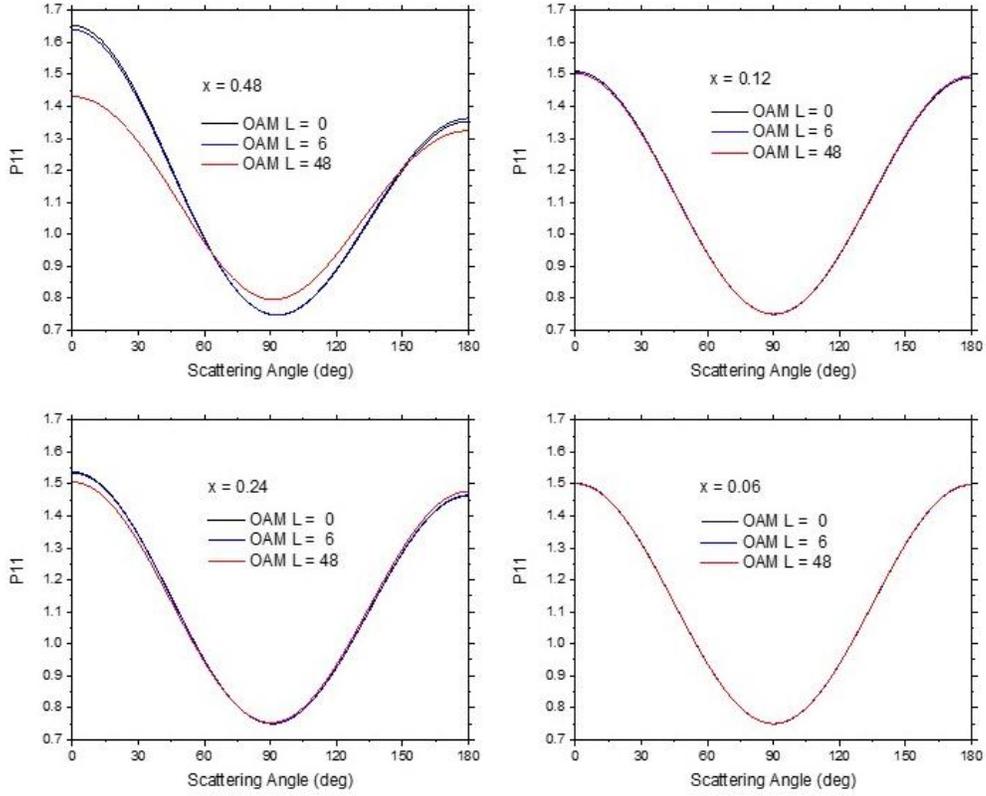


Figure 2. Phase function P_{11} of spherical particles with a refractive index of 1.33 and size parameter $x = 0.48, 0.24, 0.12,$ and 0.06 , respectively, illuminated by a Laguerre-Gaussian beams with $p = 0$. The laser waist $w_0 = 400\Delta s$, where $\Delta s = \lambda / 200$. The laser OAM topological charge numbers are $L = 0, 6,$ and 48 . The distance of the central axis of the beam from the particle center is $d = 200\Delta s$.

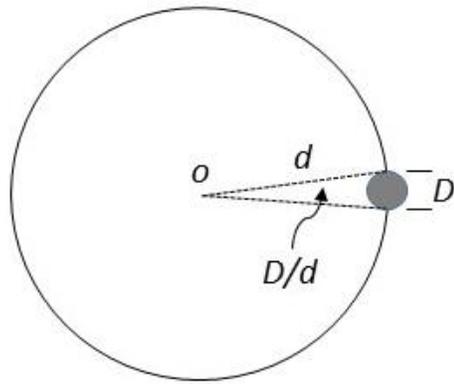


Figure 3. Illustration of a cross cut of a particle and an incident OAM laser beam. In this figure, D denotes particle size, d is the distance from the beam's central axis to the center of the particle.