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Title: **Benchmarking Mixed Mode Failure in Progressive Damage and Failure Analysis Methods**

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ABSTRACT

The verification and validation of progressive-damage-analysis finite element methods are difficult but critical tasks to undertake during their development. Verification exercises assess whether a predictive analysis tool produces results that are consistent with the fundamental concepts and assumptions of the tool under evaluation. Ideally, closed-form analytical solutions can be derived for which method verification results can be compared. Problems selected for computational tool verification are often simple and isolate individual features of the tool. In the case of progressive damage finite element methods, verifications should be performed to evaluate the ability of the model to predict the initiation of damage and its growth through the finite element mesh under a variety of conditions.

Mabson et al. proposed a test case of a unidirectional, fiber-reinforced plate with a center crack subjected to tensile loads to evaluate matrix crack propagation predictions. The problem was modeled using the Abaqus Hashin continuum damage mechanics (CDM) model for fiber-reinforced composites. Different combinations of matrix strengths and element sizes were used in the simulations, and the results were compared to a closed-form solution based on linear elastic fracture mechanics (LEFM). It was determined that the Abaqus CDM model could predict the LEFM solution of Mode I cracks only when the finite element mesh density met specific requirements based on the material properties.

This paper presents closed-form LEFM solutions for a center notch mixed mode (CNMM) verification problem. Parametric finite element analyses were developed using progressive damage analysis methods of both the Discrete Damage Mechanics (DDM) and CDM classes. The progressive damage analysis methods applied in the analyses of the CNMM problem include CompDam and the Floating Node Method. Analyses were conducted with various mode mixities and element sizes to verify that the damage models were working as intended and to identify any limits of applicability.

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INTRODUCTION

While the use of composite materials is widespread across the commercial aerospace industry, it is necessary to be able to accurately predict the structural strength of components composed of fiber-reinforced materials to enable new and innovative structural designs. The failure processes of structural components composed of fiber-reinforced composite materials involve several different, interacting failure mechanisms. The individual failure mechanisms typically initiate at different loads and locations, and propagate at different rates. The approaches for predicting these failure events are collectively termed progressive damage and failure analysis (PDFA) methods. Within the finite element analysis framework, PDFA methods can be sorted into the discrete damage mechanics (DDM) class, in which each instance of damage is discretely included or inserted into the mesh, and the continuum damage mechanics (CDM) class, in which the stiffness properties of the original mesh are altered to represent the presence of damage below the length scale of explicit modeling.

As part of the NASA Advanced Composites Project (ACP), several PDFA methods were subjected to verification and validation evaluation in order to identify technical gaps, develop improvements, and increase confidence for critical structural applications. The verification analyses evaluated whether the methods were working as intended (e.g., mathematical formulation, coding), with analysis results compared to either closed-form analytical or benchmark solutions. The validation exercises evaluated how well the theories and assumptions of the analytical models matched the physical realities of experiments.

Two problems that were used in the ACP PDFA code verification effort were the center notch tension (CNT) [1] and center notch shear (CNS) [2]. Both the CNT (Mode I) and CNS (Mode II) problems involve a single matrix crack in an infinite, unidirectional composite plate. The crack was sufficiently long for growth to be governed by linear elastic fracture mechanics (LEFM), for which closed-form solutions for crack growth exist. A schematic of the generic center notch model is shown in Figure 1. Parametric analyses were conducted to study the effects of varying the matrix strength and the element size on the agreement of the predictions with LEFM for far-field Mode I and Mode II loading conditions. The results of these parametric analyses for two CDM-based material models and a model with a discrete layer of cohesive elements to represent the crack are published in reference [2]. It was found that there existed upper limits on the element sizes for which the analysis predictions agreed with LEFM. For increasingly large element sizes, the onset of matrix damage was delayed, and eventually the initiation of damage occurred at stress levels greater than those which LEFM predicts unstable crack growth.

This paper presents a closed-form solution for a center notch mixed mode (CNMM) problem. Parametric finite element analyses, similar to those conducted in reference [2], have been performed using PDFA methods of both the DDM and CDM classes. The PDFA methods applied herein for the analysis of the CNMM problem include CompDam [3] and the Floating Node Method (FNM) [4]. The finite element analysis results were compared with the closed-form solutions to identify acceptable ranges of damage model inputs and finite element model parameters so as to agree with the assumptions of LEFM.

LINEAR ELASTIC FRACTURE MECHANICS SOLUTIONS

The analytical LEFM solution for the fracture of an infinite plate with a central matrix crack under plane stress conditions can be written as a function of the orthotropic stiffness properties, the initial crack half-length a_0, and the material fracture toughness. As derived by Mabson et al. in reference [1], the far-field normal stress at which the central matrix crack is predicted to propagate under Mode I deformation, $\sigma_{\infty,1c}$, may be expressed as:

$$\sigma_{\infty,1c} = \beta_1 \sqrt{\frac{G_{1c}}{\pi a_0}} \tag{1}$$

$$\beta_1 = \left(\frac{4G_{12}E_1E_2^{3/2}}{2G_{12}E_1^{1/2} + E_1E_2^{1/2} - 2\nu_{12}G_{12}E_2^{1/2}} \right)^{1/4} \tag{2}$$

where the subscripts 1 and 2 represent the fiber and matrix material directions, respectively, G_{1c} is the Mode I matrix fracture toughness, and E_1, E_2, G_{12}, and ν_{12} are the orthotropic stiffness properties. Similarly, the far-field shear stress at which the central matrix crack is predicted to propagate under Mode II deformation, $\tau_{\infty,2c}$, has the form:

$$\tau_{\infty,2c} = \pm\beta_2 \sqrt{\frac{G_{2c}}{\pi a_0}} \tag{3}$$

$$\beta_2 = \left(\frac{4G_{12}E_1^2E_2^{1/2}}{2G_{12}E_1^{1/2} + E_1E_2^{1/2} - 2\nu_{12}G_{12}E_2^{1/2}} \right)^{1/4} \tag{4}$$

where G_{2c} is the Mode II matrix fracture toughness [2].

The failure stresses given by Equations (1) and (3) are independent of the material strengths (σ_c for Mode I and τ_c for Mode II). As discussed in reference [2], the LEFM solutions in Equations (1) and (3) serve as upper bounds for valid PDFA predictions of far-field failure stress for Mode I and Mode II loading, respectively.

In order to solve for the far-field stresses at which LEFM predicts failure for a center notch specimen with combined Mode I and Mode II loading, it is necessary to define variables representing the degree of mode mixity and the resulting mixed-mode fracture toughness G_c. The mode mixity B is here defined as the Mode II energy release rate G_2 as a fraction of the total energy release rate G_{total}:

$$B = G_2/G_{\text{total}} \tag{5}$$

where G_{total} is the sum of the Mode I and Mode II energy release rates, G_1 and G_2, respectively. The Benzeggagh-Kenane law [5] is used to define the relationship between the mode mixity B and the mixed-mode fracture toughness G_c:

$$G_c = G_{1c} + (G_{2c} - G_{1c})B^{\eta} \tag{6}$$

where η is a numerical parameter selected to fit a set of experimental mixed-mode fracture toughness data. At failure, Equation (6) equals G_{total}, and the Mode I and Mode II components of the energy release rate at failure, G_{1*} and G_{2*}, respectively, can be solved for, yielding:

$$G_{1*} = (1 - B)[G_{1c} + (G_{2c} - G_{1c})B^{\eta}] \tag{7}$$

$$G_{2*} = B[G_{1c} + (G_{2c} - G_{1c})B^{\eta}] \tag{8}$$

Substituting G_{1*} and G_{2*} into Equations (1) and (3) for G_{1c} and G_{2c}, respectively, yields the far-field stresses at which failure occurs under mixed-mode deformation as a function of B:

$$\sigma_{\infty,mc} = \beta_1 \sqrt{\frac{(1 - B)[G_{1c} + (G_{2c} - G_{1c})B^{\eta}]}{\pi a_0}} \tag{9}$$

$$\tau_{\infty,mc} = \pm\beta_2 \sqrt{\frac{B[G_{1c} + (G_{2c} - G_{1c})B^{\eta}]}{\pi a_0}} \tag{10}$$

PROGRESSIVE DAMAGE FINITE ELEMENT MODELS

The center notch finite element models presented here are based on those described in references [1] and [2]. While the two different PDFA methods featured here have unique requirements regarding finite element model structure and organization, an effort was made to have the individual models and boundary conditions be equivalent.

The PDFA methods applied herein included CompDam and the FNM. CompDam is a CDM-based material model implemented as a VUMAT user material subroutine for Abaqus/Explicit [6]. The CompDam material model utilizes the deformation gradient decomposition method (DGD) [3] and represents the initiation and evolution of matrix cracks with cohesive laws to accurately track the orientation of matrix crack surface normals as the matrix cracks open. As a material model, CompDam can be used with different element types, but is typically applied with either reduced-integration C3D8R solid elements or CPS4R plane stress elements. CompDam_DGD version 1.0.1 was used for all of the CompDam analyses presented herein. The source code repository for CompDam_DGD can be found at https://github.com/nasa/CompDam_DGD. The FNM is a discrete damage modeling method capable of representing multiple evolving discontinuities in solids [4]. The FNM represents discontinuities with equivalent fidelity to re-meshing techniques without the need to update the initial model geometry. The FNM is implemented as a UEL user-defined element for Abaqus/Standard. Implementations of the FNM have made use of the virtual crack closure technique and cohesive surfaces for representing matrix damage evolution. The analyses presented herein used cohesive surfaces for predicting matrix crack evolution. The FNM software used in this study follows the implementations described in references [7] and [8]. It is available within the United States as part of the Floating Node Method Composites Simulation Toolbox (FNMCST) and can be requested at https://software.nasa.gov/software/LAR-19000-1.

A schematic of the model configuration is shown in Figure 1. The models were originally built using the Abaqus Scripting Interface to allow for easy variation of the plate width W, plate height H, initial crack length $2a_0$, and element size L_e. The majority of the model area was defined using a linear elastic orthotropic material (green area in Figure 1). The PDFA methods were applied only in the refined mesh region (blue area in Figure 1) that surrounds the crack (red region in Figure 1). For the CompDam analyses, two-dimensional, reduced-integration plane stress CPS4R elements were used so as to maintain continuity with previous work [2]. The FNM analyses were conducted with three-dimensional, 48-node user elements. The in-plane model geometry and discretization were the same for the CompDam and FNM analyses.

The initial crack in the models was represented using the damage modeling formulation under evaluation. That is, the crack was not explicitly contained in the initial model geometry. For the CompDam analyses, the initial crack was represented by setting the state variable representing matrix damage to 1.0 (i.e., fully damaged) as an initial condition in the row of elements spanning the crack length. For the FNM analyses, the initial crack was represented by inserting an FNM crack in the row of elements containing the initial crack, as shown in Figure 2, and setting the matrix damage state variable to fully failed for the elements within the span of the crack.

Displacement-based boundary conditions were applied along the perimeter of the models. The applied boundary conditions were a superposition of the continuum mechanics definitions of simple tension in the matrix direction and pure shear deformation, as was applied in the analyses presented in reference [2]. Both the u and v nodal displacements were prescribed along the top and bottom edges of the models when using the FNM and CompDam approaches, as oriented in Figure 3. For the CompDam models, the u and v nodal displacements were also prescribed along the horizontal edges of the models. The magnitudes of the prescribed displacements depended on the x and y nodal coordinates and indirectly on the mode mixity through Equations (9) and (10), such that:

$$u = \frac{1}{2}\frac{\tau_{\infty,mc}}{G_{12}}(y - y_0) \tag{11}$$

$$v = \frac{1}{2}\frac{\tau_{\infty,mc}}{G_{12}}(x - x_0) + \frac{\sigma_{\infty,mc}}{E_2}(y - y_0) \tag{12}$$

where x_0 and y_0 are reference coordinates, located at the center of the CNMM geometry. The peak applied displacements specified in the models was 50% greater than the displacements predicted by Equations (11) and (12) to ensure that crack growth was observed.

Material properties from the literature for IM7/8552 [9] were used in each of the CNMM analyses, and are shown in Table 1. These material properties were also used in the previous center notch studies published in reference [2].

TABLE 1. IM7/8552 MATERIAL PROPERTIES [9]

Symbol	Description	Value	Units
E_1	Young's modulus, fiber dir.	171,420	MPa
E_2	Young's modulus, matrix dir.	9080	MPa
G_{12}	Shear modulus	5290	MPa
ν_{12}	Poisson ratio	0.32	-
σ_c	Strength, Mode I	62.3	MPa
τ_c	Strength, Mode II	92.3	MPa
G_{1c}	Fracture toughness, Mode I	0.277	kJ/m^2
G_{2c}	Fracture toughness, Mode II	0.788	kJ/m^2
η	Benzeggagh-Kenane exponent	2.07	-

For each CNMM analysis, the far-field stresses at which damage initiated and at which final failure occurred were recorded. The far-field stresses were obtained by taking the reaction forces in the x- and y-directions on the top edge of the specimen and dividing by the undeformed cross sectional area. The stresses obtained from this approach agreed with the stresses taken from the element in the top-left corner of the mesh, as oriented in Figure 1, as was done in [2]. Damage initiation was defined as the first instance of a nonzero matrix damage variable occurring in an element ahead of one of the original crack tips. For the explicit CompDam analyses, final failure was defined as when the matrix damage variable in an element immediately ahead of one of the initial crack tips reached the value of 1.0 (i.e., fully degraded). For the implicit FNM analyses, final failure was defined as two-piece failure, or, when a global load-drop occurred based on the nodal reaction forces on the model periphery. The far-field stresses at which final failure occurred were compared with the corresponding LEFM analytical solutions, i.e., Equations (9) and (10).

PARAMETRIC STUDY

Results from several analyses were compared to the corresponding LEFM solutions, with the results reported in the next section. The model parameters that were varied included the mode mixity B and the element size L_e. For all analyses, the plate dimensions were 127 mm long by 127 mm wide by 0.183 mm thick, with a 25.4-mm-long center notch.

Pure Mode I and Mode II analyses have been conducted for CompDam previously, and the results are reported and discussed in reference [2]. In addition to the mixed-mode cases, FNM analyses were conducted for the pure Mode I and Mode II loading configurations, with the results reported in the next section.

RESULTS

For an infinite, unidirectional fiber-reinforced plate subjected to simple tension loading in the matrix direction (i.e., Mode I loading) with an initial crack of length

25.4 mm, the analytical LEFM solution in Equation (1) predicts failure at a far-field normal stress of 9.3 MPa for the IM7/8552 material properties in Table 1. Similarly, for pure shear loading (i.e., Mode II), Equation (3) predicts failure at a far-field shear stress of ±32.6 MPa. These LEFM solutions are independent of material strength and element size, and depend only on the material stiffness, fracture toughness, and the problem geometry. Along with the LEFM solutions, the predictions for FNM for pure Mode I and Mode II loading for a range of element sizes are shown in Figures 4a and 4b, respectively. The pure mode FNM analyses were conducted with element sizes ranging from 0.1 mm to 5.0 mm for Mode I and 0.1 mm to 12.0 mm for Mode II.

For both the Mode I and Mode II loading conditions, as element size was increased, the far-field stress at which damage initiated increased. Damaged initiated early for analyses conducted with smaller elements as a result of the sharper representation of the crack tip stress concentrations. The initiation stress versus element size data are well fit by square root functions, plotted as the blue dashed lines, Figures 4a and 4b. The far-field stress at which final failure occurred, shown in red, was fairly constant with respect to element size until the far-field stress corresponding to damage initiation exceeded the LEFM failure stress prediction. Error between the LEFM and predicted far-field failure stresses was less than 10% up to element sizes of 0.4 mm and 5.0 mm for Mode I and Mode II, respectively. The data in Figures 4a and 4b is replotted over a smaller element size range in Figures 4c and 4d, for clarity. The intersection of the damage initiation and LEFM predictions was observed for both the Mode I and Mode II loading cases. These intersections occurred at element sizes of approximately 2.2 mm and 8.6 mm, respectively. Final failure predictions from analyses conducted with element sizes larger than these limits were not converged with respect to element size.

Both the Mode I and Mode II element size limits, as defined as the intersection of the LEFM and damage initiation curves, for the FNM are much less restrictive than was observed for CompDam in reference [2], where the element size limits were approximately 0.5 mm and 0.4 mm for Mode I and Mode II, respectively. A key observation in reference [2] was that the Mode II element size limit related to the center notch LEFM predictions was more restrictive than the recommendation of Turon et al. [10] to maintain at least three elements within the fracture process zone (FPZ) and the element size snap-back limit described by Maimí et al. [11]. The discrete representation of the cohesive surface representing the crack in the FNM creates a much larger stress concentration than in CDM-based methods where the crack and the adjacent material is represented by a single integration point. The larger stress concentration causes cohesive damage to initiate earlier, and allows for analyses run with larger elements to agree with LEFM predictions. The FNM results here are more similar to the discrete cohesive element results in reference [2] than the CDM-based method results.

The CNMM LEFM predictions are shown with the CompDam and the FNM results in Figures 5 and 6, respectively, for mixed-mode ratios of 0.25, 0.50, and 0.75. The mixed-mode analyses were conducted with element sizes in the range of 0.1 mm to 0.5 mm. The LEFM far-field normal and shear stresses for the evaluated mode mixities are listed in Table 2 and are plotted as a black, dotted line in the figures. Both methods accurately predicted the LEFM far-field failure normal and shear stresses for element sizes that yield an adequate stress concentration to cause damage initiation at far-field stresses below the LEFM failure stresses. For element sizes below the limits corresponding to the pure mode center notch studies, the far-field failure stresses were constant with varying element sizes. For the CNMM CompDam results in Figure 5, the element size limits seem to vary linearly with mode mixity from the previously observed Mode I limit of 0.5 mm to the Mode II limit of 0.4 mm.

TABLE 2. LEFM PREDICTIONS FOR FAR-FIELD FAILURE STRESSES

B	$\sigma_{\infty,mc}$ **[MPa]**	$\tau_{\infty,mc}$ **[MPa]**
0.00	9.3	0.0
0.25	8.4	10.2
0.50	7.9	16.4
0.75	6.6	23.8
1.00	0.0	32.6

CLOSING REMARKS

Ensuring that finite element model results are converged with respect to element size is a key step in the verification of any analysis. For nonlinear analyses involving the initiation and evolution of damage, it is good practice to verify that the results are converged with respect to element size both prior to the initiation of damage and during its evolution. Herein, a simple finite element model configuration with closed-form LEFM solutions to verify the performance of damage models for single- and mixed-mode loading configurations was presented. The described verification approach evaluates how the results from a progressive damage modeling method compare to LEFM for damage growth from an existing long crack through a mesh, and can be used to identify dependencies on material inputs, element sizes, and other advanced damage modeling approach or finite element model features.

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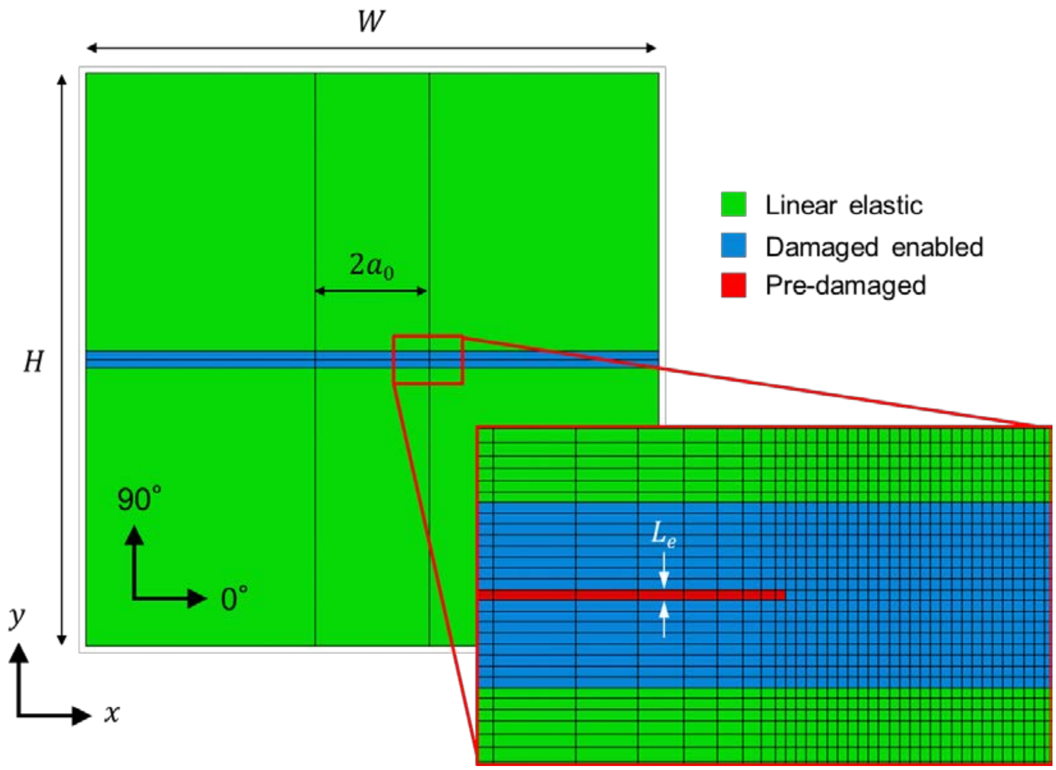

Figure 1. Center notch mixed-mode model schematic.

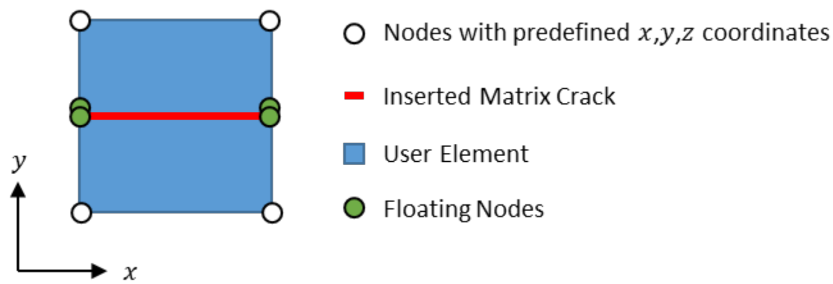

Figure 2. FNM element with inserted matrix crack.

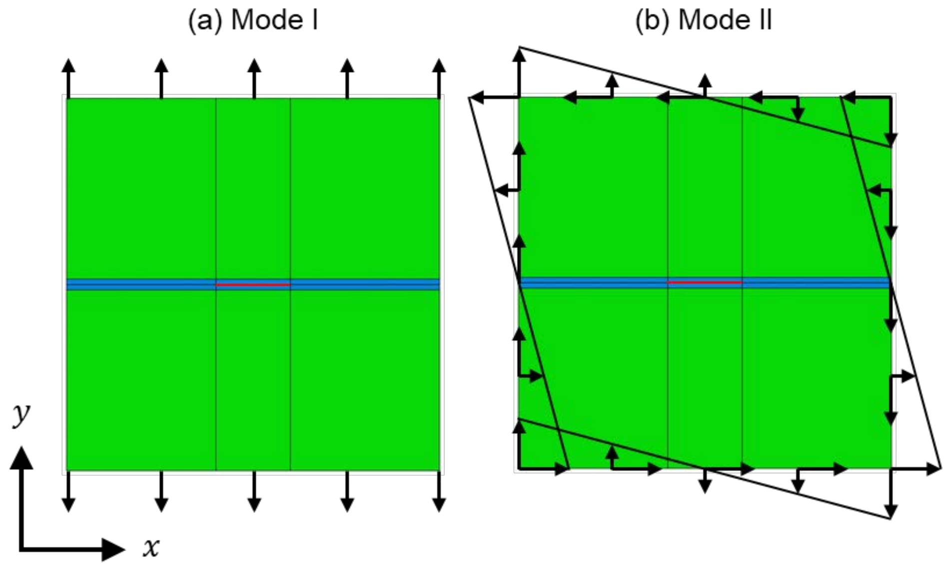

Figure 3. Boundary condition components for (a) pure Mode I and (b) pure Mode II.

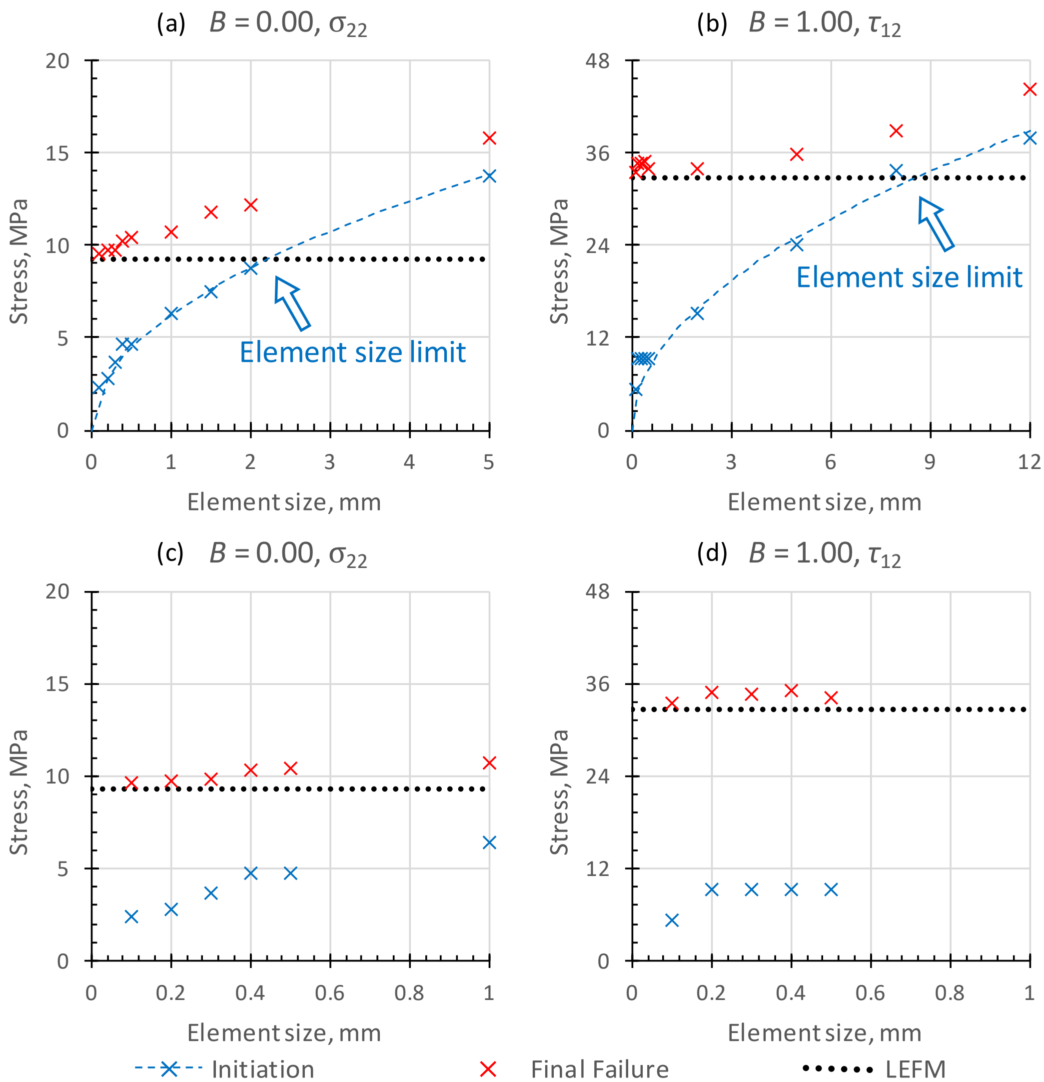

Figure 4. Center notch model predictions using FNM for (a) Mode I and (b) Mode II. The plotted stresses are the far-field stresses. The dashed initiation curves are fit through the analysis data. The same plots are reproduced in (c) and (d) with the range of elements sizes along the horizontal axis reduced to sizes more typical of damage analyses.

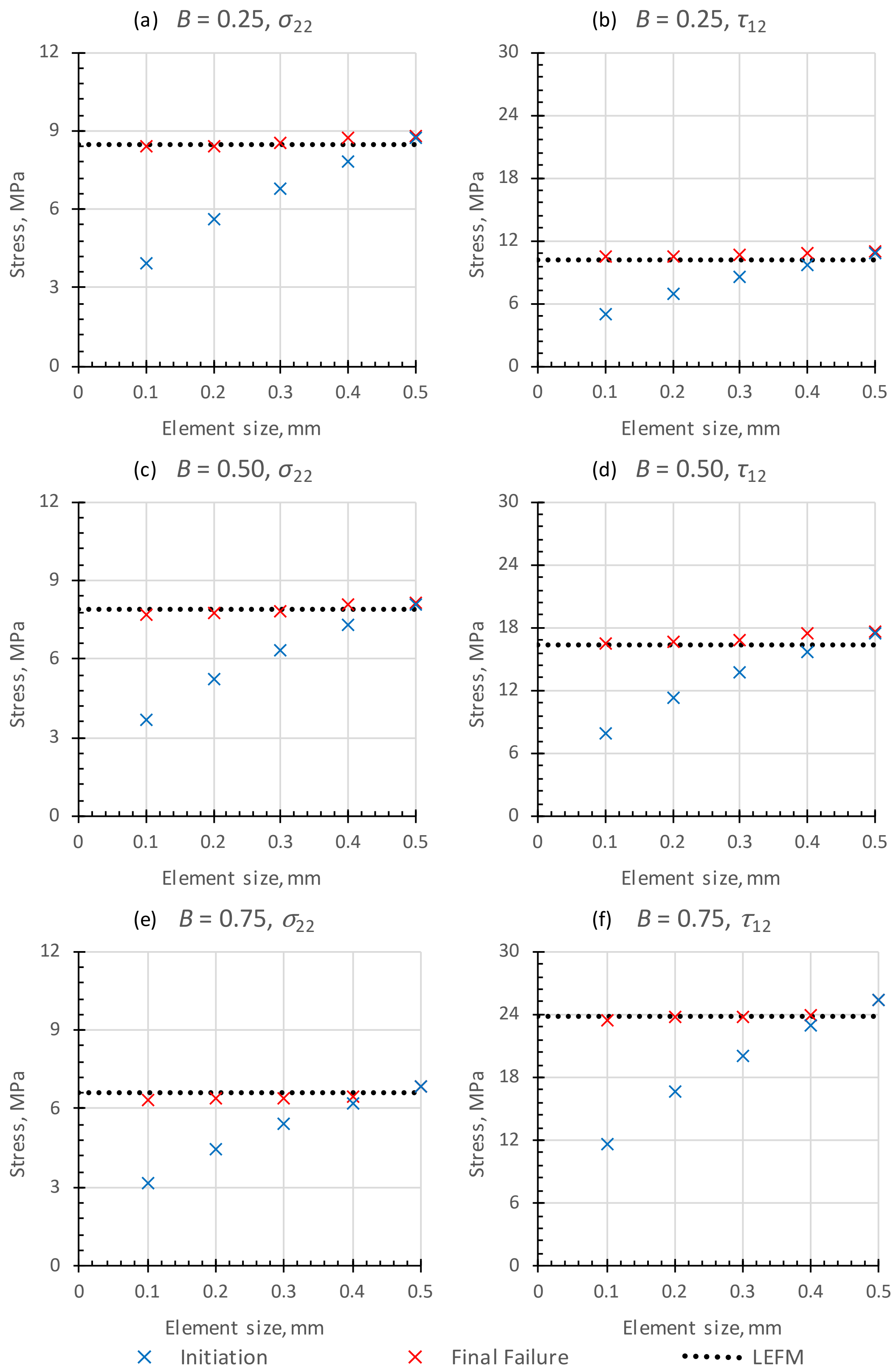

Figure 5. CNMM predictions using CompDam compared to LEFM solutions. The plotted stresses are the far-field stresses.

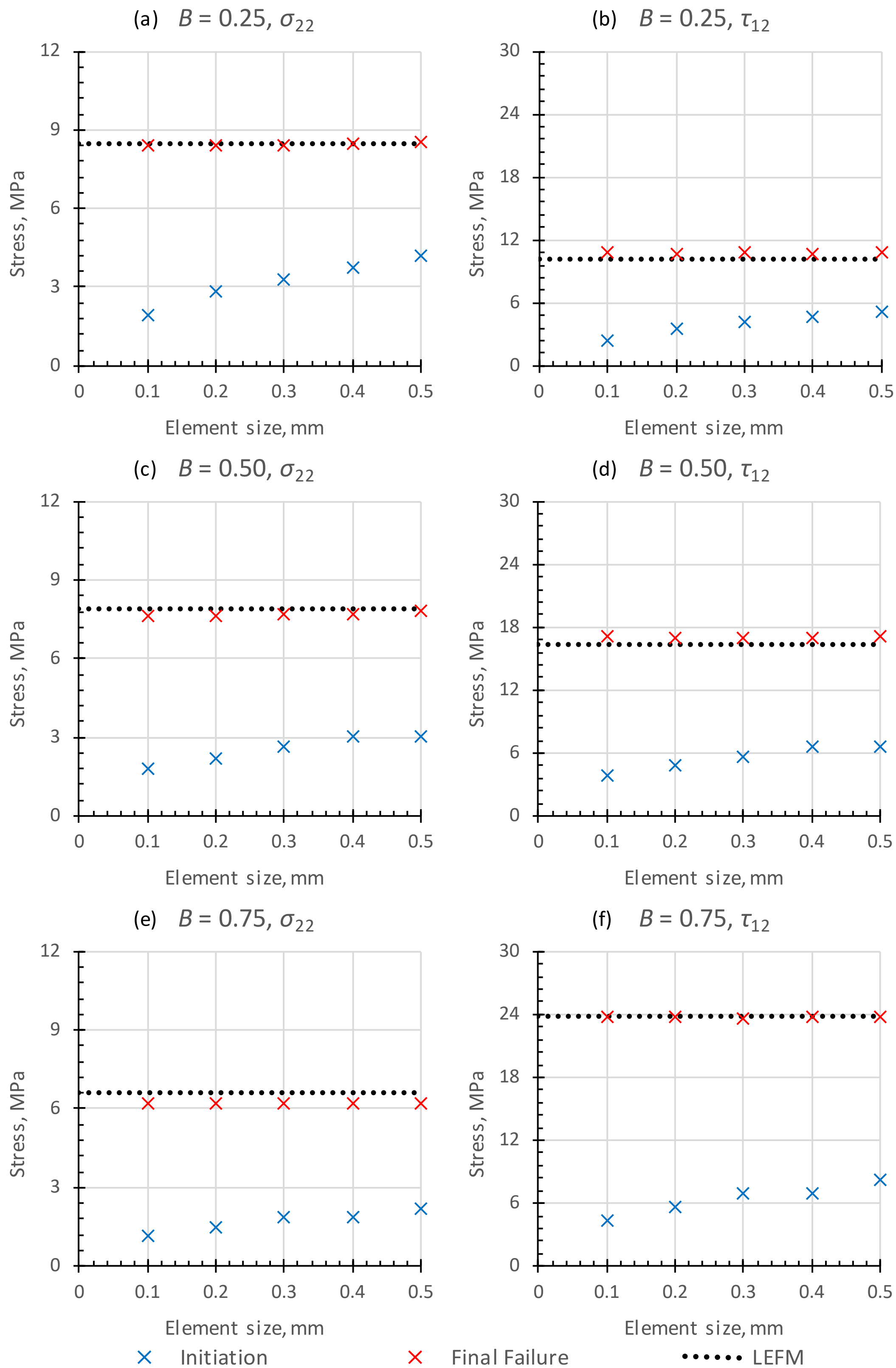

Figure 6. CNMM predictions using FNM compared to LEFM solutions. The plotted stresses are the far-field stresses.