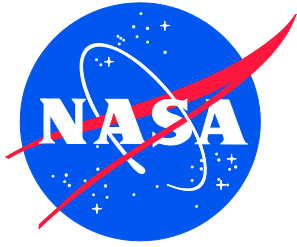


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NESC-RP-15-01037



# Application of System Identification to Parachute Modeling

## *Appendices*

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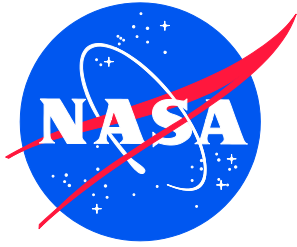
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October 2019

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# **NASA Engineering and Safety Center Technical Assessment Report**

## **Volume II: Appendices**

### **Application of System Identification to Parachute Modeling**

**August 29, 2019**

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# **Appendix A. Verification Case 1 – Constant Density Descent**

## **A.1 Introduction**

A series of exact analytical solutions is developed for a capsule with two parachutes to verify CAPDYN and FAST results for terminal conditions. The case described herein is for the capsule and two parachutes acting as a fixed system in terminal descent. In this simplest case, the density is modeled as a constant defined by the initial release altitude. The analytical solution predicts the system terminal dynamic pressure, velocity, and altitude for comparison with CAPDYN and FAST simulation results.

### **A.1.1 Initial Conditions**

The initial conditions are set such that the system descends at a constant velocity with no motion in the horizontal direction.

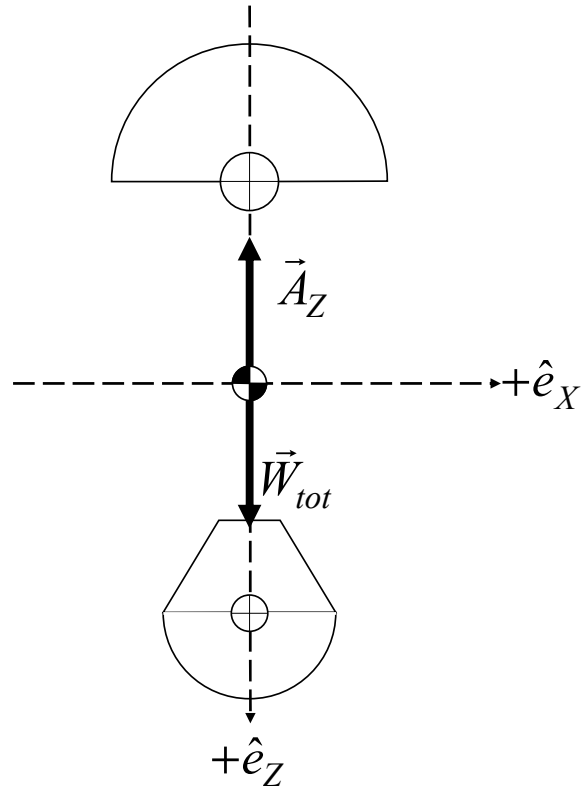
### **A.1.2 Simplifying Assumptions**

The following simplifying assumptions are necessary for Verification Case 1:

- Both parachutes have identical physical characteristics, aerodynamic models, and dimensions.
- Proximity effects are ignored.
- Density is held constant at the 5,000-ft altitude value.
- There is a single riser line attach point on the load.
- The motion is limited to the vertical axis for all three bodies.
- The two parachutes will occupy the same space.
- There is no wind.
- There are no capsule aerodynamic forces or moments.
- All out-of-plane parachute aerodynamic coefficients are set to zero.
- There is no enclosed air mass included in this simulation.

### A.1.3 Free-body Diagram: Inertial Axis System

The free-body diagram for this case is shown in Figure A-1.



*Figure A-1. Free-body Diagram, Case 1*

### A.1.4 Derivation of Equations

Newton's Law:

$$m_{\text{tot}} \ddot{z} \hat{e}_z = \sum \vec{F}_{\text{external}} = \vec{W}_{\text{tot}} + \vec{A}_z$$

$$\vec{W}_{\text{tot}} = W_{\text{tot}} \hat{e}_z$$

$$\vec{A}_z = -qSC_z \hat{e}_z$$

$$\ddot{z} = \frac{W_{\text{tot}} - qSC_z}{m_{\text{tot}}}$$

Equilibrium:

$$\ddot{z} = 0 = W_{\text{tot}} - qSC_z$$

Terminal conditions:

$$W_{\text{tot}} = q_{\text{term}} SC_z$$

$$q_{\text{term}} = \frac{W_{\text{tot}}}{SC_z} = \frac{1}{2} \rho V_{\text{term}}^2$$

$$V_{\text{term}} = \sqrt{\frac{2q_{\text{term}}}{\rho}}$$

Equation of motion:

$$\ddot{Z} = 0$$

Solution:

$$\begin{aligned}\dot{Z} &= V_{\text{term}} \\ Z &= Z_0 + V_{\text{term}}t\end{aligned}$$

### A.1.5 Analytical Solution

The rigid system of the capsule and two parachutes descends in vertical motion at the initial terminal conditions with no change and no other motion about any axis. The descent velocity and dynamic pressure will be constant, and the vertical position is a linear function of time.

## A.2 Physical Characteristics

For purposes of comparison of the analytical solutions with CAPDYN and FAST results, the following physical characteristics and initial conditions of the capsule and parachutes are assumed:

Parachutes (each):

$$\begin{aligned}W &= 328.087 \text{ lb} \\ \text{Reference area} &= 10,562.9 \text{ ft}^2 \\ \text{Drag coefficient, } C_z &= 0.85 \\ C_x &= 0\end{aligned}$$

Capsule:

$$W = 20,862.9 \text{ lb}$$

Totals:

$$\begin{aligned}W_{\text{tot}} &= 2(328.087) + 20,862.9 = 21,519.07 \text{ lb} \\ S_{\text{ref}} \text{ (two parachutes)} &= 21,125.8 \text{ ft}^2 \\ C_z &= 0.85\end{aligned}$$

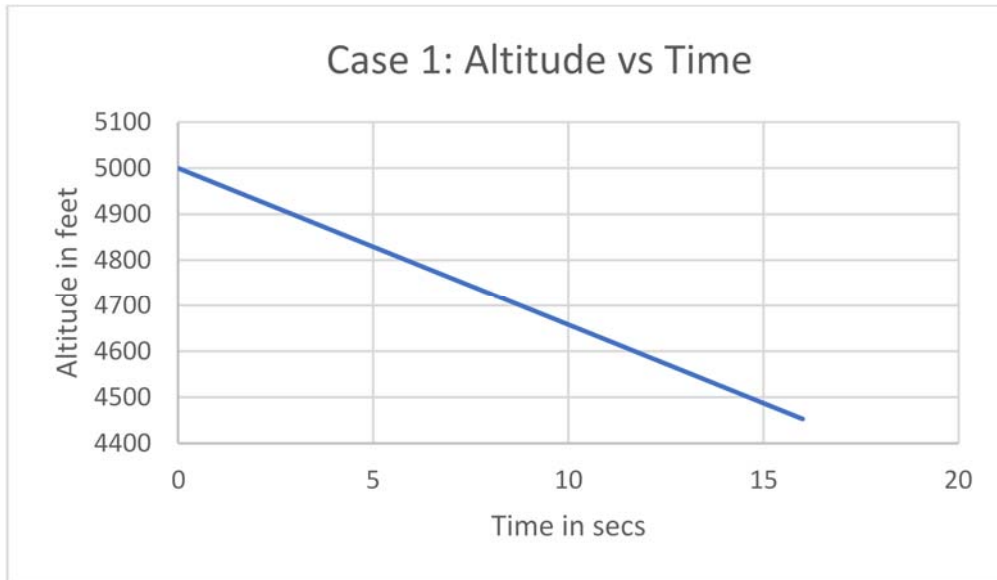
Flight conditions:

$$\begin{aligned}\text{Altitude} &= 5,000 \text{ ft} \\ \text{Air density} &= 2.05 \times 10^{-3} \text{ slugs/ft}^3 \text{ (assume constant)}\end{aligned}$$

## A.3 Results

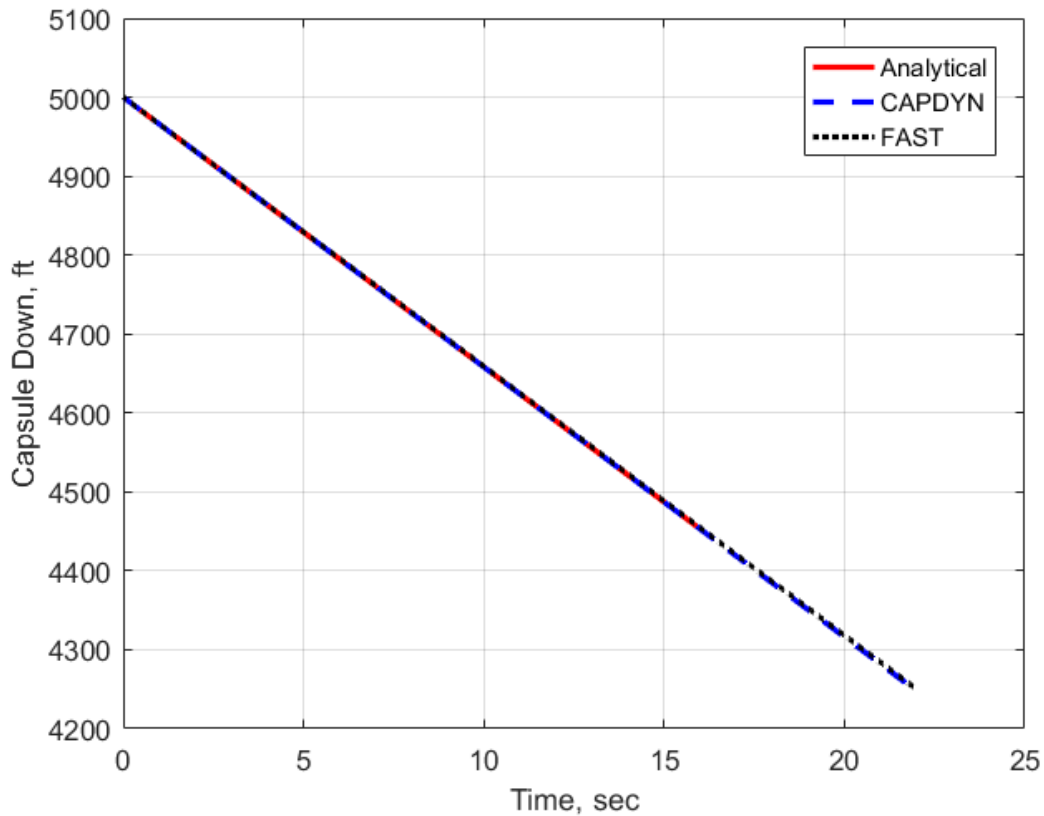
$$\begin{aligned}\text{Terminal dynamic pressure} &= 1.198 \text{ psf} \\ \text{Terminal velocity} &= 34.19 \text{ fps}\end{aligned}$$

Altitude versus time is shown in Figure A-2.



**Figure A-2. Altitude versus Time, Case 1**

This simple case assumes the atmospheric density, descent velocity, and dynamic pressure are constant with no out-of-plane motion. Thus, altitude versus time is a linear relationship. Results from CAPDYN and FAST are co-plotted with the analytical model in Figure A-3 showing excellent agreement.



**Figure A-3. Altitude versus Time Comparisons, Case 1**

## **Appendix B. Verification Case 2 – Exponential Density Descent**

### **B.1 Introduction**

A series of exact analytical solutions is developed for a capsule with two parachutes to verify CAPDYN and FAST results for descent in an atmosphere where density changes with altitude. The case described herein is for the capsule and two parachutes acting as a fixed system in descent with density that varies exponentially. In this variation on Case 1 (Appendix A), the density is modeled by the 1976 Standard Atmosphere. The analytical solution predicts the system dynamic pressure, vertical velocity, and altitude as a function of time for comparison with CAPDYN and FAST simulation results.

#### **B.1.1 Initial Conditions**

The initial conditions are set such that the system descends vertically at a constant dynamic pressure with no motion in the horizontal direction. As the system descends, density slowly increases and velocity slowly decreases to maintain a constant dynamic pressure.

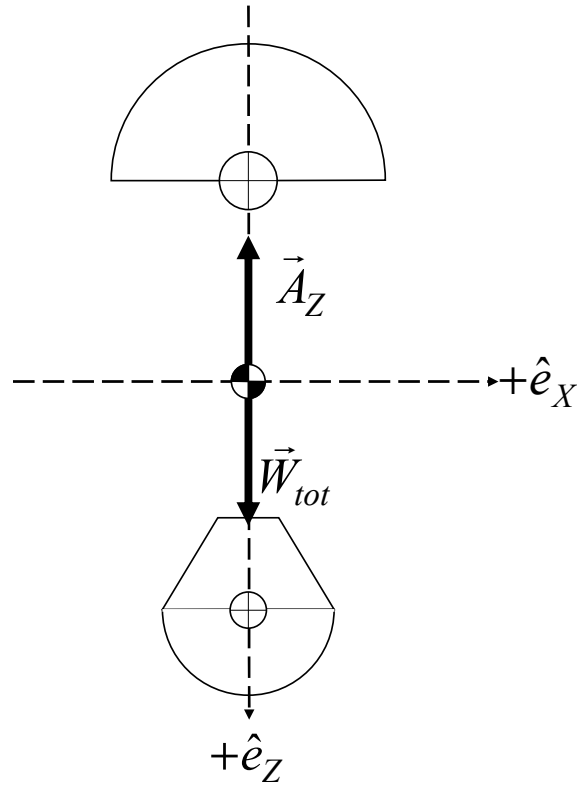
#### **B.1.2 Simplifying Assumptions**

The following simplifying assumptions are necessary for Verification Case 2:

- Both parachutes have identical physical characteristics, aerodynamic models, and dimensions.
- Proximity effects are ignored.
- There is a single riser line attach point on the load.
- Density is modeled as an exponential curve fit to the 1976 Standard Atmosphere.
- For a small time interval,  $\Delta t$ , density is assumed to be constant and vertical acceleration is assumed to be zero.
- The motion is limited to the vertical axis for all three bodies.
- The two parachutes will occupy the same space.
- There is no wind.
- There are no capsule aerodynamic forces or moments.
- All out-of-plane parachute aerodynamic coefficients are set to zero.
- There is no enclosed air mass included in this simulation.

### B.1.3 Free-body Diagram: Inertial Axis System

The free-body diagram for this case is shown in Figure B-1.



*Figure B-1. Free-body Diagram, Case 2*

### B.1.4 Derivation of Equations

Newton's Law:

$$m_{\text{tot}}\ddot{Z}\hat{e}_Z = \sum \vec{F}_{\text{external}} = \vec{W}_{\text{tot}} + \vec{A}_Z$$

$$\vec{W}_{\text{tot}} = W_{\text{tot}}\hat{e}_Z$$

$$\vec{A}_Z = -qSC_Z\hat{e}_Z$$

$$\ddot{Z} = \frac{W_{\text{tot}} - qSC_Z}{m_{\text{tot}}}$$

Equilibrium: Per the assumption, the very small acceleration is assumed to be zero during each small time interval. Thus,

$$\ddot{Z} = 0 = W_{\text{tot}} - qSC_Z$$

Terminal conditions during a small time interval:

$$W_{\text{tot}} = q_{\text{term}}SC_Z$$

$$q_{\text{term}} = \frac{W_{\text{tot}}}{SC_Z} = \frac{1}{2}\rho V_{\text{term}}^2$$

$$V_{\text{term}} = \sqrt{\frac{2q_{\text{term}}}{\rho}}; \quad \rho = \rho(H) = Ae^{-BH}$$

where

$$H = \text{altitude (ft)}$$

$$A = 0.002377 \text{ slugs/ft}^3$$

$$B = 0.0000299 \text{ (ft}^{-1}\text{)}$$

At the beginning of each small time interval, density is evaluated at the current altitude and held constant during the interval.

Equation of motion:

$$\ddot{Z} = 0$$

Solution:

$$\dot{Z} = V_{\text{term}}$$

$$Z = Z_0 + V_{\text{term}}\Delta t$$

### B.1.5 Analytical Solution

During each small time interval, the rigid system of the capsule and two parachutes descends in vertical motion at constant dynamic pressure, there is no other motion about any axis, and the vertical position is a linear function of time. At the beginning of each new interval, the density increases slightly and the descent velocity decreases slightly compared with the previous interval. In the results that follow,  $\Delta t = 5$  s.

## B.2 Physical Characteristics

For purposes of comparison of the analytical solutions with the CAPDYN and FAST results, the following physical characteristics and initial conditions of the capsule and parachutes are assumed:

Parachutes (two):

$$W = 328.087 \text{ lb}$$

$$\text{Reference area} = 10,562.9 \text{ ft}^2$$

$$\text{Drag coefficient, } C_Z = 0.85$$

$$C_X = 0$$

Capsule:

$$W = 20,862.9 \text{ lb}$$

Totals:

$$W_{\text{tot}} = 2(328.087) + 20,862.9 = 21,519.07 \text{ lb}$$

$$S_{\text{ref}} \text{ (two parachutes)} = 21,125.8 \text{ ft}^2$$

$$C_Z = 0.85$$

Flight initial conditions:

$$\text{Initial altitude} = 5,000 \text{ ft}$$

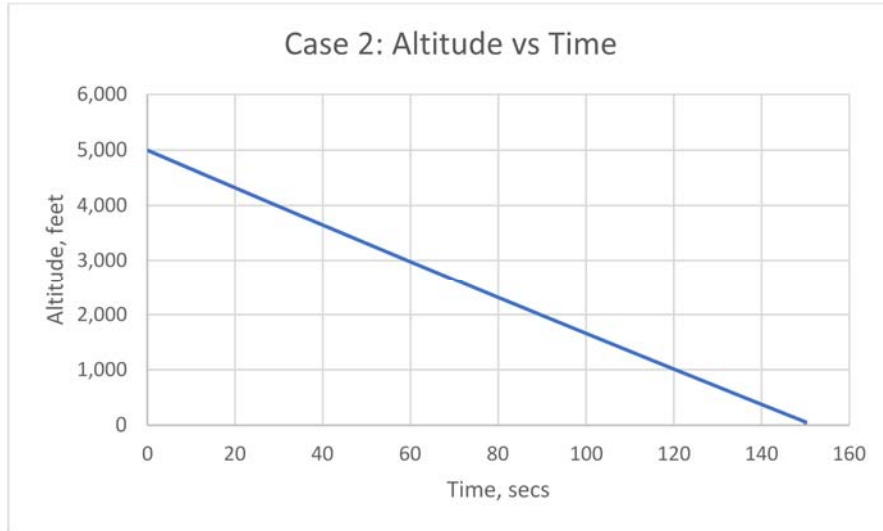
$$\text{Air density} = 2.05 \times 10^{-3} \text{ slugs/ft}^3 \text{ at } 5,000 \text{ ft}$$

### B.3 Results

Terminal dynamic pressure = 1.198 psf

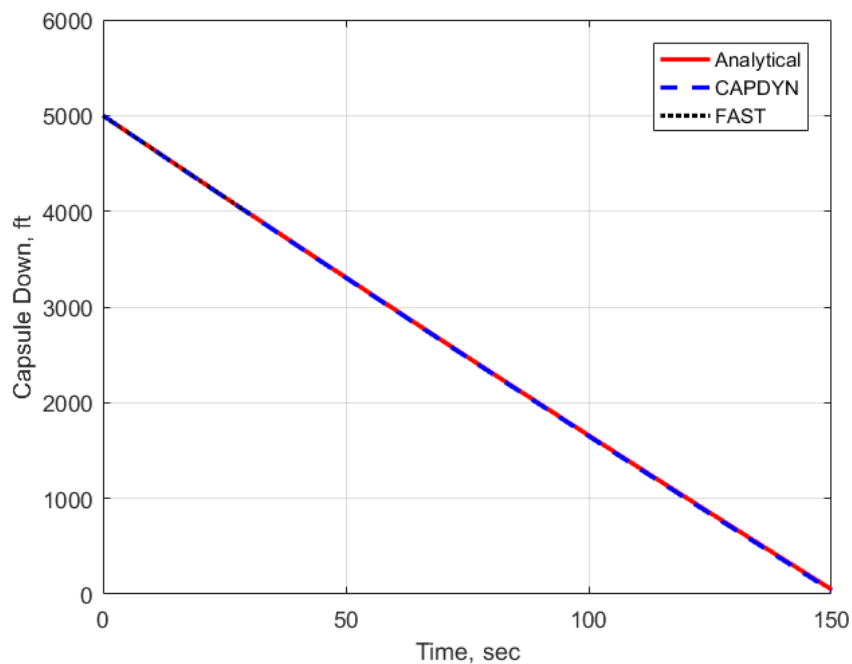
Terminal velocity = 34.19 fps at 5,000 ft

The altitude versus time is shown in Figure B-2.



**Figure B-2. Altitude versus Time, Case 2**

Case 2 is similar to Case 1, except the atmospheric density varies with altitude. As a result of the density change, the velocity slowly decreases as the system approaches sea level. The small, slow density change has an almost linear effect on the altitude versus time. Results from CAPDYN and FAST are co-plotted with the analytical model in Figure B-3, showing excellent agreement between the three approaches.



**Figure B-3. Altitude versus Time Comparisons, Case 2**



# **Appendix C. Verification Case 3 – Vertical Wind Shear, Constant Density**

## **C.1 Introduction**

A series of exact analytical solutions is developed for a capsule with two parachutes to verify vertical wind shear effects in CAPDYN simulated results. The case described herein is for the capsule and two parachutes acting as a fixed system starting and ending in terminal descent. A vertical updraft wind shear is simulated, and the parachute system deceleration response is calculated. The analytical solution predicts the shear effect on the system deceleration and velocity.

### **C.1.1 Initial Conditions**

The initial conditions are set such that the system descends vertically at a constant dynamic pressure with no motion in the horizontal direction.

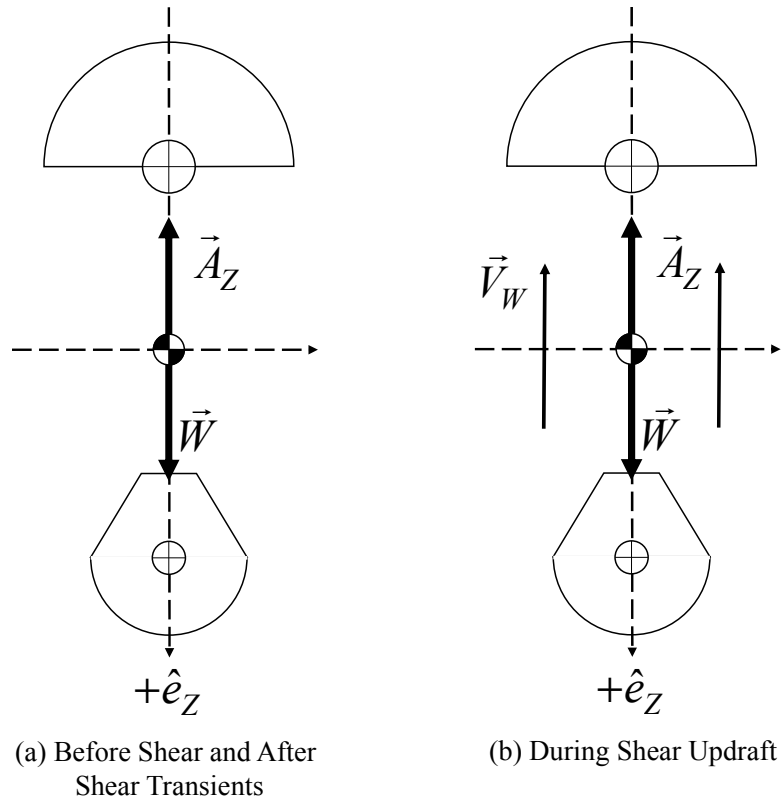
### **C.1.2 Simplifying Assumptions**

The following simplifying assumptions are necessary for Verification Case 3:

- Both parachutes have identical physical characteristics, aerodynamic models, and dimensions.
- Proximity effects are ignored.
- There is a single riser line attach point on the load.
- Density is held constant at the 5,000-ft altitude value.
- The motion is limited to the vertical axis for all three bodies.
- The two parachutes will occupy the same space.
- The simulated vertical wind shear is held constant.
- There are no capsule aerodynamic forces or moments.
- All out-of-plane parachute aerodynamic coefficients are set to zero.
- There is no enclosed air mass included in this simulation.

### C.1.3 Free-body Diagram: Inertial Axis System

The free-body diagram is shown in Figure C-1.



*Figure C-1. Free-body Diagram, Case 3*

### C.1.4 Derivation of Equations

Newton's Law:

$$\sum \vec{F}_{\text{external}} = m_{\text{tot}} \vec{a} = \vec{W} + \vec{A}_Z$$

$$\vec{W} = W \hat{e}_z$$

$$\vec{A}_Z = -qSC_Z \hat{e}_z$$

$$\ddot{Z} = \frac{W - qSC_Z}{m_{\text{tot}}}$$

Note that  $m_{\text{tot}}$  does not include enclosed or apparent air mass.

Equilibrium conditions:

Before shear and after equilibrium with shear:

$$m_{\text{tot}} \ddot{Z} = 0 = W - qSC_Z$$

At vertical wind shear initial conditions:

$$q = \frac{1}{2}\rho(V_{\text{term}} + V_W)^2 = \left(\frac{1}{2}\rho\right)V^2$$

$$A_Z = qSC_Z$$

$$m_{\text{tot}}\ddot{Z} = m_{\text{tot}}\ddot{Z}_{\text{term}} = A_Z - A_{Z_{\text{term}}}$$

$$\ddot{Z} = \frac{A_Z - A_{Z_{\text{term}}}}{m_{\text{tot}}} = \frac{SC_Z}{m_{\text{tot}}}(q - q_{\text{term}})$$

$$\ddot{Z} = \frac{SC_Z}{m_{\text{tot}}}\left(\frac{1}{2}\rho\right)(V^2 - V_{\text{term}}^2) = k(V^2 - V_{\text{term}}^2)$$

where

$$k = \left(\frac{SC_Z}{2m_{\text{tot}}}\right)\rho$$

Descent velocity post shear transients:

$$V_{\text{term}_{\text{wind}}} = V_{\text{term}} + V_W$$

### C.1.5 Analytical Solution

Before and after the shear transient effects, the rigid system of capsule and two parachutes descend in vertical motion at the terminal dynamic pressure with no change and no other motion about any axis. A constant vertical updraft wind shear is simulated, causing the system to decelerate. The shear results in the system initial inertial terminal velocity eventually decreasing by the magnitude of the wind shear.

## C.2 Physical Characteristics

For purposes of comparison of the analytical solutions with CAPDYN results, the following physical characteristics and initial conditions of the capsule and parachutes are assumed:

Parachutes (each):

$$W = 328.087 \text{ lb}$$

$$\text{Reference area} = 10,562.9 \text{ ft}^2$$

$$\text{Drag coefficient, } C_Z = 0.85$$

Capsule:

$$W = 20,862.9 \text{ lb}$$

Totals:

$$W_{\text{tot}} = 2(328.087) + 20,862.9 = 21,519.07 \text{ lb}$$

$$S_{\text{ref}} \text{ (two parachutes)} = 21,125.8 \text{ ft}^2$$

$$C_Z = 0.85$$

Flight initial conditions:

Initial altitude = 5,000 ft

Air density =  $2.05 \times 10^{-3}$  slugs/ft<sup>3</sup> at 5,000 ft (assume constant)

Wind shear = 10 fps, vertically up

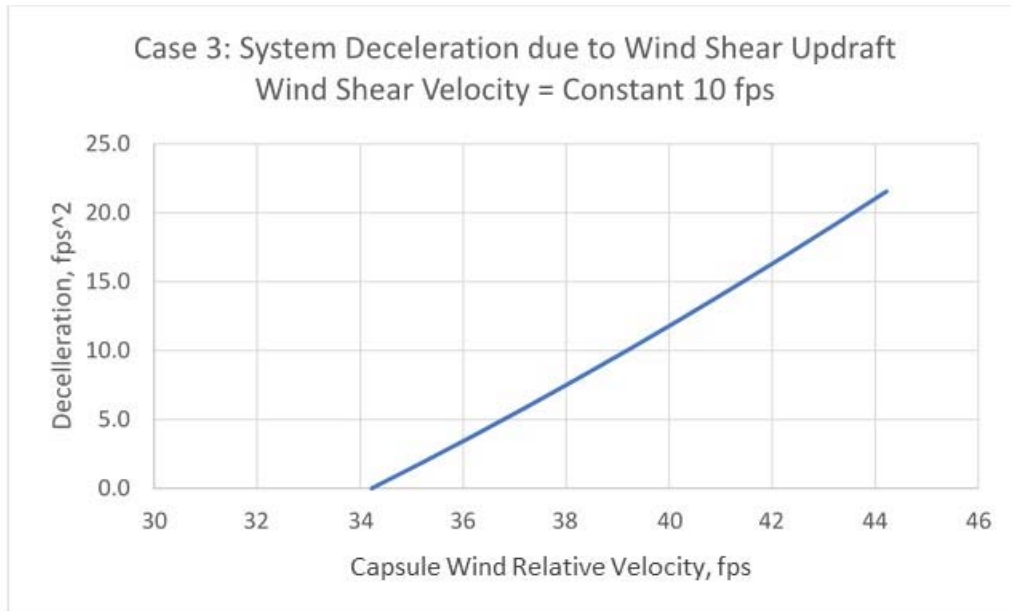
### C.3 Results

Terminal dynamic pressure = 1.198 psf

Terminal wind relative velocity = 34.19 fps at 5,000 ft

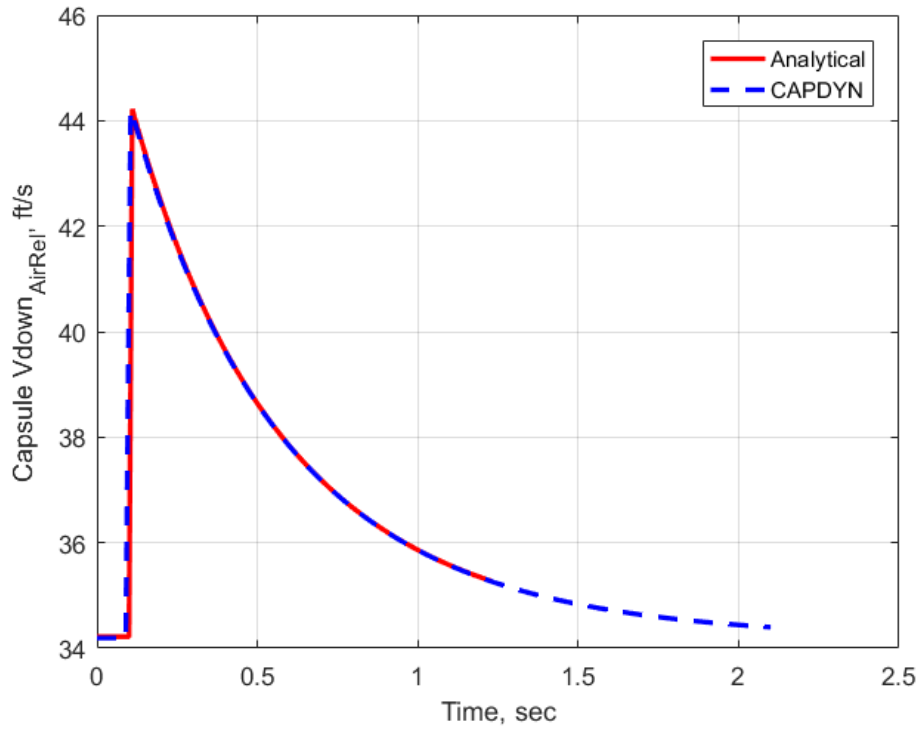
Final terminal inertial velocity = 24.19 fps

Deceleration versus velocity is shown in Figure C-2.



**Figure C-2. System Deceleration Due to Wind Shear Updraft, Case 3**

Initially, the system is in terminal equilibrium descent with a 34.2-fps velocity. The 10-fps wind updraft causes the relative velocity at the parachute to go immediately to 44.2 fps, which creates excess drag and the deceleration of 22 fps<sup>2</sup>. As time goes on, the wind relative velocity at the parachute decreases, and the acceleration approaches zero. The wind relative system velocity becomes 34.2 fps, while the inertial velocity is 24.2 fps. Results from CAPDYN are co-plotted with the analytical model in Figure C-3 with excellent agreement.



**Figure C-3. Capsule Air Relative Velocity Down Comparison, Case 3**

## **Appendix D. Verification Case 4 – Steady-state Glide**

### **D.1 Introduction**

A series of exact analytical solutions is developed for a capsule with two parachutes to verify CAPDYN and FAST results. The case described herein is for the capsule and two parachutes acting as a fixed system starting and ending in a steady-state glide. Initial conditions are chosen such that equilibrium is obtained in both the vertical and horizontal directions. The analytical solution predicts the glide path for the simulated L/D.

#### **D.1.1 Initial Conditions**

The initial conditions are set such that the system descends at a constant flight path and constant dynamic pressure. Density is set at the 5,000-ft level and held constant.

#### **D.1.2 Simplifying Assumptions**

The following simplifying assumptions are necessary for Verification Case 4:

- Both parachutes have identical physical characteristics, aerodynamic models, and dimensions.
- Proximity effects are ignored.
- There is a single riser line attach point on the load.
- Density is held constant at the 5,000-ft altitude value.
- The motion is limited to the vertical and horizontal plane for all three bodies.
- The two parachutes will occupy the same space.
- There is no wind.
- The small effect of the horizontal velocity on dynamic pressure is assumed to be negligible.
- There are no capsule aerodynamic forces or moments.
- All out-of-plane parachute aerodynamic coefficients are set to zero.
- There is no enclosed air mass included in this simulation.

#### **D.1.3 Free-body Diagram: Inertial Axis System**

Figure D-1 shows the free-body diagram of the system, with the external forces, the angles, and the axis system.

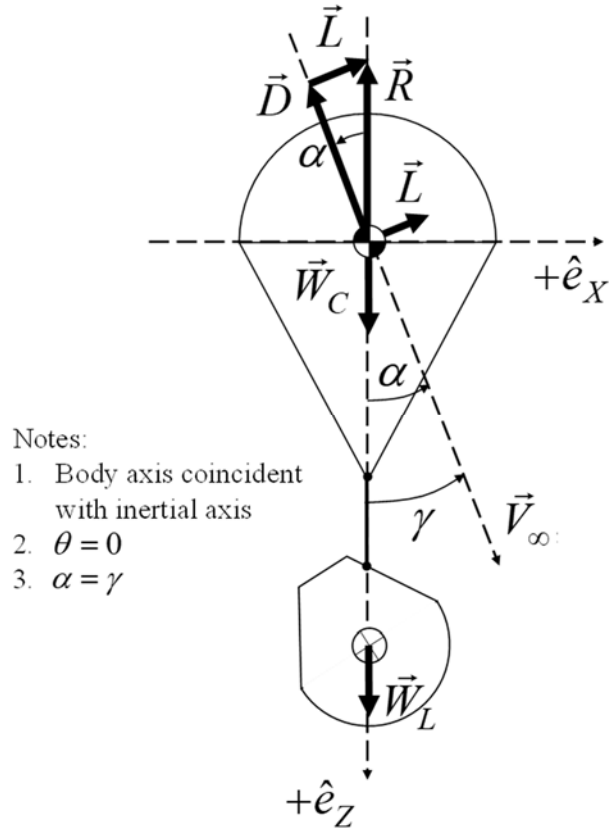


Figure D-1. Free-body Diagram, Case 4

#### D.1.4 Derivation of Equations

Newton's Law:

$$\begin{aligned}\Sigma \vec{F}_{\text{external}} &= m_{\text{tot}} \vec{a} = \vec{W}_{\text{tot}} + \vec{R} \\ \vec{W}_{\text{tot}} &= W_{\text{tot}} \hat{e}_Z \\ \vec{R} &= \vec{L} + \vec{D}\end{aligned}$$

Equilibrium glide constraints:

$$\begin{aligned}\ddot{X} &= 0 = \ddot{Z} \\ m_{\text{tot}} \ddot{X} &= L \cos \alpha - D \sin \alpha = 0 \\ m_{\text{tot}} \ddot{Z} &= -(D \cos \alpha + L \sin \alpha) + W_{\text{tot}} = 0\end{aligned}$$

Horizontal motion in equilibrium:

$$\begin{aligned}m_{\text{tot}} \ddot{X} &= L \cos \alpha - D \sin \alpha = 0, & \frac{L}{D} &= \frac{\sin \alpha}{\cos \alpha} \\ \frac{L}{D} &= \tan \alpha, & \alpha &= \tan^{-1} \left( \frac{L}{D} \right)\end{aligned}$$

Vertical motion in equilibrium:

$$m_{\text{tot}}\ddot{Z} = -(D \cos \alpha + L \sin \alpha) + W_{\text{tot}} = 0$$

$$D \cos \alpha + L \sin \alpha = W_{\text{tot}}$$

$$\text{Let } D \cos \alpha + L \sin \alpha = R = W_{\text{tot}}$$

$$R = q_{\text{term}} S C_R$$

$$L = q_{\text{term}} S C_L$$

$$D = q_{\text{term}} S C_D$$

$$C_R = \sqrt{C_L^2 + C_D^2}$$

$$\frac{L}{D} = \frac{C_L}{C_D}$$

$$q_{\text{term}} = \frac{W_{\text{tot}}}{S C_R}$$

$$V_{\text{term}} = \sqrt{\frac{2q_{\text{term}}}{\rho}} = V_{\infty}$$

Solution for equilibrium glide:

#### Horizontal Component

$$\ddot{X} = 0$$

$$\dot{X} = V_{\text{term}} \sin \alpha$$

$$X = X_0 + \dot{X}t$$

#### Vertical Component

$$\ddot{Z} = 0$$

$$\dot{Z} = V_{\text{term}} \cos \alpha$$

$$Z = Z_0 + \dot{Z}t$$

### D.1.5 Analytical Solution

For the entire simulation, the rigid system of the capsule and two parachutes descends with a constant glide path at a constant terminal dynamic pressure with no change and no other motion about any axis. Acceleration is zero in both the vertical and horizontal directions. The constant glide path angle is as predicted for the simulated L/D.

### D.2 Physical Characteristics

For comparison of the analytical solutions with CAPDYN and FAST results, the following physical characteristics and initial conditions of the capsule and parachutes are assumed:

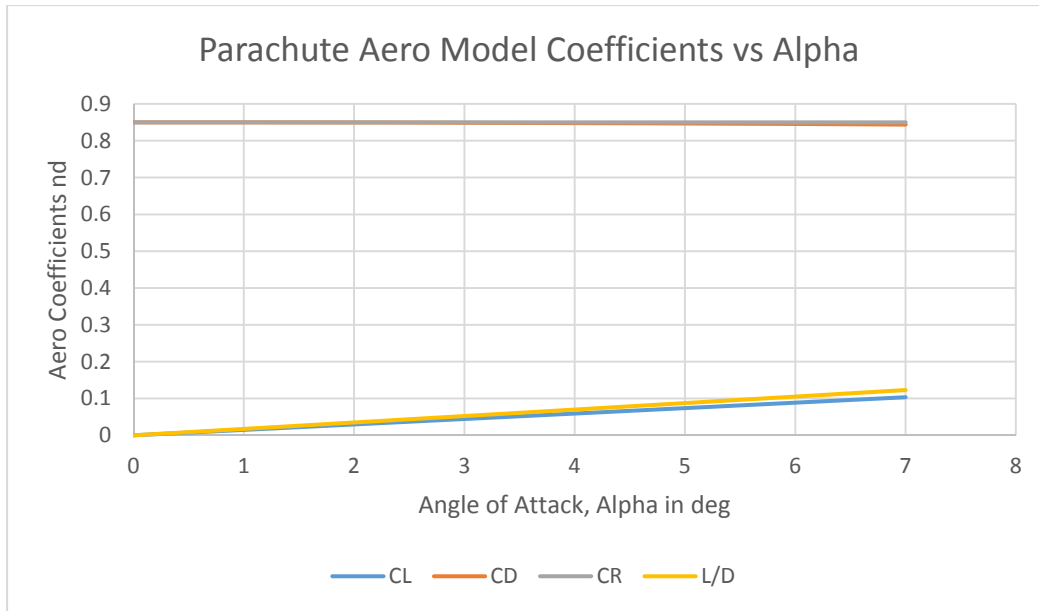
Parachutes (each):

$$W = 328.087 \text{ lb}$$

$$\text{Reference area} = 10,562.9 \text{ ft}^2$$

The aerodynamic model is shown in Figure D-2.





**Figure D-2. Parachute Aerodynamic Model, Case 4**

Capsule:

$$W = 20,862.9 \text{ lb}$$

Totals:

$$W_{\text{tot}} = 2(328.087) + 20,862.9 = 21,519.07 \text{ lb}$$

$$S_{\text{ref}} (\text{two parachutes}) = 21,125.8 \text{ ft}^2$$

Flight initial conditions:

$$\text{Altitude} = 5,000 \text{ ft}$$

$$\text{Air density} = 2.05 \times 10^{-3} \text{ slugs/ft}^3 \text{ (constant)}$$

$$\text{Terminal velocity} = 34.19 \text{ fps}$$

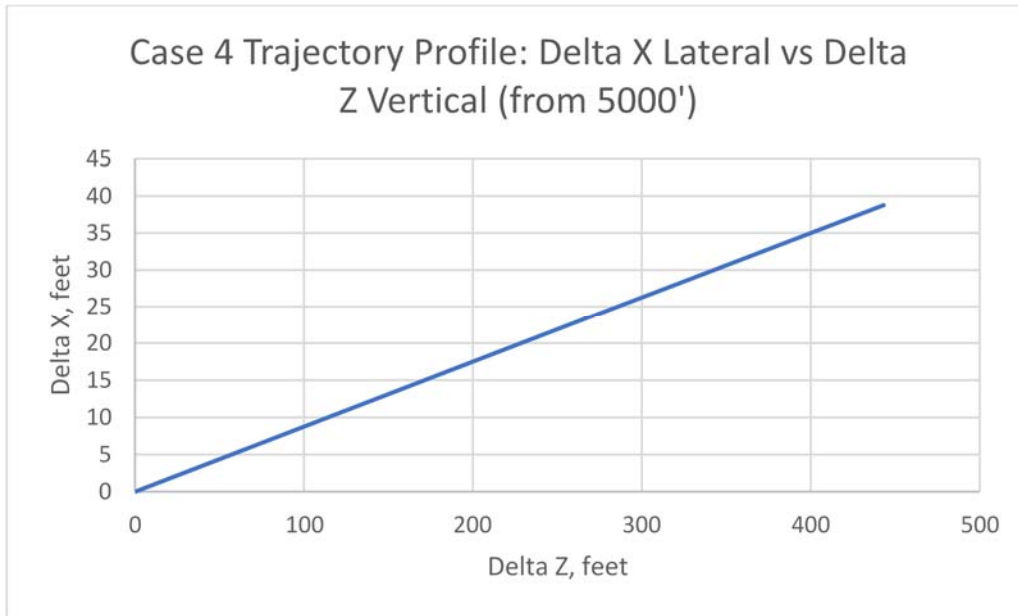
$$\alpha = \gamma = 5 \text{ degrees}$$

$$\theta = 0 \text{ degrees}$$

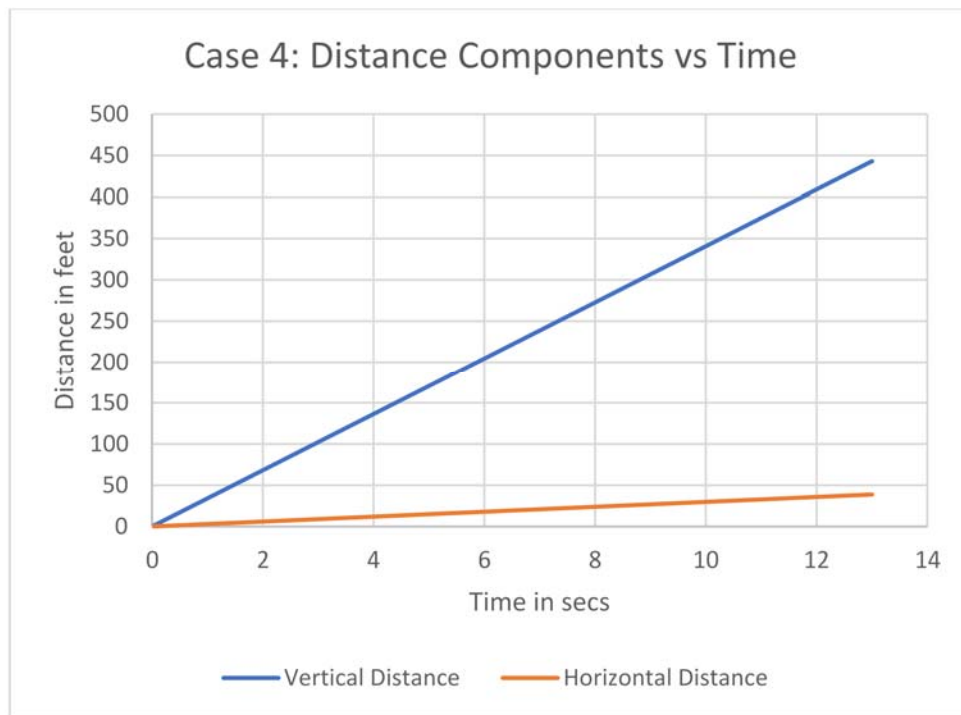
$$L/D = 0.0875$$

### D.3 Results

Figure D-3 describes the lateral versus vertical motion for equilibrium glide. Figure D-4 describes horizontal and vertical distances versus time.

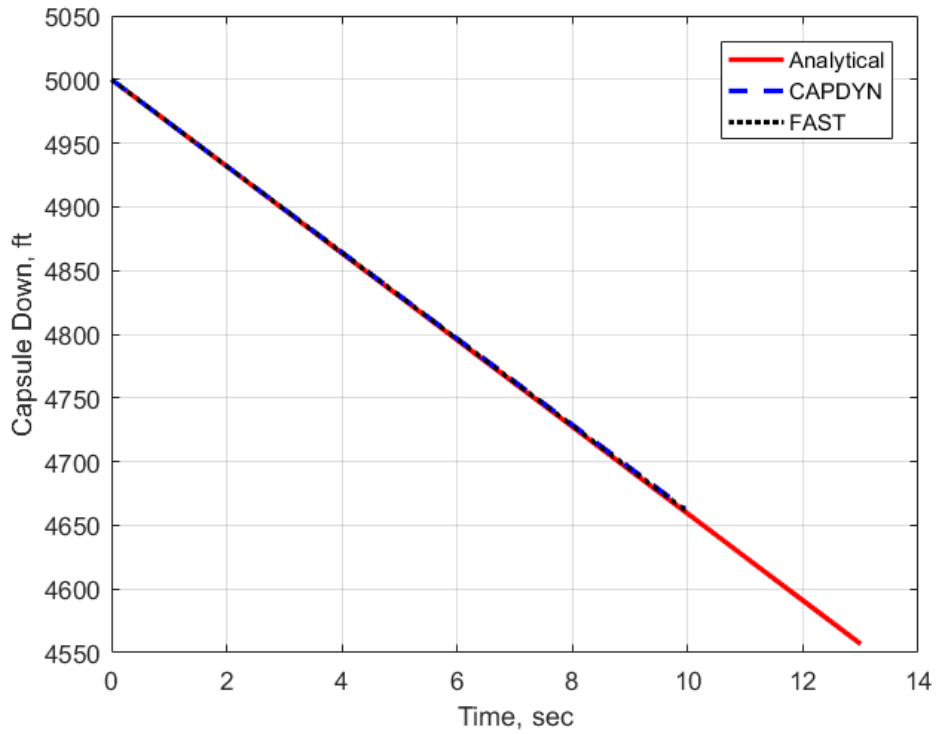


**Figure D-3. Case 4 Trajectory Profile**

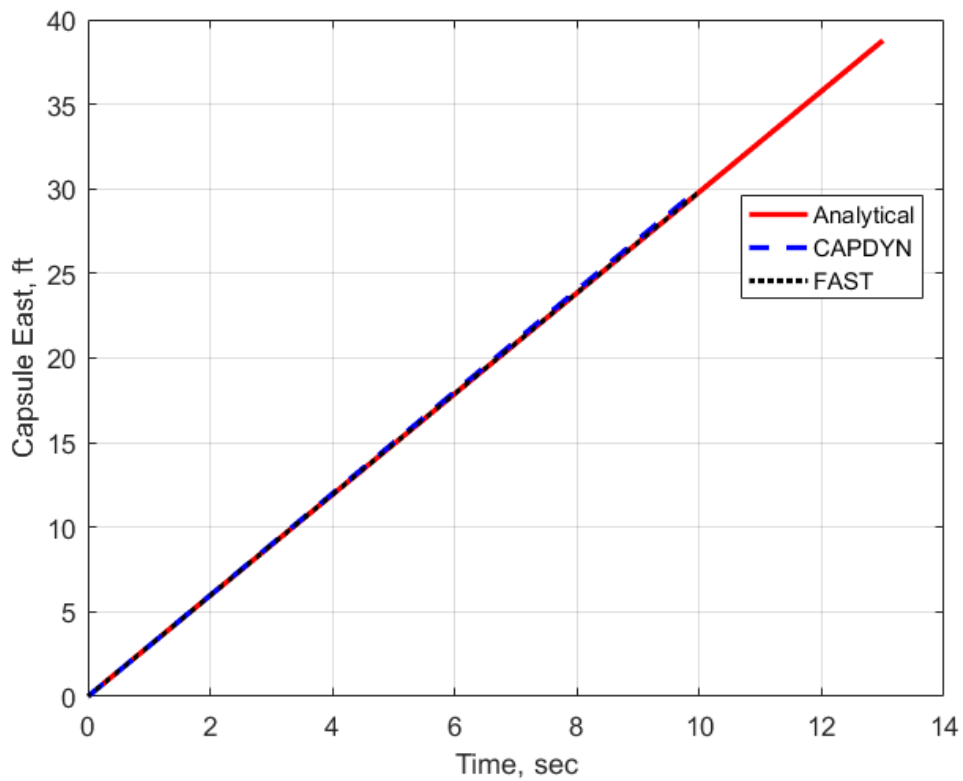


**Figure D-4. Distance Components versus Time, Case 4**

The initial conditions are selected such that the system is in equilibrium both vertically and horizontally. The accelerations are zero, and the velocity components are constant—thus, the linear distance versus time. The terminal dynamic pressure is a constant 1.198 psf. Capsule Down and East positions from CAPDYN and FAST are co-plotted with the analytical model in Figures D-5 and D-6. Results from the three approaches are nearly identical.



**Figure D-5. Capsule Altitude (ft) versus Time Comparison, Case 4**



**Figure D-6. Capsule East Position (ft) versus Time Comparisons, Case 4**

## **Appendix E. Verification Case 5 – Horizontal Wind Shear, Constant Density**

### **E.1 Introduction**

A series of exact analytical solutions are developed for a capsule with two parachutes to verify CAPDYN results. The case described herein is for the capsule and two parachutes acting as a fixed system impacted by a horizontal wind shear. Initial conditions are chosen such that equilibrium is obtained before the shear occurs. The analytical solution predicts the effect of the shear on the system horizontal motion.

#### **E.1.1 Initial Conditions**

The initial conditions are set such that the system is descending vertically at a constant dynamic pressure and velocity. Density is set at the 5,000-ft level and held constant. A horizontal constant wind shear is simulated at the initiation of the simulation.

#### **E.1.2 Simplifying Assumptions**

The following simplifying assumptions are necessary for Verification Case 5:

- Both parachutes have identical physical characteristics, aerodynamic models, and dimensions.
- Proximity effects are ignored.
- There is a single riser line attach point on the load.
- Density is held constant at the 5,000-ft altitude value.
- Horizontal velocity component is assumed to have a negligible impact on dynamic pressure.
- The motion is limited to the vertical and horizontal plane for all three bodies.
- The two parachutes will occupy the same space.
- The wind shear starts and continues at a constant horizontal velocity magnitude.
- There are no capsule aerodynamic forces or moments.
- All out-of-plane parachute aerodynamic coefficients are set to zero.
- There is no enclosed air mass included in this simulation.

### E.1.3 Free-body Diagram: Inertial Axis System

Figure E-1 shows the free-body diagram of the system, with the external forces and the axis system.

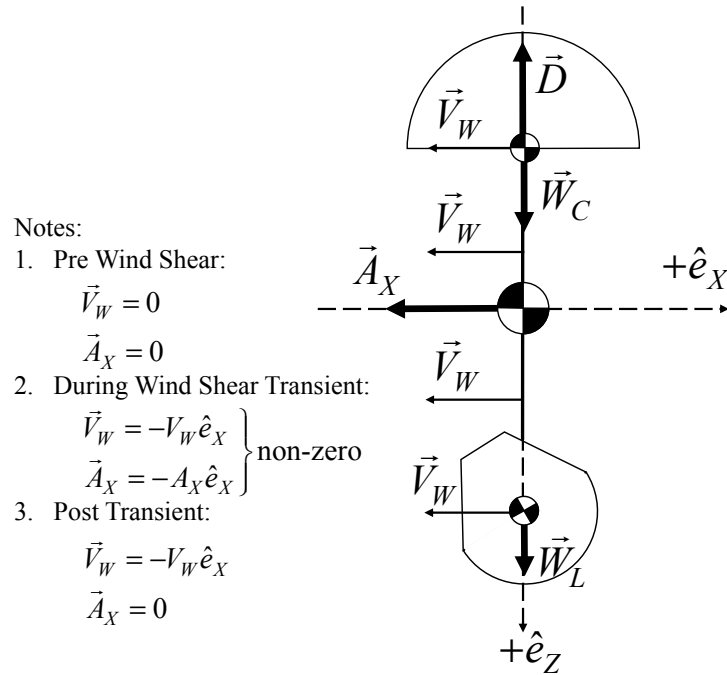


Figure E-1. Free-body Diagram, Case 5

### E.1.4 Derivation of Equations

#### E.1.4.1 Pre Wind Shear: Vertical Descent

Newton's Law:

$$\sum \vec{F}_{\text{external}} = m\vec{a} = \vec{W}_C + \vec{W}_L + \vec{D} = m\ddot{Z}\hat{e}_Z$$

$$\vec{W}_{\text{tot}} = \vec{W}_C + \vec{W}_L = W_{\text{tot}}\hat{e}_Z$$

$$\vec{D} = -qSC_Z\hat{e}_Z$$

Equilibrium conditions:

$$\ddot{Z} = 0$$

$$\ddot{X} = \dot{X} = X = 0$$

$$q_{\text{term}} = \frac{W_{\text{tot}}}{SC_Z}$$

$$V_{\text{term}} = \sqrt{\frac{2q_{\text{term}}}{\rho}}$$

### E.1.4.2 Post Transient: Final Steady-state Conditions

Newton's Law:

$$\sum \vec{F}_{\text{external}} = m\vec{a} = \vec{W}_C + \vec{W}_L + \vec{D} = m\vec{\ddot{Z}} = \vec{0} = m\vec{\ddot{X}} = \vec{0}$$

Equilibrium conditions:

$$\ddot{Z} = 0$$

$$\ddot{X} = 0$$

Vertical:

$$q_{\text{term}} = \frac{W_{\text{tot}}}{SC_Z}; V_{\text{term}} = \sqrt{\frac{2q_{\text{term}}}{\rho}} \text{ (wind relative)}$$

Horizontal:

$$\dot{X} = -V_W, \quad X = -V_W t + X_0$$

### E.1.4.3 During Wind-shear Transient

Initial conditions at shear initiation:

$$t = 0$$

$$X_0 = 0, \quad \dot{X}_0 = 0, \quad \ddot{X} = -\frac{A_X}{m}$$

$$Z_0 = Z_{IC}, \quad \dot{Z}_{IC} = V_{\text{term}}, \quad \ddot{Z}_0 = 0$$

Newton's Law, horizontal:

$$m\ddot{X} = -A_X = -qSC_X$$

Aerodynamic model:

$$C_X = C_{X_\alpha} \alpha = C_{X_\alpha} \frac{\dot{X} + V_W}{\dot{Z}}$$

$$\dot{Z} \approx \text{constant} = V_{\text{term}}$$

$$\ddot{X} = \left( \frac{-qS}{mV_{\text{term}}} C_{X_\alpha} \right) (\dot{X} + V_W)$$

where

$$k = \frac{qS}{mV_{\text{term}}} C_{X_\alpha}$$

#### E.1.4.4 Equation of Motion

$$\ddot{X} + k\dot{X} = -kV_W$$

#### E.1.4.5 Solution from Calculus

$$\dot{X} = Ae^{-kt} - V_W$$

Using the boundary conditions:

$$X = V_W \left[ \frac{1}{k} (1 - e^{-kt}) - t \right]$$

$$\dot{X} = V_W (e^{-kt} - 1)$$

$$\ddot{X} = -kV_W e^{-kt}$$

#### E.1.5 Analytical Solution

For the entire simulation, the system is acted upon by a horizontal wind shear. The vertical motion is near constant, while the horizontal motion reacts to the shear. The horizontal inertial velocity starts at zero and exponentially approaches the wind speed.

### E.2 Physical Characteristics

For purposes of comparison of the analytical solutions with CAPDYN results, the following physical characteristics and initial conditions of the capsule and parachutes are assumed:

Parachutes (each):

$$W = 328.087 \text{ lb}$$

$$\text{Reference area} = 10,562.9 \text{ ft}^2$$

$$C_Z = 0.85$$

$$C_X = C_{X_\alpha} \alpha$$

$$C_{X_\alpha} = 0.573 \text{ per rad}$$

Capsule:

$$W = 20,862.9 \text{ lb}$$

Totals:

$$W_{\text{tot}} = 2(328.087) + 20,862.9 = 21,519.07 \text{ lb}$$

$$S_{\text{ref}} (\text{two parachutes}) = 21,125.8 \text{ ft}^2$$

Flight initial conditions:

$$\text{Initial altitude} = 5,000 \text{ ft}$$

$$\text{Air density} = 2.05 \times 10^{-3} \text{ (constant) slugs/ft}^3$$

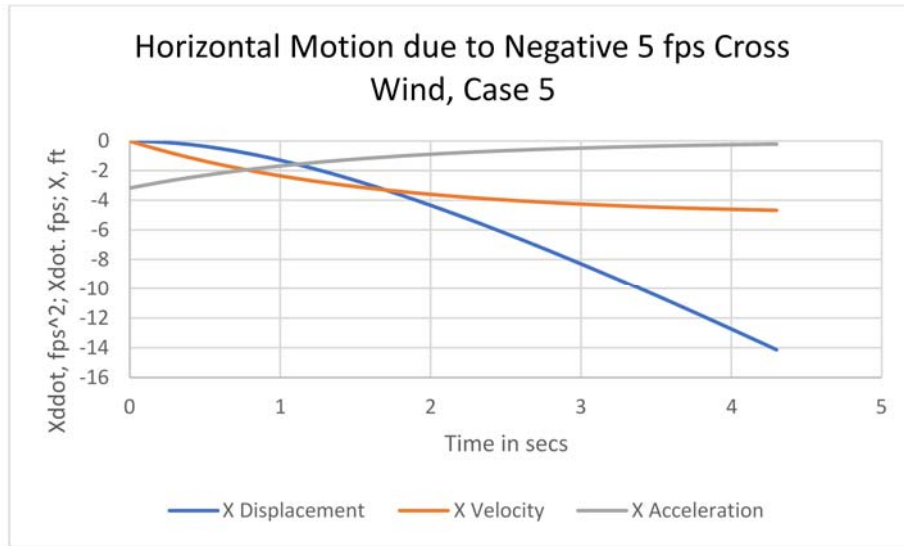
$$\text{Terminal velocity} = 34.19 \text{ fps}$$

$$\text{Terminal dynamic pressure} = 1.198 \text{ psf}$$

$$\text{Wind shear velocity} = -5 \text{ fps}$$

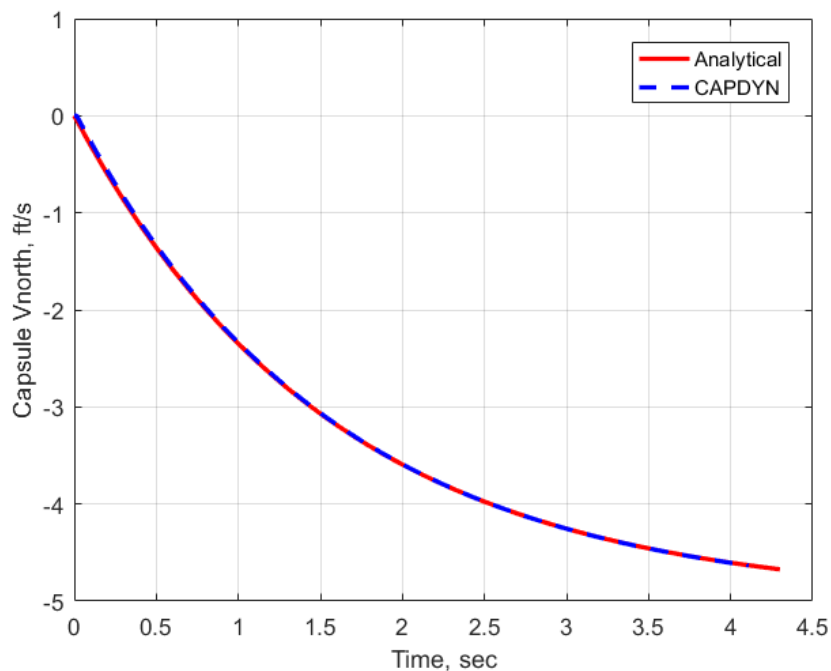
### E.3 Results

The horizontal motion is shown in Figure E-2.



**Figure E-2. Horizontal Motion from 5-fps Crosswind, Case 5**

The initial conditions are selected assuming the system is in equilibrium, terminal vertical velocity with zero horizontal motion. A 5-fps horizontal wind shear is simulated at time  $t = 0$ , causing a horizontal acceleration that results in the system horizontal velocity increasing with time. The system exponentially approaches the wind magnitude as the horizontal acceleration approaches zero. The vertical motion is only marginally affected. Similar to Case 3, FAST was unable to participate in this case due to certain limitations. Results from CAPDYN are co-plotted with the analytical model in Figure E-3 showing excellent agreement.



**Figure E-3. Capsule Velocity North (ft/s) Comparison**

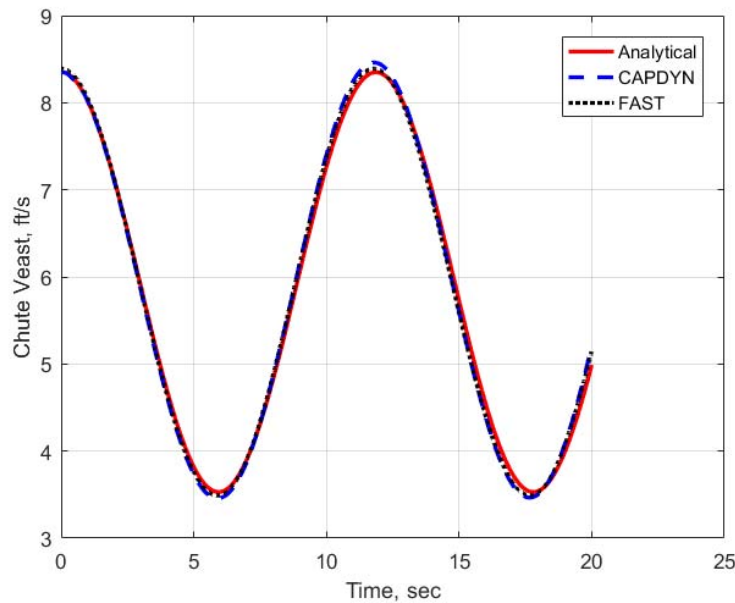


## Appendix F. Verification Case 6 – Pendulum Motion

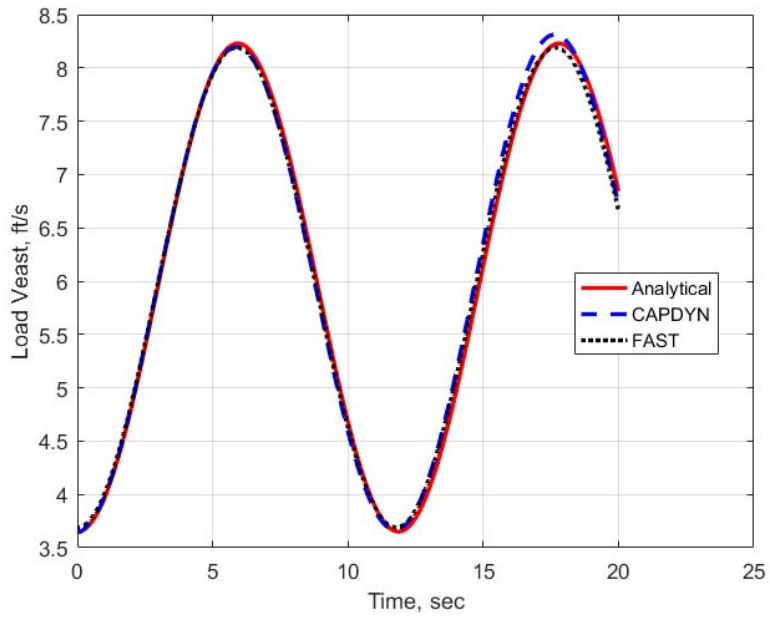
Solutions to Equations (6.3.1-5) and (6.3.1-7) are provided in this section. The system is undergoing pendulum motion while translating toward the East and falling. Time histories of the horizontal (East component) velocities of the payload and parachutes, along with swing angle, are shown. The assumed values for  $C_{N\alpha}$ ,  $C_A$ , and  $q_\infty$  are 0.85, 0.85, and 1.20 psf, respectively. The trim angle of attack is 10 degrees. The simulations conducted for this check case involve no wind disturbance and no aerodynamic force applied to the capsule. Results from CAPDYN and FAST are co-plotted with the analytical model. The initial values of the swing angle and its time derivatives are  $\theta_o = 0$  and  $\dot{\theta}_o = 0.02$  rad/s.

### F.1 Undamped Pendulum Motion ( $C_{N\alpha} = 0$ )

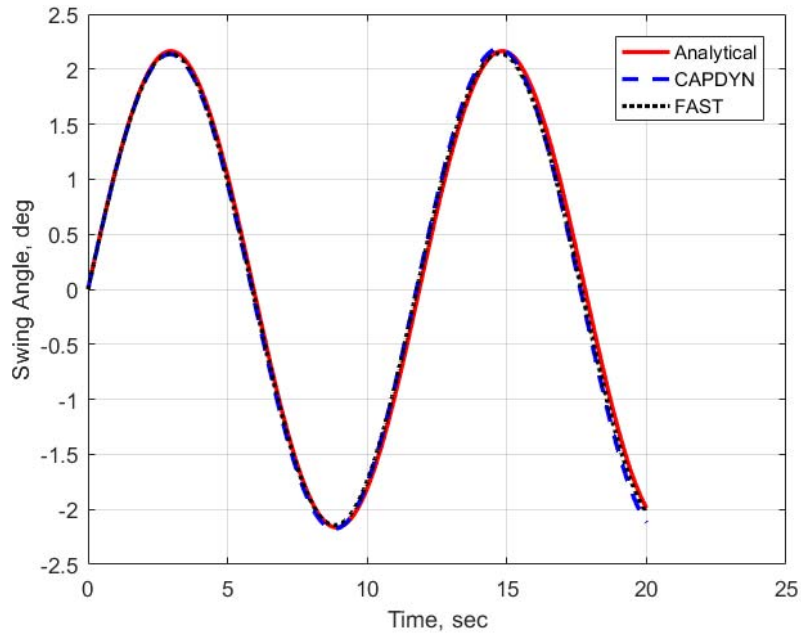
This section shows the results for the undamped pendulum motion. The parachute and load East velocity components are shown in Figures F-1 and F-2. The pendulum swing angle is shown in Figure F-3. The CAPDYN and FAST results show excellent agreement with the analytical solution. The slight discrepancies are likely due to numerical integration schemes.



**Figure F-1. Parachute Velocity East ( $C_{N\alpha} = 0$ )**



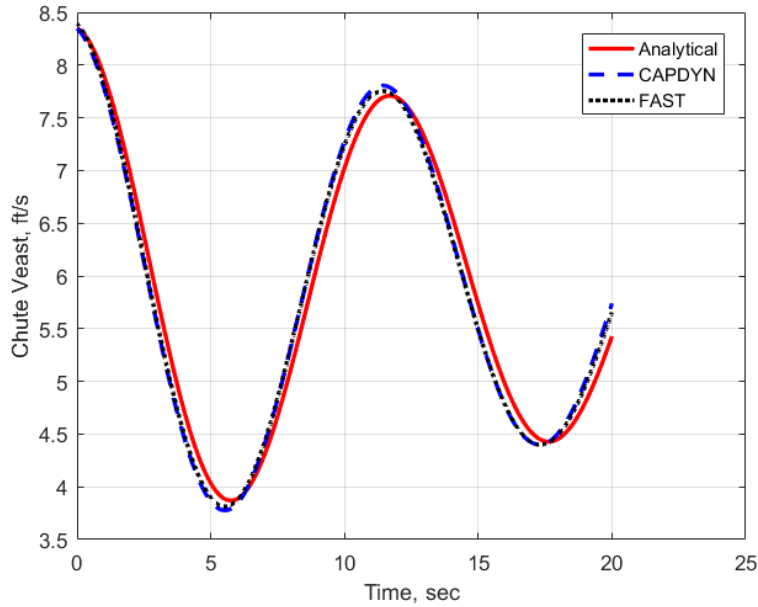
**Figure F-2. Load Velocity East ( $C_{N\alpha} = 0$ )**



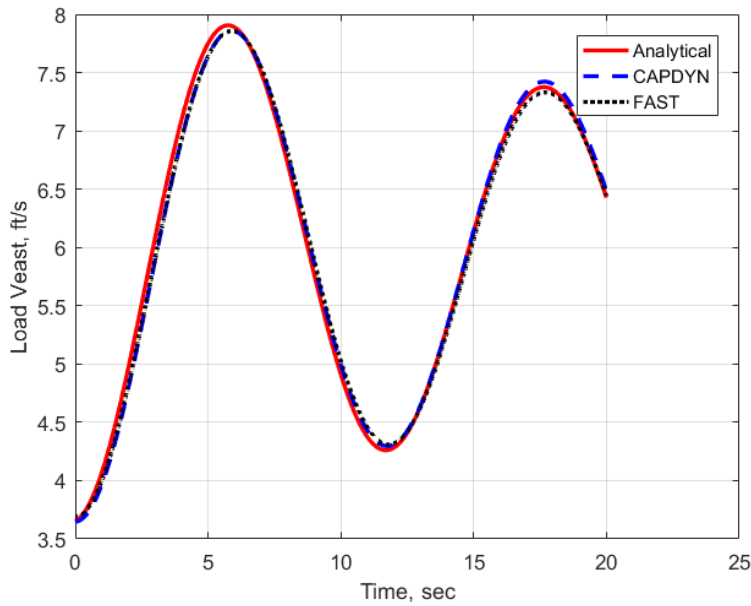
**Figure F-3. Swing Angle ( $C_{N\alpha} = 0$ )**

## F.2 Damped Pendulum Motion ( $C_{N\dot{\alpha}} = 0.304$ )

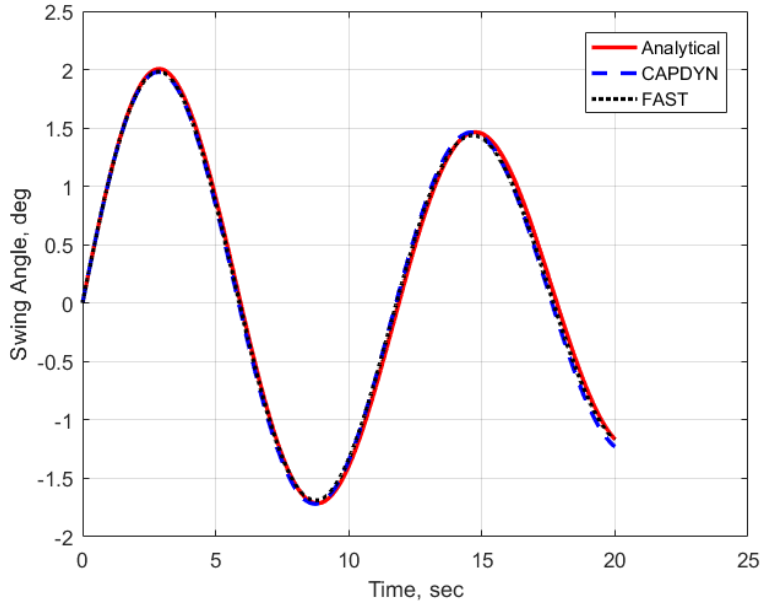
This section shows the results for the damped pendulum motion. The parachute and load East velocity components are shown in Figures F-4 and F-5. The pendulum swing angle is shown in Figure F-6. The CAPDYN and FAST results show excellent agreement with the analytical solution. The slight discrepancies are likely due to numerical integration schemes.



**Figure F-4. Parachute Velocity East ( $C_{N\dot{\alpha}} = 0.304$ )**



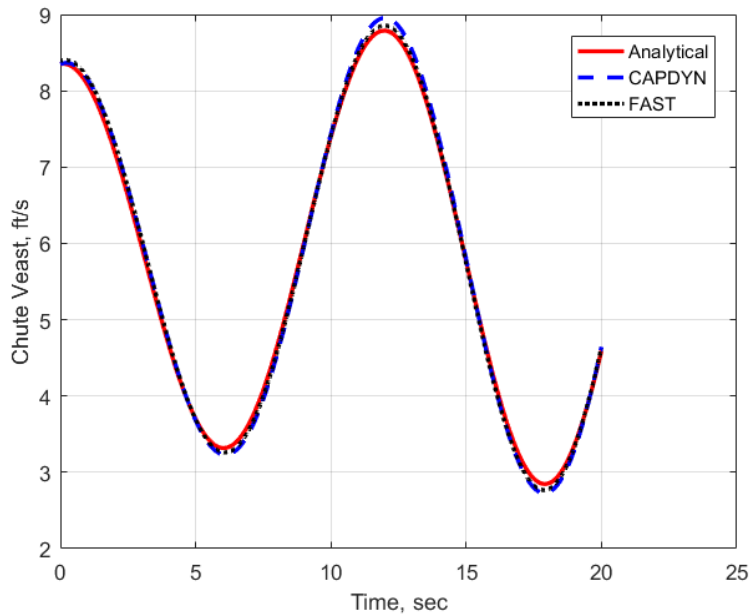
**Figure F-5. Load Velocity East ( $C_{N\dot{\alpha}} = 0.304$ )**



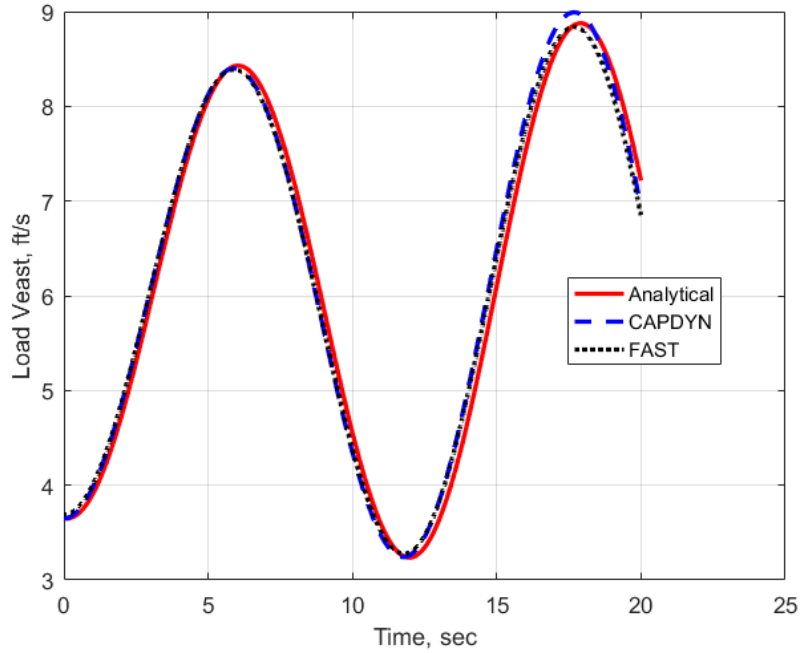
**Figure F-6. Swing Angle ( $C_{N\dot{\alpha}} = 0.304$ )**

### F.3 Divergent Pendulum Motion ( $C_{N\dot{\alpha}} = -0.160$ )

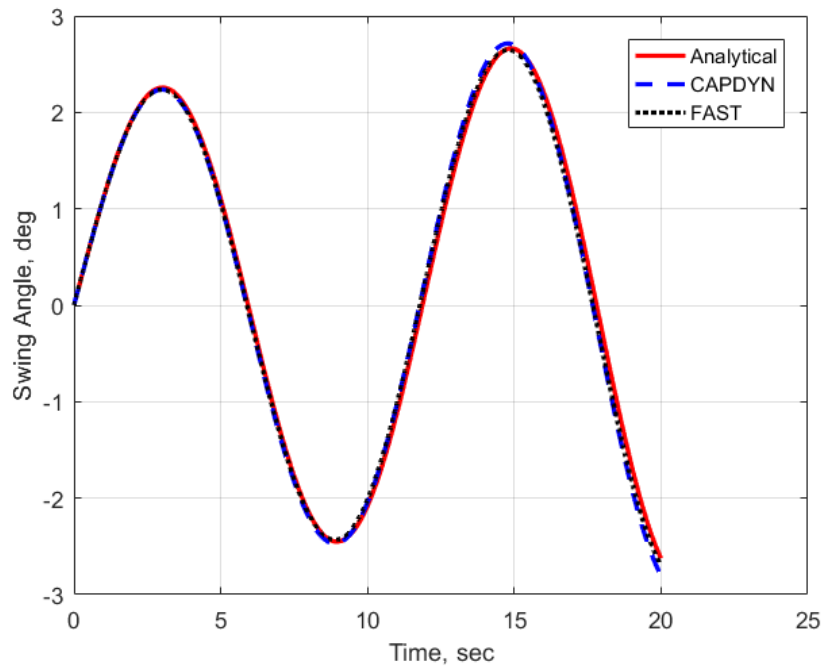
This section shows the results for the damped pendulum motion. The parachute and load East velocity components are shown in Figures F-7 and F-8. The pendulum swing angle is shown in Figure F-9. The CAPDYN and FAST results show excellent agreement with the analytical solution. The slight discrepancies are likely due to numerical integration schemes.



**Figure F-7. Parachute Velocity East ( $C_{N\dot{\alpha}} = -0.160$ )**



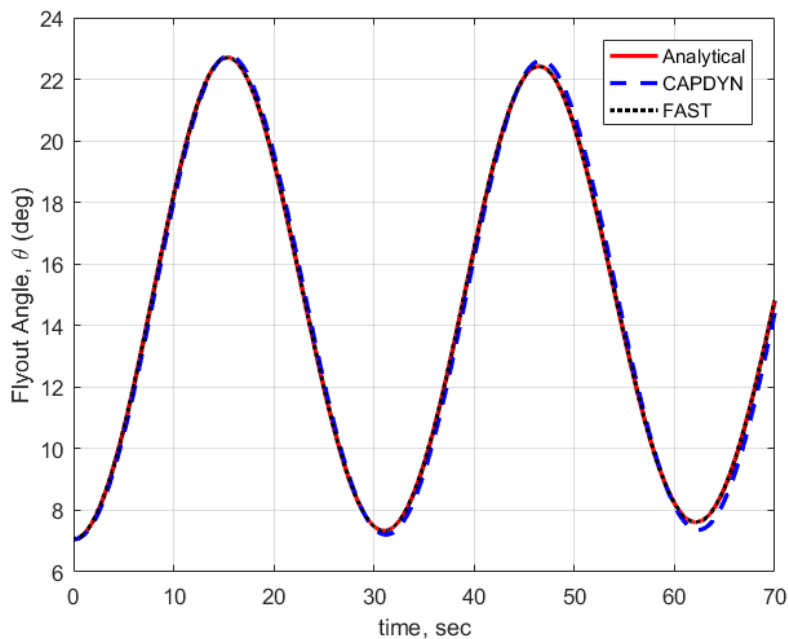
**Figure F-8. Load Velocity East ( $C_{N\alpha} = -0.160$ )**



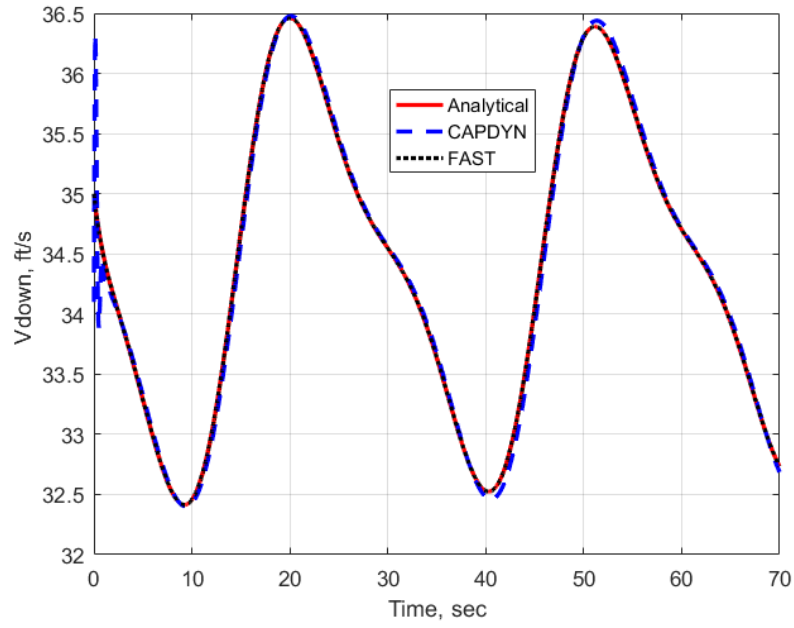
**Figure F-9. Swing Angle ( $C_{N\alpha} = -0.160$ )**

## Appendix G. Verification Case 7 – Flyout Motion

Solutions to the coupled equations of motion in Equation (6.3.2-1) are provided in this section. The system is undergoing the symmetric scissors motion while falling at the same time. Time histories of the flyout angle and Down velocity are shown. The assumed values for  $C_{N\alpha}$ ,  $C_A$ , and  $q_\infty$  are 0.225, 0.85, and 1.24 psf, respectively. The trim angle of attack is 13.33 degrees. The simulations conducted for this check case involve no wind disturbance and no aerodynamic force applied to the capsule. Results from CAPDYN and FAST are co-plotted with the analytical model. Figure G-1 shows the flyout of one of the parachutes. Velocity of the capsule in the Down direction is shown in Figure G-2. Note that the transient in the CAPDYN  $V_{\text{down}}$  solution is due to line tension initialization and can be mitigated by carefully selecting the initial conditions. The results from the three approaches are otherwise identical.



**Figure G-1. Flyout Motion: Parachute 1 Flyout Angle**



**Figure G-2. Flyout Motion: Capsule  $V_{down}$**

## Appendix H. Verification Case 8 – Maypole

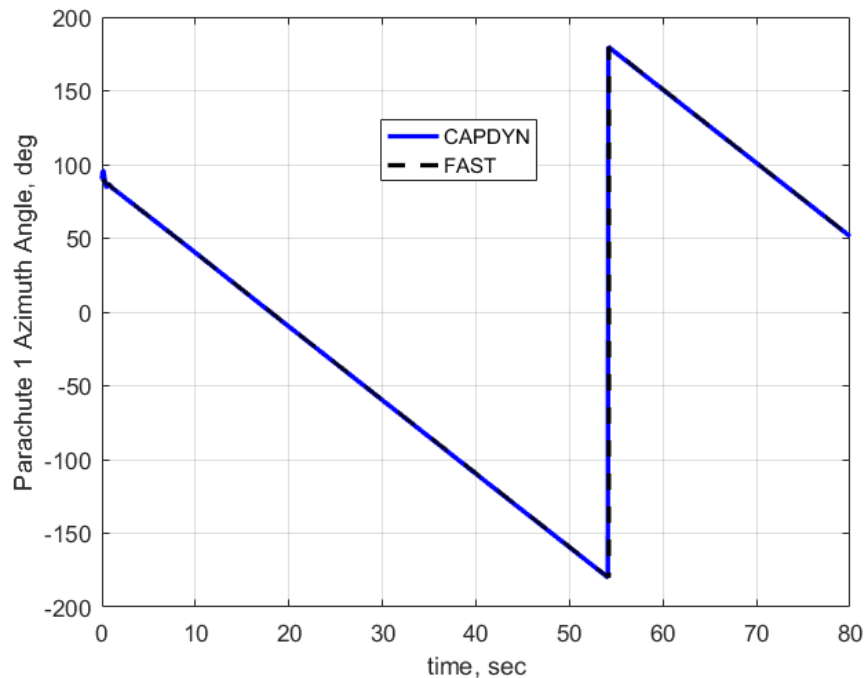
The analytical development in Section 6.3.2 indicates that the normal force coefficient is dependent on the orbiting radius and angular velocity,  $\Omega$ . For this check case, the team settled on an orbiting radius of 40 ft and a value of 5 deg/s for  $\Omega$ . Using Equation (6.3.2-22),  $C_N$  was determined to be 0.01. The simulations conducted for this check case involve no wind disturbance and no aerodynamic force applied to the capsule. The azimuth angle,  $\chi$ , is computed as follows:

$$\chi = \text{atan2}(\Delta r_{\text{east}}, \Delta r_{\text{north}}) \quad (\text{H-1a})$$

$$\Delta r_{\text{east}} = r_{\text{chute, east}} - r_{\text{capsule, east}} \quad (\text{H-1b})$$

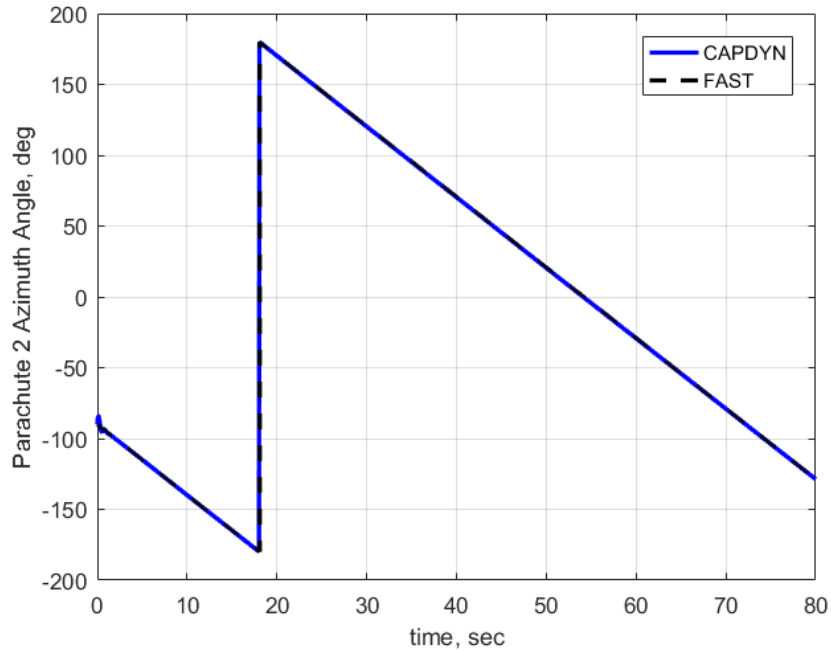
$$\Delta r_{\text{north}} = r_{\text{chute, north}} - r_{\text{capsule, north}} \quad (\text{H-1c})$$

Time histories for the parachute azimuth angles are shown in Figures H-1 and H-2. Velocities of Parachute 1 in the East and North are shown in Figures H-3 and H-4. The analytical approach was used to prescribe the aerodynamics required to maintain the maypole motion, which was subsequently implemented into CAPDYN and FAST. The results from the two simulations are nearly identical.

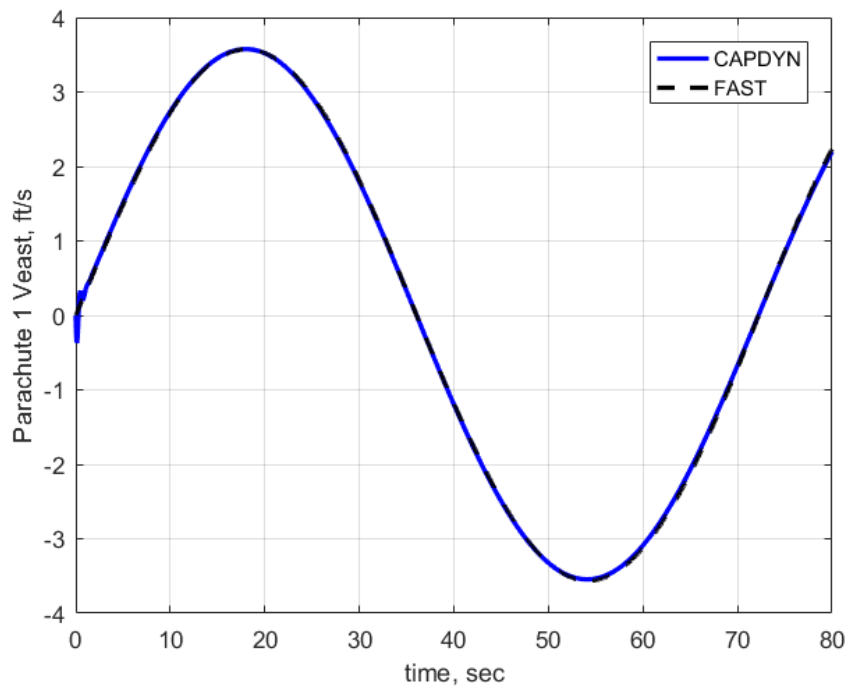


**Figure H-1. Maypole Motion: Parachute 1 Azimuth**

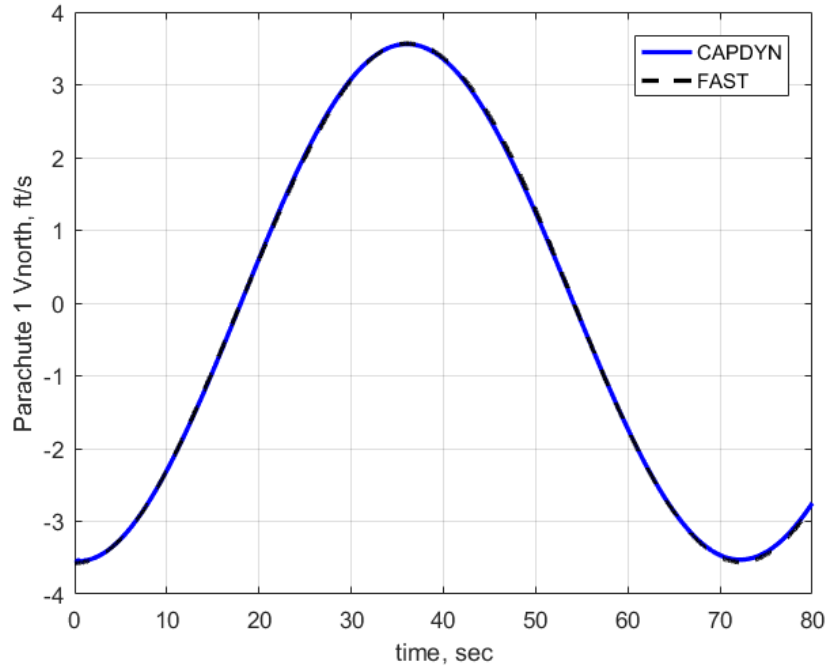




**Figure H-2. Maypole Motion: Parachute 2 Azimuth**



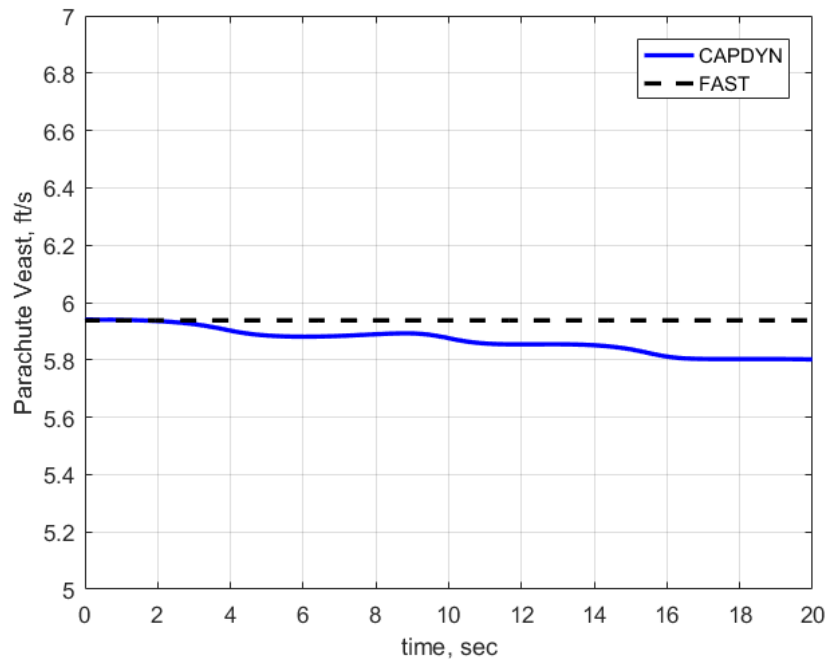
**Figure H-3. Maypole Motion: Parachute 1 V<sub>east</sub>**



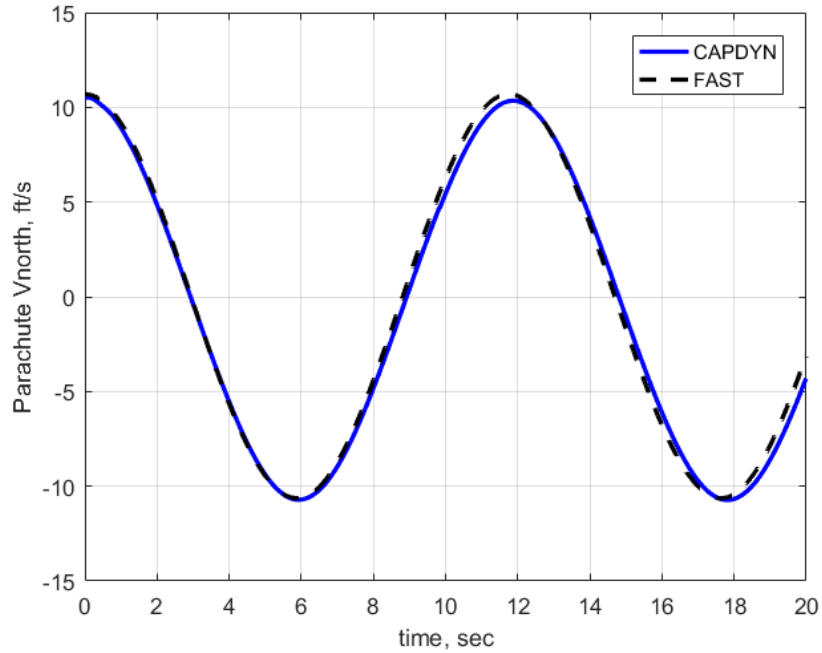
**Figure H-4. Maypole Motion: Parachute 1  $V_{north}$**

## Appendix I. Verification Case 9 – Nonplanar Pendulum Motion

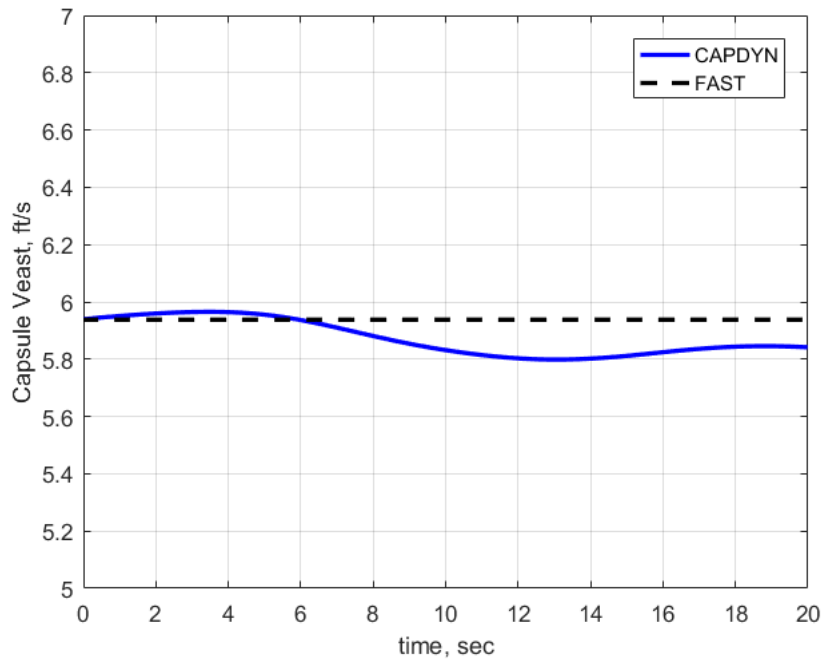
To assess the full capability of CAPDYN, the system is prescribed to undergo a nonplanar pendulum motion. Similar to the planar case, the parachutes are treated as a single particle. The system undergoes undamped pendulum motion ( $C_{N\alpha} = 0$ ) in the North-Down plane while translating and falling in the East-Down plane with velocities of 5.94 and 34.17 ft/s at the center of mass, respectively. Time histories of the North and East velocity components of the parachutes, with swing angles, are shown. The assumed values for  $C_{N\alpha}$ ,  $C_A$ , and  $q_\infty$  are 0.85, 0.85, and 1.20 psf, respectively. The simulations conducted for this check case involve no wind disturbance and no aerodynamic force applied to the capsule. The initial values of the swing angle and its time derivatives are  $\theta_o = 0$  and  $\dot{\theta}_o = 0.08726$  rad/s. Due to the complexity of the motion, only CAPDYN and FAST results are available. See Appendix K for the treatment of the aerodynamics model in CAPDYN to produce this motion. The parachute velocities in the North and East directions are shown in Figures I-1 and I-2. The capsule North and East velocity components are shown in Figures I-3 and I-4. Due to numerical issues in CAPDYN, there appears to be a small (but steady) drift in the  $V_{\text{east}}$  for the parachutes and the capsule. The swing-angle time history is shown in Figure I-5. Otherwise, the comparisons show excellent agreement.



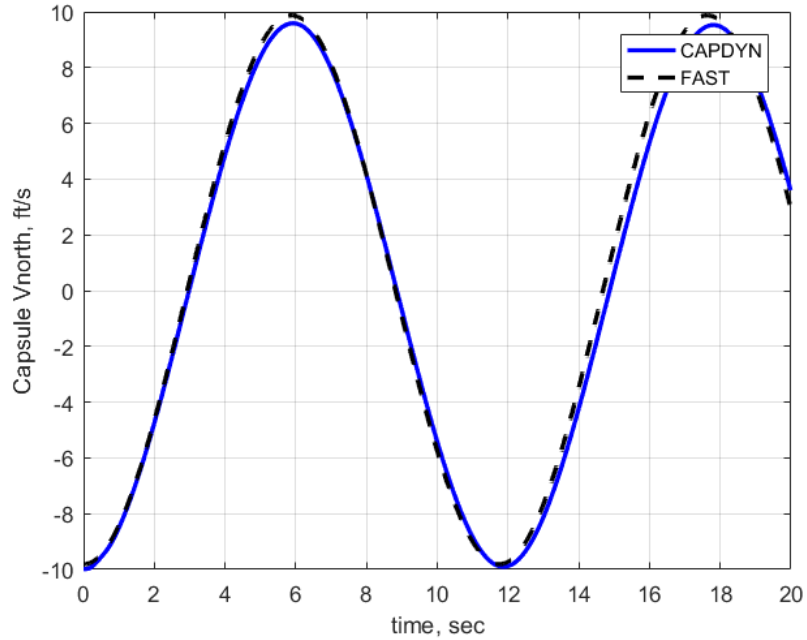
**Figure I-1. Nonplanar Pendulum Motion: Parachute Velocity East**



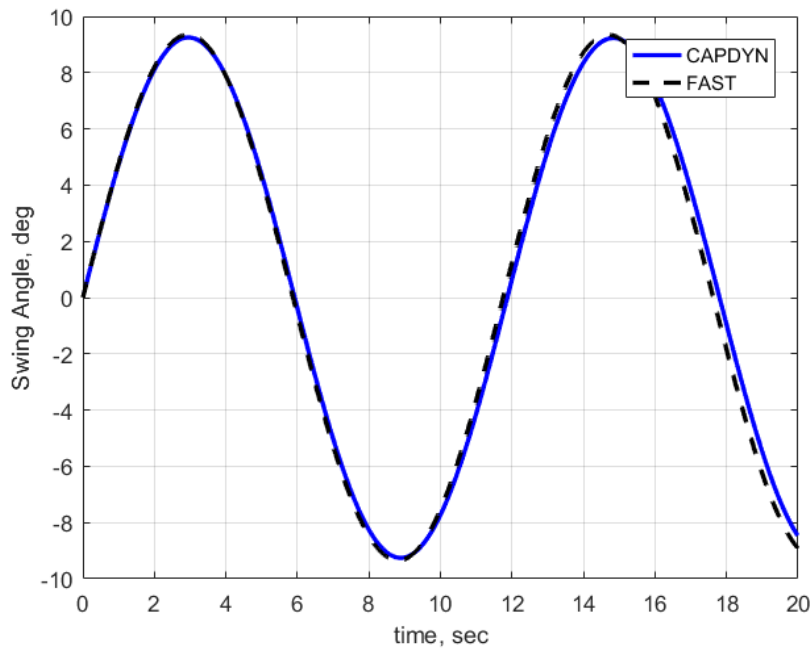
**Figure I-2. Nonplanar Pendulum Motion: Parachute Velocity North**



**Figure I-3. Nonplanar Pendulum Motion: Load Velocity East**



**Figure I-4. Nonplanar Pendulum Motion: Load Velocity North**



**Figure I-5. Nonplanar Pendulum Motion: Swing Angle North-Down Plane**

## Appendix J. Verification Case 10 – Nonplanar Flyout Motion

To further assess the full capability of CAPDYN and the computation of proximity variables (e.g.,  $D_{prox}$  and  $\phi_{prox}$ ), the system is prescribed to undergo a nonplanar scissors motion. Similar to the planar case, the system undergoes undamped scissors motion in the North-Down plane while translating and falling in the East-Down plane with velocities of 5.94 and 34 ft/s, respectively. Time histories of the NED velocities of the payload and parachutes with  $\phi_{prox}$  are shown. The assumed values for  $C_{N\alpha}$ ,  $C_A$ , and  $q_\infty$  are 0.225, 0.85, and 1.24 psf, respectively. The trim angle of attack is 13.33 degrees. The simulations conducted for this check case involve no wind disturbance and no aerodynamic force applied to the capsule. Due to the complexity of the motion, only CAPDYN and FAST results are available. A similar procedure (as the nonplanar pendulum motion) is used in CAPDYN in determining the special aerodynamics model required for this prescribed motion. The proximity aero angles,  $\phi_{prox1}$  and  $\phi_{prox2}$ , are

$$\phi_{prox1} = \text{atan 2} \left( \left( \Delta r(2)_{\frac{p1}{p2}} \right)_{p1}, \left( \Delta r(3)_{\frac{p1}{p2}} \right)_{p1} \right)$$

$$\phi_{prox2} = \text{atan 2} \left( \left( \Delta r(2)_{\frac{p2}{p1}} \right)_{p2}, \left( \Delta r(3)_{\frac{p2}{p1}} \right)_{p2} \right)$$

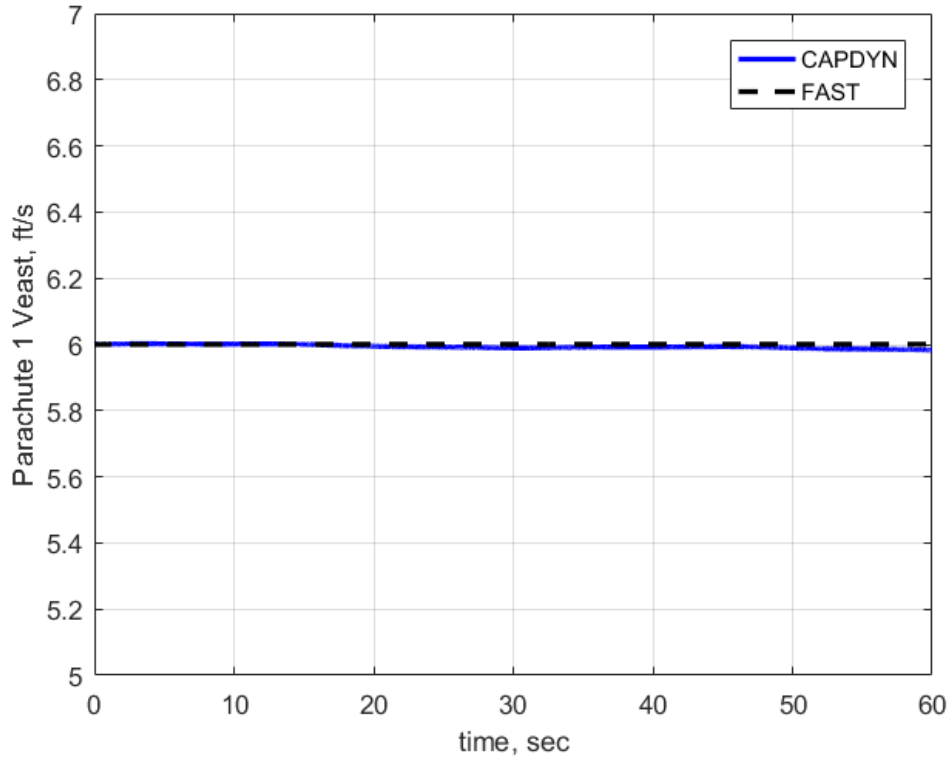
where  $(\Delta r_{p2/p1})_{p2}$  and  $(\Delta r_{p1/p2})_{p1}$  are the relative position vectors between the parachutes in the NED frame, computed as

$$(\Delta r_{p2/p1})_{NED} = r_{p2_{NED}} - r_{p1_{NED}}$$

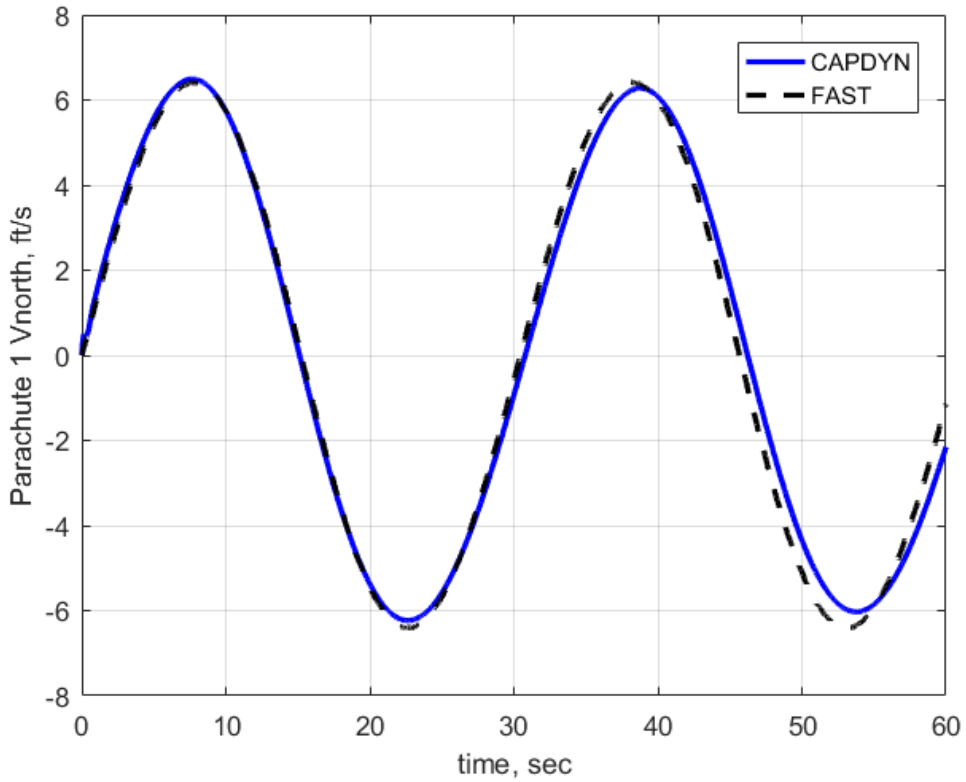
$$(\Delta r_{p1/p2})_{NED} = r_{p1_{NED}} - r_{p2_{NED}}$$

converted to the wind axes of  $p2$  and  $p1$ , respectively.

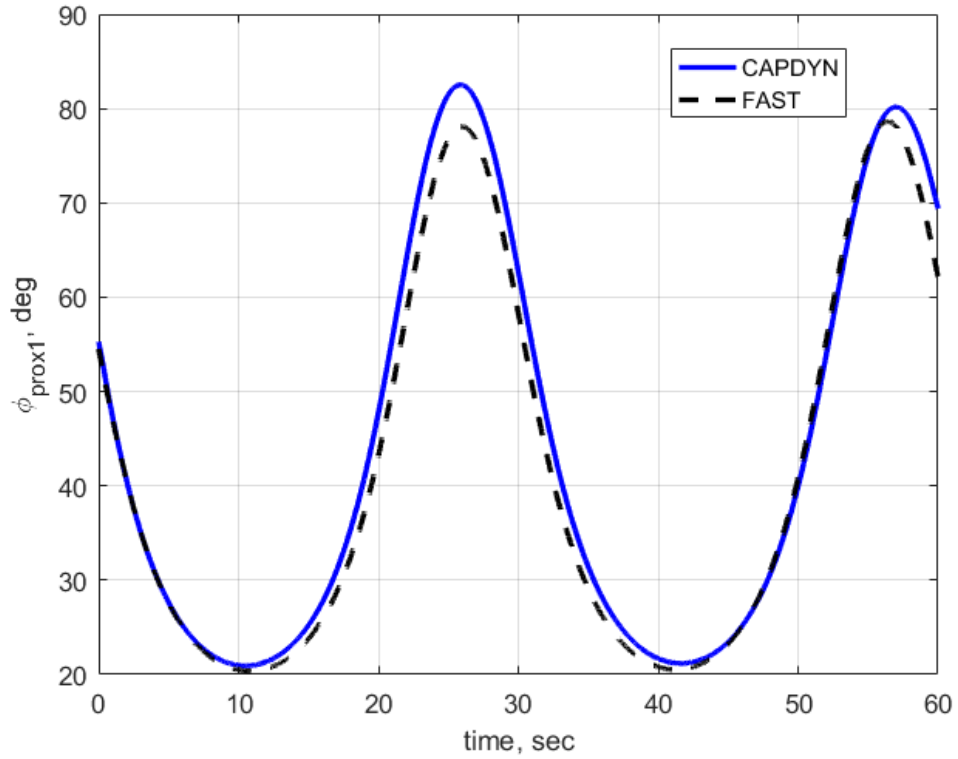
East and North velocity components for parachute 1 are shown in Figures J-1 and J-2. The proximity angles are shown in Figures J-3 and J-4. The flyout angles are shown in Figures J-5 and J-6. The simulations show excellent agreement. The small amplitude and phase discrepancies in some of the outputs may be due to slight parameter differences and/or numerical integration schemes.



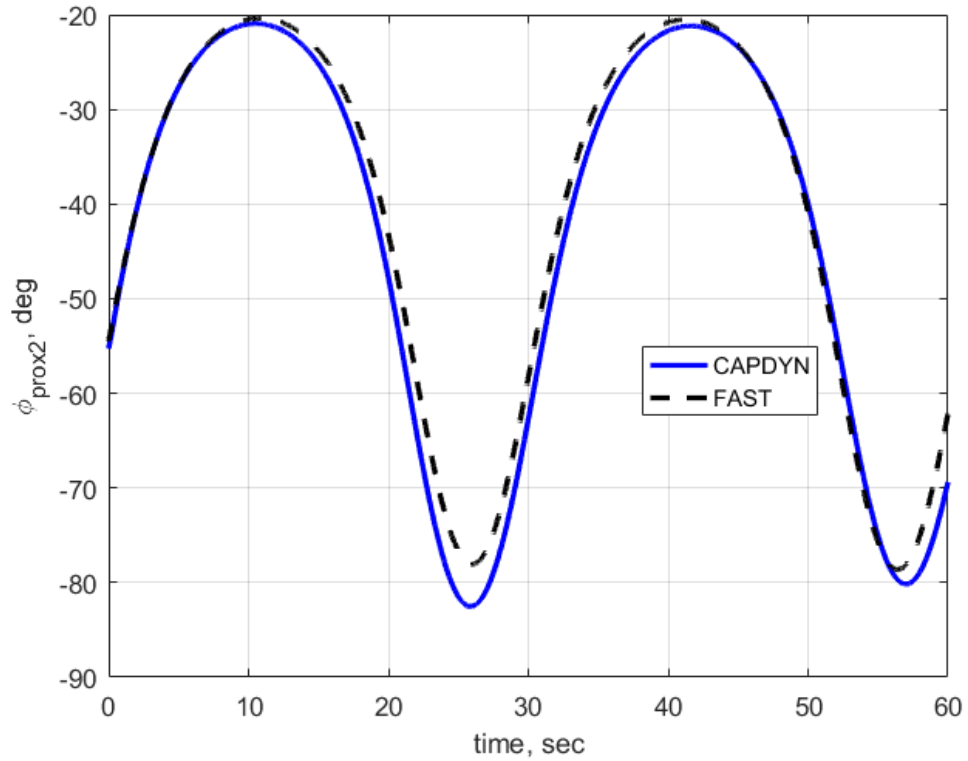
**Figure J-1. Nonplanar Scissors Motion: Parachute 1 Velocity East**



**Figure J-2. Nonplanar Scissors Motion: Parachute 1 Velocity North**

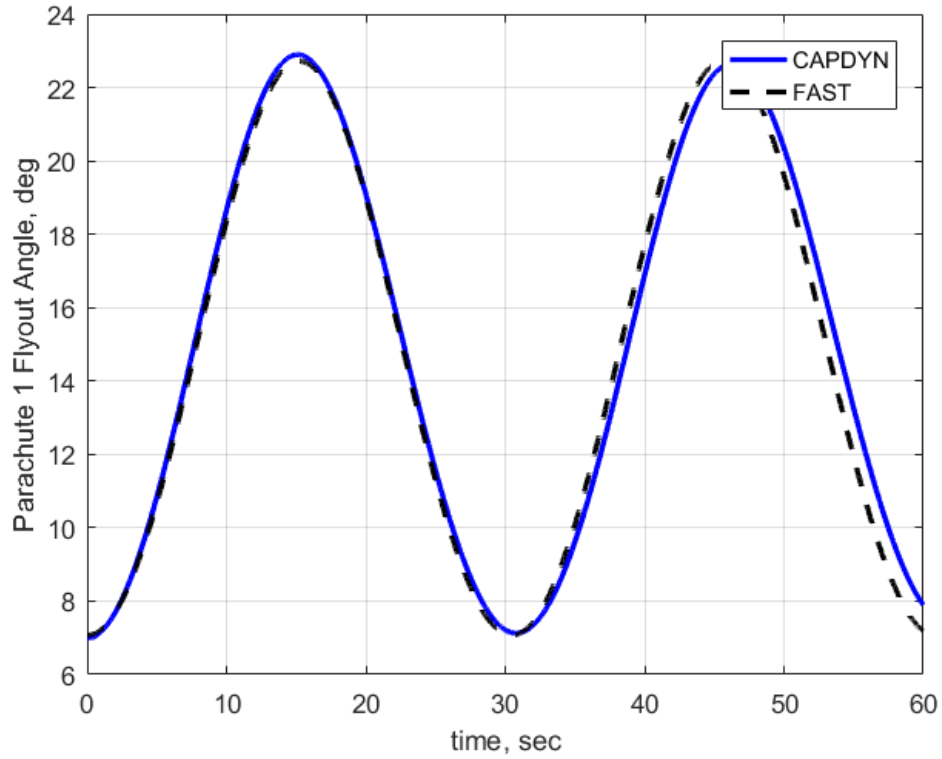


**Figure J-3. Nonplanar Scissors Motion:  $\phi_{prox1}$**

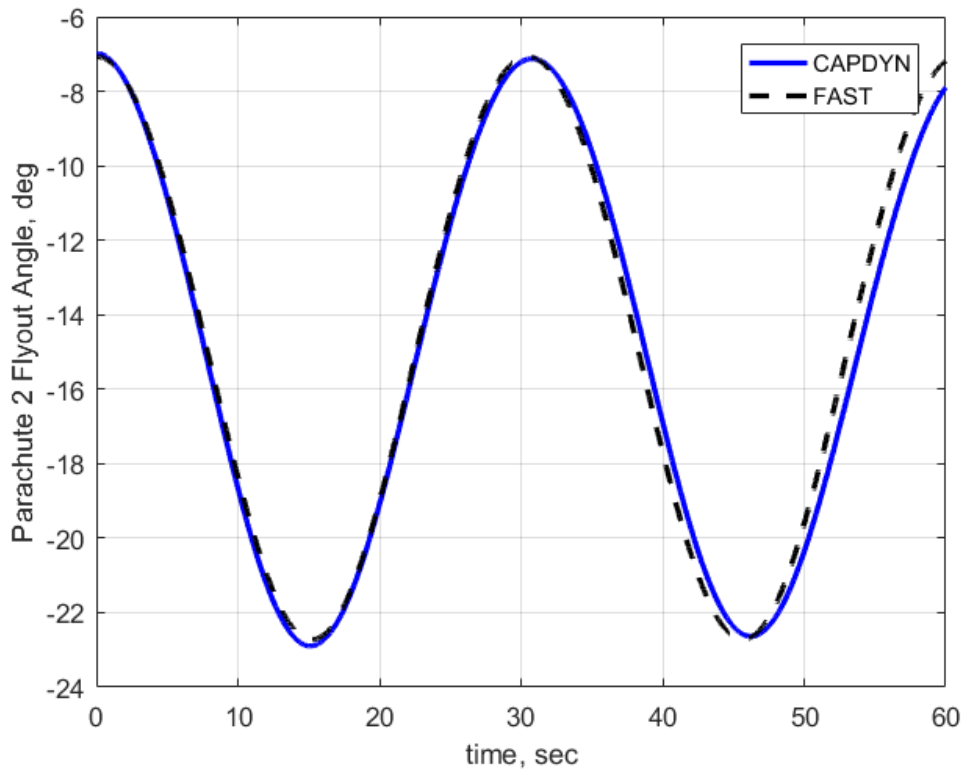


**Figure J-4. Nonplanar Scissors Motion:  $\phi_{prox2}$**





**Figure J-5. Nonplanar Scissors Motion: Parachute 1 Flyout Angle North-Down Plane**

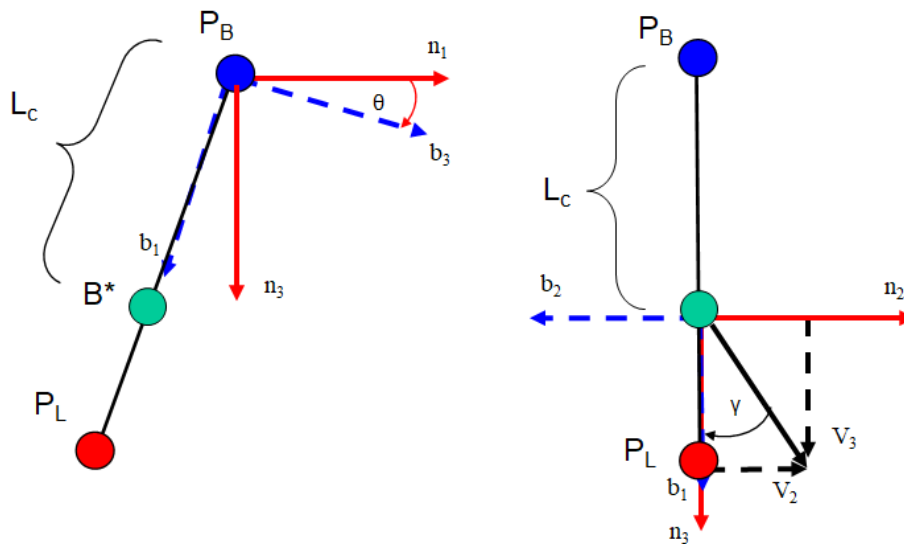


**Figure J-6. Nonplanar Scissors Motion: Parachute 2 Flyout Angle North-Down Plane**

## Appendix K. Prescribed Aerodynamics Required for Nonplanar Pendulum Motion in CAPDYN

Due to the time-varying nature of the wind axis system in CAPDYN, described in Section 6.3.3, a prescribed aerodynamics model (in the CAPDYN wind axis system) is required for the system to maintain pendulum motion in the North-Down plane while translating in the East-Down plane. Equations K-1 through K-14 derive the required aerodynamics model to produce the nonplanar pendulum motion in CAPDYN. Note that the same procedures can be followed to produce the nonplanar flyout motion in CAPDYN as described in Appendix J.

The following derivations describe the aerodynamics required in CAPDYN's aero axis to maintain the pendulum motion in the  $\hat{n}_1$  and  $\hat{n}_3$  planes while translating in  $\hat{n}_2$  and  $\hat{n}_3$  with constant velocities. The parachutes are assumed to be on top of one another. Figure K-1 is a schematic of the nonplanar pendulum motion.



**Figure K-1. Nonplanar Pendulum Motion**

Table K-1 shows the direction cosine matrix between the inertial (NED) frame to the parachute body frame. Figure K-2 illustrates a top view. Table K-2 shows the direction cosine matrix between a frame that the aerodynamics coefficients are prescribed with the parachute body frame.

**Table K-1. Direction Cosine Matrix (body frame to inertial frame)**

	$\hat{b}_1$	$\hat{b}_2$	$\hat{b}_3$
$\hat{n}_1$	$-\sin \theta$	$0$	$\cos \theta$
$\hat{n}_2$	$0$	$-1$	$0$
$\hat{n}_3$	$\cos \theta$	$0$	$\sin \theta$

Unit vector from  $P_B$  (parachute particle) to  $P_L$  (load particle),  $\hat{\mathbf{i}}$ :

$$\hat{\mathbf{i}} = -\sin \theta \hat{\mathbf{n}}_1 + \cos \theta \hat{\mathbf{n}}_3 \quad (\text{K-1})$$

Desired velocity of the parachute particle in NED, where  $V_2$  is a constant:

$${}^N\mathbf{v}^{PB} = L_c \dot{\theta} \cos \theta \hat{\mathbf{n}}_1 + V_2 \hat{\mathbf{n}}_2 + (V_3 + L_c \dot{\theta} \sin \theta) \hat{\mathbf{n}}_3 \quad (\text{K-2})$$

Construct the  $\hat{\mathbf{j}}$  axis as per CAPDYN:

$$\hat{\mathbf{j}} = \frac{-\hat{\mathbf{i}} \times {}^N\mathbf{v}^{PB}}{|-\hat{\mathbf{i}} \times {}^N\mathbf{v}^{PB}|} \quad (\text{K-3})$$

$$\hat{\mathbf{j}} = \frac{V_2 \cos \theta \hat{\mathbf{n}}_1 - (L_c \dot{\theta} + V_3 \sin \theta) \hat{\mathbf{n}}_2 + V_2 \sin \theta \hat{\mathbf{n}}_3}{\sqrt{V_2^2 + (L_c \dot{\theta} + V_3 \sin \theta)^2}} \quad (\text{K-4})$$

Construct the  $\hat{\mathbf{k}}$  axis:

$$\hat{\mathbf{k}} = \hat{\mathbf{i}} \times \hat{\mathbf{j}} \quad (\text{K-5})$$

$$\hat{\mathbf{k}} = \frac{(L_c \dot{\theta} \cos \theta + V_3 \sin \theta \cos \theta) \hat{\mathbf{n}}_1 + V_2 \hat{\mathbf{n}}_2 + (L_c \dot{\theta} \sin \theta + V_3 \sin^2 \theta) \hat{\mathbf{n}}_3}{\sqrt{V_2^2 + (L_c \dot{\theta} + V_3 \sin \theta)^2}} \quad (\text{K-6})$$

Aerodynamic coefficient as defined in the CAPDYN aero frame, c (note that

$\hat{\mathbf{c}}_1 = \hat{\mathbf{i}}$ ,  $\hat{\mathbf{c}}_2 = \hat{\mathbf{j}}$ ,  $\hat{\mathbf{c}}_3 = \hat{\mathbf{k}}$ ):

$$C_F = -C_{A,c} \hat{\mathbf{i}} + C_{Y,c} \hat{\mathbf{j}} - C_{N,c} \hat{\mathbf{k}} \quad (\text{K-7})$$

To maintain constant velocity in  $\hat{\mathbf{n}}_2$ :

$$C_F \cdot \hat{\mathbf{n}}_2 = 0 \quad (\text{K-8})$$

$$0 = \frac{C_{Y,c}(L_c \dot{\theta} + V_3 \sin \theta) + C_{N,c} V_2}{\sqrt{V_2^2 + (L_c \dot{\theta} + V_3 \sin \theta)^2}} \quad (\text{K-9})$$

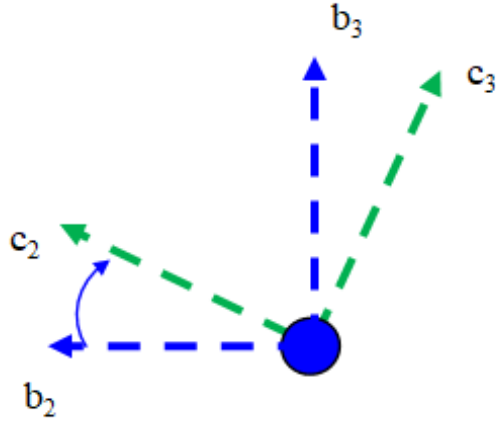
Solve for  $C_{Y,c}$  in terms of  $C_{N,c}$ :

$$C_{Y,c} = -C_{N,c} \frac{V_2}{L_c \dot{\theta} + V_3 \sin \theta} \quad (\text{K-10})$$

Desired aerodynamics in the b frame:

$$C_F = -C_{A,b} \hat{\mathbf{b}}_1 + 0 \hat{\mathbf{b}}_2 - C_{N,b} \hat{\mathbf{b}}_3 \quad (\text{K-11})$$

The b frame to the CAPDYN aero frame, c, is a rotation about  $\hat{\mathbf{b}}_1$  by  $\phi$ .



**Figure K-2. Nonplanar Pendulum Motion (top view)**

**Table K-2. Direction Cosine Matrix (aero frame to body frame)**

	$\hat{\mathbf{c}}_1$	$\hat{\mathbf{c}}_2$	$\hat{\mathbf{c}}_3$
$\hat{\mathbf{b}}_1$	1	0	0
$\hat{\mathbf{b}}_2$	0	$\cos \phi$	$-\sin \phi$
$\hat{\mathbf{b}}_3$	0	$\sin \phi$	$\cos \phi$

$C_{N,b}$  can be expressed in terms of  $C_{Y,c}$  and  $C_{N,c}$ :

$$C_{N,b} = -C_{Y,c} \sin \phi + C_{N,c} \cos \phi \quad (\text{K-12})$$

Use Equations (K-10) and (K-12) to arrive at Equation (K-13) to express  $C_{N,c}$  in terms of  $C_{N,b}$ :

$$C_{N,c} = \frac{C_{N,b}(V_3 \sin \theta + L_C \dot{\theta})}{V_2 \sin \phi + (V_3 \sin \theta + L_C \dot{\theta}) \cos \phi} \quad (\text{K-13})$$

For check case 9, assume a linear aero model with no damping:

$$C_{N,b} = C_{N_\alpha} \theta \quad (\text{K-14})$$

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The NASA Engineering and Safety Center (NESC) was requested to determine the feasibility of using modern system identification techniques for developing high-fidelity parachute models from parachute drop-test flight data, using full-scale Orion Capsule Parachute Assembly System (CPAS) drop-test pendulum-mode flight data as a test case. This document contains the Appendices to the main report.

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