High-fidelity Simulations of ArcJets

Nagi N. Mansour

November 14th, 2019

ARCHeS team: Jeremie Meurisse, Magnus Haw, Joey Schulz, Sergio Fraile, Sander Visser,



Stardust entry Jan 15, 2006



Arc Jet Experiment on meteorites



NASA Entry Vehicles / Missions Supported by Ames





High enthalpy arc-jets



Needed: Capability to perform realistic simulations

Bruce Walter Arc Heater visualization



1D study of the mini-Arc (with J.B. Scoggins & J. Jimenez-Miro)



1D study of the mini-Arc (Note from J.B. Scoggins)



Figure 1: Damkohler number associated with the O2 dissociation reaction for different inlet temperatures.



Equilibrium formulation for applications of interest (high pressure ~ 9atm, high temperature ~10,000 K)





Characteristics:

- » High pressure
- » High temperature
- >>> Variable Air/Ar mixture
- >>> Strong Imposed voltage drop (constant current)
- >>> Ballast at the electrodes to ensure uniform current
- >>> External B to force current to rotate around the electrode





Characteristics:

- » High pressure
- » High temperature
- >>> Variable Air/Ar mixture
- >>> Strong Imposed voltage drop (constant current)
- >>> Ballast at the electrodes to ensure uniform current
- >>> External B to force current to rotate around the electrode

Compressible Navier-Stokes Equations: Equilibrium formulation with variable elemental fraction

$$\begin{split} \frac{\partial \rho_i}{\partial t} + \boldsymbol{\nabla} \cdot (\rho_i \boldsymbol{u} + \boldsymbol{F}_i^d) &= 0 & i \in [1, N_e] \\ \frac{\partial \rho \boldsymbol{u}}{\partial t} + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{u} \boldsymbol{u}) &= -\boldsymbol{\nabla} p + \boldsymbol{\nabla} \cdot \bar{\tau} + \boldsymbol{J} \times \boldsymbol{B} \\ \frac{\partial \rho E}{\partial t} + \boldsymbol{\nabla} \cdot (\rho H \boldsymbol{u}) + \boldsymbol{\nabla} \cdot (\boldsymbol{q}_d) &= \boldsymbol{\nabla} \cdot (\bar{\tau} \cdot \boldsymbol{u} - \boldsymbol{q}) + \sigma |\boldsymbol{E}|^2 + \boldsymbol{u} \cdot (\boldsymbol{J} \times \boldsymbol{B}) - \boldsymbol{\nabla} \cdot \boldsymbol{q}_{rad} \\ \rho &= \sum_{i}^{N_e} \rho_i = \sum_{k}^{N_e} \rho_k & Z_i = \frac{\rho_i}{\rho} & Z_i^x = \frac{\rho_i M^m}{\rho M_i^m} & i \in [1, N_e] \\ H &= E + \frac{p}{\rho} = e + \frac{|\boldsymbol{u}|^2}{2} + \frac{p}{\rho} \end{split}$$

13

Compressible Navier-**Stokes Equations:** Equilibrium formulation with variable elemental fraction

$$\begin{split} \mathbf{F}_{i}^{d} &= \rho_{i} \boldsymbol{v}_{i} = \left(\frac{\partial F_{i}^{d}}{\partial p}\right) \boldsymbol{\nabla} p + \left(\frac{\partial F_{i}^{d}}{\partial T}\right) \boldsymbol{\nabla} T + \sum_{j}^{N_{e}} \left(\frac{\partial F_{i}^{d}}{\partial Z_{j}^{x}}\right) \boldsymbol{\nabla} Z_{j}^{x} \qquad i, j \in [1, N_{e}] \\ \left(\frac{\partial F_{i}^{d}}{\partial p}\right) &= -\sum_{k}^{N_{s}} \rho_{k} \frac{M_{i}^{m}}{M_{k}^{m}} \nu_{ki} \sum_{l}^{N_{s}} D_{kl} \left(\frac{\partial X_{l}}{\partial p} + \frac{X_{l} - Y_{l}}{p}\right) \qquad i, j \in [1, N_{e}] \& k, l \in [1, N_{s}] \\ \left(\frac{\partial F_{i}^{d}}{\partial T}\right) &= -\sum_{k}^{N_{s}} \rho_{k} \frac{M_{i}^{m}}{M_{k}^{m}} \nu_{ki} \sum_{l}^{N_{s}} D_{kl} \left(\frac{\partial X_{l}}{\partial T} + \frac{\kappa_{l}^{r}}{T}\right) \qquad i, j \in [1, N_{e}] \& k, l \in [1, N_{s}] \\ \left(\frac{\partial F_{i}^{d}}{\partial Z_{i}^{x}}\right) &= -\sum_{k}^{N_{s}} \rho_{k} \frac{M_{i}^{m}}{M_{k}^{m}} \nu_{ki} \sum_{l}^{N_{s}} D_{kl} \left(\frac{\partial X_{l}}{\partial Z_{i}^{x}}\right) \qquad i, j \in [1, N_{e}] \& k, l \in [1, N_{s}] \end{split}$$

 $i, j \in [1, N_e] \& k, l \in [1, N_s]$

Transport properties from MUTATION++

V. Giovangigli, Multicomponent Flow Modeling, Birkhäuser, Boston, 1999

References J.B. Scoggins, T.E. Magin, Development of Mutation++: multicomponent thermodynamic and transport property library for ionized plasmas written in C++, AIAA Pap. 2966 (2014) 14



Figure 12: Specific heat from Mutation + + library for air (left) and Argon (right) in function of p and T.





Characteristics:

- » High pressure
- » High temperature
- >>> Variable Air/Ar mixture

>>> Strong Imposed voltage drop (constant current)

- >>> Ballast at the electrodes to ensure uniform current
- >>> External B to force current to rotate around the electrode

$$\begin{split} \boldsymbol{\nabla}\times\boldsymbol{B} &= \mu_0 \boldsymbol{J} \\ \text{Set } \boldsymbol{B} &= \boldsymbol{\nabla}\times\boldsymbol{A} \\ \text{MHD} \\ \text{approximation} \quad \sigma \frac{\partial \boldsymbol{A}}{\partial t} - \frac{1}{\mu_0} \nabla^2 \boldsymbol{A} - \sigma \boldsymbol{u} \times (\boldsymbol{\nabla}\times\boldsymbol{A}) + \sigma \boldsymbol{\nabla}\psi = 0 \\ \text{where } \psi \text{ is a Coulomb potential determined so that:} \\ \boldsymbol{\nabla}\cdot\boldsymbol{A} &= 0 \end{split}$$

$$oldsymbol{E} = oldsymbol{E}_{imp} + oldsymbol{E}_g$$

 $oldsymbol{E}_{imp} = -oldsymbol{
abla}\phi_{imp}$
 $oldsymbol{
abla} \cdot (oldsymbol{J}_{imp}) = 0$

Splitting the Electric field

$$\boldsymbol{\nabla} \cdot (\boldsymbol{J}_{imp}) = 0 \quad \Rightarrow \quad \left| \boldsymbol{\nabla} \cdot (-\sigma \boldsymbol{\nabla} \phi_{imp}) = 0 \right|$$







Characteristics:

- » High pressure
- » High temperature
- >>> Variable Air/Ar mixture
- >>> Strong Imposed voltage drop (constant current)
- >>> Ballast at the electrodes to ensure uniform current
- >>> External B to force current to rotate around the electrode



$$-I_{e_i}^c = \int_{S_{e_i}^c} \sigma \,\nabla\phi \cdot \mathbf{n} \, ds = \frac{1}{R_b} (\phi_{e_i} - \phi_G)$$
$$\phi_{e_i}^n = \phi_G - R_b \int_{S_{e_i}^c} \sigma \,\nabla\phi^{n-1} \cdot \mathbf{n} \, ds$$





Characteristics:

- » High pressure
- » High temperature
- >>> Variable Air/Ar mixture
- >>> Strong Imposed voltage drop (constant current)
- >>> Ballast at the electrodes to ensure uniform current
- \gg External ${\bf B}$ to force current to rotate around the electrode

Electromotive force





Figure 6: Electrode with Internal Magnetic Drive.

Figure 7: B_e for an anode with 4 electrodes.

Biot-Savart

$$\boldsymbol{B_e} = \sum_{i}^{N_e} \frac{\mu_0 I_{e_i}}{4\pi} \oint \frac{d\boldsymbol{l_i} \times \boldsymbol{r_i}}{|\boldsymbol{r_i}|^3}$$

22





Characteristics:

- » High pressure
- » High temperature
- >>> Variable Air/Ar mixture
- >>> Strong Imposed voltage drop (constant current)
- >>> Ballast at the electrodes to ensure uniform current
- \gg External ${\bf B}$ to force current to rotate around the electrode



$$\boldsymbol{\Omega} \cdot \nabla I_{\nu} = \kappa_{\nu} (S_{\nu} - I_{\nu})$$
$$\boldsymbol{Q} = \int_{\nu} \int_{\Omega} I_{\nu} \vec{\Omega} \, d\Omega \, d\nu$$
$$\nabla \cdot \boldsymbol{Q} = \int_{\nu} \int_{\Omega} \kappa_{\nu} (S_{\nu} - I_{\nu}) \, d\Omega \, d\nu$$



Radiation transfer Computing the line-by-line (LBL) opacity of a mixture

$$\kappa_{mix} = X \ \kappa_{air} + Y \ \kappa_{argon}$$

$$X = \frac{A}{B\left(\frac{C-D}{E} + \frac{A}{B}\right)} \quad \text{and} \quad Y = \frac{C-D}{E\left(\frac{C-D}{E} + \frac{A}{B}\right)} = 1 - X$$

$$A = \sum n(N, N^{+}, N_{2}, N_{2}^{+}, O, O^{+}, O_{2}, O_{2}^{+}, NO, NO^{+})_{mix}$$

$$B = \sum n(N, N^{+}, N_{2}, N_{2}^{+}, O, O^{+}, O_{2}, O_{2}^{+}, NO, NO^{+})_{air}$$

$$C = \sum n(Ar, Ar^{+})_{mix} \qquad D = \sum n(Ar, Ar^{+})_{air} \qquad E = \sum n(Ar, Ar^{+})_{argon}$$



Fig. 1: Spectral absorption coefficients for LTE air and argon at 10,000 K and 1 atm



Fig. 2: LBL absorption coefficient for a mixture with a mol fraction of 0.5 air + 0.5 argon at 15,000 K and 10 atm computed with the reduced model (RED) vs NEQAIR (NEQ)

25

Radiation transfer Binning the opacity into tables

$$\frac{\partial I_{\lambda}(\widehat{\boldsymbol{n}},s)}{\partial s} = \kappa_{\lambda}(s) \left[B_{\lambda}(T) - I_{\lambda}(\widehat{\boldsymbol{n}},s) \right]$$
$$\frac{\partial}{\partial s} \overline{I}_{b} = \overline{\kappa_{\lambda}}B_{\lambda}|_{b} - \overline{\kappa_{\lambda}}\overline{I_{\lambda}}|_{b} \quad \text{where,} \quad \overline{f}_{b} = \int_{\lambda_{1}}^{\lambda_{2}} f_{\lambda} \, d\lambda$$

 $\overline{\kappa_{\lambda}I_{\lambda}}|_{b} \approx \tilde{\kappa}_{b} \overline{I}_{b}$ Planck-averaged $\tilde{\kappa}_{b} = \kappa_{P_{b}} = \frac{\int_{\lambda_{1}}^{\lambda_{2}} \kappa_{\lambda} B_{\lambda} d\lambda}{\int_{\lambda_{1}}^{\lambda_{2}} B_{\lambda} d\lambda}$ Rosseland-averaged $\tilde{\kappa}_{b} = \kappa_{R_{b}} = \frac{\int_{\lambda_{1}}^{\lambda_{2}} \frac{\partial B_{\lambda}}{\partial T} d\lambda}{\int_{\lambda_{1}}^{\lambda_{2}} \frac{\partial B_{\lambda}}{\partial T} d\lambda}$

Blended-average

$$\tilde{\kappa}_b = \kappa_{PR_b} = \sqrt{\kappa_{P_b} \, \kappa_{R_b}}$$



Fig. 3: Comparison of the different band averaging models for air at 10,000 K and 10 atm 26

Radiative

$$\int_{V} \mathbf{\Omega} \cdot \nabla \bar{I}_{b} \, dV = \int_{V} \nabla \cdot (\mathbf{\Omega} \bar{I}_{b}) \, dV = \int_{S} \bar{I}_{b} \mathbf{\Omega} \cdot d\mathbf{S} = \int_{V} \kappa_{\nu} (\bar{S}_{b} - \bar{I}_{b}) \, dV$$
transport

$$\sum_{k} \bar{I}_{b,k} (\Omega_{i}) \mathbf{\Omega} \cdot \Delta \mathbf{S}_{k} = \bar{\kappa}_{b} (\bar{S}_{b,c} - \bar{I}_{b,c} (\Omega)) \, V_{c}$$
up-winding

$$\sum_{k \in \mathbf{\Omega}_{i} \cdot \Delta \mathbf{S}_{k} < 0} \bar{I}_{b,k} (\Omega_{i}) \mathbf{\Omega} \cdot \Delta \mathbf{S}_{k} + \sum_{k \in \mathbf{\Omega} \cdot \Delta \mathbf{S}_{k} > 0} \bar{I}_{b,c} (\Omega) \mathbf{\Omega} \cdot \Delta \mathbf{S}_{k} = \bar{\kappa}_{b,c} (\bar{S}_{b,c} - \bar{I}_{b,c} (\Omega_{i})) \, V_{c}$$

$$\bar{I}_{b,c}(\Omega_i) = \frac{-\sum_{k \in \mathbf{\Omega} \cdot \Delta \mathbf{S}_k < 0} I_{b,k}(\Omega_i) \mathbf{\Omega} \cdot \Delta \mathbf{S}_k + \bar{\kappa}_{b,c} S_{b,c} V_c}{\sum_{k \in \vec{\Omega} \cdot \Delta \mathbf{S}_k > 0} \mathbf{\Omega} \cdot \Delta \mathbf{S}_k + \bar{\kappa}_{b,c} V_c}$$





Animation Courtesy: Alejandro Alvarez

$$\boldsymbol{Q} = \int_{\nu} \int_{\Omega} I(\boldsymbol{\Omega}, \nu) \ d\Omega \ d\nu \approx \sum_{b} \sum_{\Omega_{i}} w_{i} \bar{I}_{b}(\Omega_{i})$$

28

ARCHeS

MHDSurface current interaction3D radiationSurface responseAir-Ar chemistrySwirling and ballast BC

OpenFOAMModernModularFinite-volumeIO, MPI

Turbulence







33





$\begin{array}{l} \text{Temperature} \\ \text{High pressure} \not \rightarrow T_h = T_e = T \end{array}$





Velocity





Highly unsteady and turbulent flow



3D Radiation





Imposed Electric Field

 $\nabla\cdot\sigma\nabla\phi=0$





Induced Magnetic Field $\nabla^2 \mathbf{B} = \mu_0 \nabla \sigma \times \nabla \phi$





External Magnetic Field

$$\boldsymbol{B_e} = \sum_{i}^{N_e} \frac{\mu_0 I_{e_i}}{4\pi} \oint \frac{d\boldsymbol{l_i} \times \boldsymbol{r_i}}{|\boldsymbol{r_i}|^3}$$





Arc instabilities in ARCHeS



Arc behavior for air only



Arc behavior for air/Ar mixture



Arc reattachment



Summary

- * Priority physics models have been implemented
- **Working tool is being tested for:**
 - ✤ Mini-ARC
 - **₩ HyMETS**
 - ₩ IHF & AHF

Next Challenges

- Current numerical scheme not optimized for nearly incompressible flow – an all-speed formulation is needed
- # Electrode boundary conditions
- # Melt of the electrodes