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Structure Deformation Calculation Program Based on Displacement Theory for Shape Predictions

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PATENT PROTECTION NOTICE

The method for structure deformed shape predictions using *Displacement Theory* to transform distributed surface strains into structure deformed shapes described in this NASA technical report is protected under *Method for Real-Time Structure-Shape Sensing*, U.S. Patent No. 7,520,176, issued April 21, 2009. Therefore, those interested in using the method (with the accompanying program) should contact NASA Technology Transfer Office at NASA Armstrong Flight Research Center, Edwards, California for more information.

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Abstract

Separated programs were written in $C/C++$ to validate the Displacement Transfer Functions. The Structure Deformation Calculation Program was written to combine all of the programs to calculate deformed shapes of a structure using surface strain data and structural geometrical parameters. Users do not need to know the material properties, nor the complex internal structures geometry because the Displacement Theory is purely geometrical in nature. Users only need to know the structure types as defined in this report and information such as the structure length, depth factors, number of strain sensors, and the surface strains measured at the strain-sensing stations installed on the structures. Depending on the structure type, an applicable Displacement Transfer Function will be used. This program requires two input files created by users; the recorded strain data file in comma-separated values format and the structure geometry data file in text format. The program will output the out-of-plane deflections, slopes, cross-sectional twist angles, and depth factors if applicable. All output files are created in comma-separated values format. A section in this report describes step-by-step procedures on how to use the Structure Deformation Calculation Program for structure deformed shape calculations.

Nomenclature

Introduction

Traditionally, the wing deflections can be measured during ground testing by using position transducers or a photogrammetry system. For in-flight deflection measurements, those methods cannot be used. One technique is to use the electro-optical flight deflection measurement systems, which are composed of onboard cameras and several wing mounted targets. Such systems can provide wing deflection information during the flight, but can be too heavy for lightweight flying vehicle applications.

After the invention of the Displacement Theory which contains different Displacement Transfer Functions (refs. 1–12), a patented technology called, "Method for Real-Time Structure Shape-Sensing," U.S. Patent Number 7,520,176 (ref. 2), was granted. The shape-sensing technology is to use the Displacement Transfer Functions to transform distributed surface strains into structure deformed shapes. This structure shape-sensing technology is quite attractive for the in-flight deformed shape monitoring of flight vehicles for flight control and maintaining flight safety. In addition, the real time wing shape monitored could then be input to the aircraft control system for aero-elastic wing shape control.

The objective of this technical memorandum is to provide users some guidance on how to use the Structure Deformation Calculation Program to calculate the deformed shape of a structure based on the Displacement Theory and Displacement Transfer Functions (refs. 1–12). Users need to prepare two files, the recorded measured surface strain data in a comma-separated values (csv) file and the required geometrical information in a text (txt) file. Depending on the structure type, the program will create several csv output files that contain the out-of-plane deflections y_i , slopes θ_i , cross-sectional twist angles ϕ_i , and depth factors c_i if applicable.

Shape Prediction Technical Background

The structure shape prediction using the Displacement Transfer Functions to transform the distributed surface strains into structure deformed shapes was reported in many National Aeronautic and Space Administration (NASA) technical reports (refs. 1–12). The following sections only cover what are related to the Structure Deformation Calculation Program. To understand more about the Displacement Theory and Displacement Transfer Functions, users can read the NASA technical reports listed in the reference section.

Key Terminologies

A surface line, along which the strain-sensing stations are to be discretely distributed, is called a *strainsensing line*. The surface strains are to be measured at those strain-sensing stations and recorded. The region between any two adjacent strain-sensing stations is called the *domain*. The structure depth-wise cross section along the strain-sensing line is called the *embedded beam* (not to be confused with the traditional isolated Euler-Bernoulli beam). The distances from the embedded beam neutral axis to the strain-sensing stations along the lower strain-sensing line are called the *depth factors*. When the data of bending surface strains, domain lengths, depth factors, and number of strain-sensing stations are input into the appropriate Displacement Transfer Functions, the deformed shape of each embedded beam can be calculated.

Theoretical Background

In the formulations of the Displacement Transfer Functions (refs. 1–12), each embedded beam was first discretized into multiple small domains with domain junctures matching the strain-sensing stations. Such a discretization approach allowed the surface strain distribution along each strain-sensing line to be represented with a piecewise-linear function. The piecewise-linear approach enabled piecewise integrations of the embedded-beam curvature equation to yield the Displacement Transfer Functions, which geometrically relate the surface strains to the out-of plane deflections along the embedded beam.

For structure shape calculations using the Displacement Transfer Functions, surface strain data and depth factors of an embedded beam are needed. Based on the type of structure geometry and loading conditions, users can select the proper strain-sensing line system and structure type for their structures. If the depth factors are unknown, extra strain-sensing line(s) is/are required. The Structure Deformation Calculation Program covers seven structure types that have depth factors known and depth factors unknown.

Depth Factors

The depth factors of a structure are important variables in Displacement Transfer Functions. The depth factors c_i , along with strains ε_i and domain lengths Δl_i , are used in the calculations of the vertical deflections y_i , slopes θ_i , and cross-sectional twist angles ϕ_i if applicable at the *i*-th strain-sensing location. For some structures, it is difficult to know the depth factors; therefore, extra strain-sensing lines are needed.

Depth Factors Known

Structure types 1 and 6 for a one-line system applied to a cantilever beam are shown in the type 1 and type 6 sections of this report. Structure type 2 for a two-end supported tubular beam is shown in the type 2 section. Since the depth factors are known, only one strain-sensing line on the lower surface is needed for bending shape prediction analysis.

Structure type 3 for a two-line system on the lower surface for combined bending and torsion or on the side and lower surfaces for combined horizontal and vertical bending is shown in the type 3 section. The two-line system includes tapered un-swept and swept wing boxes. If the depth factor is known, only two strain-sensing lines along the lower front and lower rear edges are needed. For this type of structure, the local cross-sectional twist angles can be calculated.

Structure type 5 for a square thin plate (finite-element model) subjected to a point load at the plate center, inducing two-dimensional bending under different edge conditions (four edges clamped or simply

supported) is shown in the type 5 section. Because the depth factors are known, only the multi parallel strain-sensing lines on the lower surface are needed.

Depth Factors Unknown

Structure type 4 of a four-line system with two lines on the lower surface and two lines on the upper surface for shape calculations of structures under combined bending and torsion is shown in the type 4 section. The four-line system is the most suitable sensing system for slender aircraft wings, for which the two neutral axes are unknown and are always subjected to both bending and torsion loadings. Two upper strain-sensing lines are needed for calculations of unknown depth factors. If the depth factors are known, the upper surface lines are not required.

The depths at the beam root and beam tip $\{h_0, h_n\}$ at the front of the embedded beam are known, and the local depth h_i can be calculated as shown in equation (1a).

$$
h_i = h_0 - (h_0 - h_n) \frac{x_i}{l} \qquad ; \quad (i = 1, 2, 3, \dots, n)
$$
 (1a)

The depths at the beam root and beam tip $\{h'_0, h'_n\}$ at the rear of the embedded beam are known, and the local depth h'_i can be calculated as shown in equation (1b).

$$
h'_{i} = h'_{0} - (h'_{0} - h'_{n}) \frac{x_{i}}{l} \qquad ; \quad (i = 1, 2, 3, ..., n)
$$
 (1b)

The values of calculated h_i and the bending strains $\{\varepsilon_i, \bar{\varepsilon}_i\}$ where $\bar{\varepsilon}_i$ are the bending strains of the front upper surface are used to calculate c_i as shown in equation (2a).

$$
c_i = \frac{|\varepsilon_i|}{|\varepsilon_i| + |\bar{\varepsilon}_i|} h_i; \qquad \bar{c}_i = h_i - c_i; \qquad ; \quad (i = 1, 2, 3, \dots, n)
$$
 (2a)

The values of calculated h'_i at the rear and the bending strains $\{\varepsilon'_i, \overline{\varepsilon}'_i\}$ where $\overline{\varepsilon}'_i$ are the bending strains of the rear upper surface are used to calculate c_i' as shown in equation (2b).

$$
c'_{i} = \frac{|\varepsilon'_{i}|}{|\varepsilon'_{i}| + |\bar{\varepsilon}'_{i}|} h'_{i}; \qquad \bar{c}'_{i} = h'_{i} - c'_{i} ; \quad (i = 1, 2, 3, ..., n)
$$
 (2b)

For a nonuniform large bending structure of a two-line system on lower and upper surfaces, an extra upper strain-sensing line is required to calculate the depth factors as shown in type 7 section.

List of the Shifted Displacement Transfer Functions

Based on the piecewise-linear representations of both depth factor c_i and surface strain ε_i where $i = 1, 2, 3, ..., n$, the Shifted Displacement Transfer Functions (refs. 1, 3) were formulated to transform the surface strains ε_i into slopes and vertical deflections $\{tan\theta_i, y_i\}$ along the embedded beam. The Shifted Displacement Transfer Functions for vertical deflections have the following different mathematical forms formulated for different types of structures (nonuniform, slightly nonuniform, and uniform).

Vertical Deflection for Cantilever Embedded Beams

There are three Shifted Displacement Transfer Functions for a cantilever embedded beam where $(y_0 = tan\theta_0 = 0).$

Nonuniform Shifted Displacement Transfer Functions

The depth factors are not equal $(c_{i-1} \neq c_i)$, (refs. 1, 3). The slope equation (in recursive form) is shown as equation (3a):

$$
\tan \theta_i = (\Delta l)_i \left[\frac{\varepsilon_{i-1} - \varepsilon_i}{c_{i-1} - c_i} + \frac{\varepsilon_{i-1}c_i - \varepsilon_i c_{i-1}}{(c_{i-1} - c_i)^2} \log \frac{c_i}{c_{i-1}} \right] + \tan \theta_{i-1} \qquad (3a)
$$

The vertical deflection equation (in recursive form) is shown as equation (3b):

$$
y_{i} = (\Delta l)^{2}_{i} \left[\frac{\varepsilon_{i-1} - \varepsilon_{i}}{2(c_{i-1} - c_{i})} - \frac{\varepsilon_{i-1}c_{i} - \varepsilon_{i}c_{i-1}}{(c_{i-1} - c_{i})^{3}} \left(c_{i} \log \frac{c_{i}}{c_{i-1}} + (c_{i-1} - c_{i}) \right) \right] + y_{i-1} + (\Delta l)_{i} \tan \theta_{i-1}
$$
(3b)
;
(i = 1,2,3,...,n)

Equations (3a) and (3b) are used for structure types 1, 3, and 4.

Slightly Nonuniform Shifted Displacement Transfer Functions

The depth factors are almost equal $(c_{i-1} \rightarrow c_i)$, (refs. 1, 3). The slope equation (in recursive form) is shown as equation (4a):

$$
\tan \theta_i = \frac{(\Delta l)_i}{2c_{i-1}} \left[\left(2 - \frac{c_i}{c_{i-1}} \right) \varepsilon_{i-1} + \varepsilon_i \right] + \tan \theta_{i-1} \qquad (4a)
$$
\n
$$
\vdots \quad (i = 1, 2, 3, \dots, n)
$$

The vertical deflection equation (in recursive form) is shown as equation (4b):

$$
y_i = \frac{(\Delta l)_i^2}{6c_{i-1}} \left[\left(3 - \frac{c_i}{c_{i-1}} \right) \varepsilon_{i-1} + \varepsilon_i \right] + y_{i-1} + (\Delta l)_i \tan \theta_{i-1} \n; \quad (i = 1, 2, 3, ..., n)
$$
\n(4b)

Equations (4a) and (4b) are used for structure types 1, 3, and 4.

Uniform Shifted Displacement Transfer Functions

The depth factors are equal $(c_{i-1} = c_i = c)$, (ref. 1). The slope equation (in recursive form) is shown as equation (5a):

$$
\tan \theta_i = \frac{(\Delta l)_i}{2c} (\varepsilon_{i-1} + \varepsilon_i) + \tan \theta_{i-1}
$$
\n(5a)\n
\n
$$
\frac{(i - 1, 2, 3, ..., n)}{(i - 1, 2, 3, ..., n)}
$$

The vertical deflection equation (in recursive form) is shown as equation (5b):

$$
y_i = \frac{(\Delta l)_i^2}{6c} (2\varepsilon_{i-1} + \varepsilon_i) + y_{i-1} + (\Delta l)_i \tan \theta_{i-1}
$$

$$
; (i = 1,2,3,...,n)
$$
 (5b)

Vertical Deflection for Two-End Supported Embedded Beams

The vertical deflection y_i^B of the two-end supported embedded beam (simply supported or fixed) can be calculated from equation (6) (ref. 1):

$$
y_i^B = y_i - \frac{x_i}{\underbrace{l}_{\text{Shift factor}} y_n}
$$
\n
$$
= \frac{1}{6} \sum_{j=1}^i \frac{(\Delta l)_j^2}{c_{i-j}} \left\{ \left[3(2j-1) - (3j-2) \frac{c_{i-j+1}}{c_{i-j}} \right] \varepsilon_{i-j} + (3j-2) \varepsilon_{i-j+1} \right\} - \frac{x_i}{l} y_n
$$
\n
$$
\tag{6}
$$
\n
$$
\tag{6}
$$
\n
$$
\tag{7}
$$
\n
$$
\tag{7}
$$

In equation (6), y_i is the vertical deflection of the slightly nonuniform cantilever embedded beam (applicable to the limit case of uniform embedded beams). The mathematical expression of y_i in equation (6) was obtained by combining the slope equation (4a) and the deflection equation (4b) into a single equation. The shift factor $(x_i/l) y_n$ appearing in equation (6) is to proportionally shift the cantilever deflection curve of y_i and convert it to the deflection curve of the two-end supported beam with zero deflection $y_i^B = 0$ at the beam tip $i = n$ (second support point).

List of Curved Displacement Transfer Functions

For large bending deformations of highly flexible slender structures, one must understand that the actual (true) deflection \hat{y}_i of a material point at $x = x_i$ is a curved distance traced by the same material point from its initial un-deformed position to its final deformed position. Thus, the conventional vertical deflection y_i is merely the vertical component of the curved true deflection \hat{y}_i (refs. 3, 11). The Curved Displacement Transfer Functions have the following different mathematical forms for different types of structures (nonuniform, slightly nonuniform, and uniform).

Large Deflection for Cantilever Embedded Beam

Just like the small bending deformations, the large bending deformations have three Curved Displacement Transfer Functions for a cantilever embedded beam for which $\hat{y}_i = \theta_0 = 0$.

Nonuniform Curved Displacement Transfer Functions

The depth factors are not equal $(c_{i-1} \neq c_i)$, (ref. 11). The slope equation (in recursive form) is shown in equation (7a):

$$
\theta_{i} = (\Delta l)_{i} \left[\frac{\varepsilon_{i-1} - \varepsilon_{i}}{c_{i-1} - c_{i}} + \frac{\varepsilon_{i-1} c_{i} - \varepsilon_{i} c_{i-1}}{(c_{i-1} - c_{i})^{2}} \log \frac{c_{i}}{c_{i-1}} \right] + \theta_{i-1} \qquad (7a)
$$

The curved deflection equation (in recursive form) is shown in equation (7b):

$$
\widehat{y_i} = (\Delta l)_i^2 \left[\frac{\varepsilon_{i-1} - \varepsilon_i}{2(c_{i-1} - c_i)} - \frac{\varepsilon_{i-1}c_i - \varepsilon_i c_{i-1}}{(c_{i-1} - c_i)^3} \left(c_i \log \frac{c_i}{c_{i-1}} + (c_{i-1} - c_i) \right) \right] + \widehat{y_{i-1}} + (\Delta l)_i \theta_{i-1}
$$
\n(7b)

; $(i = 1,2,3,...,n)$

Slightly Nonuniform Curved Displacement Transfer Functions

The depth factors are almost equal $(c_{i-1} \to c_i)$, (ref. 3). The slope equation (in recursive form) is shown in equation (8a):

$$
\theta_{i} = \frac{(\Delta l)_{i}}{2c_{i-1}} \left[\left(2 - \frac{c_{i}}{c_{i-1}} \right) \varepsilon_{i-1} + \varepsilon_{i} \right] + \theta_{i-1}
$$
\n(8a)\n
\n
$$
\vdots \quad (i = 1, 2, 3, ..., n)
$$

The curved deflection equation (in recursive form) is shown in equation (8b):

$$
\widehat{y_i} = \frac{(\Delta l)_i^2}{6c_{i-1}} \left[\left(3 - \frac{c_i}{c_{i-1}} \right) \varepsilon_{i-1} + \varepsilon_i \right] + \widehat{y_{i-1}} + (\Delta l)_i \theta_{i-1}
$$
\n(8b)\n
\n
$$
\vdots \quad (i = 1, 2, 3, ..., n)
$$

Uniform Curved Displacement Transfer Functions

The depth factors are equal $(c_{i-1} = c_i = c)$, (ref. 11). The slope equation (in recursive form) is shown in equation (9a):

$$
\theta_{i} = \frac{(\Delta l)_{i}}{2c} (\varepsilon_{i-1} + \varepsilon_{i}) + \theta_{i-1} \qquad (9a)
$$
\n
$$
\vdots \quad (i = 1, 2, 3, ..., n)
$$

The curved deflection equation (in recursive form) is shown in equation (9b):

$$
\widehat{y_i} = \frac{(\Delta l)_i^2}{6c} (2\varepsilon_{i-1} + \varepsilon_i) + \widehat{y_{i-1}} + (\Delta l)_i \theta_{i-1}
$$
\n(9b)\n
\n
$$
\widehat{y_i} = \frac{(\Delta l)_i^2}{6c} (2\varepsilon_{i-1} + \varepsilon_i) + \widehat{y_{i-1}} + (\Delta l)_i \theta_{i-1}
$$

Large Deflection for Two-end Supported Embedded Beam

The curved deflection \hat{y} i B of the two-end supported embedded beam (simply supported or fixed) can be calculated from equation (10), which enforces zero deflection at the beam tip ($i = n$) of the cantilever be calculated from equation (10), which emotics zero
embedded beam using shifting factor $(x_i/l)\hat{y}_n$ (ref. 1):

$$
\widehat{y}_{i}^{B} = \widehat{y}_{i} - \frac{x_{i}}{l} \widehat{y}_{n}
$$
\n
$$
= \frac{1}{6} \sum_{j=1}^{i} \frac{(\Delta l)_{j}^{2}}{c_{j-1}} \left\{ \left[3(2j-1) - (3j-2) \frac{c_{i-j+1}}{c_{i-j}} \right] \mathcal{E}_{i-j} + (3j-2) \mathcal{E}_{i-j+1} \right\} - \frac{x_{i}}{l} \widehat{y}_{n}
$$
\n(10)\n
\n(11)\n
\n(12)\n
\n(12)\n
\n(13)\n
\n(14)\n
\n(15)\n
\n(16)\n
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\n(11)\n

In equation (10), \hat{y} i B is the curved deflection of a slightly nonuniform cantilever embedded beam. Equation (10) was obtained by combining the slope angle equation (8a) and the curved deflection equation (8b) into a single equation, and is applicable to the limit case of uniform embedded beams.

It is important to mention that, if $\{\tan q_i, y_i\}$ in equations (3) – (5) are replaced respectively with $\{\theta_i, \hat{y}_i\}$, then equations (3) – (5) become equations (7) – (9) for the shape calculations of structures under geometrical nonlinear large deformations (ref. 11).

Cross-Sectional Twist Angle

For structure types 3 and 4 that have front and rear strain-sensing lines, the cross-sectional twist angle at the strain-sensing station *i*, $x = x_i$, is calculated using equation (11).

$$
\phi_i = \sin^{-1} \left\{ \frac{y_i - y'_i}{d_i} \text{ or } \frac{\widehat{y_i} - \widehat{y'_i}}{d_i} \right\} \tag{11}
$$
\n
$$
\vdots \quad (i = 1, 2, 3, ..., n)
$$

Procedure to Use the Program

In order to use the Structure Deformation Calculation Program, users are required to have Microsoft Visual Studio software (Microsoft Corporation, Redmond, Washington) or any server that can compile $C/C++$ to compile this program. Users need to create two input files, a strain data file and a geometry file in the required formats. The arrangements of the data in these files are different depending on the structure type as defined in the next section.

Preparation of the Strain Data File

The strain data file must be in csv format with the first line containing the header of "time" and names of the strain-sensing stations. The second line to the last line should contain the time and the measured surface strains ε_i of each strain-sensing station *i* on each strain-sensing line from the fixed end ε_0 to the free end ε_n . The time format in this file will be copied to the output files. If there are multiple strain-sensing lines, strain data on one line must finish before starting strain data on the next line.

Preparation of the Geometry file

The geometry file must be prepared in txt format with spaces or tab delimiters between two values. This file contains the geometry data that the program will use to calculate deformations. The distances in this report are measured in inches, but users can use any units they want as long as they are consistent. This file has some or all of the following elements.

- 1. Structure type from 1 to 7.
- 2. Total length of the structure in inches l.
- 3. Domain length in inches. The domain length, $(\Delta l)_i = \Delta l_i$, is the distance between two adjacent strain sensors *i*-1 and *i* on a strain-sensing line; Δl_i can be constant or variable.
- 4. Total number of strain-sensing stations installed on the structure. If the structure has multiple strainsensing lines, the number of stations installed on each strain-sensing line must be the same. The domain lengths Δl_i between two adjacent sensors *i*-1 and *i* on each strain-sensing line must also be

the same; for example, Δl_i on line $1 = \Delta l_i$ on line $2 = \Delta l_i$ on line k: $\Delta l_{1i} = \Delta l_{2i} = \Delta l_{ki}$.

- 5. Depth factors in inches (c_i can be known or unknown).
- 6. Chord-wise distances in inches (d_i) for structures that have front and rear strain-sensing lines).
- 7. Depths in inches (h_0 at the fixed end and h_n at the free end for structures that have lower and upper strain-sensing lines).

Running the program

After creating two required input files, users can run this the Structure Deformation Calculation Program. The program will prompt the user for three following inputs.

\$ Enter strain data filename:

\$ Enter geometry filename:

\$ Enter structure type:

Users must enter inputs to the above prompts in order to run the program. The program will always calculate vertical deflections y_i , slopes θ_i , and determine the maximum and minimum deflections for each strain-sensing station. Different structure configurations in the formulations of the Displacement Theory and Displacement Transfer Functions (refs. 1-12) are categorized into seven structure types in this program.

Depending on the structure type, the program will also calculate the depth factors c_n and/or the crosssectional twist angles ϕ_i .

Output files created by the program

The program will use the name of the strain data file to create the names of the output files in csv format by appending it with _Deflections, _Slopes, _Deflections_MaxMin, _DepthFactors, and _TwistedAngles. For example, if the strain data filename is N13.csv, the output files are N13_Deflections.csv, N13_Slopes.csv, N13_Deflections_MaxMin.csv, N13_DepthFactors.csv, and N13_TwistAngles.csv. The first row in the deflection file and slope file is labeled exactly the same as the first row in the strain data file. The first column in the deflection file and slope file is exactly the same as the first column in the strain data file. All structure types will have the deflection and maximum minimum deflection files. The deflections are measured in inches and the slopes and twist angles are measured in degrees. When finishing, the program will print out a complete message and also the names of the output files.

Structure Types

With the intention to make the Structure Deformation Calculation Program easy to use, one-line, twoline, and four-line systems, with known and unknown depth factors, with vertical and curved deflections, and with short and long lengths are categorized into seven structure types. The structure types cover the range from the simplest one-line uniform cantilever beam with known depth factors to a complicated fourline doubly tapered wing with unknown depth factors. Each structure type requires different formats of the strain and geometry files and has different output files. Dependent on the structure type, a correct Transfer Function is used in the program to calculate deflections, slopes, cross-sectional twist angles, and depth factors if applicable.

Type 1 – Cantilever Embedded Beam

For a cantilever embedded beam with strain-sensing stations distributed along the bottom strain-sensing line, the depth factors are known, and no torsion is involved. The one-line system can be used for shape prediction analysis. The cantilever embedded beam is the simplest structure type.

Type 1 Structure

Figure 1(a) shows a uniform cantilever beam with $c_0 = c_n$, and figure 1(b) shows a tapered cantilever or nonuniform beam with $c_0 > c_n$.

Figure 1(a). Type 1 structure of a uniform cantilever beam.

Figure 1(b). Type 1 structure of a tapered cantilever or nonuniform beam.

Type 1 Strain Data File

For type 1 strain data file, recorded strains must be arranged as shown in figure $1(c)$. The SG_0 is always the strain-sensing station at the fixed end, and SG_n is always the strain-sensing station at the freeend. In figure $1(c)$, SG_n is SG_16.

- The first line is the header containing the title "time" and names of the strain-sensing station starting from the fixed end.
- The second line to the last line must contain the times and measured strains at stations SG_0, SG_1, …, SG_n.
- The first column contains the times that can be in any time format.
- The columns after the first column contain the measured strains at stations $SG_0, SG_1, ..., SG_n$.

190071

Type 1 Geometry File

For type 1 geometry files, users must prepare the geometry file in txt format as shown in figure 1(d) or figure $1(e)$.

190072

Figure 1(d). Type 1 geometry file of a uniform cantilever beam constant domains.

190073

Figure 1(e). Type 1 geometry file of a tapered cantilever beam variable domains.

This file has two lines: Line 1:

- The first field is the structure type.
- The second field is the structure length.
- The third field is the number of strain-sensing stations counting from the fixed end.
- The fourth field is the depth factor c_0 at the fixed end.
- The fifth field is the depth factor c_n at the free end.

Line 2 for *Δlⁱ* domain:

- \bullet 1 is for constant domain; after 1 is nothing as shown in figure 1(d).
- 2 is for variable domain; after 2 are Δl_1 , Δl_2 , …, Δl_n as shown in figure 1(e).

Type 1 Deflection File

After starting the program, users need to enter the strain data filename, the geometry filename, and structure type as 1. The program will compare the entered structure type 1 with the structure type programmed in the geometry file. If they are equal to 1, the program will calculate the deflections and save the results in a deflection file as shown in figure 1(f).

190074

Type 1 Slope File

Similar to the deflections, the program will calculate the slopes. The results will be saved in a slope file as shown in figure $1(g)$.

190075

Figure $1(g)$. Type 1 slope file.

Max Min Deflection File for All Structure Types

After calculating deflections for all strain-sensing stations, the max min deflections are determined and written in the output max min deflection file. Users should center the data columns so that the data are more readable. This file is always created for all structure types with the format as shown in figure 1(h).

190076

Figure 1(h). Maximum and minimum deflection file.

Type 1 Output Files

Type 1 does not have depth factors nor twist angles. Type 1 output files are a deflection file, a slope file, and a max min deflection file.

Type 2 – Two-end Supported Beam

A cantilever beam with a two-end supported beam is installed with strain-sensing stations distributed along the bottom strain-sensing line. In this case, the load P is applied in the middle of the structure. The slopes will not be calculated.

Type 2 Structure

Figure 2(a) shows a two-end simply supported beam and figure 2(b) shows a two-end fixed beam. An additional case is one end fixed and other end simply supported.

Figure 2(b). Type 2 structure of a beam with two-end fixed beam.

Type 2 Strain Data File

The strain data file is prepared in csv format similar to type 1 as shown in figure 2(c).

190079

Figure 2(c). Type 2 strain data file.

Type 2 Geometry File

The geometry file is prepared in txt format similar to the type 1 above. The geometry file is shown in figures $1(d)$ and $1(e)$.

Type 2 Deflection File

After starting the program, users need to enter the strain data filename, the geometry filename, and the structure type as 2. The program will compare the entered structure type 2 with the structure type programmed in the geometry file. If they are equal to 2, the program will calculate the deflections and save the results in a deflection file as shown in figure 2(d).

190080

Type 2 Output Files

Type 2 only have deflections. Type 2 output files are a deflection file and a max min deflection file.

Type 3 – Tapered Wing Box and Two-line System

A tapered wing box with two strain-sensing lines where strain-sensing stations are distributed along front and rear bottom lines. Twist angles will be calculated in this case. Any two strain-sensing lines can be used as long as they are in the same vertical or horizontal plane. The domain lengths for strain-sensing station *i* on two strain-sensing lines must be the same; for example, $\Delta l_i = \Delta l_{1i}$ on line 1 = Δl_{2i} on line 2.

Type 3 Structure

Figure 3(a) shows a wing box with two lower strain-sensing lines.

Figure 3(a). Type 3 Structure of a tapered wing box two-line system.

Type 3 Strain Data File

Users need to prepare the strain data file in csv format as shown in figure 3(b). The strain values on one line must be completed before starting on the other line.

190082

Figure 3(b). Type 3 strain data file.

Type 3 Geometry File

Users need to prepare the geometry file as shown in figures 3(c) and 3(d). This file has three lines.

190083

Figure 3(c). Type 3 geometry file constant domains.

Figure 3(d). Type 3 geometry file variable domains.

Line 1:

- The first field is the structure type.
- The second field is the structure length.
- The third field is the total number of strain-sensing stations.
- The fourth field is the chore-wise distance d_0 at the fixed end.
- The fifth field is the chore-wise distance d_n at the free end.

Line 2 for *Δlⁱ* domain:

- \bullet 1 is for constant domain; after 1 is nothing as shown in figure 3(c).
- 2 is for variable domain; after 2 are Δl_1 , Δl_2 , …, Δl_n as shown in figure 3(d).

Line 3 for depth factors:

- If the beam depth tapers down linearly from the fixed end to the free end, enter 1. After 1, enter the depth factor c_0 and c_n for the front line, c_0' and c_n' for the rear line as shown in figure 3(c).
- If the beam depth does not taper down linearly from the fixed end to the free end, enter 2. After 2, enter the depth factors in the order of the strain sensors in the strain data file as shown in figure 3(d), c_0 for lf_sg0, …, c_n for lf_sg8, c_0' for lr_sg0, …, c_n' for lr_sg8.

Type 3 Deflection File

After starting the program, users need to enter the strain data filename, the geometry filename, and the structure type as 3. The program will compare the entered structure type 3 with the structure type programmed in the geometry file. If they are equal to 3, the program will calculate the deflections and save the results in a deflection file as shown in figure 3(e).

190085

190084

Type 3 Slope File

Similar to the deflections, the program will calculate the slopes. The results will be saved in a slope file as shown in figure 3(f).

190086

Figure 3(f). Type 3 slope file.

Type 3 Cross-sectional Twist Angle File

Similar to the deflections and slopes, the program will calculate the twist angles. The results will be saved in a twist angle file as shown in figure 3(g).

190087

Figure 3(g). Type 3 twist angle file.

Type 3 Output Files

Type 3 does not have depth factors. Type 3 output files are a deflection file, a slope file, a twist angle file, and a max min deflection file.

Type 4 – Doubly Tapered Wing and Four-line System

A doubly tapered wing with four strain-sensing lines where strain-sensing stations are distributed along two front lines and two rear lines. Depth factors and twist angles will be calculated in this case. The domain lengths for strain-sensing station *i* on four strain-sensing lines must be the same; for example, $\Delta l_i = \Delta l_{li}$ on line $1 = \Delta l_{2i}$ on line $2 = \Delta l_{3i}$ on line $3 = \Delta l_{4i}$ on line 4. Type 4 is the most complicated type; users need to prepare the strain data file and the geometry file carefully.

Type 4 Structure

Figure 4(a) shows a doubly tapered wing with a four-line system. The two extra lines must be added to determine depth factors c_i . After running this structure type one time, users have the depth factors c_i created by this program. Then, users can use structure type 3 with a two-line system.

The lower front strains are $(\varepsilon_0, \varepsilon_1, \varepsilon_2, ..., \varepsilon_n)$. The upper front strains are $(\bar{\varepsilon}_0, \bar{\varepsilon}_1, \bar{\varepsilon}_2, ..., \bar{\varepsilon}_n)$. The lower rear strains are $(\varepsilon'_0, \varepsilon'_1, \varepsilon'_2, ..., \varepsilon'_n)$. The upper rear strains are $(\bar{\varepsilon}_0', \bar{\varepsilon}_1', \bar{\varepsilon}_2', ..., \bar{\varepsilon}_n')$.

Figure 4(a). Type 4 structure of a doubly tapered wing four-line system.

Type 4 Strain Data File

Users need to prepare a single strain data file in csv format as shown in figure 4(b). The first strain sensor on each line must always be located at the fixed end. The strain values on one line must be completed before starting on the next line. The order of strains need to be exactly as shown in figure 4(b). The top half containing strain data for the front starts from column B and the bottom half containing strain data for the rear starts from column T.

190089

Figure 4(b). Type 4 strain data file.

Type 4 Geometry File

Users need to prepare the Geometry file in txt format as shown in figure 4(c).

190090

Figure 4(c). Type 4 geometry File.

Line 1:

- The first field is the structure type.
- The second field is the structure length.
- The third field is the total number of strain-sensing stations.
- The fourth field is the separation distance from the front and the rear at the fixed end, d_0 .
- The fifth field is the separation distance from the front and the rear at the free end, d_n .
- The sixth field is the beam depth at the front fixed end, h_0 .
- The seventh field is the beam depth at the front free end, h_n .
- The eighth field is the beam depth at the rear fixed end, h_0' .
- The ninth field is the beam depth at the rear free end, h_n' .

Line 2 for *Δlⁱ* domain:

- \bullet 1 is for constant domain; nothing after 1 as shown in figure 1(d).
- 2 is for variable domain; after 2 are $\Delta l_1, \Delta l_2, \ldots, \Delta l_n$ as shown in figure 4(c).

Type 4 Deflection File

After starting the program, users need to enter the strain data filename, the geometry filename, and the structure type as 4. The program will compare the entered structure type 4 with the structure type programmed in the geometry file. If they are equal to 4, the program will calculate the deflections and save the results in a deflection file as shown in figure 4(d).

190091

Figure 4(d). Type 4 deflection file.

Type 4 Slope File

Similar to the deflections, the program will calculate the slopes. The results will be saved in a slope file as shown in figure 4(e).

190092

Figure 4(e). Type 4 slope file.

Type 4 Depth Factor File

The program will calculate the depth factors for type 4. The results will be saved in a depth factor file as shown in figure 4(f). Structure type 4 does not have depth factors c_i ; therefore, a four-line system is used. After the c_i are calculated from this program, users can use structure type 3 with a two-line system.

190093

Figure 4(f). Type 4 depth factor file.

Type 4 Twist Angle File

The program will also calculate the twist angles for type 4. The results will be saved in a twist angle file as shown in figure $4(g)$. Users can change the title names LwrStation $0, \ldots, UprStation 8$ to whatever names that make sense to them.

 ϵ G M \overline{N} \circ \circ ion_2 LwrStat n_0 LwrSta ion_3 LwrStation_4 LwrStat 8:23:15:101 0 0.034923 0.115094 0.222468 0.358599 0.528832 0.735983 0.989195 1.305771 0 0.035029 0.11537 0.222871 0.35914 0.52951 0.736809 0.989891 1.30708 8:23:15:301 $\mathbf{0}$ 0.03471 0.114253 0.220524 0.355022 0.523041 0.727263 0.976731 1.288961 $0.000068 \hspace{0.2cm} 0.013298 \hspace{0.2cm} 0.050638 \hspace{0.2cm} 0.118045 \hspace{0.2cm} 0.220974 \hspace{0.2cm} 0.362442 \hspace{0.2cm} 0.55148 \hspace{0.2cm} 0.806163$ 8:23:15:501 0 0.034496 0.113412 0.218579 0.351444 0.517251 0.718543 0.964267 1.272151 0 0.000457 0.01544 0.055593 0.127114 0.235582 0.384204 0.582198 0.847397 0 0.034282 0.112572 0.216634 0.347866 0.51146 0.709823 0.951803 1.255341 0 0.000981 0.017583 0.060548 0.136184 0.250189 0.405967 0.612916 0.888632 5 8:23:15:701 8:23:15:901 0 0.034069 0.111731 0.21469 0.344289 0.50567 0.701103 0.939339 1.238532 0 0.001506 0.019725 0.065504 0.145254 0.264797 0.42773 0.643634 0.929867 7 8:23:16:101 0 0.033855 0.110891 0.212745 0.340711 0.499879 0.692383 0.926875 1.221722 0 0.00203 0.021867 0.070459 0.154323 0.279405 0.449493 0.674352 0.971103 N4_TwistAngles (F)

190094

Figure 4(g). Type 4 twist angle file.

Type 4 Output Files

The program creates the most output files for this type. Type 4 output files are a deflection file, a slope file, a depth factor file, a twist angle file, and a max min deflection file.

Type 5 – Thin Uniform Plate

For a uniform plate $(c_0 = c_n)$, the strain sensor system requires multiple parallel strain-sensing lines across the two opposite edges as shown in figure 5(a). The four edges of the plate can be either fixed or simply supported. The load is applied somewhere in the center of the plate. The plate must be very thin and the depth factor is very small compared to the length. The domain lengths for strain-sensing station *i* on every strain-sensing line must be the same; for example, $\Delta l_i = \Delta l_{1i}$ on line $1 = \Delta l_{2i}$ on line $2 = \Delta l_{3i}$ on line $3 = \Delta l$ *ki* on line k. Similar to type 2, the slopes will not be calculated.

Type 5 Structure

Figure 5(a) shows a thin uniform plate with parallel strain-sensing lines with undeformed and deformed shapes.

Figure 5(a). Type 5 structure of a very thin plate.

Type 5 Strain Data File

Users need to prepare the strain data file in csv format with strain values on one line which must be completed before starting on the next line. Lines must start from one end across to the other end as shown in figure 5(b); for example, line 1, line 2, …, line k. For more details of how to arrange the type 5 strain data file, users can refer to the type 1 strain data file and the type 4 strain data file in the report.

190096

Figure 5(b). Type 5 strain data file for three lines.

Type 5 Geometry File

The geometry file should be prepared in txt format as shown in figure 5(c). This file has two lines: Line 1:

- The first field is the structure type.
- The second field is the length of the strain-sensing line.
- The third field is the number of strain-sensing stations.
- The fourth field is the plate thickness.
- The fifth field is the number of strain-sensing lines on the plate.

Line 2 for *Δlⁱ* domain:

- \bullet 1 is for constant domain; after 1 is nothing as shown in figure 1(d).
- 2 is for variable domain; after 2 are Δl_1 , Δl_2 , …, Δl_n as shown in figure 5(c).

190097

190098

Figure 5(c). Type 5 geometry file.

Type 5 Deflection File

After starting the program, users need to enter the strain data filename, the geometry filename, and the structure type as 5. The program will compare the entered structure type 5 with the structure type programmed in the geometry file. If they are equal to 5, the program will calculate the deflections and save the results in a deflection file as shown in figure 5(d).

	B		D	E		G	H			K		M	N	\circ	P	O	\mathbb{R}		
time											Line1 SGO Line1 SG1 Line1 SG2 Line1 SG3 Line1 SG4 Line1 SG5 Line1 SG6 Line1 SG8 Line2 SG0 Line2 SG1 Line2 SG2 Line2 SG3 Line2 SG4 Line2 SG5 Line2 SG6 Line2 SG6 Line2 SG7 Line2 SG8								
8:23:15:101		-0.033		$-0.06402 - 0.08976 - 0.10206$			$-0.08976 - 0.06402$	-0.033			-0.0345	-0.06665	-0.09306	-0.10559	-0.09306	-0.06665	-0.0345		
8:23:15:301		-0.033	-0.06403		$-0.08977 - 0.10207$		$-0.08977 - 0.06403$	-0.033	Ω		-0.0345 Ω	-0.06665	-0.09307	-0.1056	-0.09307	-0.06665	-0.0345		
4 8:23:15:501		$0 - 0.03302$	-0.06405	-0.08981	-0.10211	-0.08981	-0.06405	-0.03302	Ω		$0 - 0.03451$	-0.06667	-0.09309	-0.10562	-0.09309	-0.06667	-0.03451		
8:23:15:701		$0 - 0.03302$	-0.06406	-0.08981	-0.10212	-0.08981	-0.06406	-0.03302			$0 -0.03451$	-0.06667	-0.0931	-0.10563	-0.0931	-0.06667	-0.03451		
6 8:23:15:901		$0 - 0.03302$		$-0.06407 - 0.08982$	-0.10213	-0.08982	-0.06407	-0.03302	$\mathbf{0}$		$0 - 0.03452 - 0.06668$		-0.09311	-0.10564	-0.09311	-0.06668	-0.03452		$0 -$
\mathbf{A}		U	v	W	X		z	AA	AB	AC	AD	AE	AF	AG	AH	AI	A	AK	
1 time						Line3 SG0 Line3 SG1 Line3 SG2 Line3 SG3 Line3 SG4 Line3 SG5 Line3 SG6 Line3 SG7 Line3 SG8													
2 8:23:15:101						-0.0336 -0.06507 -0.09108 -0.10347 -0.09108 -0.06507		-0.0336											
3 8:23:15:301		$0 - 0.03361$	-0.06508	-0.09109	-0.10349	-0.09109	-0.06508	-0.03361											
4 8:23:15:501						0 -0.03361 -0.06509 -0.09111 -0.10351 -0.09111 -0.06509 -0.03361													
5 8:23:15:701		$0 - 0.03362$		$-0.06511 - 0.09113$	-0.10353	$-0.09113 - 0.06511 - 0.03362$			$^{\circ}$										
6 8:23:15:901		$0 - 0.03362$		$-0.06511 - 0.09114$		$-0.10354 -0.09114 -0.06511 -0.03362$			$\mathbf{0}$										
	N5 Deflections	$(+)$									\pm 4								

Figure 5(d). Type 5 deflection file for three lines.

Type 5 Output Files

Type 5 does not have slopes, nor twist angles. The output files are a deflection file and a max min deflection file.

Type 6 – Long Beam with Known Depth Factors

Type 6 is the same as type 1 where the length of the structure is very long compared to the width and known depth factors c_i . For large deformations, $tan\theta_i$ is replaced by θ_i in the Displacement Transfer Function used for type 1.

Type 6 Structure

Figure 6(a) shows a nonuniform long cantilever beam with undeformed and deformed shapes.

Figure 6(a). Type 6 structure of a long beam with known c_i .

Type 6 Strain Data File

For type 6, recorded strains must be arranged as type 1 and is shown in figure 6(b). For more details, refer to the type 1 strain data file in the report.

190100

Figure 6(b). Type 6 strain data file.

Type 6 Geometry File

For type 6, users need to prepare the geometry file in txt format similar to type 1. For more details, refer to the type 1 geometry file shown in figures 1(d) and 1(e).

Type 6 Deflection File

After starting the program, users need to enter the strain data filename, the geometry filename, and the structure type as 6. The program will compare the entered structure type 6 with the structure type programmed in the geometry file. If they are equal to 6, the program will calculate the deflections and save the results in a deflection file as shown in figure 6(c).

190101

Figure 6(c). Type 6 deflection file.

Type 6 Slope File

Similar to the deflections, the program will calculate the slopes. The results will be saved in a slope file as shown in figure 6(d).

190102

Type 6 Output Files

Type 6 does not have depth factors, nor twist angles. Type 6 output files are a deflection file, a slope file, and a max min deflection file.

Type 7 – Long Beam with Unknown Depth Factors

Type 7 is the same as type 1 where the length of the structure is very long compared to the width and unknown depth factors c_i . For this type, an extra strain-sensing line on the upper surface is needed to calculate the depth factors c_i . For large deformations, $tan\theta_i$ is replaced by θ_i in the Displacement Transfer Function used for type 1.

Type 7 Structure

Figure 7(a) shows a long cantilever beam with two strain-sensing lines.

Figure 7(a). Type 7 Structure of a long beam with unknown c_i .

Type 7 Strain Data File

Users need to prepare the strain data file in csv format as shown in figure 7(b). The two strain sensors lf_sg0 and uf_sg0 are always at the fixed end. The strain values on one line must be completed before starting on the other line; normally the lower front strain-sensing line is first followed by the upper front strain-sensing line.

190104

Figure 7(b). Type 7 strain data file.

Type 7 Geometry File

For type 7, users need to prepare the Geometry file in txt format as shown in figures 7(c) and 7(d). This file has two lines:

Line 1:

- The first field is the structure type.
- The second field is the structure length.
- The third field is the number of strain-sensing stations.
- The fourth field is the wing root depth h_0 at the front.
- The fifth field is wing tip depth h_n at the front.

Line 2 for *Δlⁱ* domain:

- \bullet 1 is for constant domain; after 1 is nothing as shown in figure 7(c).
- 2 is for variable domain; after 2 are Δl_1 , Δl_2 , ..., Δl_n as shown in figure 7(d).

190105

Figure 7(c). Type 7 geometry file constant domains.

190106

Figure 7(d). Type 7 geometry file variable domains.

Type 7 Deflection File

After starting the program, users need to enter the strain data filename, the geometry filename, and the structure type as 7. The program will compare the entered structure type 7 with the structure type programmed in the geometry file. If they are equal to 7, the program will calculate the deflections and save the results in a deflection file as shown in figure 7(e).

190107

Figure 7(e). Type 7 deflection file.

Type 7 Slope File

Similar to the deflections, the program will calculate the slopes. The results will be saved in a slope file as shown in figure 7(f).

190108

Figure 7(f). Type 7 slope file.

Type 7 Depth Factor File

Similar to the deflections and slopes, the program will calculate the depth factors. The results will be saved in a depth factor file as shown in figure $7(g)$. After the c_i are calculated from this program, users can use structure type 6 with only one strain-sensing line.

190109

Figure 7(g). Type 7 depth factor file.

Type 7 Output Files

Type 7 does not have twist angles. Type 7 output files are a deflection file, a slope file, a depth factor file, and a max min deflection file.

Final Remarks

There have been many NASA/TPs and NASA/TMs written and published about the Displacement Theory throughout the years. The Displacement Transfer Functions were derived for many structure types. The shape prediction accuracy of the Displacement Theory was analytically validated by finite-element analysis of the Ikhana wing (General Atomics Aeronautical Systems Inc., Poway, California) (ref. 13). The Displacement Theory was also experimentally validated using real-time strain data recorded from the ground loads tests performed in the Flight Load Laboratory at the NASA Armstrong Flight Research Center with full-scale Global Observer (AeroVironment Inc., Monrovia, California) aircraft wings (ref.14) and the GIII (Gulfstream Aerospace, Savannah, Georgia) swept wing structure (ref. 15). In order for users to apply the Displacement Transfer Functions without requiring deep knowledge of the Displacement Theory, the Structure Deformation Calculation Program was written and completed. This program will output the outof-plane deflections, slopes, cross-sectional twist angles, and depth factors based on the structure type. The outputs of this program can be plotted for all strain-sensing stations in one time slice, one strain-sensing station in all time slices, or all strain-sensing stations in all time slices. This program is versatile and can be applied to a wide range of structures such as aircraft and spacecraft (wing, tail, and fuselage), ships (slab, plate, beam, and truss), skyscrapers, radio towers, bridges, and windmills. The data outputs by the program can be used to monitor the integrity of a structure, and appropriate actions would be made if the structure shows weakness that may cause serious safety issues.

Appendix A: Program Flowchart

The Structure Deformation Calculation Program is written for 7 structure types, it is important that users know their structure types. Each structure type requires different geometry information and different strain arrangement. The program flowchart is displayed in figures A1 and A2.

Figure A1. Flowchart of the start of the Structure Deformation Calculation Program.

Figure A2. Flowchart of the end of the Structure Deformation Calculation Program.

Appendix B: Program Header File

ifstream geoFile; ofstream outFile; ofstream thetaFile; ofstream phiFile; ofstream maxminFile; ofstream cFile; string inputFile; string ingeoFile; string outputFile; string outthetaFile; string outphiFile; string outmaxminFile; string outcFile; vector<string> stationNames, tMax, tMin; vector<double> epsilon, deltaL, x, y, yB, yMax, yMin; vector<double> theta, tan_theta, phi, sin_phi; vector<double> c, d, h; const int MAX $LINK = 500000$

char *token, *t; char $*$ nextToken = NULL; char inBuff[MAX_LINE];

double C, C0, Cn, C0_prime, Cn_prime, D0, Dn, H0, Hn, H0_prime, Hn_prime, strain, length, const_deltaL, H_ratio, Hprime_ratio;

int calC = FALSE, checked = OK ;

unsigned int i, j, k, n, structureType, structType, cType, domainType, numLines, nStations, numStations, noStations, first_time = 1 , phiCreated = 0 , $cCreated = 0$, lineNum = 0;

Appendix C: Program Code in C++


```
* 4 for doubly wing box with four-line system & unknown c *
* 5 for thin uniform plate *
* 6 for curved deformation of long tapered cantilever beam *
* 7 for curved deformation of long nonlinear beam *
* ** NOMENCLATURE used in the program: *
* ** C: depth factor of uniform beam, in. *
* c[i]: depth factors at strain-sensing station i, x=xi, in. *
* C0, c[0]: value of c[i] at fixed end (beam root) strain-sensing station, in. *
* Cn, c[n]: value of c[i] at free end (beam tip) strain-sensing station, in. *
* d: chord-wise distance between two span-wise parallel strain lines, in. *
* d[i]: chord-wise distance between front strain-sensing stations i and rear strain-sensing * 
* stations i', in. *
* deltaL[i]: distance between strain-sensing stations on a same strain-sensing line i-1 \& i, in. *
* x[i]: distance from the fixed end to the i-th strain-sensing station, in. *
* y[i]: deflection at strain-sensing station i, in. *
* theta[i], \theta[i]: slope of deformed beam at strain-sensing station i, deg
* phi[i], Φ[i]: cross-sectional twist angle at strain-sensing station i, deg *
* ** REVISION HISTORY: *
* ** Initial Release: September 13, 2017 *
* ** Revisions: *
* ********************************************************************************/
#include "DisplacementCalculation.h"
int CalcDisplacement()
{
  double term1, term2, term3, term4;
  // Clear epsilon arrays
  epsilon.clear();
  // Read a line of data in the Strain input file
  while (inFile.getline(inBuff, MAX_LINE))
  {
    // Read time for the current time slice
    token = strtok_s(inBuff, " ,\t\n", &nextToken);
    // First value is time
    if (token)
    {
      // Save time
      t = token:
      // Write time to deflection output file
      outFile << t <<",";
```

```
// Write time to slope output file
    if ((structureType != 2) && (structureType != 5))
    {
        thetaFile << t <<",";
    }
    // Write time to twist angle output file
    if ((structureType == 3) \parallel (structureType == 4))
        phiFile << t <<",";
}
// Read the input strains for the current time slice
while (token)
{
    token = strtok_s(0, ",\t\n", &nextToken);
    if (token)
    {
        strain = atof(token);
        epsilon.push_back(strain);
        y.push_back(0);
        yB.push_back(0);
        if ((structureType != 2) && (structureType != 5))
        {
            theta.push_back(0);
            tan_theta.push_back(0);
        }
        if ((structureType == 3) \parallel (structureType == 4))
        {
            phi.push_back(0);
            sin_phi.push_back(0);
        }
    }
}
switch (structureType)
{
case 1: // uniform or tapered cantilever beam
    // Set deflection and slope at the fixed end
    y[0] = 0.0;theta[0] = 0.0;
    // Write to deflection and slope output files
    outFile << fixed << setprecision(6) << y[0];
    thetaFile << fixed << setprecision(6) << theta[0];
```

```
// Uniform
    if (C0 == Cn){
        for (i = 1; i < numStations; i++)
         {
            // Eq. (5a) in this paper or Eq. (24) in NASA/TP-2009-214643
            term1 = epsilon[i - 1] + epsilon[i];tan\_theta[i] = (delta[i] / (2.0*C0)) * term1 + tan\_theta[i - 1];theta[i] = atan(tan_theta[i]) * 180.0 / PI;
            / Eq. (5b) in this paper or Eq. (26) in NASA/TP-2009-214643
            term2 = (2.0*epsilon[<i>i</i> - 1]) + epsilon[i];
            y[i] = (delta[i] * delta[i] / (6.0 * CO)) * term2 + y[i - 1] +deltaL[i] * tan_theta[i - 1];
            // Write to deflection and slope output files
            outFile << "," << fixed << setprecision(6) << y[i];
            thetaFile << "," << fixed << setprecision(6) << theta[i];
        }
    }
    else if ((c[1] / CO > TPR_RATION)&& (Cn / c[n-1] > TPR_RATION){
    // Slightly Tapered
        for (i = 1; i < numStations; i++)
        {
            / Eq. (4a) in this paper or Eq. (14a) in NASA/TP-2015-218464
            term1 = (2.0 - (c[i] / c[i - 1])) * epsilon[i - 1] + epsilon[i];
            tan\_theta[i] = (delta[i] / (2.0 * c[i - 1])) * term1 + tan\_theta[i - 1];theta[i] = atan(tan_theta[i]) * 180.0 / PI;
            / Eq. (4b) in this paper or Eq. (14b) in NASA/TP-2015-218464
            term2 = (3.0 - (c[i] / c[i - 1])) * epsilon[i - 1] + epsilon[i];
            y[i] = (delta[i] * delta[i] / (6.0 * c[i - 1])) * term2 + y[i - 1] +deltaL[i] * tan theta[i - 1];
            // Write to deflection and slope output files
            outFile << "," << fixed << setprecision(6) << y[i];
            thetaFile << "," << fixed << setprecision(6) << theta[i];
         }
}
else
    {
        // Nonuniform 
        for (i = 1; i < numStations; i++){
            // Eq. (3a) in this paper or Eq. (13a) in NASA/TP-2015-218464
            term1 = (epsilon[i - 1] - epsilon[i]) / (c[i - 1] - c[i]);term2 = (epsilon[i - 1] * c[i] - epsilon[i] * c[i - 1]) *
                       log(c[i] / c[i - 1]) / pow((c[i - 1] - c[i]), 2);tan\_theta[i] = deltaL[i] * (term1 + term2) + tan\_theta[i - 1];theta[i] = atan(tan_theta[i]) * 180.0 / PI;
```

```
// Eq. (3b) in this paper or Eq. (13b) in NASA/TP-2015-218464
             term3 = (epsilon[i - 1] * c[i] - epsilon[i] * c[i - 1])pow((c[i - 1] - c[i]), 3);term4 = c[i] * log(c[i] / c[i - 1]) + (c[i - 1] - c[i]);
             y[i] = delta[i] * delta[i] * ((term1 / 2.0) - term3*term4) +y[i - 1] + delta[i] * tan\_theta[i - 1];// Write to deflection and slope output files
             outFile << "," << fixed << setprecision(6) << y[i];
             thetaFile << "," << fixed << setprecision(6) << theta[i];
        }
    }
    break;
case 2: // two-end supported
    // Set values at the selected fixed end
    y[0] = 0.0;
    yB[0] = 0.0;// Write to deflection output file
    outFile << fixed << setprecision(6) << yB[0];
    // Calculate deflection y
    for (i = 1; i < numStations; i++){
        term1 = 0.0;term2 = 0.0;/ Eq. (6) in this paper or Eq. (36) in NASA/TP-2009-214643
        for (i = 1; j \le i; j++){
             term2 = (1.0 / c[i - i]) *
                 ((3.0 * (2.0 * j - 1.0) - ((3.0 * j - 2.0) * c[i - j + 1] / c[i - j])) * epsilon[i - j]+ (3.0<sup>*</sup>j - 2.0) * epsilon[i - j + 1];term1 += term2;}
        y[i] = deltaL[i] * deltaL[i] * term1 / 6.0;}
    // Calculate deflection yB
    for (i = 1; i < numStations; i++){
        // yB = y - the correction termyB[i] = y[i] - (x[i] / length * y[n]);// Write to deflection output file
        outFile << "," << fixed << setprecision(6) << yB[i];
```

```
}
break;
```

```
case 3: // two-line system
    // Calculate deflections and slopes
    for (i = 0; j < 2; j++){
        // Set values at the fixed end
        y[j^*nStations] = 0.0;theta[i^*nStations] = 0.0;
        // Write to deflection and slope output files
        outFile << fixed << setprecision(6) << y[j*nStations];
        thetaFile << fixed << setprecision(6) << theta[j*nStations];
        for (i = j^*nStations + 1; i < (j^*nStations + nStations); i++){
             // Eq. (3a) in this paper or Eq. (13a) in NASA/TP-2015-218464
             term1 = (epsilon[i - 1] - epsilon[i]) / (c[i - 1] - c[i]);term2 = (epsilon[i - 1] * c[i] - epsilon[i] * c[i - 1]) * log(c[i] / c[i - 1]) /pow((c[i - 1] - c[i]), 2);tan_theta[i] = deltaL[i - j*nStations] * (term1 + term2) + tan_theta[i - 1];
             theta[i] = atan(tan_theta[i]) * 180.0 / PI;
        // Eq. (3b) in this paper or Eq. (13b) in NASA/TP-2015-218464
             term3 = (epsilon[i - 1] * c[i] - epsilon[i] * c[i - 1]) / pow((c[i - 1] - c[i]), 3);
             term4 = c[i] * log(c[i] / c[i - 1]) + (c[i - 1] - c[i]);y[i] = deltaL[i - j^*nStations] * deltaL[i - j^*nStations] * (term1 / 2.0) -term3*term4) + y[i - 1] + deltaL[i - j*nStations] * tan_theta[i - 1];
             // Write to deflection and slope output files
             outFile << "," << fixed << setprecision(6) << y[i];
             thetaFile << "," << fixed << setprecision(6) << theta[i];
        }
        // Write "," in output files
        outFile << ",";
        thetaFile << ",";
    }
    // Calculate twist angles
    CalcTwistAngles();
    break;
case 4: // 4-line system
    \mathcal{U} Calculate c[i]
    if (calC =FALSE)
    {
        CalculateC();
```

```
// Calculate deflections and slopes
    for (j = 0; j < 4; j++){
        // Set values at the fixed end
        y[i*nStations] = 0.0;theta[i^*nStations] = 0.0;
        // Write to deflection and slope output files
        outFile << fixed << setprecision(6) << y[i*nStations];
        thetaFile << fixed << setprecision(6) << theta[j*nStations];
        for (i = j^*nStations + 1; i < (j^*nStations + nStations); i++)\left\{ \right.// Eq. (3a) in this paper or Eq. (13a) in NASA/TP-2015-218464
             term1 = (epsilon[i - 1] - epsilon[i]) / (c[i - 1] - c[i]);
             term2 = (epsilon[i - 1] * c[i] - epsilon[i] * c[i - 1]) * log(c[i] / c[i - 1]) /
                                                            pow((c[i - 1] - c[i]), 2);tan\_theta[i] = delta[i - j^*nStations] * (term1 + term2) + tan\_theta[i - 1];theta[i] = atan(tan_theta[i]) * 180.0 / PI;
             / Eq. (3b) in this paper or Eq. (13b) in NASA/TP-2015-218464
             term3 = (epsilon[i - 1] * c[i] - epsilon[i] * c[i - 1]) / pow((c[i - 1] - c[i]), 3);
             term4 = c[i] * log(c[i] / c[i - 1]) + (c[i - 1] - c[i]);y[i] = deltaL[i - j*nStations] * deltaL[i - j*nStations] * (term1 / 2.0) -term3*term4) + y[i - 1] + deltaL[i - j*nStations] * tan_theta[i - 1];
             // Write to deflection and slope output files
             outFile << "," << fixed << setprecision(6) << y[i];
             thetaFile << "," << fixed << setprecision(6) << theta[i];
         }
        // Write "," in output files
        outFile << ",";
        thetaFile << ",";
    }
    // Calculate twist angles
    CalcTwistAngles();
    break;
case 5: // Thin uniform plate
    // Calculate deflections
    for (i = 0; j < numLines; j++){
```

```
// Set values at the selected fixed end
y[i*nStations] = 0.0;yB[j*nStations] = 0.0;
```

```
// Write to deflection output file
    outFile << fixed << setprecision(6) << yB[j*nStations];
    // Calculate y deflections
    for (i = j^*nStations + 1; i < (j^*nStations + nStations); i++){
        term1 = 0.0:
        term2 = 0.0;
        / Eq. (6) in this paper or Eq. (36) in NASA/TP-2009-214643
        for (k = 1; k \leq (i - j^* n \text{Stations}); k++){
            term2 = (1.0 / C) *
                 ((3.0 * (2.0 * k - 1.0) - (3.0 * k - 2.0)) * epsilon[i - k]+(3.0*k - 2.0) * epsilon[i - k + 1];term1 += term2;}
        y[i] = delta[i - j * nStations] * delta[i - j * nStations] * term1 / 6.0;}
    // Calculate yB deflections
    for (i = j^*nStations + 1; i < (j^*nStations + nStations); i++){
        // yB = y - the correction termyB[i] = y[i] - (x[i-j*nStations] / length * y[n + j*nStations]);// Write to deflection output file
        outFile << "," << fixed << setprecision(6) << yB[i];
    }
    // Write "," in output file
    outFile << ",";
}
break;
```
case 6: // curved deformation of 1 line long tapered beam

// Set deflection and slope at the fixed end $y[0] = 0.0;$ theta[0] = 0.0 ;

// Write to deflection and slope output files outFile $<<$ fixed $<<$ setprecision(6) $<<$ y[0]; thetaFile $<<$ fixed $<<$ setprecision(6) $<<$ theta[0];

// Calculate curved deflections of tapered beam

// Uniform if $(C0 == Cn)$

```
{
    for (i = 1; i < numStations; i++)
    {
        // Eq. (9a) in this paper or Eq. (18a) in NASA/TP-2017-219406
        term1 = epsilon[i - 1] + epsilon[i];theta[i] = (deltaL[i] / (2.0 * CO)) * term1 + theta[i - 1];
        / Eq. (9b) in this paper or Eq. (18b) in NASA/TP-2017-219406
        term2 = (2.0*epsilon[<i>i</i> - 1]) + epsilon[<i>i</i>];y[i] = (delta[i] * delta[i] / (6.0 * CO)) * term2 + y[i - 1] +deltaL[i] * theta[i - 1];
        // Write to deflection and slope output files
        outFile << "," << fixed << setprecision(6) << y[i];
        thetaFile << "," << fixed << setprecision(6) << theta[i] * 180.0 / PI;
    }
}
else if ((c[1] / CO > TPR\_RATIO) \&& (Cn / c[n - 1] > TPR\_RATIO)){
    // Slightly Nonuniform Tapered
    for (i = 1; i < numStations; i++)
    {
        / Eq. (8a) in this paper or Eq. (18) in NASA/TP-2009-214643
        term1 = (2.0 - (c[i] / c[i - 1])) * epsilon[i - 1] + epsilon[i];
        tan theta[i] = (deltaL[i] / (2.0 * c[i - 1]) * term1 + theta[i - 1];
        // Eq. (8b) in this paper or Eq. (21) in NASA/TP-2009-214643
        term2 = (3.0 - (c[i] / c[i - 1])) * epsilon[i - 1] + epsilon[i];
        y[i] = (delta[i] * delta[i] / (6.0 * c[i - 1])) * term2 + y[i - 1] +deltaL[i] * theta[i - 1];
        // Write to deflection and slope output files
        outFile << "," << fixed << setprecision(6) << y[i];
        thetaFile << "," << fixed << setprecision(6) << theta[i] * 180.0 / PI;
    }
}
else
{
    // Nonuniform Curved Deformation
    for (i = 1; i < numStations; i++){
        // Eq. (7a) in this paper or Eq. (18a) in NASA/TP-2017-219406
        term1 = (epsilon[i - 1] - epsilon[i]) / (c[i - 1] - c[i]);term2 = (epsilon[i - 1] * c[i] - epsilon[i] * c[i - 1])*log(c[i] / c[i - 1]) /
                                                                pow((c[i - 1] - c[i]), 2);theta[i] = deltaL[i] * (term1 + term2) + theta[i - 1];
        // Eq. (7b) in this paper or Eq. (18b) in NASA/TP-2017-219406
        term3 = (epsilon[i - 1] * c[i] - epsilon[i] * c[i - 1]) / pow((c[i - 1] - c[i]), 3);
        term4 = c[i] * log(c[i] / c[i - 1]) + (c[i - 1] - c[i]);
```

```
y[i] = deltaL[i] * deltaL[i] * ((term1 / 2.0) - term3*term4) + y[i - 1] +deltaL[i] * theta[i - 1];
        // Write to deflection and slope output files
        outFile << "," << fixed << setprecision(6) << y[i];
        thetaFile << "," << fixed << setprecision(6) << theta[i] * 180.0 / PI;
    }
}
break;
```
case 7: // curved deformation for 2 lines, lower and upper; unknown c

```
// Calculate c[i]
if (calC == FALSE){
    CalculateC();
}
```
// Calculate nonlinear large deflections and slopes for a long nonuniform structure for $(j = 0; j < 2; j++)$

```
{
```

```
// Set values at the fixed end
y[i*nStations] = 0.0;theta[j*nStations] = 0.0;
```
thetaFile $<<$ ",";

```
// Write to deflection and slope output files
outFile << fixed << setprecision(6) << y[j*nStations];
thetaFile << fixed << setprecision(6) << theta[j*nStations];
for (i = j^*nStations + 1; i < (j^*nStations + nStations); i++){
    / Eq. (7a) in this paper or Eq. (18a) in NASA/TP-2017-219406
    term1 = (epsilon[i - 1] - epsilon[i]) / (c[i - 1] - c[i]);
    term2 = (epsilon[i - 1] * c[i] - epsilon[i] * c[i - 1])*log(c[i] / c[i - 1]) /
                                                            pow((c[i - 1] - c[i]), 2);theta[i] = deltaL[i - j*nStations] * (term1 + term2) + theta[i - 1];
    // Eq. (7b) in this paper or Eq. (18b) in NASA/TP-2017-219406
    term3 = (epsilon[i - 1] * c[i] - epsilon[i] * c[i - 1]) / pow((c[i - 1] - c[i]), 3);
    term4 = c[i] * log(c[i] / c[i - 1]) + (c[i - 1] - c[i]);y[i] = delta[i - i * nStations] * delta[i - i * nStations] * (term1 / 2.0) -term3*term4) + deltaL[i - j*nStations] * theta[i - 1] + y[i - 1];
    // Write to deflection and slope output files
    outFile << "," << fixed << setprecision(6) << y[i];
    thetaFile << "," << fixed << setprecision(6) << theta[i] * 180.0 / PI;
}
// Write "," in output files
outFile << ",";
```

```
}
    break;
} // switch
// Put the end of line to output files
outFile << endl;
if ((structureType != 2) && (structureType != 5))
{
    thetaFile << endl;
}
if ((structureType == 3) \parallel (structureType == 4))
    phiFile << endl;
// Initialize vectors yMax, yMin, tMax and tMin for the first time
if (first_time == 1)
{
    if ((structureType != 2) && (structureType != 5))
    {
        for (i = 0; i < numStations; i++){
            yMax[i] = y[i];yMin[i] = y[i];tMax[i] = t;tMin[i] = t;}
    }
    else
    {
        for (i = 0; i < numStations; i++){
            yMax[i] = yB[i];yMin[i] = yB[i];tMax[i] = t;tMin[i] = t;}
    }
    first_time = 0;}
// Determin max and min deflections
DetermineMaxMin();
```
// Clear vectors epsilon.clear();

```
y.clear();
        yB.clear();
        theta.clear();
        phi.clear();
    }
    return(OK);
}
int GetUserInputs()
{
   // Get required data from user
    cout << "$ Enter Strain Data filename: ";
    cin >> inputFile;
    // Open input file 
    inFile.open(inputFile.c_str(), ios::in);
    if (inFile.fail())
    {
        cerr << "Could not open " << inputFile << "!\n";
        cerr << "Please check your Strain Data filename!\n\n";
        checked = ERROR;
        return(ERROR);
    }
    cout << "$ Enter Geometry filename: ";
    cin >> ingeoFile;
   // Open geometry input file 
    geoFile.open(ingeoFile.c_str(), ios::in);
    if (geoFile.fail())
    {
        cerr << "Could not open " << ingeoFile << "!\n";
        cerr << "Please check your Geometry filename!\n\n";
        checked = ERROR;
        return(ERROR);
    }
    cout << "\nStructure Type:\n";
    cout << "1 for uniform or tapered beam with 1-line system.\ln";
    cout << "2 for two-end supported.\n";
    cout << "3 for 2-line system.\n";
    cout << "4 for 4-line system.\n";
    cout << "5 for thin uniform plate.\n";
    cout << "6 for curved deformation of long tapered beam.\n";
cout << "7 for curved deformation of long nonlinear beam.\ln";
// Prompt for structure type from user
    cout << "\n$ Enter structure type: ";
    cin >> structureType;
```

```
if ((structureType < 1) \| (structureType > 7))
```

```
{
       cout << "\nThe entered structure type is not valid! Must be from 1 to 7!\n";
       cerr << "Please rerun the program and enter a valid structure type!\n\n";
        checked = ERROR;
       return(ERROR);
    }
   return(OK);
}
int ReadGeometryFile()
{
   // Check structureType to load geometry data correctly
    switch (structureType)
    {
    case 1: // uniform & tapered
    case 2: // two-end supported
    case 6: // curved deformation of long tapered beam
        ReadType1_2_6();
       break;
    case 3: // 2 lines and known c
        ReadType3();
       break;
    case 4: // 4 lines and unknown c
        ReadType4();
       break;
    case 5: // 2-point supported
        ReadType5();
       break;
    case 7: // nonuniform curved deflection, 2 lines, lower and upper, unknown c
        ReadType7();
       break;
    } // switch structureType
   // Check structure type
    if (structType != structureType)
    {
       cerr << "\nThe entered structure type is different from the one in the Geometry file!\n";
       cerr << "Please Check Geometry File " << ingeoFile << "!\n\n";
        checked = ERROR;
        return(ERROR);
```

```
}
   // Initialize vectors yMax, yMin, tMax, and tMin
    for (i = 0; i < numStations; i++)
    {
        yMax.push_back(0);
        yMin.push_back(0);
        tMax.push_back("0");
        tMin.push_back("0");
    }
    return(OK);
}
int ReadType1_2_6()
{
    lineNum = 0;// Read the 1st line of geometry file
    while (geoFile.getline(inBuff, MAX_LINE))
    {
        lineNum++;
        switch (lineNum)
        {
        // Read data
        case 1: // Read line No. 1
            // Read structure type
            token = strtok_s(inBuff, " ,\t\n", &nextToken);
            structType = <i>atoi</i>(token);// Read structure length
            token = strtok_s(0, ",\t\n", &nextToken);
            length = atof(token);// Read number of strain-sensing stations
            token = strtok_s(0, ",\t\n", &nextToken);
            numStatistics = <i>atoi</i>(token);// Read depth factor at the fixed end C0
            token = strtok_s(0, ",\t\n", &nextToken);
            CO = \text{atof}(\text{token});// Read depth factor at the free end Cn
            token = strtok_s(0, ",\t\n", &nextToken);
            Cn = \text{atof}(\text{token});// Calculate n
            n = numStations - 1;break;
```
case 2: // Read line No. 2

```
// Read domain type
token = strtok_s(inBuff, " ,\t\n", &nextToken);
domainType = atoi(token);// Push the index of c, deltaL, and x
c.push_back(C0);
deltaL.push_back(0);
x.push_back(0);
if (domainType == VAR\_DOMAIN){
    // Read deltaL
    token = strtok_s(0, ",\t\n", &nextToken);
    while (token)
    {
        deltaL.push_back(atof(token));
        c.push_back(0);
        x.push_back(0);
        // read next deltaL
        token = strtok_s(0, ",\t\n", &nextToken);
    }
    // Calculate Xi and Ci for variable domain
    for (i = 1; i < numStations; i++)
    {
        x[i] = x[i - 1] + delta[i];c[i] = CO - (CO - Cn) * (x[i] / length);}
}
else
{
    const<sub>_delta</sub>L = length / double(n);
    for (i = 1; i < numStations; i++)
    {
        deltaL.push_back(const_deltaL);
        x.push_back(0);
        c.push_back(0);
    }
    // Calculate Xi and Ci for constant domain
    for (i = 1; i < numStations; i++)
    {
        x[i] = (double) i * const\_deltaL;c[i] = C0 - (C0 - Cn) * (x[i] / length);}
}
```

```
break;
        } // switch (lineNum)
    } // while
    // Close geoFile
    geoFile.close();
    return(OK);
int ReadType3()
    lineNum = 0;// read the 1st line of geometry file
    while (geoFile.getline(inBuff, MAX_LINE))
    {
        lineNum++;
        switch (lineNum)
        {
        // Read data
        case 1: // Read line No. 1 
            // Read structure type
            token = strtok_s(inBuff, " ,\t\n", &nextToken);
            structType = <i>atoi</i>(token);// Read structure length
            token = strtok_s(0, ",\t\n", &nextToken);
            length = atof(token);// Read number of strain-sensing stations
            token = strtok_s(0, ",\t\n", &nextToken);
            numStations = <math>atoi(token)</math>;// Read chord-wise distant at root D0
            token = strtok_s(0, ",\t\n", &nextToken);
            D0 = \text{atof}(\text{token});// Read chord-wise distant at tip Dn
            token = strtok_s(0, ",\t\n", &nextToken);
            Dn = \text{atof}(\text{token});// Calculate n
            n\text{Statistics} = \text{numStatistics} / 2;n = nStations - 1;
            break;
```
{

case 2: // Read line No. 2

```
// Read domain type
token = strtok_s(inBuff, ",\t\n", \&nextToken;
domainType = atoi(token);// Push the index of deltaL and x
deltaL.push_back(0);
x.push_back(0);
if (domainType == VAR\_DOMAIN){
    // Read deltaL
    token = strtok_s(0, ",\t\n", &nextToken);
    while (token)
    {
        deltaL.push_back(atof(token));
        x.push_back(0);
        // read next deltaL
        token = strtok_s(0, ",\t\n", &nextToken);
    }
    // Calculate Xi for variable domain
    x[0] = 0.0;for (i = 1; i < nStations; i++){
        x[i] = x[i - 1] + delta[i];}
}
else
{
    const_deltaL = length / double(n);
    for (i = 1; i < nStations; i++){
        deltaL.push_back(const_deltaL);
        x.push_back(0);
    }
    // Calculate Xi for constant domain
    x[0] = 0.0;for (i = 1; i < nStations; i++){
        x[i] = ((double)i) * const\_deltaL;}
}
break;
```

```
case 3: // Read line No. 3
```
// Read cType token = strtok_s(inBuff, ",\t\n", $\&nextToken$;

```
cType = <i>atoi</i>(token);// Check if tapered
if (cType == TAPERED){
    // Read C0
    token = strtok_s(0, ",\t\n", &nextToken);
    CO = \text{atof}(\text{token});// Read Cn
    token = strtok_s(0, ",\t\n", &nextToken);
    Cn = \text{atof}(\text{token});// Read C0_prime
    token = strtok_s(0, ",\t\n", &nextToken);
    CO\_prime = \text{atof}(\text{token});// Read Cn_prime
    token = strtok_s(0, ",\t\n", &nextToken);
    Cn-prime = atof(token);
    // Initialize c, & x vectors
    for (i = 0; i < numStations; i++)
    {
         c.push_back(0);
         x.push_back(0);
    }
    // Calculate Ci for the front
    c[0] = C0;for (i = 1; i < nStations; i++){
         c[i] = C0 - (C0 - Cn)<sup>*</sup> (x[i] / length);
    }
    // Calculate Ci for the rear
    c[nStations] = C0_prime;
    for (i = nStations + 1; i < numStations; i++)
    {
        c[i] = CO\_prime - (CO\_prime - Cn\_prime) * (x[i-nStations] / length);}
}
else
{
    \frac{1}{2} Read c[i]
    token = strtok_s(0, ",\t\n", &nextToken);
    while (token)
    {
         c.push_back(atof(token));
         // read next c
```

```
token = strtok_s(0, ",\t\n", &nextToken);
                }
            }
            break;
        } // switch (lineNum)
    } // while getline
    // Close geoFile
    geoFile.close();
    // Initialize vectors d & phi
    for (i = 0; i < nStations; i++){
        d.push_back(0);
        phi.push_back(0);
    }
    return(OK);
}
int ReadType4()
{
    lineNum = 0;// Read the 1st line of geometry file
    while (geoFile.getline(inBuff, MAX_LINE))
    {
        lineNum++;
        switch (lineNum)
        {
        // Read data
        case 1: // Read line No. 1
            // Read structure type
            token = strtok_s(inBuff, ",\t\n", &nextToken);
            structType = <i>atoi</i>(token);// Read structure length
            token = strtok_s(0, ",\t\n", &nextToken);
            length = atof(token);// Read number of strain-sensing stations
            token = strtok_s(0, ",\t\n", &nextToken);
            numStatistics = <i>atoi</i>(token);// Read chord-wise distant at root D0
            token = strtok_s(0, ",\t\n", &nextToken);
            D0 = \text{atof}(\text{token});
```
// Read chord-wise distant at tip Dn token = strtok_s(0, ",\t\n", &nextToken); $Dn = \text{atof}(\text{token});$

// Read wing root depth at front H0 token = strtok_s(0, $\sqrt{\pi}$,\t\n", &nextToken); $H0 = \text{atof}(\text{token});$

// Read wing tip depth at front Hn token = strtok_s(0, ",\t\n", &nextToken); $Hn = \text{atof}(\text{token});$

// Read wing root depth at rear H0_prime token = strtok_s(0, \cdot , \t\n", &nextToken); $H0$ _prime = atof(token);

// Read wing tip depth at rear Hn_prime token = strtok_s(0, ",\t\n", &nextToken); Hn _{prime} = atof(token);

```
// Initialize variables
n\text{Statistics} = \text{numStatistics} / 4;noStations = numStations / 2;n = nStations - 1;
break;
```

```
case 2: // Read line No. 2
```

```
// Read domain type
token = strtok_s(inBuff, " ,\t\n", &nextToken);
domainType = atoi(token);
```

```
// Push the index of deltaL and x
deltaL.push_back(0);
x.push_back(0);
```

```
if (domainType == VAR\_DOMAIN){
    // Read deltaL
    token = strtok_s(0, " \, \cdot \mid \cdot \mid n", \, & nextToken);while (token)
    {
        deltaL.push_back(atof(token));
        x.push_back(0);
        // read next deltaL
        token = strtok_s(0, ",\t\n", &nextToken);
    }
```

```
// Calculate Xi for variable domain
for (i = 1; i < nStations; i++)
```

```
{
                    x[i] = x[i - 1] + deltaL[i];}
            }
            else
            {
                const_deltaL = length / double(n);
                for (i = 1; i < nStations; i++){
                    deltaL.push_back(const_deltaL);
                    x.push_back(0);
                }
                // Calculate Xi for constant domain
                for (i = 1; i < nStations; i++){
                    x[i] = ((double)i) * const\_deltaL;}
            }
            break;
        } // switch lineNum
    } // while getline
    // Close geoFile
    geoFile.close();
   // Initialize vectors h and c
    for (i = 0; i < numStations; i++)
    {
        h.push_back(0);
        c.push_back(0);
    }
    // Initialize vectors d and phi
    for (i = 0; i < noStations; i++){
        d.push_back(0);
        phi.push_back(0);
    }
    return(OK);
}
int ReadType5()
\{lineNum = 0;// read the 1st line of geometry file
    while (geoFile.getline(inBuff, MAX_LINE))
    {
```

```
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```

```
lineNum++;
switch (lineNum)
// Read data
case 1: // Read line No. 1
    // Read structure type
    token = strtok_s(inBuffer, " \cdot \cdot \cdot Rn", & nextToken);structType = <i>atoi</i>(token);// Read structure length
    token = strtok_s(0, " ,\t\n", &nextToken);
    length = atof(token);// Read number of strain-sensing stations
    token = strtok_s(0, ",\t\n", &nextToken);
    numStations = atoi(token);
    // Read depth factor of the thin plate
    token = strtok_s(0, " ,\t\n", &nextToken);
    C = \text{atof}(\text{token});// Read number of lines
    token = strtok_s(0, ",\t\n", &nextToken);
    numLines = atoi(token);
    // Calculate n
    nStations = numStations / numLines;
    n = nStations - 1;
    break;
case 2: // Read line No. 2 
    // Read domain type
    token = strtok_s(inBuff, " ,\t\n", &nextToken);
    domainType = atoi(token);// Push the index of deltaL and x
    deltaL.push_back(0);
    x.push_back(0);
    if (domainType == VAR_DOMAIN)
    {
        // Read deltaL
        token = strtok_s(0, ",\t\n", &nextToken);
        while (token)
        {
            deltaL.push_back(atof(token));
            x.push_back(0);
```

```
// Read next deltaL
                    token = strtok_s(0, " ,\t\n", &nextToken);
                }
                // Calculate Xi for variable domain
                for (i = 1; i < nStations; i++){
                    x[i] = x[i - 1] + delta[i];}
            }
            else
            {
                const\_deltaL = length / double(n);for (i = 1; i < nStations; i++){
                    deltaL.push_back(const_deltaL);
                    x.push_back(0);
                }
                // Calculate Xi for constant domain
                for (i = 1; i < nStations; i++){
                    x[i] = (double)i * const\_deltaL;}
            }
            break;
        } // switch (lineNum)
    } // while
   // Close geoFile
    geoFile.close();
   return(OK);
int ReadType7()
   lineNum = 0;// Read the 1st line of geometry file
    while (geoFile.getline(inBuff, MAX_LINE))
    {
        lineNum++;
        switch (lineNum)
        {
        // Read data
        case 1: // Read line No. 1
```
// Read structure type $token = strtok_s(inBuffer, " \cdot \cdot \cdot Rn", & nextToken);$ $structType = *atoi*(token);$

// Read structure length token = strtok_s(0, \sqrt{n} , \t\n", &nextToken); $length = atof(token);$

// Read number of strain-sensing stations token = strtok_s(0, ",\t\n", &nextToken); $numStations = $atoi(token)$;$

// Read wing root depth at front H0 token = strtok_s(0, \sqrt{n} , \t\n", &nextToken); $H0 = \text{atof}(\text{token});$

// Read wing tip depth at front Hn token = strtok_s(0, ",\t\n", &nextToken); $Hn = \text{atof}(\text{token});$

```
// Initialize variables
n\text{Statistics} = \text{numStatistics} / 2;n = nStations - 1;
```
break;

```
case 2: // Read line No. 2
```

```
// Read domain type
token = strtok_s(inBuff, ",\t\n", &nextToken);
domainType = atoi(token);
```

```
// Push the index of deltaL and x
deltaL.push_back(0);
x.push_back(0);
```

```
if (domainType == VAR\_DOMAIN){
    // Read deltaL
    token = strtok_s(0, " \, \cdot \mid \cdot \mid n", \, & nextToken);while (token)
    {
        deltaL.push_back(atof(token));
        x.push_back(0);
        // read next deltaL
        token = strtok_s(0, ",\t\n", &nextToken);
    }
```

```
// Calculate Xi for variable domain
for (i = 1; i < nStations; i++)
```

```
{
        x[i] = x[i - 1] + deltaL[i];}
}
else
{
    const_deltaL = length / double(n);
    for (i = 1; i < nStations; i++){
        deltaL.push_back(const_deltaL);
        x.push_back(0);
    }
    // Calculate Xi for constant domain
    for (i = 1; i < nStations; i++){
        x[i] = ((double)i) * const\_deltaL;}
}
```
break;

} // switch lineNum

```
} // while getline
   // Close geoFile
    geoFile.close();
   // Initialize vectors h and c
    for (i = 0; i < numStations; i++)
    {
        h.push_back(0);
        c.push_back(0);
    }
    return(OK);
}
int CreateOutputFiles()
{
    unsigned int loc, loc1;
    // Create deflection output file
    outputFile = inputFile;loc = outputFile.find(".");
    outputFile.insert(loc, "_Deflections");
```

```
// Open deflection output file 
outFile.open(outputFile.c_str(), ios::out);
```

```
if (outFile.fail())
{
    cerr << "Could not open " << outputFile << endl;
    return(ERROR);
}
// Create slope (angle theta) output file
if ((structureType != 2) && (structureType != 5))
{
    outthetaFile = inputFile;outthetaFile.insert(loc, "_Slopes");
    // Open slope output file 
    thetaFile.open(outthetaFile.c_str(), ios::out);
    if (thetaFile.fail())
    {
        cerr << "Could not open " << outthetaFile << endl;
        return(ERROR);
    }
}
// Create max and min deflection output file
outmaxminFile = outputFile;
loc1 = \text{outmaxminFile.find}("");
outmaxminFile.insert(loc1, "_MaxMin");
// Open max and min deflection output file
maxminFile.open(outmaxminFile.c_str(), ios::out);
if (maxminFile.fail())
{
    cerr << "Could not open " << outmaxminFile << endl;
    return(ERROR);
}
maxminFile << "SG Name, Time at Max Deflection, Max Deflection, Time at Min Deflection, 
                Min Deflection\n";
if ((structureType == 3) \parallel (structureType == 4))
{
    // Create twist angle (phi) output file
    outputFile = inputFile;outphiFile.insert(loc, "_TwistAngles");
    // Open twist angle output file
    phiFile.open(outphiFile.c_str(), ios::out);
    if (phiFile.fail())
    {
        cerr << "Could not open " << outphiFile << endl;
        return(ERROR);
    }
    phiCreated = 1;
}
```

```
if ((structureType == 4) \parallel (structureType == 7))
{
    // Create depth factor output file
    outcFile = inputFile;outcFile.insert(loc, "_DepthFactors");
    // Open depth factor output file
    cFile.open(outcFile.c_str(), ios::out);
    if (cFile.fail())
    {
        cerr << "Could not open depth factor file " << outcFile << endl;
        checked = ERROR;return(ERROR);
    }
    // Initialize cCreated
    cCreated = 1:
}
// Read 1st line of input file
inFile.getline(inBuff, MAX_LINE);
// Write to output files
outFile << inBuff << endl;
if ((structureType != 2) && (structureType != 5))
{
    thetaFile << inBuff << endl;
}
// Read the 1st name of the 1st line
token = strtok_s(inBuff, " ,\t\n", &nextToken);
if ((structureType == 3) \parallel (structureType == 4))
{
    phiFile << token << ",";
}
// Read strain-sensing station names
token = strtok_s(0, ",\t\n", &nextToken);
while (token)
{
    if ((structureType == 4) \parallel (structureType == 7))
    {
        cFile \ll token << ", ";
    }
    stationNames.push_back(token);
    token = strtok_s(0, ",\t\n", &nextToken);
```

```
// Done reading the first title line
    if (structureType == 3)
    {
        for (i = 0; i < nStations; i++){
            phiFile << "Station_" << i<< ",";
        }
        phiFile << endl;
    }
    else if (structureType == 4)
    {
        // Write names for lower strain-sensing stations
        for (i = 0; i < nStations; i++){
            phiFile << "LwrStation_" << i << ",";
        }
        // Write names for upper strain-sensing stations
        for (i = nStations; i < noStations; i++){
            phiFile << "UprStation_" << (i - nStations) << ",";
        }
        phiFile << endl;
        cFile << endl;
    }
    else if (structureType == 7)
    {
        cFile << endl;
    }
    return(OK);
}
void CalculateC()
{
    // Initialize h[0] and h[n]h[0] = H0;h[n] = Hn;H_{ratio} = Hn / H0;// Calculate front c[i] using Eqs. (1a & 2a)
    for (i = 0; i < n; i++){
        // Lower front
        h[i] = H0 - (H0 - Hn)*(x[i] / length);c[i] = abs(epsilon[i]) * h[i] / (abs(epsilon[i]) + abs(epsilon[i - nStations]));// Upper front
        c[i + nStations] = h[i] - c[i];}
```

```
c[n] = H ratio*c[0];
    c[n + nStations] = H\_ratio * c[nStations];// if structure type 4, need to do the rear
    if (structureType == 4)
    {
        // Calculate rear c[i] using Eqs. (1b \& 2b)
        h[noStations] = H0 prime;
        h[noStations + n] = Hn prime;
        Hprime_ratio = Hn_prime / H0_prime;
        for (i = noStations; i < (noStations + n); i++){
            // Lower rear
            h[i] = H0_prime - (H0_prime - Hn_prime)*(x[i - noStations] / length);
            c[i] = abs(epsilon[i]) * h[i] / (abs(epsilon[i]) + abs(epsilon[i - nStation[i]);
            // Upper rear
            c[i + nStations] = h[i] - c[i];}
        c[noStations + n] = Hprime\_ratio * c[noStations];c[noStations + n + nStations] = Hprime\_ratio * c[noStations + nStations];}
    for (i = 0; i < numStations; i++){
        cFile \ll c[i] \ll ", ";
    }
   cFile << endl;
    cFile.close();
    calC = TRUE;void CalcTwistAngles()
   // Initialize d[0] and d[n]
    d[0] = D0;d[n] = Dn;switch (structureType)
    {
    case 3: // 2 lines
    {
        // Set twist angle at the root to 0.0
        phi[0] = 0.0;phiFile << fixed << setprecision(6) << phi[0];
        // Eq. (11) in this paper or Eq. (38) in NASA/TP-2009-214643
```

```
// Calculate twist angle phi 
        for (i = 1; i < nStations; i++){
            d[i] = D0 - (D0 - Dn)*(x[i] / length);sin\_phi[i] = (y[i] - y[i + nStations]) / d[i];phi[i] = a\sin(\sin\phi hi[i]) * 180.0 / PI;
            phiFile << "," << fixed << setprecision(6) << phi[i];
        }
        break;
    }
    case 4: // 4 lines
    {
        // Set twist angle at lower root to 0.0
        phi[0] = 0.0;phiFile << fixed << setprecision(6) << phi[0];
        // Eq. (11) in this paper or Eq. (38) in NASA/TP-2009-214643
        // Calculate lower twist angle phi
        for (i = 1; i < nStations; i++){
            d[i] = D0 - (D0 - Dn)*(x[i] / length);sin\phi[i] = (y[i] - y[i + noStations]) / d[i];phi[i] = a\sin(\sin\phi\phi\sin[i])^*180.0 / PI;phiFile << "," << fixed << setprecision(6) << phi[i];
        }
        // Set twist angle at upper root to 0.0
        phi[n\text{Stations}] = 0.0;phiFile << "," << fixed << setprecision(6) << phi[nStations];
        // Calculate upper twist angle phi
        for (i = (1 + nStatus); i < noStations; i++){
            sin\_phi[i] = (y[i] - y[i + noStations]) / d[i - nStations];phi[i] = a\sin(\sin\phi hi[i]) * 180.0 / PI;
            phiFile << "," << fixed << setprecision(6) << phi[i];
        }
        break;
    } // case 4
    } // switch structureType
void DetermineMaxMin()
    for (i = 1; i < numStations; i++)
    {
        // Check for max and min deflections
        if ((structureType != 2) && (structureType != 5))
```

```
{
           if (y[i] \ge yMax[i]){
                tMax[i] = t;yMax[i] = y[i];}
           else if (y[i] < yMin[i]){
                tMin[i] = t;yMin[i] = y[i];}
        }
       else
        {
           if (yB[i] \ge yMax[i]){
                tMax[i] = t;yMax[i] = yB[i];}
           else if (yB[i] < yMin[i]){
                tMin[i] = t;yMin[i] = yB[i];}
        }
    }
}
void WriteMaxMinFile()
{
   // Write yMax and yMin to file
    for (i = 0; i < numStations; i++)
    {
        maxminFile << fixed;
       maxminFile << stationNames[i] << "," << tMax[i] << "," << setprecision(6) << yMax[i]
           << "," << tMin[i] << "," << setprecision(6) << yMin[i] << endl;
    }
}
void PrintOutputFilenames()
{
   // Print out successful messages
    cout << "\n$ Displacement Calculation program completed successfully!\n" << endl;
    cout << "$ Output files are listed below:\ln" << endl;
    cout << "$ Deflection file: " << outputFile << endl;
    if ((structureType != 2) && (structureType != 5))
    {
       cout << "$ Slope file: " << outthetaFile << endl;
    }
```
```
if (cCreated == 1){
        cout << "$ Depth Factor file: " << outcFile << endl;
    }
    if (phiCreated == 1)
    {
        cout << "$ Twist Angle file: " << outphiFile << endl;
    }
    cout << "$ Max and Min Deflection file: " << outmaxminFile << endl << endl;
}
void CloseFiles_ClearVectors()
{
   // Close all files
    outFile.close();
    maxminFile.close();
    if ((structureType != 2) && (structureType != 5))
    {
        thetaFile.close();
    }
    if ((structureType == 3) \parallel (structureType == 4))
    {
        phiFile.close();
    }
    // Clear out all vectors
    epsilon.clear();
    x.clear();
    y.clear();
    yB.clear();
    yMax.clear();
    yMin.clear();
    tMax.clear();
    tMin.clear();
    theta.clear();
    tan_theta.clear();
    phi.clear();
    sin_phi.clear();
    c.clear();
    d.clear();
    h.clear();
    deltaL.clear();}
void main()
```

```
{
   GetUserInputs();
   if (checked != ERROR)
   {
       ReadGeometryFile();
       if (checked != ERROR)
       {
           CreateOutputFiles();
           if (checked != ERROR)
           {
               // Let user know the program is running
               cout << "\n$ Displacement Calculation program is running ..." << endl;
               CalcDisplacement();
               if (checked != ERROR)
               {
                   WriteMaxMinFile();
                   PrintOutputFilenames();
               }
           }
       }
   }
   CloseFiles_ClearVectors();
}
```
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