Polar Coding for Forward Error Correction in Space Communications with LDPC Comparisons

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***Abstract*—With the surging development of optical telecommunications for space applications, the importance of error correction has become more apparent than ever. Specifically, the exploration of forward error correction code (FEC) methodologies will be instrumental in developing the standards for optical communications in space. Despite the widespread use of low-density parity-check (LDPC) codes, alternate FEC codes such as polar codes have shown immense promise in assisting space communications error correction with their ability to bypass the error floors that plague LDPC codes. Extremely promising techniques including cyclic redundancy checks (CRC), successive cancellation (SC), and successive cancellation lists (SCL) that assist polar coding in achieving the Shannon limit in a timely manner are evaluated. MATLAB simulations are conducted with AWGN and burst noise to test each technique's ability to handle noise typically encountered in space and each technique's ability to correct unexpected errors. Results of simulations for different rates and message lengths are also reported to determine each technique's ability to handle large data volumes and fix errors. Similar simulations are conducted for LDPC codes with additional tests for convolutional and no interleavers. Finally, a discussion regarding the future ability of polar codes to satisfy current missions in the place of, or in conjunction with, LDPC codes along with the merits of each FEC technique's ability to process data efficiently and handle data while maintaining adequate performance will be provided. Preliminary recommendations will be made for each technique's effectiveness for GEO related missions along with discussions regarding each technique's ability to fit within the CCSDS standards for optical communications.**

***Index Terms*—FEC, LDPC codes, polar codes, space communications, optical communications**

I. INTRODUCTION

The requirement for error free space communications has existed since the first NASA mission. Specifically, as decreased signal power became a major challenge in error free communications, FEC codes assisted in catching and correcting the resulting errors. In the earlier days of space missions, Reed-Muller codes and convolutional codes were implemented as they were common for that time-period. Convolutional codes carried onto future missions and even included Viterbi decoders for missions such as Voyager [1]. Furthermore, the addition of Reed-Soloman codes resulted in a concatenated convolutional and Reed-Soloman coding scheme that was necessary for multiple deep-space missions requiring more powerful codes [2]. Variations of such concatenated codes become common place for many missions until turbo codes were seen as substantially more useful with their lower complexity and higher coding gains [3]. The discovery of LDPC's flexibility in terms of degrees of freedom led to even better performance after design inefficiencies were addressed [3].

The goal of LDPC coding is to implement a parity-check matrix that is sparse and randomly generated. Introduced by Gallagher in the 1960s, they were not fully explored until the 1990s when their lower complexity and ability to perform near the theoretically achievable coding gain were seen as a major advantage [4]. Along with their graphical representations, LDPC codes were developed to bypass the commonly used turbo codes for higher code rates [5] [6]. Additional LDPC coding schemes, such as ones with Reed-Soloman outer coding, were developed for improved performance and better compatibility with lower code rates [7].

Recently, polar codes entered the fray with improved performance and lower complexity stemming from its block structure design and recursive nature. Described by Arikan in his 2009 work, polar codes garnered interest when it was proven they had the ability to asymptotically achieve the Shannon capacity on many channels [8]. Their highly regular structure makes them particularly useful for real world applications and 3GPPs preliminary adoption of polar codes for eMBB in 5G further exemplifies this. Polar coding's ability to bypass the "error floors" that plague LDPC coding while also maintaining low complexity and high performance make it extremely unique.

In this paper, we report on our research into the viability of polar codes for optical communications and compare them with their conventional LDPC counterparts. We have evaluated extremely promising techniques including cyclic redundancy checks (CRC), successive cancellation (SC), and successive cancellation lists (SCL) that assist polar coding in achieving the Shannon limit. These methodologies have the unique potential to overtake current LDPC standards as shown by the reported results of MATLAB simulations to test the ability of each polar coding methodology to handle a variety of variables. In addition, simulations conducted with AWGN and burst noise to test each technique's ability to handle different types of noise are reported. Further simulations are run via MATLAB and C for LDPC codes with additional tests for convolutional and no interleavers.

Finally, there is a brief discussion regarding the future

ability of polar codes to satisfy current missions by itself or in a hybrid coding scheme with LDPC codes. The merits of each FEC technique's ability to process data efficiently and handle data while maintaining adequate performance will also be provided. Preliminary recommendations will be made for each technique's effectiveness for GEO related missions along with discussions regarding each technique's ability to fit within the CCSDS standards for optical communications.

II. CODING SCHEMES

A. Polar Coding

As described in [8], the description of polar codes begins with letting $W : \mathbb{X} \rightarrow \mathbb{Y}$ denote a binary-input discrete memoryless channel with $\mathbb{X}$ and $\mathbb{Y}$ representing the binary input alphabet and the output alphabet respectively. Then, the channel transition probabilities are $W(y|x), x \in \mathbb{X}$ and $y \in \mathbb{Y}$. Specifically, polar codes utilize the polarization effect of the matrix $\mathbf{G_1} = \left(\begin{smallmatrix} 1 & 0 \\ 1 & 1 \end{smallmatrix}\right)$. A polar code with length $N = 2^n$ will establish N virtual channels with the polarization effect and the generator matrix, $\mathbf{G_n}$, can be defined as the n-fold Kronecker product of $\mathbf{G_1}$. In other words, the generator matrix can be defined as:

$$\mathbf{G_n} = \mathbf{G_1}^{\otimes n}. \tag{1}$$

With this definition and a message length of K bits (resulting in a code rate of $R = K/N$), the information bits are carried on the K most reliable polarized channels $W_N^{(j)}$ with indices $j \in \mathbb{J}$. The remaining channels are then used to transmit the "frozen bits," which is a fixed binary sequence and can be also be represented as the frozen set $\mathbb{F}$. It should be noted that $\mathbb{F}$ is the complement of the information set $\mathbb{J}$.

If we use a binary source block with K information bits, then a code block can be mapped using $N - K$ frozen bits using the following equation:

$$x_1^N = u_1^N \cdot \mathbf{G_n}, \tag{2}$$

where u_1^N is the binary input and x_1^N is the output code vector.

1) **Cyclic Redundancy Check (CRC):** Cyclic error-correcting (CRC) techniques typically add a fixed-length check value as their encoding scheme for messages. If there are k information bits and m bit CRC sequence added, then there will be $K = k+m$ bits for a K bit input block. Implementing the CRC in the source bits maintains the code rate as it is defined. Additionally, generator polynomials are typically needed, which act as the divisor in the long division operation.

An example of CRC being used in error correction is the parity bit. The parity bit has a two term generator polynomial in $x+1$ and can be classified as a 1-bit CRC. It should be noted that better performing FEC techniques typically implement CRC codes in combination with other FEC techniques, not solely by themselves.

2) **Successive Cancellation (SC):** SC codes eliminate redundancies by cutting the polar codes into smaller pieces for processing. Again, based on [8], if we let the estimate of u_1^N be denoted as $\hat{u}_1^N$, then $\hat{u}_j^N$ can be found successively after y_1^N has been received using the following equation:

$$\hat{u}_j = \begin{cases} h_j(y_1^N, \hat{u}_1^{j-1}) & j \in \mathbb{J} \\ u_j & j \in \mathbb{F} \end{cases} \tag{3}$$

where the bits are determined successively for j from 1 to N and the decision function, h_j is defined as

$$h_j = \begin{cases} 0 & \text{if } \frac{W_N^{(j)}(y_1^N, \hat{u}_1^{j-1}|0)}{W_N^{(j)}(y_1^N, \hat{u}_1^{j-1}|1)} \geq 1 \\ 1 & \text{otherwise} \end{cases} \tag{4}$$

It is further established that a decoder block error occurs if $\hat{u}_1^N \neq u_1^N$.

3) **Successive Cancellation List (SCL):** Successive cancellation lists (SCL) for polar codes were exhibited in [9] as a means to improve the performance of SC decoding. Although SC performs extremely well and approaches the Shannon capacity, it struggles with small and medium length codes in polar coding. SCLs operate by keeping a list of L survival code bits at each step instead of a single survival path implemented by SC.

If we let $\hat{u}_i$ be a random bit, the decoder makes $2L$ candidates from the original L by keeping both paths with $\hat{u}_i = 0$ and $\hat{u}_i = 1$. Moreover, L can be used as a threshold to determine when SCL discards the worst paths and, thus, improves the performance of SC.

B. Low-Density Parity-Check (LDPC) codes

Low-density parity-check (LDPC) codes implement a series of parity check equations based on binary parity check block codes. Typically, the symbols in the code satisfy m parity check equations with block codes that consist of binary vectors of fixed length n. As a result, each codeword will contain $(n-m) = k$ information digits and m check digits. Subsequently, a sparse parity matrix, H, is created whose dimensions are $m \times n$.

One of the more common ways to implement a LDPC code is to invert H to obtain the generator matrix G. Then, matrix multiplication can be utilized to encode. Due to G typically being dense, the encoding complexity is thus quadratic in codeword length. As a result, regular LDPC codes are normally implemented with irregular codes being utilized during the special cases where G is not dense. While regular LDPC codes have code digits entailed in the same number of equations and each equation has the same number of code symbols, irregular LDPC codes can stray from such properties to create better performing constructions. Furthermore, decoding will typically be done via quantized soft symbol detection at each node in the graph. Specifically, log-likelihood ratios (LLR) for each variable node result in soft decisions and are commonplace with LDPC codes.

III. Results

In this paper, we implemented three different techniques that are variations and combinations of the ones described in Section II with AWGN and burst noise. All implementations were done as close as possible to CCSDS specifications in [10] as the techniques permitted.

A. SC and SCL

As previously mentioned, standard SC techniques are very useful in eliminating redundancies. Although there are better performing techniques, SC still serves as the foundation of many of the current improvements in polar coding. We implement a basic SC framework with different codeword lengths and observed the BER performance.

Polar coding using SCL has already shown that it can outperform turbo and LDPC codes under certain parameters [11]. Additionally, it is already an improvement upon the known SC methodology as explained in Section II of this paper. For that reason, we implemented a basic SCL framework using log-likelihood-ratios (LLR) to test its BER performance with different codeword lengths.

For both SC and SCL simulations, we used a BPSK constellation along with testing both AWGN noise and burst noise to signify the worst possible noisy scenario and more accurately portray noise in space respectively. BER results are shown in Fig. 1 for various message lengths of 256 bits and the two different types of noise.

B. CRC-Aided Polar Coding

First introduced in [11], CRC-Aided Polar codes were shown to beat turbo codes in terms of performance and decreased the complexity. We implemented a similar framework with CRC $= 24$ and SCL decoding on a QPSK constellation. Furthermore, per the CCSDS recommendations in [10], we implemented a rate of $R = \frac{1}{2}$ for all tests. Finally, we primarily tested with AWGN noise to simulate the worst case scenario with relation to space applications. BER results are shown in Fig. 2 for various message lengths for AWGN noise.

C. LDPC with Convolutional Interleaver and No Interleaver

Classical LDPC codes have been implemented for many years. Instead of focusing on the previously done work with block interleavers, we simulated a conventional DVB-S2 model with LDPC coding per the new CCSDS protocols under consideration with convolutional interleavers. For a standard message length size, we tested various interleaver sizes as recommended in [10] and compared the performance under the best case scenario of no interleaver at all. That is, the convolutional interleaver size had a cap of $24,000$ binary digits for our QPSK model as anything larger would need advanced computing power. Fig. 3 shows the BER plots for two different interleaver sizes.

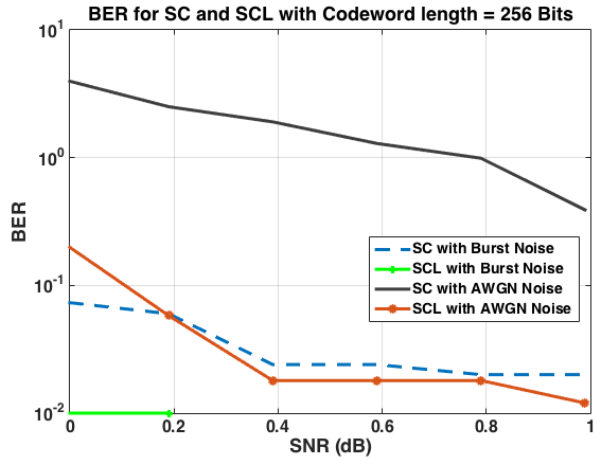

Fig. 1. BER plots for Polar coding with SC and SCL coding

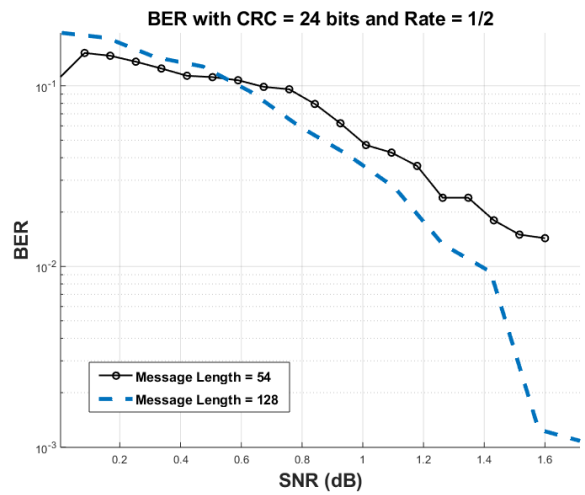

Fig. 2. BER plots for CRC-Aided Polar Coding

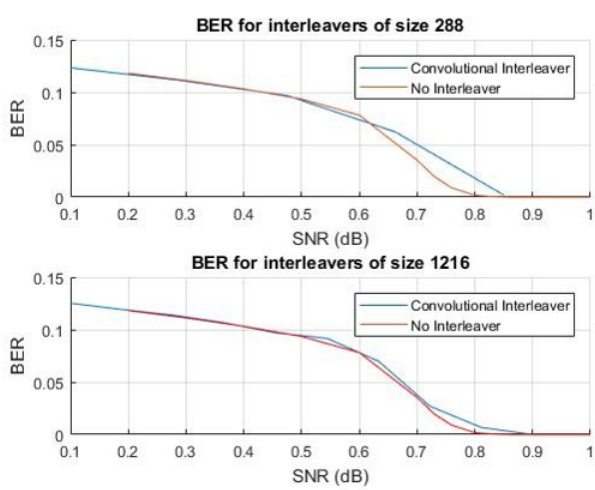

Fig. 3. BER Plots for LDPC Coding with Convolutional and No Interleaver

D. Discussion

We were able to make a number of observations regarding the performance of the different methodologies. The classical results of polar codes overcoming the error floor that plague LDPC codes were verified, but polar coding typically did so at a higher SNR.

After analyzing the simulations, it was noticed that polar coding processed data more efficiently, but had difficulty with accuracy when tested with larger data sets. This was particularly true for shorter SC polar codes. Even with burst noise, which simulates space related noise better, SC polar codes still underperformed with respect to CCSDS recommended LDPC codes with burst noise and only nominally outperformed CCSDS recommended LDPC codes with AWGN noise.

SCL polar codes performed significantly better than SC polar codes. Regardless of code length, SCL polar codes consistently outperformed SC polar codes and had promising performance when tested with burst noise. However, similar to SC, SCL codes struggled to accurately process larger data volumes. Such a characteristic is troubling given the precision and dependability needed for space communications.

CRC-Aided polar codes performed quite well when tested with all types of noise. The main drawback that was observed was the larger power necessary to obtain a meaningful BER when compared to LDPC codes. Moreover, it was observed that CRC-Aided codes struggled with accuracy with larger data sets similar to SC and SCL codes. Despite these drawbacks, CRC-Aided polar codes also have promising performance for space applications. Fig. 2 shows its performance with AWGN noise and its noticeable improvement with burst noise.

Fig. 3 displays the comparisons for the increasingly popular LDPC codes with convolutional interleaver and a standard no interleaver LDPC code in the AWGN environment. Interestingly, the simulations run for various binary digits ranging from 288 to $24,000$ exhibited that there was not a large BER improvement with a convolutional interleaver. In fact, a lack of interleaver actually contributed to better BER performance for smaller binary digits such as 288.

Throughout the majority of the simulations, BER performance of the polar coding methodologies were comparable to LDPC coding when the codeword was less than or equal to 1024 bits. As burst noise and AWGN noise were repeatedly tested, it was also observed that LDPC coding performed better with unstructured noise than polar coding. In other words, LDPC handled unpredictable errors better, but that indicates that polar coding performs better for space applications. The polar coding methodologies were able to process data significantly faster than any type of LDPC coding. The processing speed was most noticeable for any type of polar coding compared to LDPC codes with convolutional interleavers as those would need significantly more processing power.

Fig. 4 gives a visual representation of the strengths of the polar coding and LDPC coding methodologies when compared head to head.

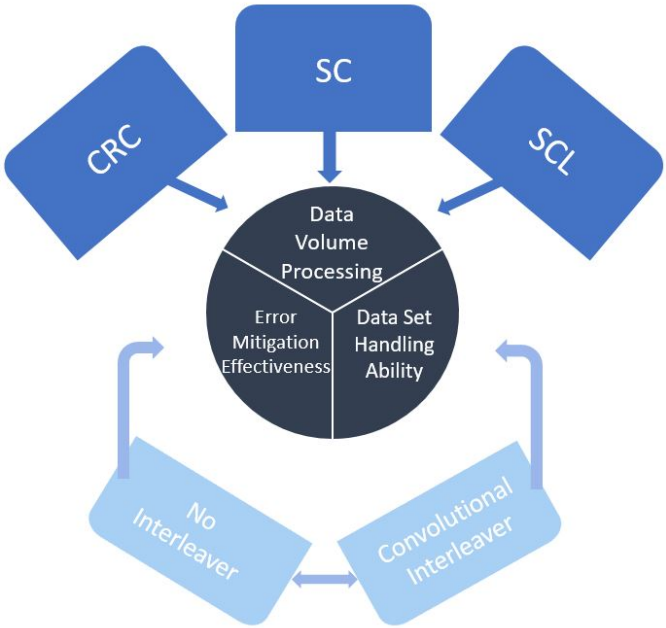

Fig. 4. Strengths of polar coding (top) and LDPC coding (bottom).

E. CCSDS Recommendation

As previously mentioned, most of the simulations were done based on different CCSDS recommendations in [10]. Thus, our results can be directly applied to the current standards being put in place. CCSDS protocols recommend codes of length 2048 and larger for optical communications, especially for GEO related missions. Furthermore, CCSDS protocols also suggest 7000 bites and larger for the $\frac{1}{2}$ rate, which is most commonly used in NASA missions. Based on these current frameworks, the polar coding methodologies mentioned in this paper would most likely not be a great fit for those space applications.

However, current missions that use RF communications and smaller slice lengths can benefit immensely from polar coding. Polar coding has immense promise considering it's a better candidate for real-time error correction with its ability to handle larger data volume and handles burst noise more effectively than LDPC codes. With that being said, it should be noted that LDPC codes are more than capable of satisfying NASAs communications criteria at the moment, but industry standards are moving towards more attractive options such as polar coding.

IV. Conclusion

In this paper, we evaluated SC, SCL, and CRC-Aided polar codes in AWGN and burst noise and compared them to LDPC codes with convolutional and no interleavers. We discussed the ability of each FEC techniques ability to satisfy the needs of current and future space communications. A brief discussion and recommendations regarding the results within the framework of CCSDS recommendations were included.

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