DESIGN OF A NOZZLE FOR THE SPYDER 2ND STAGE SOLID ROCKET MOTOR

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ABSTRACT

During the 2019 summer term, the author worked with a group of four interns to complete the preliminary design of a 2nd stage solid rocket motor for Up Aerospace's Spyder Launch Vehicle. The Spyder vehicle is a four stage, solid fuel rocket designed as part of collaboration between NASA and Up Aerospace to develop a vehicle capable of delivering a 10 kg, 6U CubeSat into a 350 km, circular, low Earth orbit. As part of the agreement, NASA is tasked with designing high performance 2nd, 3rd, and 4th stages while Up Aerospace will provide the 1st stage, derived from the first stage of the company's sub-orbital Spaceloft XL vehicle. Previous intern teams have designed the 3rd and 4th stages, which left the preliminary design of the 2nd stage motor to be completed this summer. The purpose of this report is to highlight a trade study which the author conducted to determine the nozzle geometry which would most benefit the performance of the 2nd stage motor. In this study, various nozzle parameters such as throat radius (RSI), expansion ratio, mass and their effects on the Isp and Delta V of the 2nd Stage were investigated. From this study, a nozzle geometry providing the necessary performance was chosen and implemented as part of the preliminary design of the 2nd stage motor.

INTRODUCTION

To mature the 2nd stage motor design, a trade space was needed to determine the nozzle configuration which would most benefit the performance of the 2nd stage. The trade space established did not only evaluate different expansion ratios for the same throat radius, but also investigated the possible performance gained from decreasing the throat radius to increase the expansion ratio and Isp capable of being delivered by the nozzle. Decreasing the throat radius would cause the chamber pressure to increase, consequently increasing the case and insulation mass required to safely operate a motor at higher pressures. To account for this factor, accurate estimates of inert mass first needed to be established. After doing so, the effects of varying nozzle expansion ratios, exit half angles, and subsequently length and mass were evaluated against motor and nozzle performance factors such as delta V and Isp. For this study, four throat radii ranging from 1.75" to 2.375" and consequently four different chamber pressures ranging 550 psia to 1200 psia were investigated.

BACKGROUND

To launch into Low Earth Orbit, a payload needs to be accelerated to the orbital velocity necessary to keep it from falling back to Earth. The change in velocity required between launch and orbital insertion is known as Delta V. The Delta V which a rocket or stage can deliver can be calculated using the Ideal Rocket equation,

$$\Delta V = -g_0 * I_{sp} * \ln\left(\frac{M_f}{M_i}\right) \tag{3}$$

Where g_0 is the acceleration due to gravity at the earth's surface, I_{sp} is the specific impulse of the rocket, M_i is the initial mass of the rocket, and M_f is the final mass of the rocket after burnout. From preliminary calculations beyond the scope of this paper, it was determined that 30500 ft/s of delta V would be required for a payload to be inserted into a 350 km circular orbit around the Earth. Using the known masses and Isp values of the 1^{st}, 3^{rd}, and 4^{th} stages and equation 3, the delta V of each stage was calculated. The delta V required by the 2^{nd} stage could then be found by taking the difference between the total delta V required and the delta V of the 1^{st}, 3^{rd}, and 4^{th} stages. From this, the required delta V of the 2^{nd} Stage was calculated to be 7340 ft/s.

Specific impulse is an efficiency factor of the nozzle which defines the impulse delivered by the motor per unit of propellant weight. The main variables of a nozzle's specific impulse investigated in this trade were exit cone half angle, throat radius, and expansion ratio which is affected by the throat radius. The expansion ratio, ε, of a nozzle is defined as the ratio between the nozzle exit area and throat area, and can be calculated using the equation,

$$\varepsilon = \frac{{R_{exit}}^2}{{R_{si}}^2} \tag{2}$$

Where R_{exit} is the radius of the nozzle's exit and R_{si} is the radius of the nozzle's throat. A larger expansion ratio and smaller exit half angle will increase the Isp of a nozzle by allowing the gas to expand more and by allowing more of the exhaust gas to produce thrust in the direction of the motor's central axis.

A cross section view of the 2^{nd} Stage motor with the major components annotated is provided in figure 1.

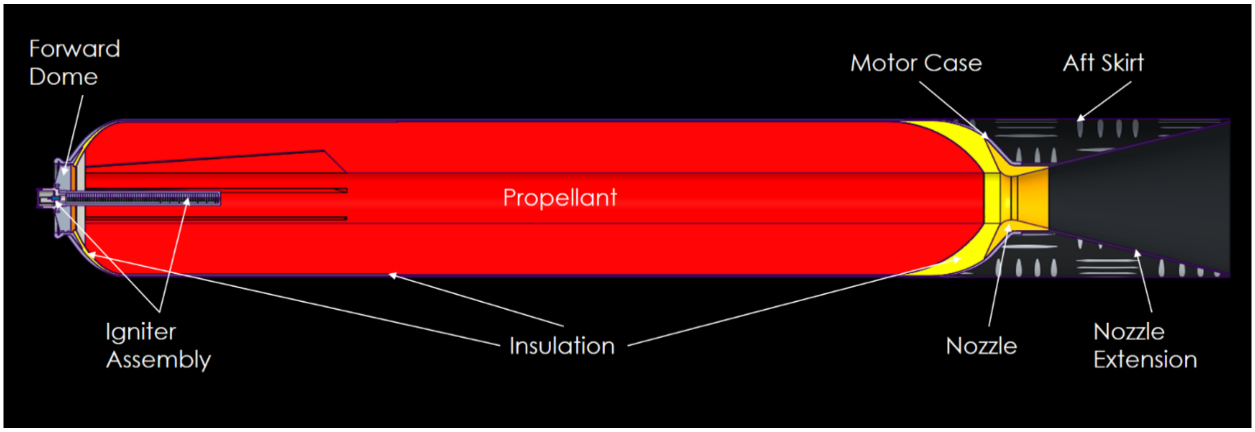

Figure 1. Cross Section of 2nd Stage Motor

For this study, the nozzle, nozzle extension, motor case, and insulation were of interest and were affected by the trade.

METHODS

To increase the usefulness of the trade study, accurate values for inert masses that scale with chamber pressure, such as motor case and insulation were necessary to predict the performance of the motor with a given nozzle geometry.

Inert Mass Estimation Methods

The inert mass of the 2nd stage motor was more accurately determined using the 3rd stage design data and preliminary 2nd stage case and insulation mass estimates previously determined before the start of the trade study. A previous trade study had investigated the increase in inert mass of the motor as a result of changing the throat radius and chamber pressure of the 2nd stage motor. Better estimates of case mass were then determined by comparing the mass estimated from the CAD model to the mass estimated during the preliminary trade investigation. The CAD of the case used for the estimation was a lengthened version of the 3rd stage design and was assumed to be an accurate representation of the mass of a 2nd stage motor case designed to operate at 550 psia, because the case wall thickness between the two models remained the same. A factor to anchor the case mass estimates with the CAD model was then calculated using the equation,

$$f_{case} = \frac{m_{case,CAD}}{m_{case,est.}} \tag{1}$$

Where $m_{case,CAD}$ is the mass of the case from the CAD model and $m_{case,est.}$ is the mass estimate of a 2nd stage motor case designed to operate at 550 psia, the same pressure at which the CAD model was design to operate at.

Table 1. Case Mass Anchoring Factor

Mass of 550 psia Case from CAD [lbm]	27
Mass of 550 psia Case from Preliminary Estimation [lbm]	14.5
Case Mass Correction Factor	1.862

As shown in table 1, the anchoring factor of the case was calculated using eqn. 1 to be 1.862. The mass estimates for the case designs at any pressure could then be multiplied by this factor in order to anchor the estimates and provide a more accurate estimate of case mass when scaling throat radius and chamber pressure.

Preliminary insulation mass estimates could also be anchored using Insulation CAD from the 3rd stage motor CAD and the same technique. The factor used to anchor the insulation mass estimates was calculated using the equation,

$$f_{insulation} = \frac{m_{insulation,CAD}}{m_{insulation,est.}} \tag{2}$$

Where $m_{insulation,CAD}$ is the mass of the insulation from the 3rd stage CAD and $m_{insulation,est.}$ is the mass estimate of the 3rd stage insulation calculated using the same methods used to estimate the 2nd stage insulation mass.

Table 2. Insulation Mass Anchoring Factor

Mass of 550 psia Case from CAD [lbm]	44.82
Mass of 550 psia Case from Preliminary Estimation [lbm]	33.9
Insulation Mass Correction Factor	1.322

The mass correcting factor could then be multiplied by the insulation mass estimates for the range of chamber pressure cases evaluated to calculate a more accurate estimate of the insulation mass. Since both the case mass and insulation masses used for this investigation were estimates, additional scaling factors were used to provide additional conservative and optimistic mass estimates.

Table 3. Case Correcting Factors

	Worst Case	Anchored Case	Optimistic Case
$f_{insulation}$	1.3881	1.322	1
f_{case}	1.955172414	1.86206897	1.768965517

The worst case factors, as shown in table 3, were calculated by increasing both the insulation and case anchoring factor by 5%. The anchored case used the original anchoring factors as shown in table 1 and table 2. The optimistic case used the original insulation mass estimates and the case mass factor was reduced by 5%. To calculate the masses for each case, the original case and insulation mass estimates were multiplied by their respective correcting factors. For each case, an additional 18 pounds of inert mass, which accounted for other inert mass, such as the forward bulkhead, igniter, etc., were added to the corrected case and insulation mass estimates to determine the total inert mass of the motor excluding the nozzle and aft skirt mass.

Nozzle Parameters Investigated and Mass Estimation Methods

The trade space began by investigating conical nozzle due to their ease of manufacturing and current implementation in the designs of the upper stage motors of the Spyder launch vehicle. The study investigated four different throat radii: 1.75”, 2”, 2.25”, and 2.375”. For each throat radius, expansion ratios ranging from 7 to the max expansion ratio possible with an exit radius of 8.38” were investigated. A radius of 8.38” was chosen as the max exit radius, because it would allow for a diametrical clearance of 0.5” between the nozzle and the inside wall of the aft skirt. For each expansion ratio, the length and mass of the nozzle extension and skirt for half angles of 10^0 through 20^0 were estimated by calculating the volume of the nozzle extension using the solids of revolution integration method and multiplying the volume by the density of carbon cloth phenolic. For each expansion ratio investigated, the Isp of nozzles for half angles of 10^0 , 12^0 , 14^0 , 15^0 , and 20^0 were calculated using the Solid Performance Program (SPP) Front-end Nozzle Optimizer GUI to consider the performance gained when decreasing nozzle half angle.

Knowing the propellant mass, stage 2 payload mass, inert mass of the 2^{nd} stage including the nozzle and skirt, and the Isp of the nozzle, the delta V of the 2^{nd} stage using a given nozzle design could be calculated using eqn. 1. With this data, stage 2 delta V with respect to nozzle length and expansion ratio was plotted to visualize the possible gains or losses between each throat radius and nozzle configuration.

RESULTS

With the data calculated using the techniques described in the Methods section, the delta V of select motor configurations were plotted with respect to the expansion ratio as shown in figure 2.

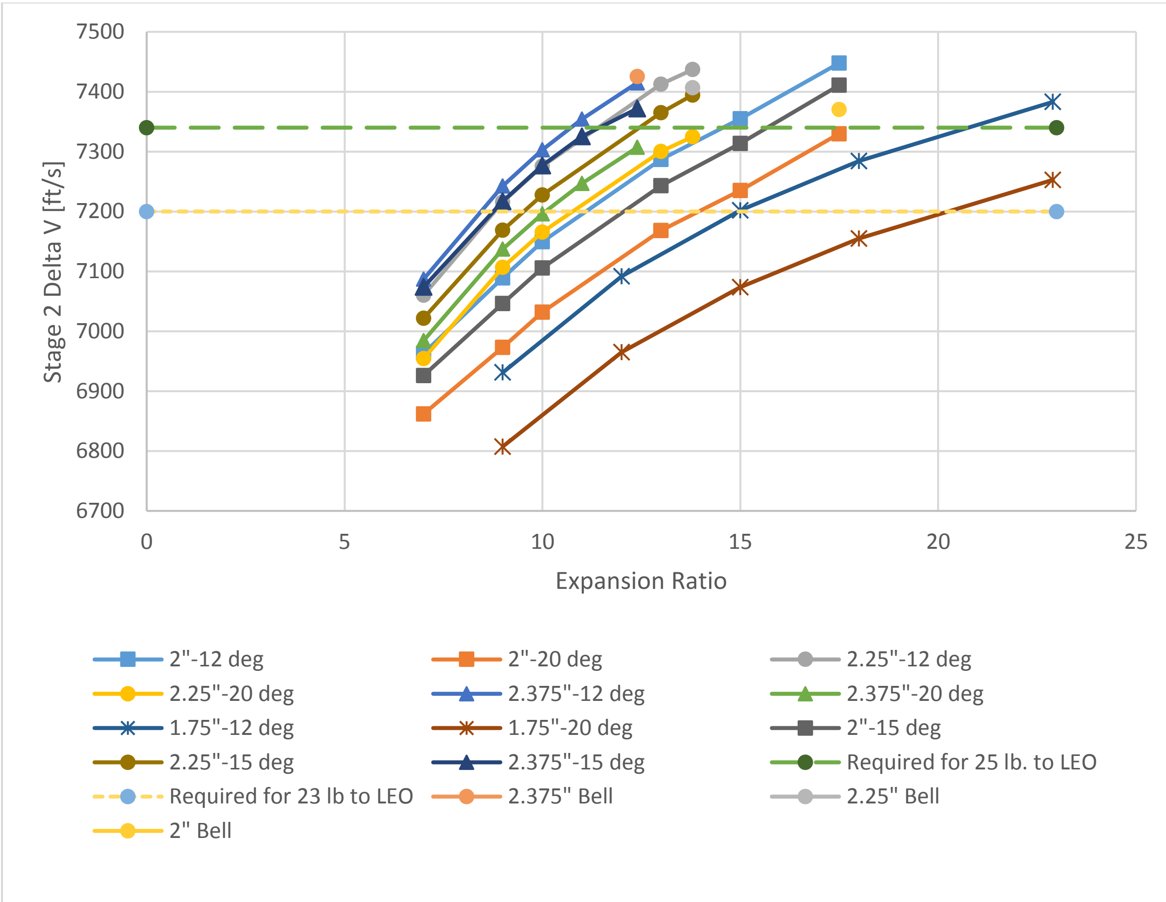

Figure 2. Stage 2 Delta V vs. Expansion Ratio

As shown in figure 2, decreasing the throat radius to increase Isp has positive returns until the throat is reduced to a radius of 1.75”. At this point, the additional case and insulation mass required to contain the higher chamber pressures negates any gains in Isp. From figure 2, the nozzle configuration which would yield the highest stage delta V would have a 2” throat radius, 13.8 expansion ratio, and 12^0 exit half angle. The delta V delivered using this 2” RSI nozzle was only marginally better than the highest performance nozzles for both the 2.25” RSI and 2.375” RSI.

Table 4. Delta V of Highest Performing Nozzles for Each RSI

Alpha [deg]	rsi [in]	ER	delta V [ft/s]	difference in delta V from rsi=1.75	difference in delta V from rsi=2	difference in delta V from rsi=2.25	Difference in delta V from rsi=2.375
12	1.75	22.9	7383.034481	0	-64.8317	-54.0141	-32.025
	2	17.5	7447.866202	64.83172159	0	10.81761	32.80669
	2.25	13.8	7437.048589	54.01410823	-10.8176	0	21.98908
	2.375	12.4	7415.059512	32.02503134	-32.8067	-21.9891	0

As evident in table 4, while the 2” throat would yield the highest delta V, it would do so by only 10.8 ft/s more than the 2.25” RSI nozzle and 32 ft/s more than the 2.375” RSI nozzle. Also of note is that each of the nozzles would be able to provide the required 7340 ft/s of delta V. A similar plot as given in figure 2 with the best and worst case scenarios included is shown in figure 3.

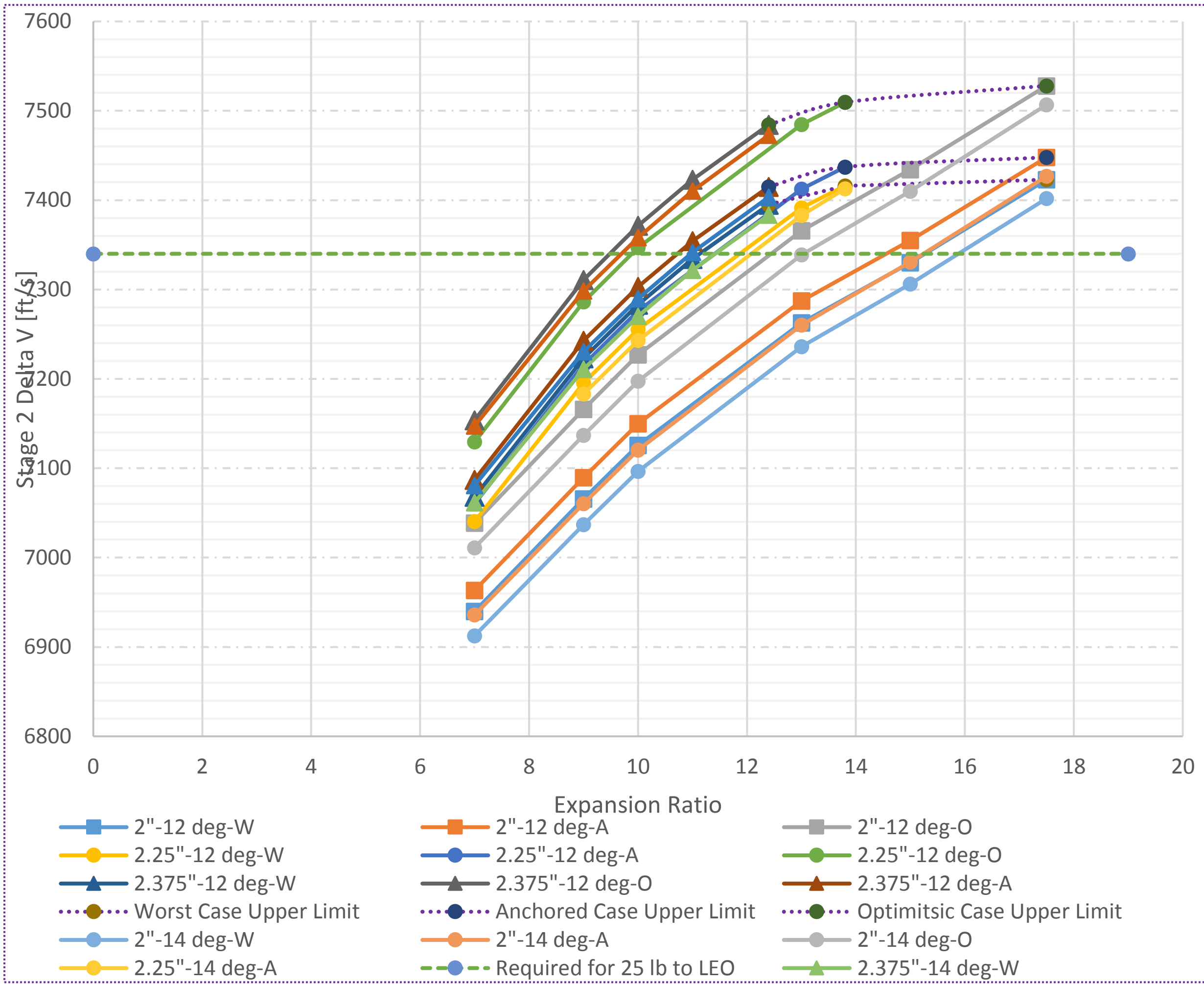

Figure 3. Delta V vs. Expansion Ratio for Optimistic, Anchored, and Worse Case Inert Mass Estimates

As evident in figure 3, the nozzles evaluated in table 4 would still be able to provide the delta V necessary to deliver 25 pounds of payload to orbit for any of the inert mass estimate cases. While the data provided in figure 2 and figure 3 proved useful into determining which expansion ratios and half angles would provide the delta V required to enter LEO, the length of these nozzle configurations and the delta V they delivered, as shown in figure 4, was also necessary to better evaluate the trade space.

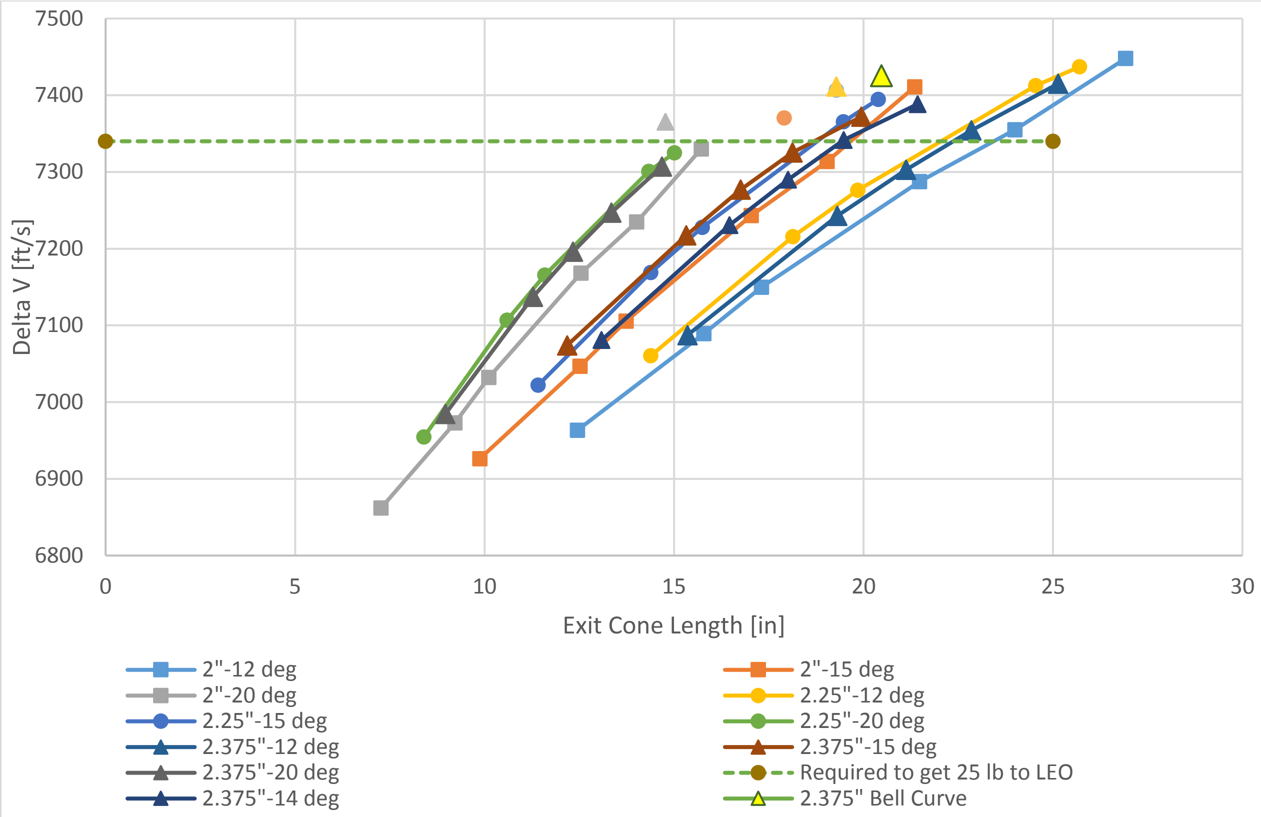

Figure 4. Stage 2 Delta V vs. Exit Cone Length

As shown in figure 4, the highest performing nozzles listed in table 4 were also the longest nozzles due to their shallow half angles.

DISCUSION

A 2.375” RSI nozzle was selected over a 2” RSI or 2.25” RSI nozzle, because the uncertainty in inert mass estimates for higher pressure motors were deemed a greater risk than the marginal increase in Delta V and Isp gained by decreasing the throat radius. In an effort to reduce nozzle length, the change in Stage 2 delta V with respect to half angle for a 2.375” RSI nozzle with an expansion ratio of 12.4 was evaluated using a backwards difference approximation. The results of these calculations are graphed in figure 5.

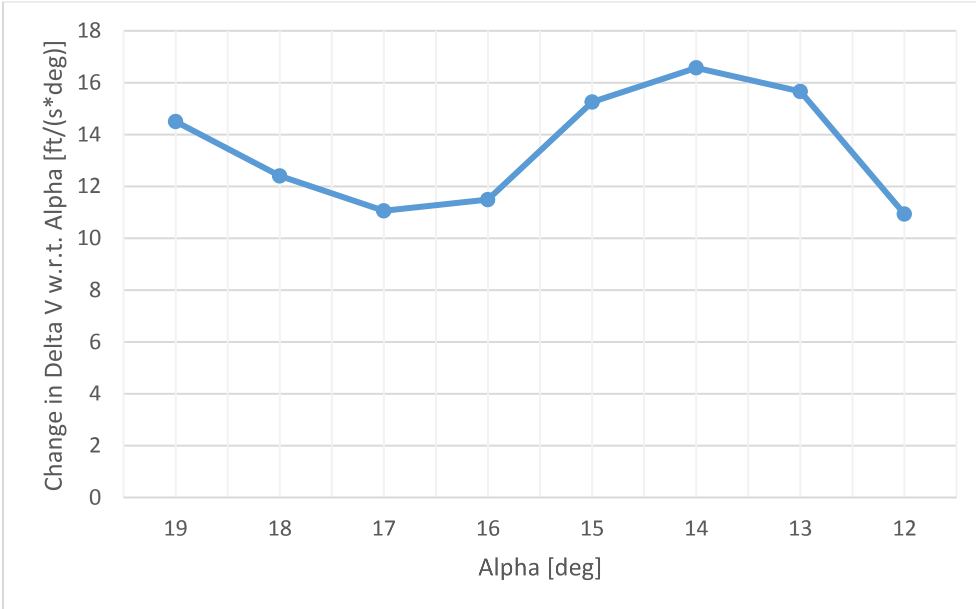

Figure 5. Change in Stage 2 Delta V w.r.t. Alpha for 2.375" RSI

As evident in figure 5, a local maxima occurs at a half angle of 14^0 , meaning that after the half angle is reduced to 14^0 the return in Stage 2 delta V begins to diminish if the half angle is reduced any further. This finding resulted in the decision to use a half angle of 14^0 to reduce nozzle length while maintaining the delta V required to deliver 25 lb. of payload to LEO.

Bell curve nozzles were also considered to increase the motor Isp or maintain the Isp while reducing nozzle length. These efforts were shortly abandoned after failing to find many possible bell curves which would have comparable performance and would not experience impingement of particles from the combustion products on the walls of the nozzle diverging section. From these findings it was determined that the efforts to develop and characterize a nozzle with acceptable particle impingement would greatly outweigh any benefits in performance gained for a motor at this scale.

From these findings, the following dimensions given in table 5 were chosen for the nozzle design.

Table 5. Stage 2 Nozzle Parameters

Throat Radius [in]	2.375
Expansion Ratio	12.04
Exit Half Angle [degrees]	14

An expansion ratio of 12.04, the maximum possible ratio for a 2.375” throat radius, was chosen after evaluation of the trade space revealed that the increase in inert mass necessary to expand the nozzle to its maximum was outweighed by the increase in performance gained by doing so. The expansion ratio is slightly reduced from the max ratio of 12.4 used in the trade study after the realization that a 12.4 expansion ratio would not allow for the necessary diametrical clearance between the outside diameter of the nozzle and the inside wall of the aft skirt as was previously thought. With this nozzle and final inert masses from the CAD model, the delta V of the current 2nd Stage design was calculated to be 7152 ft/s.

CONCLUSIONS

The results of the trade study highlighted in this report drove the final design of the nozzle for the 2nd stage motor of Spyder. Decreasing the throat radius proved to yield only marginal benefits to improving the payload capability of the launch vehicle, resulting in the decision to use a throat radius of 2.375" to operate at chambers pressures similar to that which the 3rd stage motor currently operates at. Since the nozzle of the 3rd stage motor had already undergone extensive thermal analysis, another team member scaled the 3rd stage nozzle thickness to the 2nd stage nozzle using factors which took into account the chamber pressure and burn time differences between the two motors. Using this method, a preliminary CAD model of the 2nd stage nozzle was created; however, future thermal analysis of this nozzle design will need to be conducted to ensure the nozzle will remain operational during flight and to identify points where nozzle thickness and mass can be reduced.

While the nozzle with the current inert mass values would be unable to provide the delta V necessary to get 25 lbm of payload to LEO with the current performance of the other stages, there are ways to meet this target in the future. After static testing of the current 4th, 3rd, and 2nd stage motor designs, members of the Spyder team will be able to identify areas where insulation or nozzle thickness could be reduced in order to save mass and increase the performance of the vehicle. There are also other inert components such as radios or control systems that have the potential for mass savings in order to increase the performance of the Vehicle. As a result of these potential savings, the team did not deem it necessary to attempt to increase the current nozzle efficiency of the 2nd stage motor any further to reduce the difference between the current 2nd Stage motor Delta V and the delta V required to get 25 pounds of payload to LEO that was previously calculated.

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