Uncertainty Analysis of Experimental Discharge Coefficients in Additively Manufactured Liquid Injector Elements

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  - Design Variant Prediction Interval
- Conclusion

Additive Manufacturing

Additive Rocket Engine Components

- Advantages
  - Mass Reduction
  - Part count reduction
  - Reduced assembly time
  - Reduced manufacturing costs
  - Reduced lead time
- Challenges
  - Still a maturing technology
  - Small geometric features and passages/Large scale parts
  - Manufacturing imperfections
- Several successful test programs with AM parts.



Hot-fire testing of an Aerojet Rocketdyne RL 10C-X prototype engine with 3-D printed core components. [1]



Subscale Integral injector manufactured with SLM. [2]



SLM produced integrated nozzle film coolant ring designed and tested at NASA. [3]

1. "3-D Printed RL10C-X Prototype Rocket Engine Soars Through Initial Round of Testing," Aerojet Rocketdyne Press Release, www.rocket.com/article/3-d-printed-rl10c-x-prototype-rocket-engine-soars-through-initial-round-testing, 2019.
2. Soller, S. et. al., "Design and Testing of Liquid Propellant Injectors for Additive Manufacturing," 7th European Conference for Aerospace Sciences, 2017.
3. Gradl, P., et. al. "Additive Manufacturing of Liquid Rocket Engine Combustion Devices: A Summary of Process Developments and Hot-Fire Testing Results," 54th AIAA/SAE/ASE Joint Propulsion Conference, 2018.

Objective

- Assess the manufacturer-to-manufacturer variability in flow discharge coefficients of identical parts with small flow passages.

Scope:

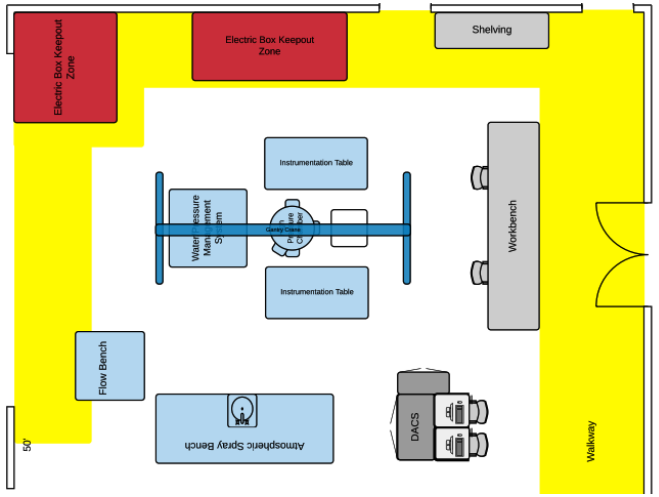
- Investigate two internal geometries:
  - 1) Radial-Fed Annulus
  - 2) Cavitating Venturi
- Investigate subtractive (baseline) and additive manufacturing (11 vendors) .
- Cold flow (water) parts over relevant operating regimes
- Perform detailed measurement uncertainty analysis
- Determine and compare differences in discharge coefficients among the manufacturing methods

Facility Overview

PRC Injector Spray Facility

Non-Reactive, cold flow environments for the study of injectors and injection processes in liquid injection devices.

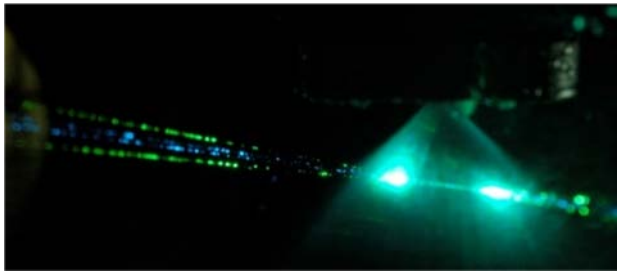
- Pressurized Spray Chamber
  - 18” Internal Diameter x 72” Tall
  - 500 psig Max Pressure
  - Four - 6” Diameter Optical Access Ports
- Atmospheric Spray Bench
- Flow Bench



- Liquid simulant flow rates up to 2 lbm/s (water and water based solutions)
- Gas flow rates up to 1 lb/s (Nitrogen/Compressed Air)
- Optical Diagnostic Access:
  - High Speed or Standard Video
  - Laser Diagnostics (PIV, PDPA)
- Common DAQ system
  - High Speed 1Ms/s
  - Integrated adjustable Low Pass Filtering
  - Temperature/Static Pressure (1000Hz)



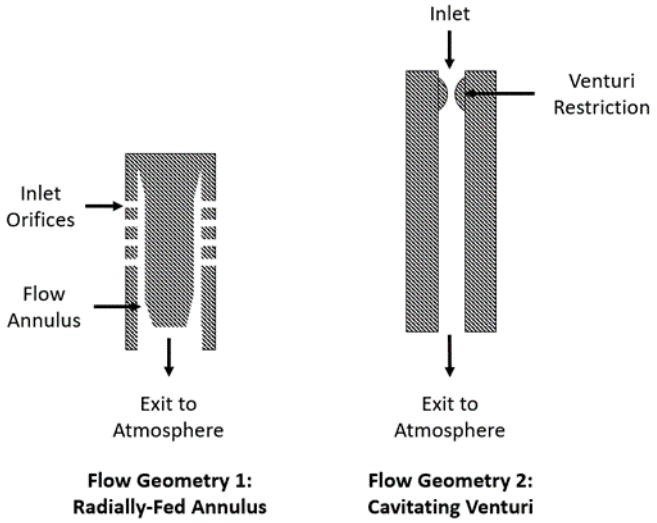
sure (left) and 500 psig (right)





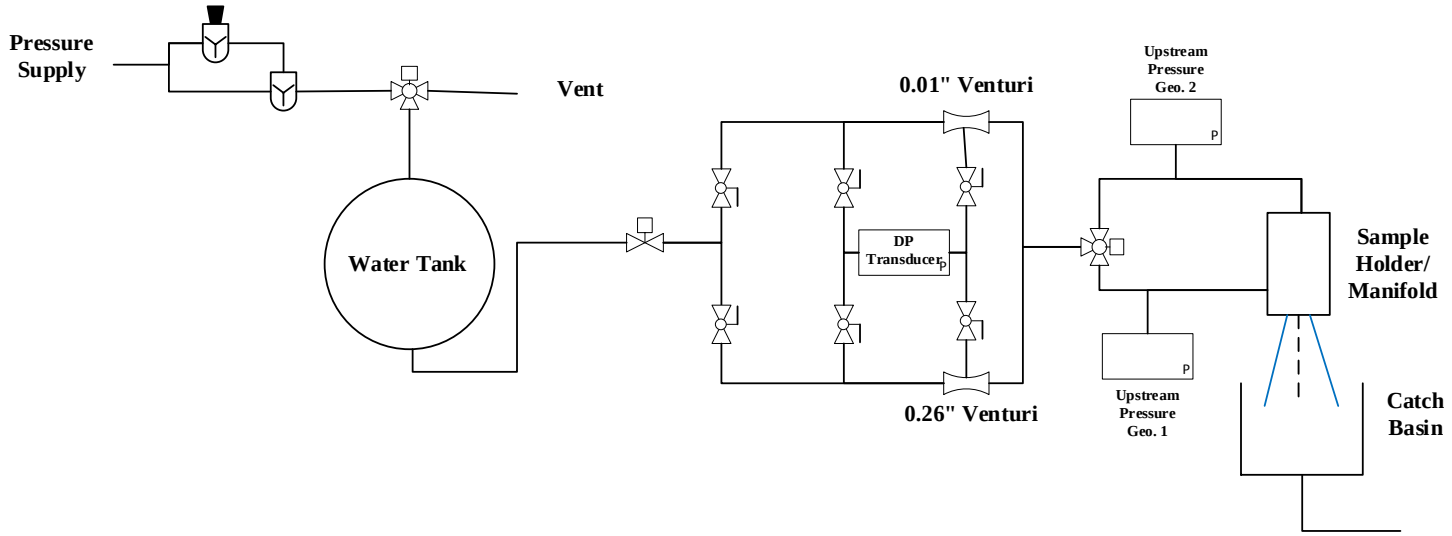
Injector Specifications

- 11 Manufacturers using SLM printers
- 4 Design Variants on the same build plate
- 45 Injectors
- Operating Conditions
  - Atmospheric Back Pressure
  - Flow Geometry 1
    - 75 psig to 550 psig
    - 0.5 lb/s to 1.6 lb/s
  - Flow Geometry 2
    - 50 psig to 1550 psig
    - 0.05 lb/s to 0.3 lb/s

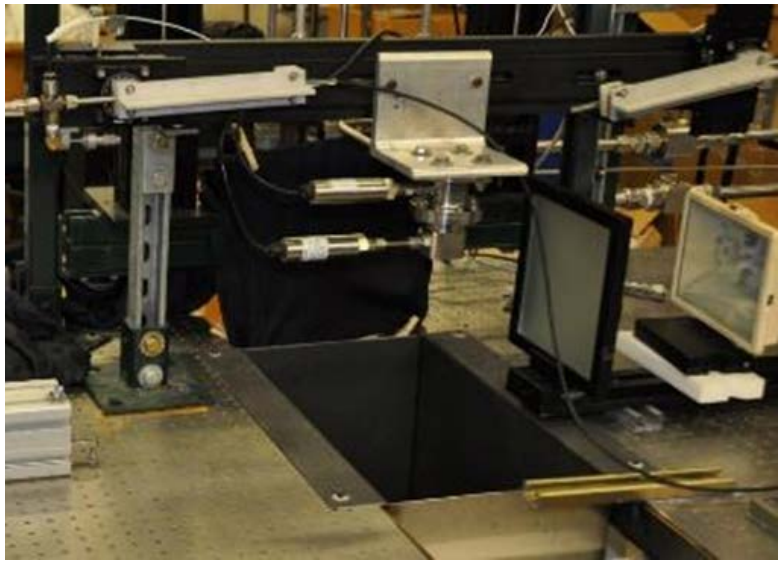


	Flow Geometry 1: Radially-Fed Annulus				Flow Geometry 2: Cavitating Venturi	
Design Variant	**Radial Hole ID (% of Baseline)**	**# of Radial Holes**	**Annulus ID (% of Baseline)**	**Annulus OD (% of Baseline)**	**Flow Duct ID (% of Baseline)**	**Venturi ID (% of Baseline)**
1	100	168	100	100	100	100
2	100	168	111.2	100	100	105.3
3	115.8	168	107.5	111.9	100	110.5
4	157.9	67	103.7	100	100	115.8
Baseline	100	168	100	100	100	100

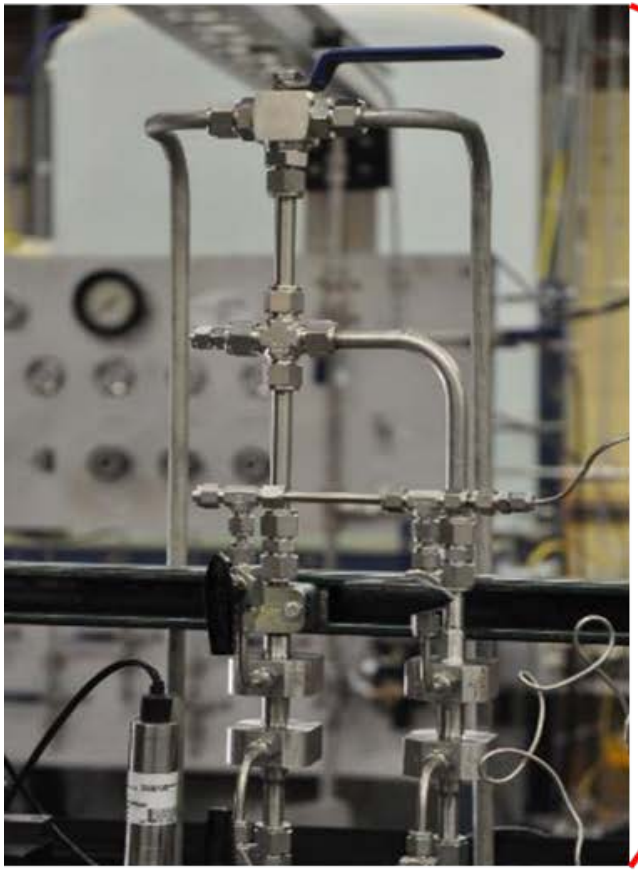
Test Setup

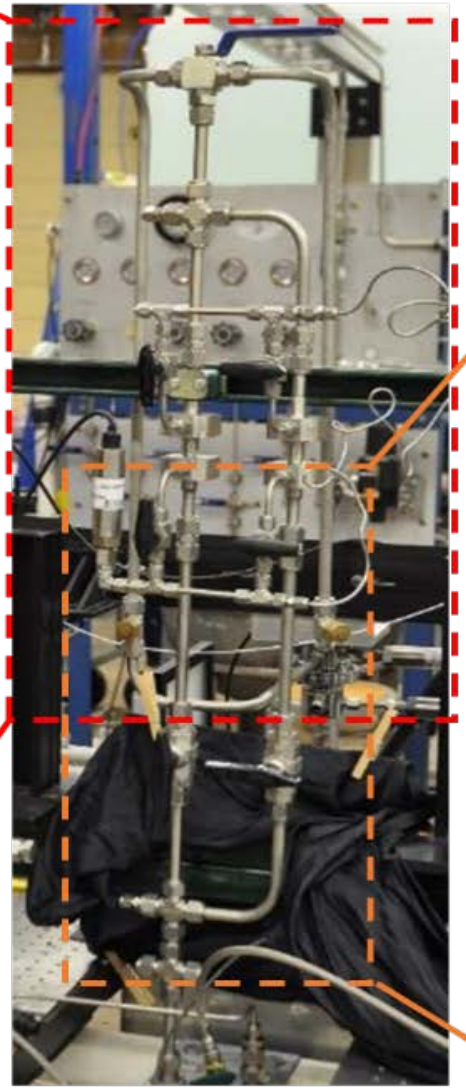


- Venturis shared common DP Transducer
- Venturi manually selected based on Flow rate
- Pressures measured at 1000Hz and averaged over 7 seconds of steady state flow



Venturis







Venturi Calibration

Establish single Discharge Coefficient, C_d, for each venturi

- In-line, end-to-end calibration
- Timed collection at steady flow
  - 18 setpoints (0.10 venturi)
  - 12 setpoints (0.26 venturi)

$$C_d = \frac{\frac{m}{t}}{\sqrt{\frac{2\rho\Delta P}{\frac{1}{\left(\frac{\pi {d_t}^2}{4}\right)^2} - \frac{1}{\left(\frac{\pi {d_1}^2}{4}\right)^2}}}}$$

m = mass of collected water
t = collection time
d_t = venturi throat diameter
d_1 = venturi inlet diameter
ρ = density
ΔP = ventrui pressure drop (inlet to throat)
σ = standard deviation
$t_{\alpha,\nu}$ = student t distribution

Calculate Setpoint Discharge Coefficient ($C_{d\ vent}$)

Calculate Setpoint Discharge Coefficient Uncertainty ($UC_{d\ vent}$)

Calculate Average Cd and mean Cd Uncertainty for Venturi

Systematic Uncertainties (B)

$$B_{C_{d_sp}} = \sqrt{\left(\frac{\partial C_d}{\partial m}B_m\right)^2 + \left(\frac{\partial C_d}{\partial t}B_t\right)^2 + \left(\frac{\partial C_d}{\partial \rho}B_\rho\right)^2 + \left(\frac{\partial C_d}{\partial \Delta P}B_{\Delta P}\right)^2}$$

$$U_{C_{d_sp}} = \sqrt{\left(B_{Cd_sp}\right)^2 + \left(\frac{1.96\sigma_{C_{d_sp}}}{\sqrt{N_{sample}}}\right)^2}$$

$$C_{d_vent} = \frac{1}{N}\sum C_{d_sp}$$

$$U_{C_{d\ vent}} = \sqrt{\sum\left(\frac{U_{C_{d_sp}}}{N_{setpoint}}\right)^2 + \left(\frac{t_{\alpha,\nu} * \sigma_{C_{d\ vent}}}{\sqrt{N_{setpoint}}}\right)^2}$$

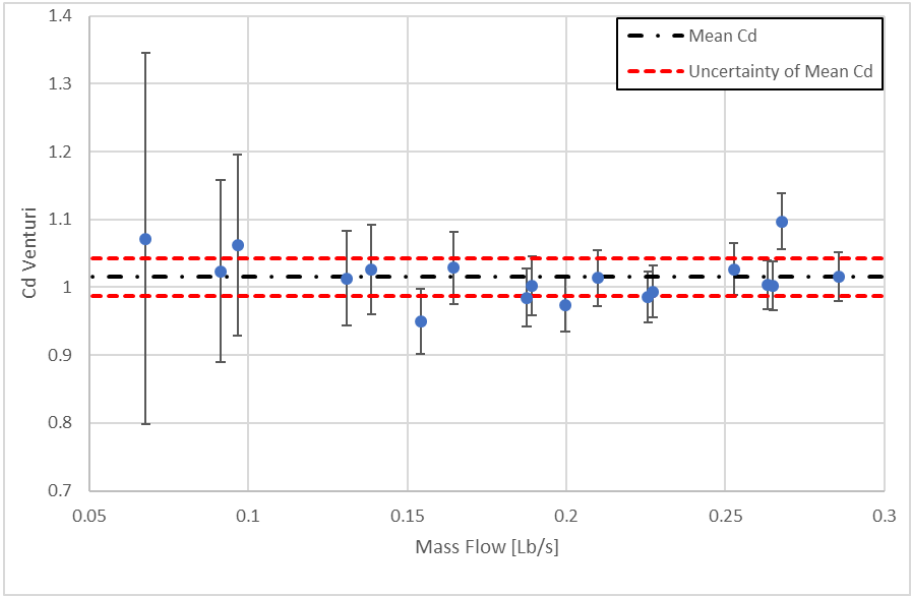
Venturi Calibration

0.100” venturi

$C_d = 1.015$

$U_{cd} = 2.7\%$

0.02 Lb/s to 0.3 Lb/s

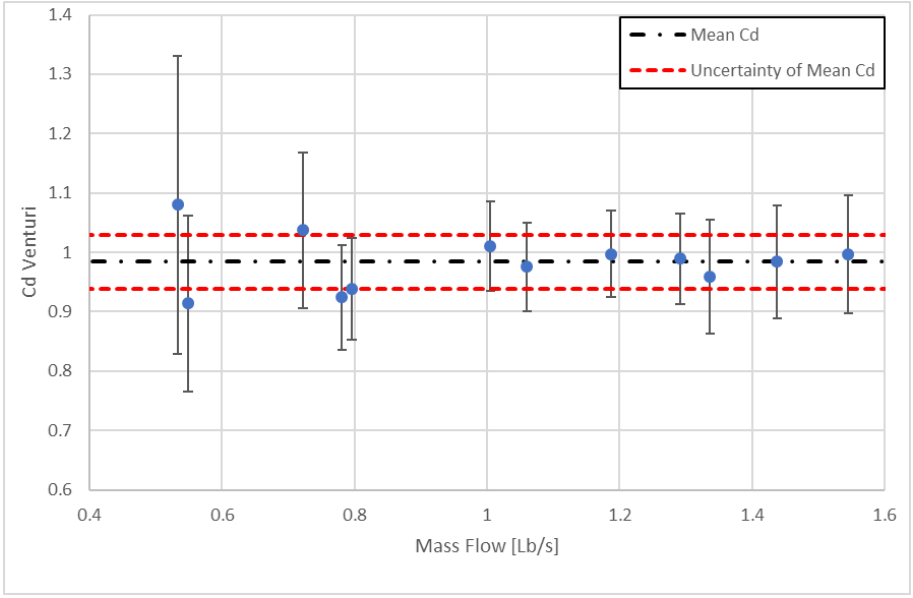


0.260” venturi

$C_d = 0.984$

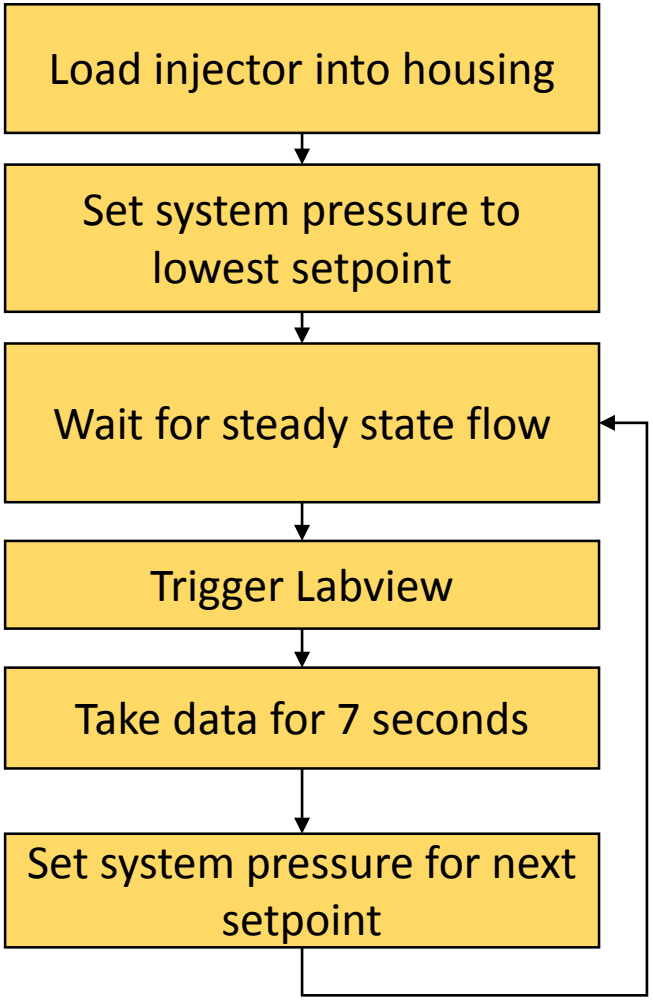
$U_{cd} = 4.6\%$

0.1 Lb/s to 2.1 Lb/s

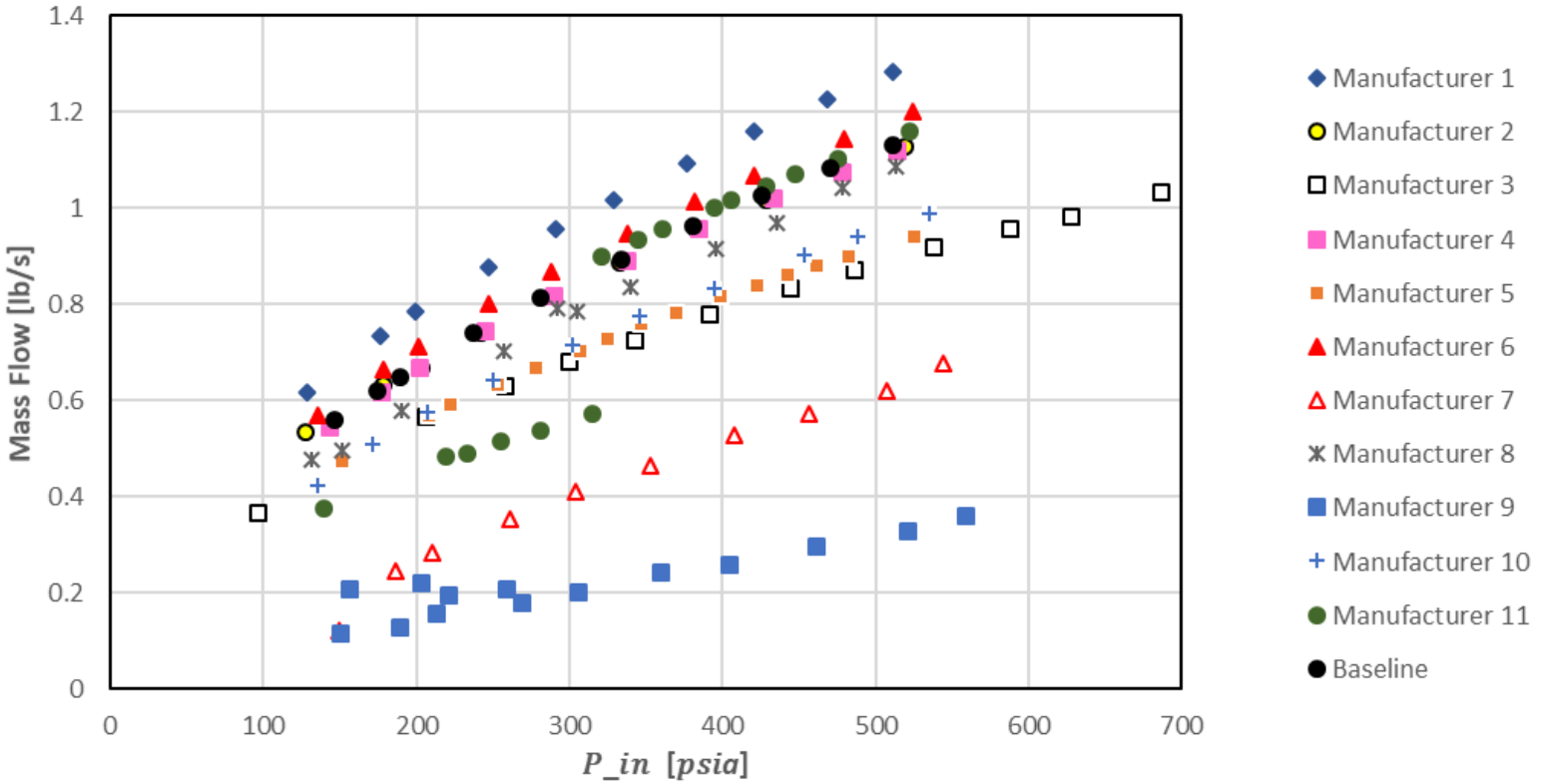


Larger uncertainty at low mass flow due to low ΔP

Testing Procedure

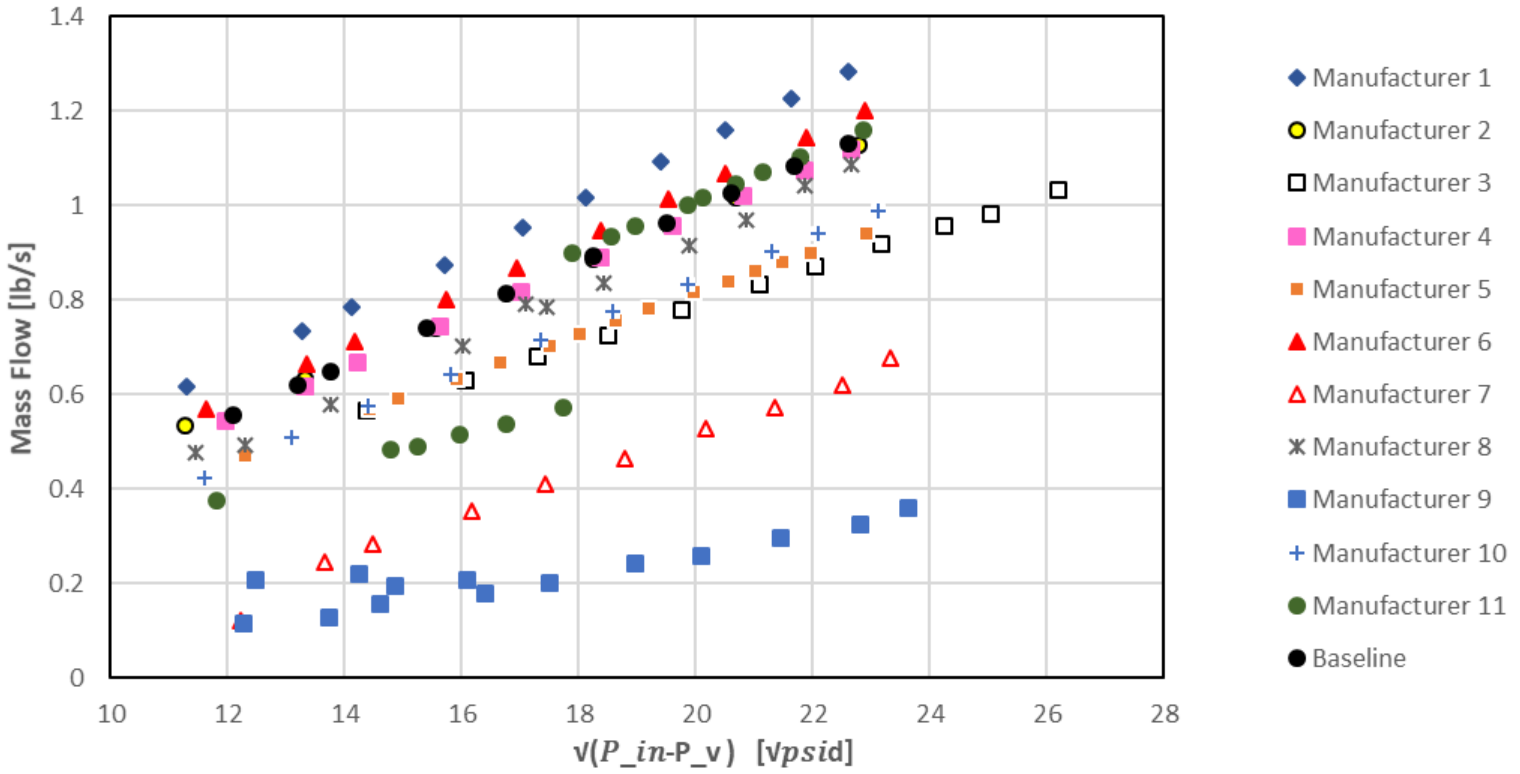


Flow Geometry 1
Design Variant 1



- Noted “Clustering”: 4 groups
- Wide range of mass flow variation at given inlet pressure
- 2 significantly lower performing test articles
- Discontinuous trend for Manufacturer 11

Flow Geometry 1
Design Variant 1



- Linear trends with square root of pressure

Individual Test Article Analysis

Calculate mass flow ($\dot{m}$) at each setpoint

$$\dot{m} = \frac{\pi}{4} C_{d\ vent}\ {d_t}^2 \sqrt{\frac{2\rho\Delta P}{1-\left(\frac{{d_t}^4}{{d_1}^4}\right)}}$$

$\dot{m}$ = mass flow rate
d_t = venturi throat diameter
d_1 = venturi inlet diameter
ρ = density
ΔP = ventrui pressure drop (inlet to throat)
$C_{d\ vent}$ = venturi discharge coefficient
P_{in} = inlet pressure
P_{cav} = vapor pressure

Calculate mass flow Uncertainty at each setpoint ($U_{\dot{m}_{setpoint}}$)

$$U_{\dot{m}_{setpoint}} = \sqrt{\left(\frac{\partial \dot{m}}{\partial C_{d\ vent}} U_{C_{d\ vent}}\right)^2 + \left(\frac{\partial \dot{m}}{\partial \rho} U_\rho\right)^2 + \left(\frac{\partial \dot{m}}{\partial \Delta P} U_{\Delta P}\right)^2 + \left(\frac{1.96\sigma_{\dot{m}_{sample}}}{\sqrt{N_{\dot{m}_{sample}}}}\right)^2}$$

Calculate Cd for injector at each setpoint (C_d)

$$C_d = \frac{\dot{m}}{A_{throat}\sqrt{2\rho(P_{in}-P_{cav})}}$$

P_{in} = inlet pressure
P_{cav} = vapor pressure

Calculate Cd for injector Uncertainty at each setpoint ($U_{C_d_sp}$)

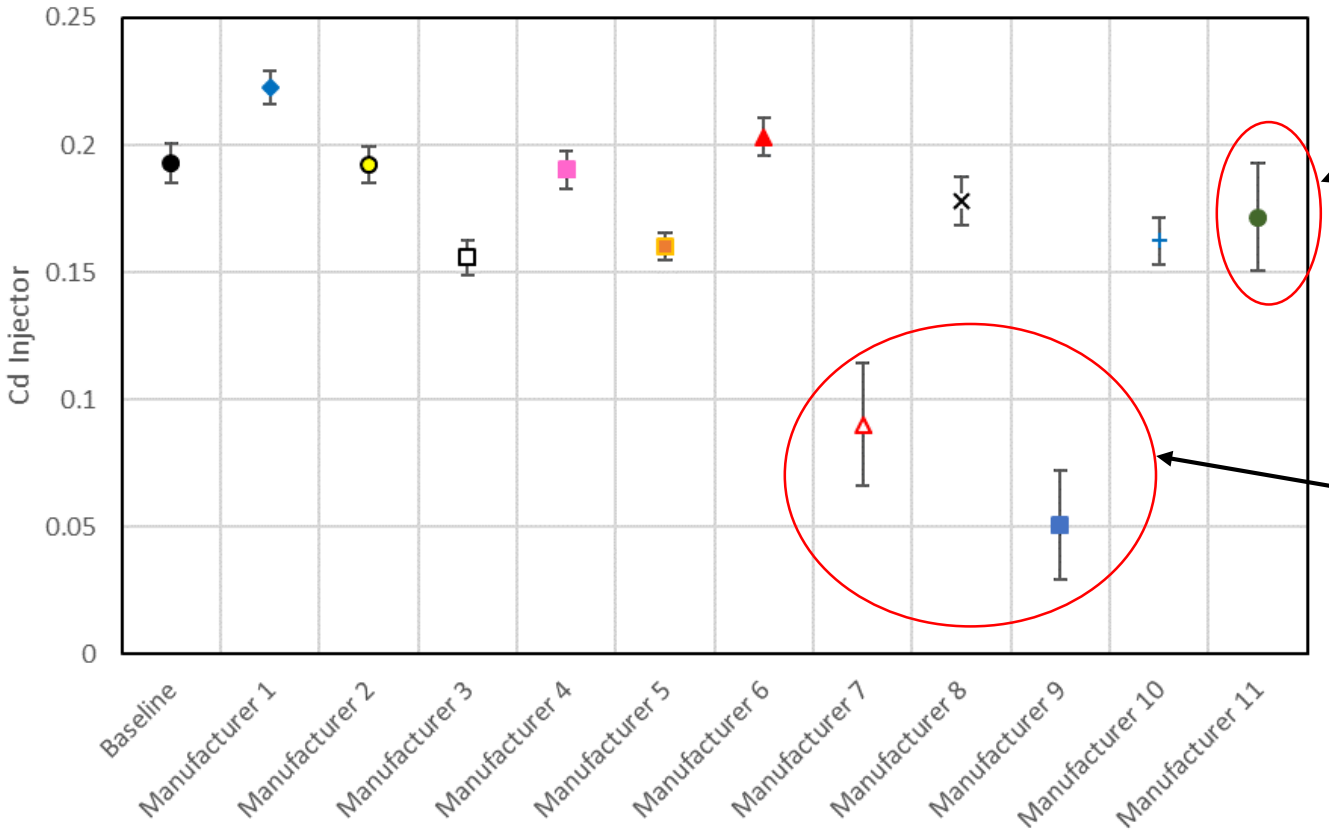
$$U_{C_d_sp} = \sqrt{\left(\frac{\partial C_{d\ inj}}{\partial \dot{m}} U_{\dot{m}}\right)^2 + \left(\frac{\partial C_{d\ inj}}{\partial \rho} B_\rho\right)^2 + \left(\frac{\partial C_{d\ inj}}{\partial P_{in}} B_{P_{in}}\right)^2 + \left(\frac{1.96\sigma_{C_d}}{\sqrt{N_{C_d}}}\right)^2}$$

Calculate Average Cd and mean Cd Uncertainty for each test article

$$U_{C_{d\ inj}} = \sqrt{\sum\left(\frac{U_{C_{d\ inj\ setpoint}}}{N_{setpoint}}\right)^2 + \left(\frac{t_{\alpha,\nu}\ \sigma_{C_{d\ inj\ setpoint}}}{\sqrt{N_{setpoint}}}\right)^2}$$

$$C_{d_inj} = \frac{1}{N}\sum C_{d_sp}$$

Flow Geometry 1
Design Variant 1

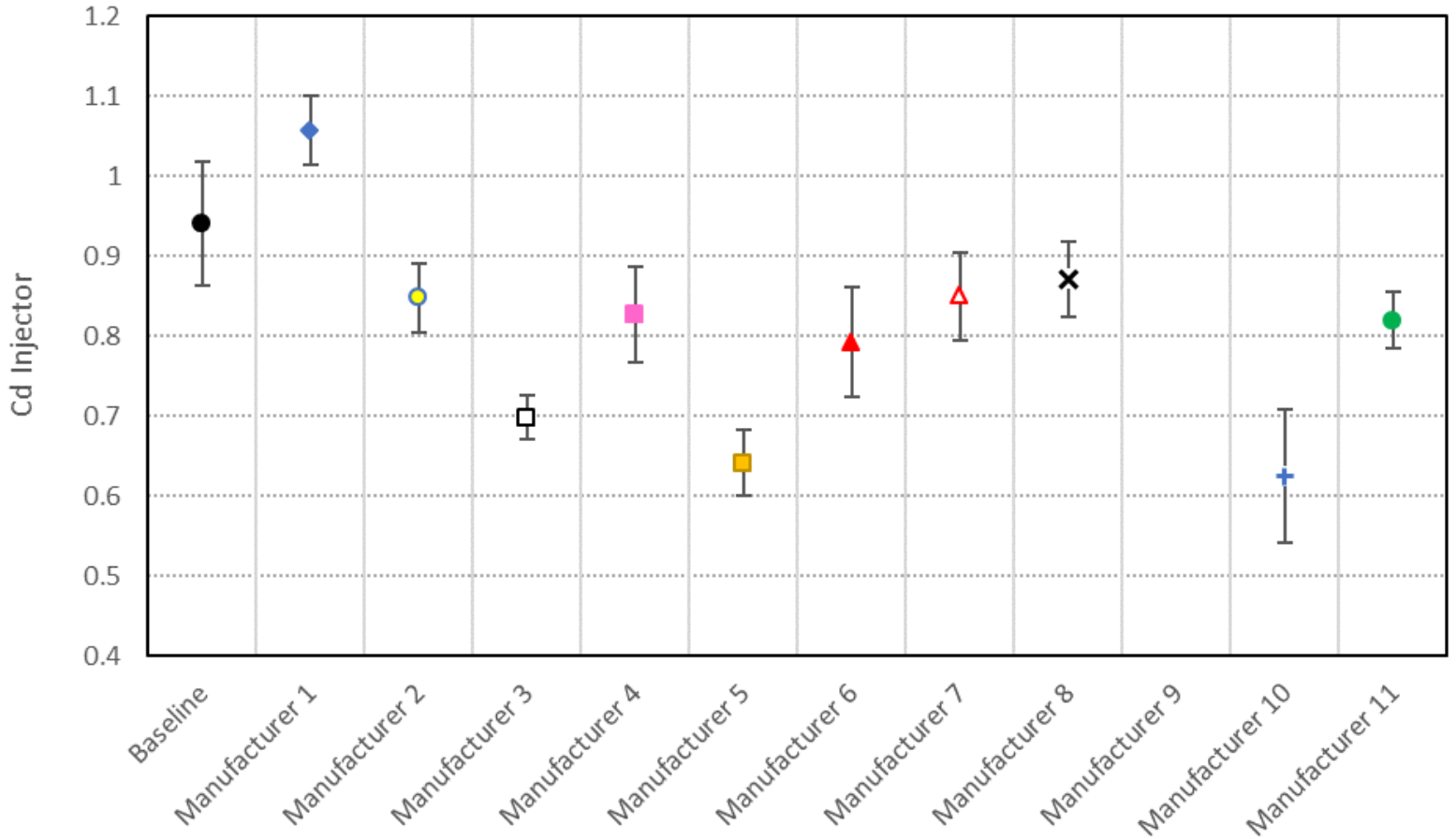


Test Article with discontinuous mass flow trend

Test Articles with significantly lower mass flow rates

- Uncertainty ranged from 3% to 6% for 9 of the test articles
- 12%-53% Uncertainty for 3 of the test articles

Flow Geometry 2 Design Variant 1



Uncertainty Range: 4%-13%

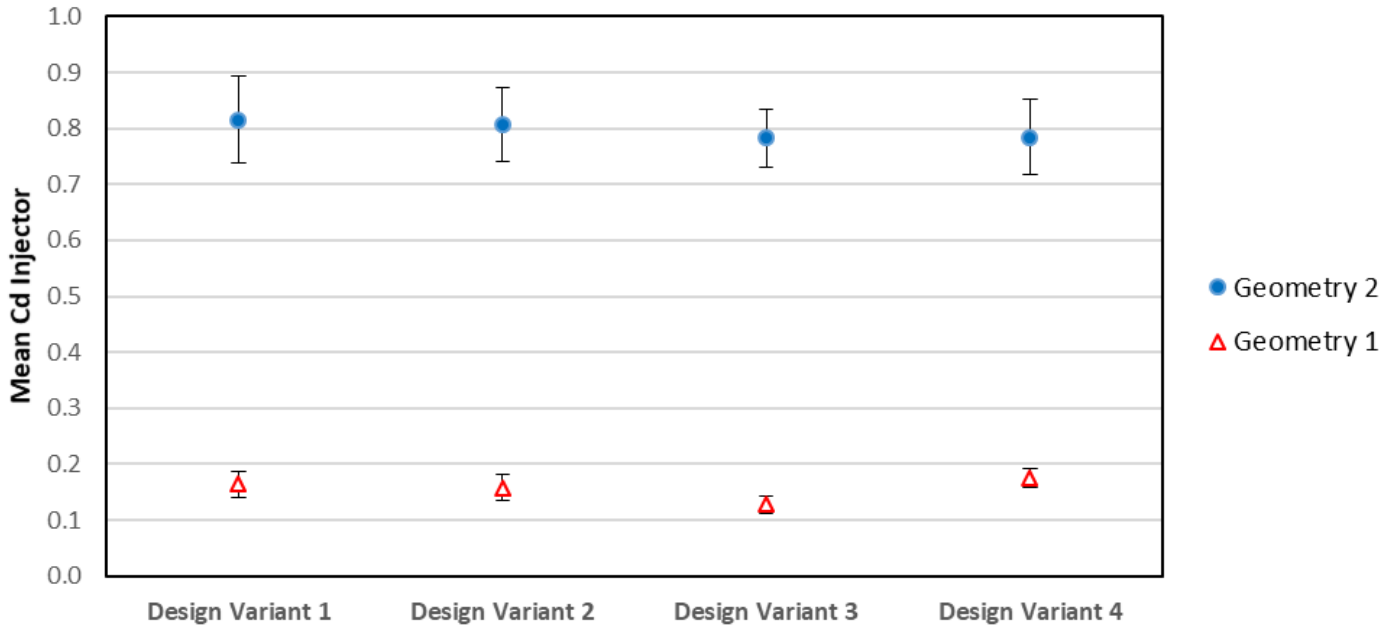
Common Design Variant Across all Manufacturers

Calculate average Cd for all manufactures for a given design variant and flow path

Calculate uncertainty of average Cd for all manufactures for a given design variant and flow path

$$C_d = \frac{1}{N}\sum C_{d_inj}$$

$$U_{C_{d\,inj}} = \sqrt{\sum \left(\frac{U_{C_{d\,inj\,setpoint}}}{N_{setpoint}}\right)^2 + \left(\frac{t_{\alpha,\nu}\,\sigma_{C_{d\,inj\,setpoint}}}{\sqrt{N_{setpoint}}}\right)^2}$$



- Geometry 1 uncertainty ranged from 10% to 14%
- Geometry 2 uncertainty ranged from 6% to 10%

Results by Manufacturer

Calculate average Cd for all design variants for a given manufacturer and flow path

↓

Calculate uncertainty of average Cd for all design variants for a given manufacturer and flow path

↓

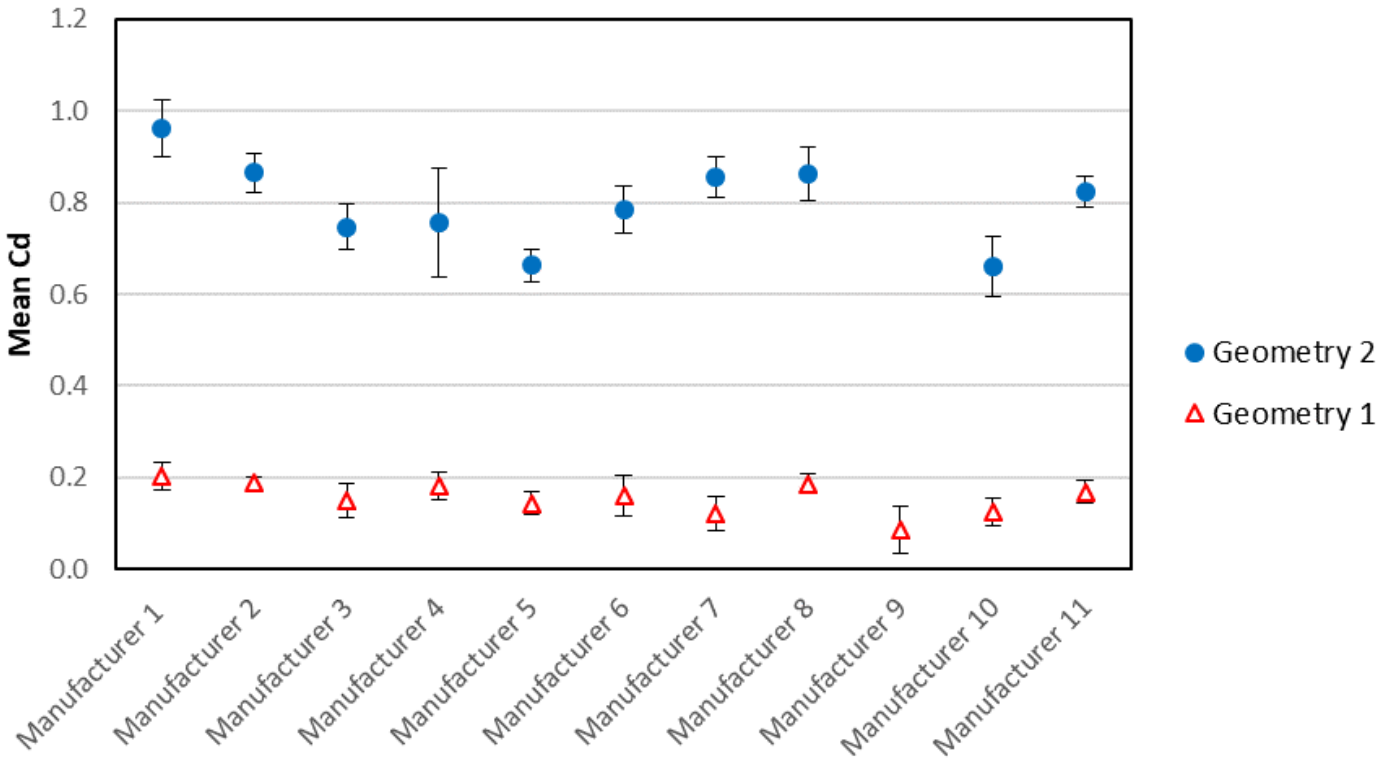
Normalize injector Cd for both flow paths

$$C_d = \frac{1}{N}\sum C_{d_inj}$$

$$U_{C_{d\,inj}} = \sqrt{\sum \left(\frac{U_{C_{d\,inj\,setpoint}}}{N_{setpoint}}\right)^2 + \left(\frac{t_{\alpha,\nu}\,\sigma_{C_{d\,inj\,setpoint}}}{\sqrt{N_{setpoint}}}\right)^2}$$

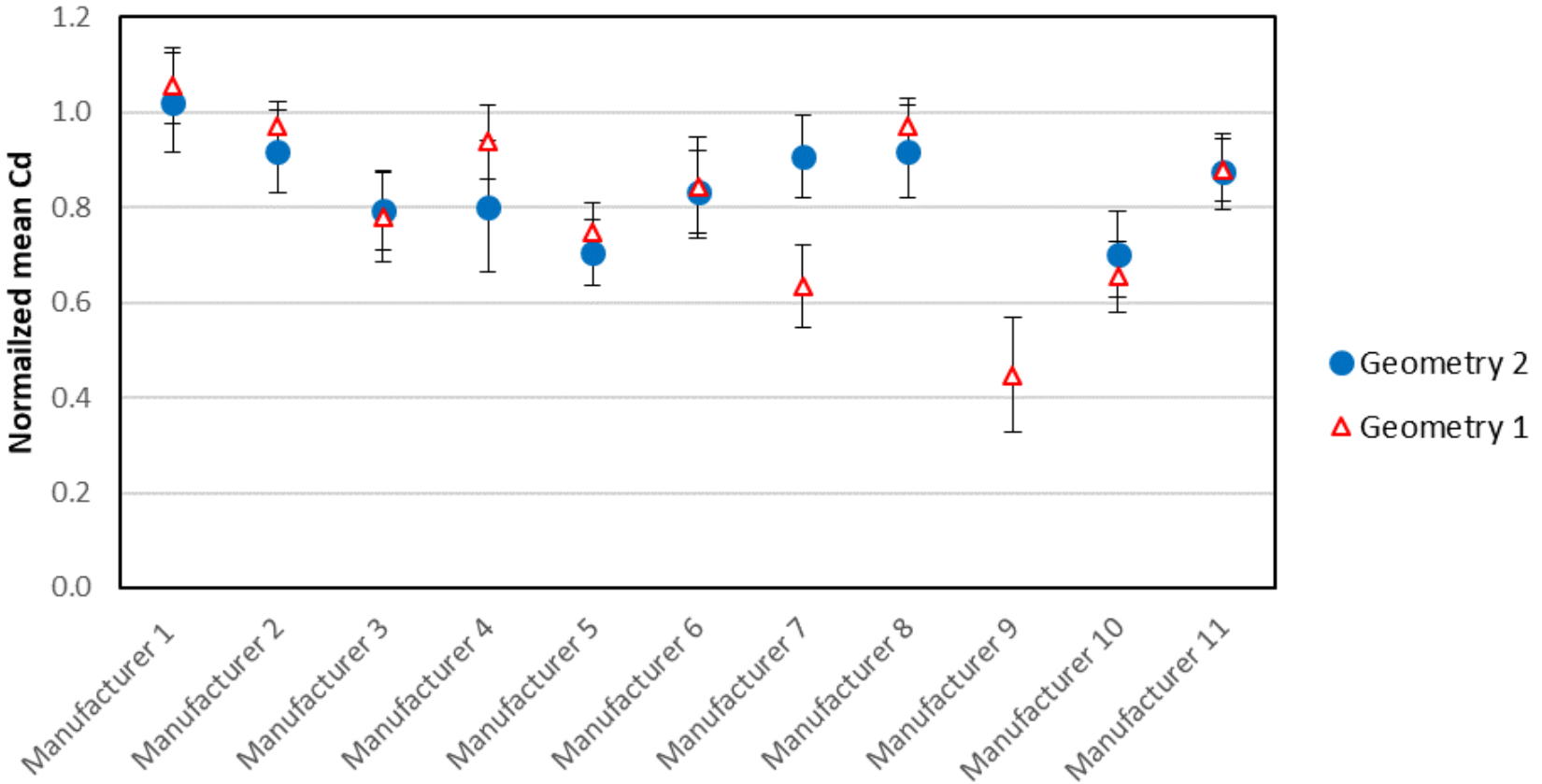
$$C_{d\,norm} = \frac{C_{d\,inj}}{C_{d\,baseline}}$$

Results by Manufacturer



- Flow geometry 1
  - 10 manufacturers ranged from 7%-30%
  - 1 manufacturer was 61 %
- Flow geometry 2
  - 9 manufacturers ranged from 4%-10%
  - 1 manufacturer was 61 %

Normalized Results by Manufacturer



Predictive Interval Analysis

Calculate average Cd for all manufacturers for a given flow path and geometry

↓

Calculate uncertainty of average Cd for all manufacturers for a given flow path and geometry

↓

Calculate the predicted Cd for a new injector from the same population

↓

Calculate the prediction interval for the new injector

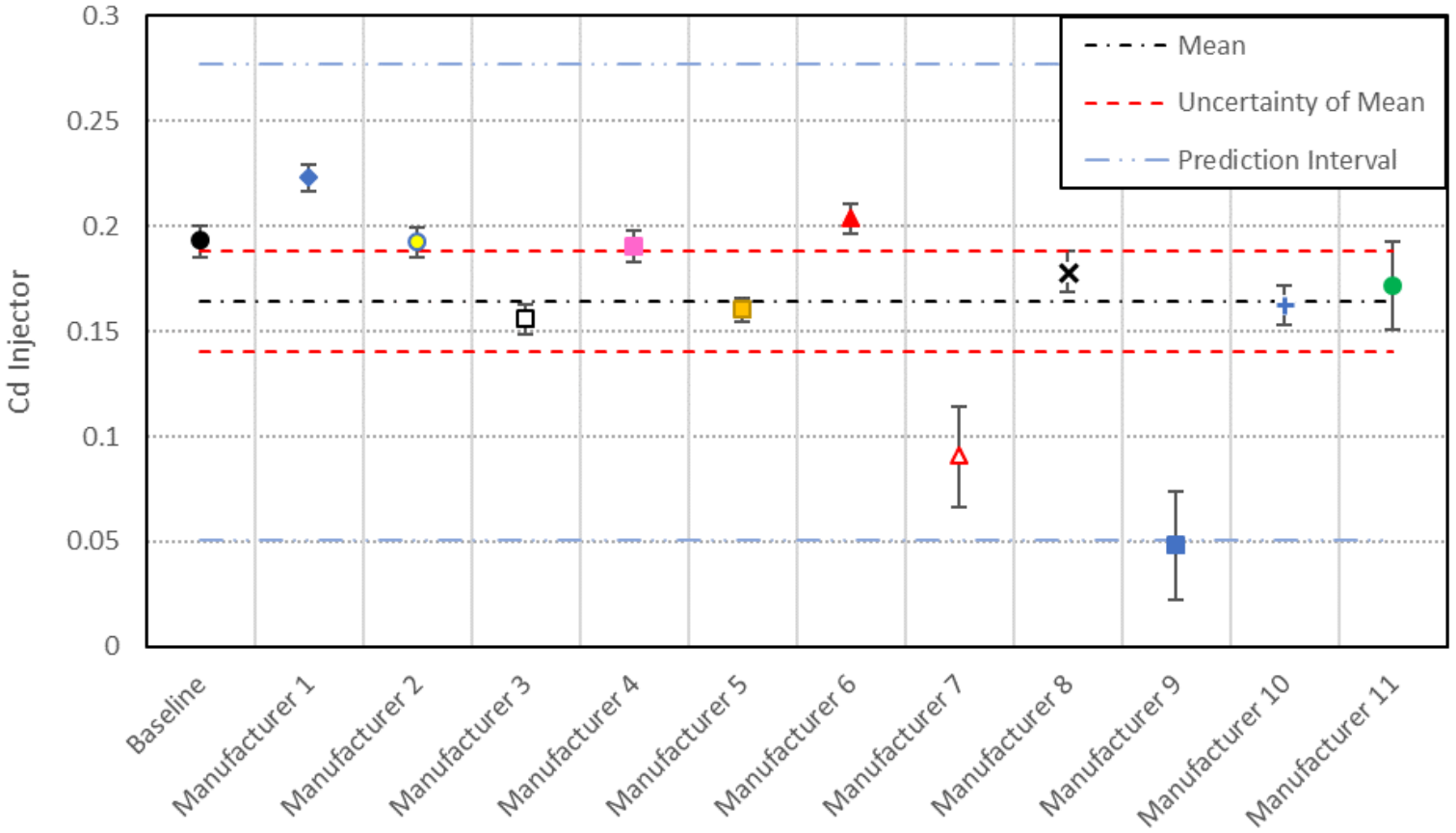
$$\overline{C_d} = \frac{1}{N}\sum C_{d_inj}$$

$$U_{C_{d\,inj}} = \sqrt{\sum\left(\frac{U_{C_{d\,inj\,setpoint}}}{N_{setpoint}}\right)^2 + \left(\frac{t_{\alpha,\nu}\,\sigma_{C_{d\,inj\,setpoint}}}{\sqrt{N_{setpoint}}}\right)^2}$$

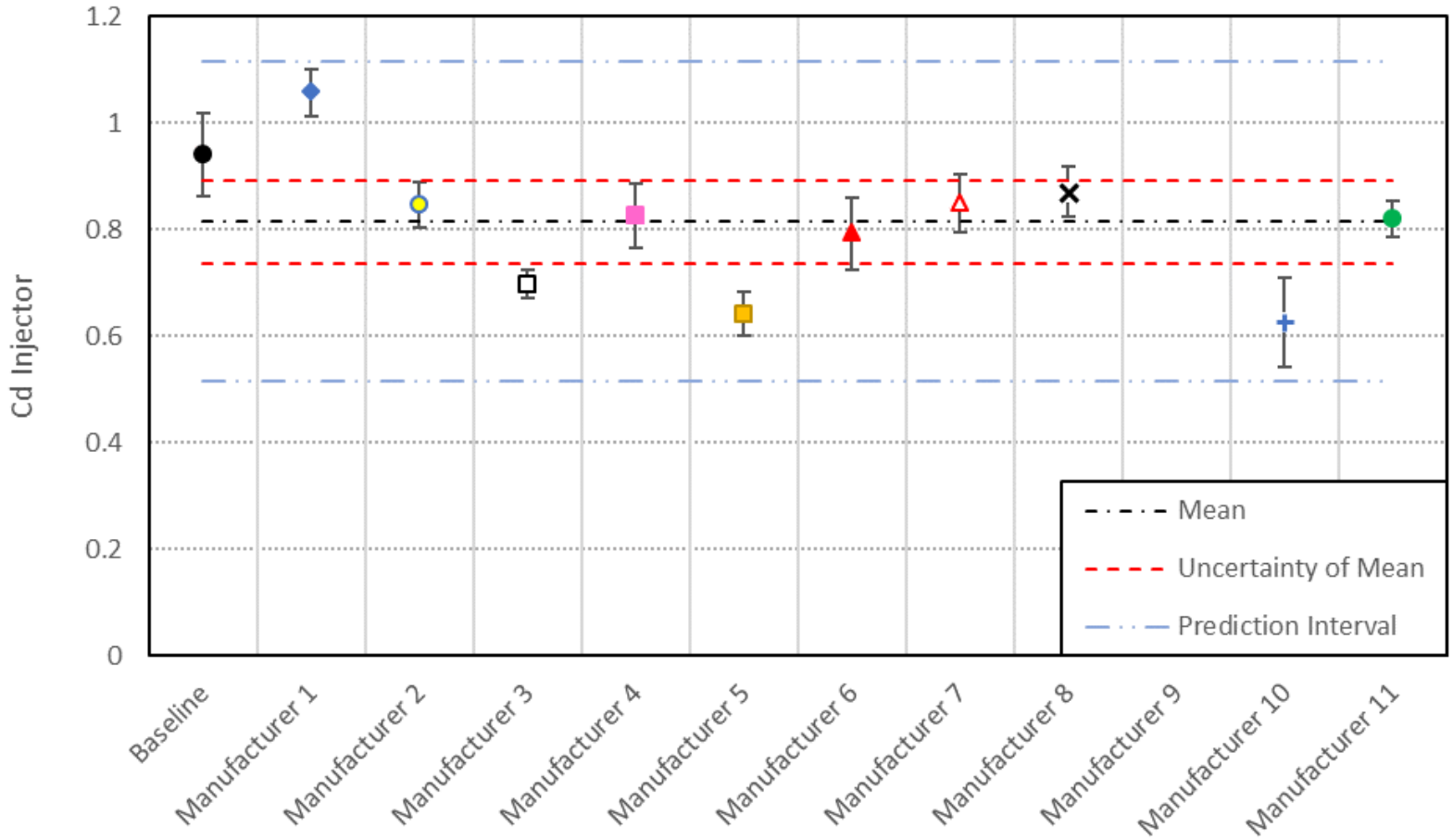
$$C_{d\,Predicted} = \overline{C_d} \pm U_{95}\sqrt{1+\frac{1}{n}}$$

$$U_{95} = \sqrt{\sum\left(\frac{U_{C_{d\,inj}}}{N}\right)^2 + \left(t_{\alpha,\nu}\,\sigma_{C_{d\,inj}}\right)^2}$$

Flow Geometry 1
Design Variant 1



Flow Geometry 2 Design Variant 1



Conclusion

- Flow Geometry1:
  - The mean CD of all injectors was 0.16
  - Significant mean CD variability depending on the manufacturer
  - Additive CD's generally lower than subtractive baseline
- Flow Geometry 2:
  - The mean CD of all injectors was 0.80
  - Significant mean CD variability depending on the manufacturer
  - Additive CD's generally lower than subtractive baseline
- Differences among CD of all injectors are generally well beyond the uncertainty bars of the CD results.
- Manufacturer is more important than slight changes in geometry