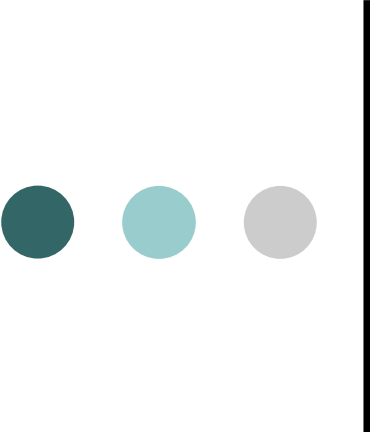




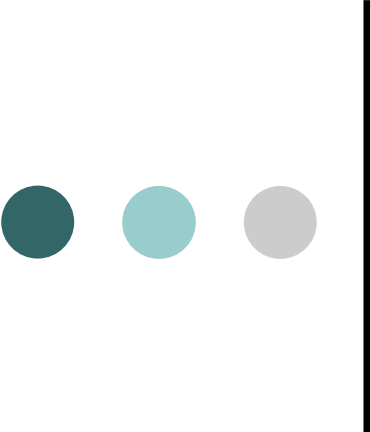
PT-Symmetry-Breaking Gyroscopes

David D. Smith

NASA Marshall Space Flight Center

Space Systems Department

“Be Precise. A lack of precision is dangerous when the margin of error is small.” - Lord Kelvin



PT-Symmetry-Breaking Gyroscopes

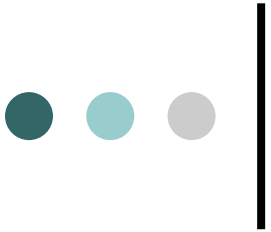
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Outline

- What is Fast Light (FL)? Why talk about FL?
- How does FL enhance the sensitivity, and possibly the precision, of optical gyros?
- Passive and Active Fast-Light gyros
- Coupled-Resonator (CR) gyros: Fast-light enhancement without a medium
- PT-Symmetry Breaking / Lasing Without Gain gyroscopes

**What do we mean by
“fast light”?**

Phase Velocity

Monochromatic Plane Wave:

$$E(z, t) = E_0 \cos \phi(z, t)$$

$$\phi(z, t) = kz - \omega t$$

Dispersion relation:

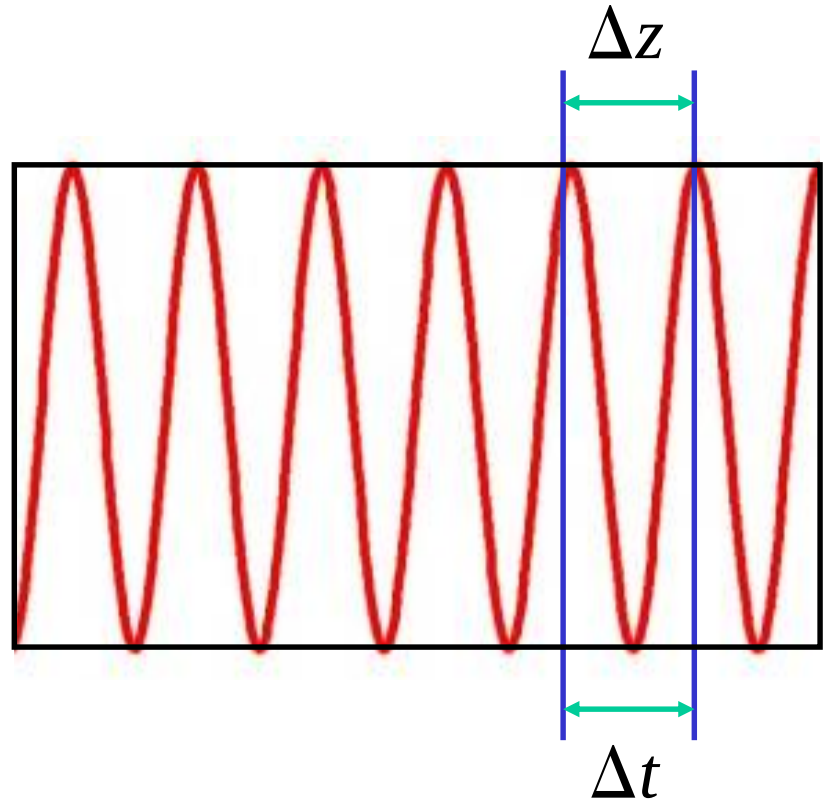
$$k = \frac{n\omega}{c}$$

Constant phase front ϕ
moves distance Δz in time Δt

$$k\Delta z = \omega\Delta t \quad \longrightarrow$$

Phase velocity:

$$v_p = \frac{\Delta z}{\Delta t} = \frac{\omega}{k} = \frac{c}{n}$$



Group Velocity

Add two monochromatic plane waves:

$$\begin{aligned} E(z, t) &\propto \cos(k_1 z - \omega_1 t) + \cos(k_2 z - \omega_2 t) \\ &= \cos(k z - \omega t) \cos(\Delta k z - \Delta \omega t) \end{aligned}$$

Rapidly
Varying

$$\omega = (\omega_1 + \omega_2) / 2$$

$$k = (k_1 + k_2) / 2$$

Phase velocity:

$$v_p = \frac{\omega}{k}$$

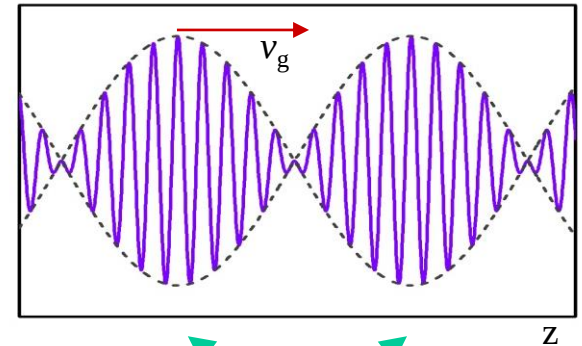
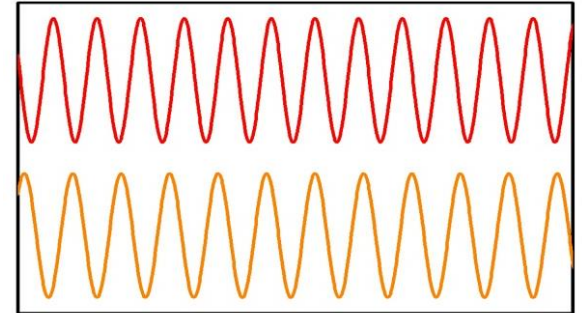
Envelope

$$\Delta \omega = (\omega_1 - \omega_2) / 2$$

$$\Delta k = (k_1 - k_2) / 2$$

Group Velocity:

$$v_g = \frac{\Delta \omega}{\Delta k}$$



Wave Groups

$$v_g = v_p \text{ if } n \text{ constant}$$

Slow and Fast Light

Multiple Fourier (frequency) components experience different index of refraction and add up in phase at peak of pulse, which propagates at

Group Velocity:

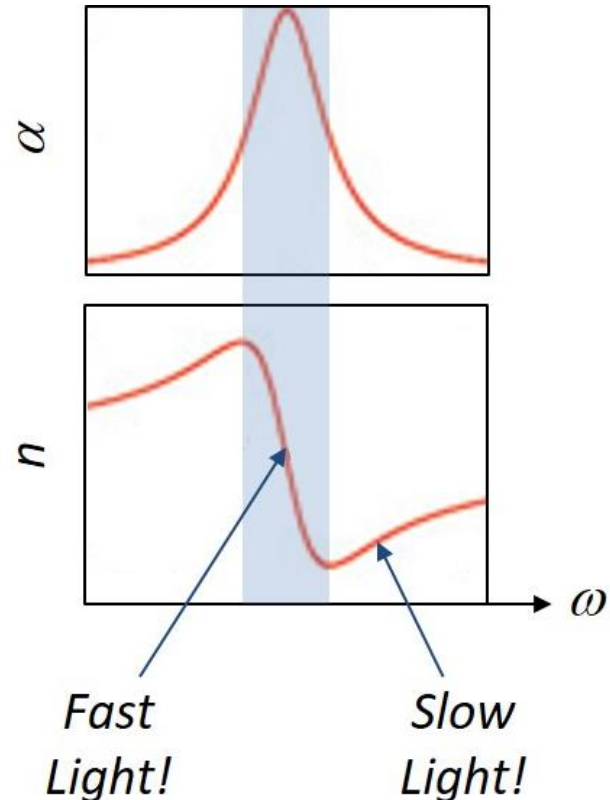
$$v_g = \frac{d\omega}{dk} = \frac{c}{n + \omega \frac{dn}{d\omega}} = \frac{c}{n_g}$$

$$\frac{dn}{d\omega} > 0 \quad \begin{array}{l} \text{Normal Dispersion} \\ \text{Slow Light } (n_g > 1) \end{array}$$

$$\frac{dn}{d\omega} < 0 \quad \begin{array}{l} \text{Anomalous Dispersion} \\ \text{Fast Light } (n_g < 1) \end{array}$$

$n_g = 0 \rightarrow \text{Critical Fast Light Condition!}$

Absorption Coefficient



Pulse leaves medium at same moment it enters!

**Why are we interested
in fast-light?**

LOST in SPACE FOREVER

Oh, the pain! Save me,
William! Oh why did I
throw out those **FAST
LIGHT GYROS.**





Relevance



- **Critical Need:** *“NASA’s future missions show a diverse set of navigational challenges that cannot be supported with current methods. Onboard autonomous navigation and maneuvering techniques are critical.”* NASA Technology Roadmap, TA-05
- **Problem:** Current inertial sensors limited in precision and require periodic updates.

External signals (GPS, DSN, Star-trackers, XNAV) limited by large delays, low flux, and can be spoofed, blocked, misidentified, or unavailable.

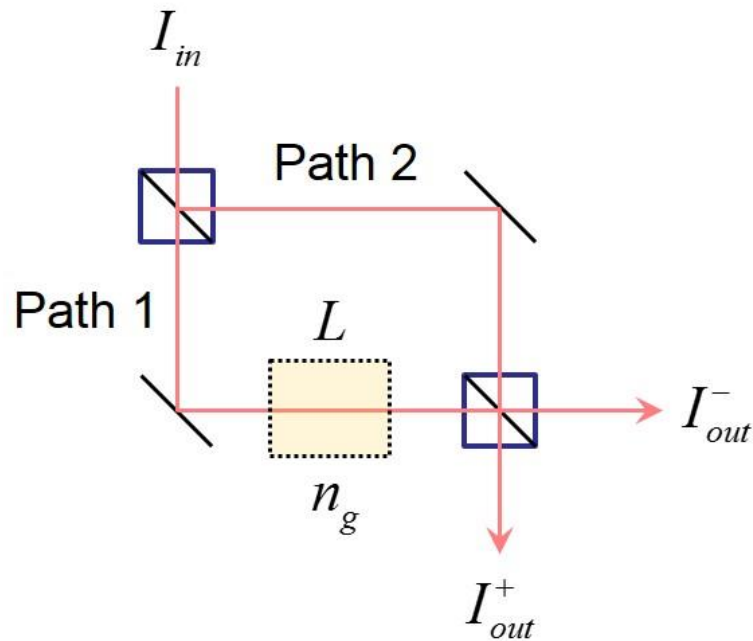
Usual methods to boost precision are to increase: (a) size (problematic in spaceflight), or (b) integration time (not useful for rapid accelerations).
- **Conclusion:** *Fundamental improvements needed in precision of inertial sensors - smaller, faster, more precise gyros that require fewer updates!*

How does it work?

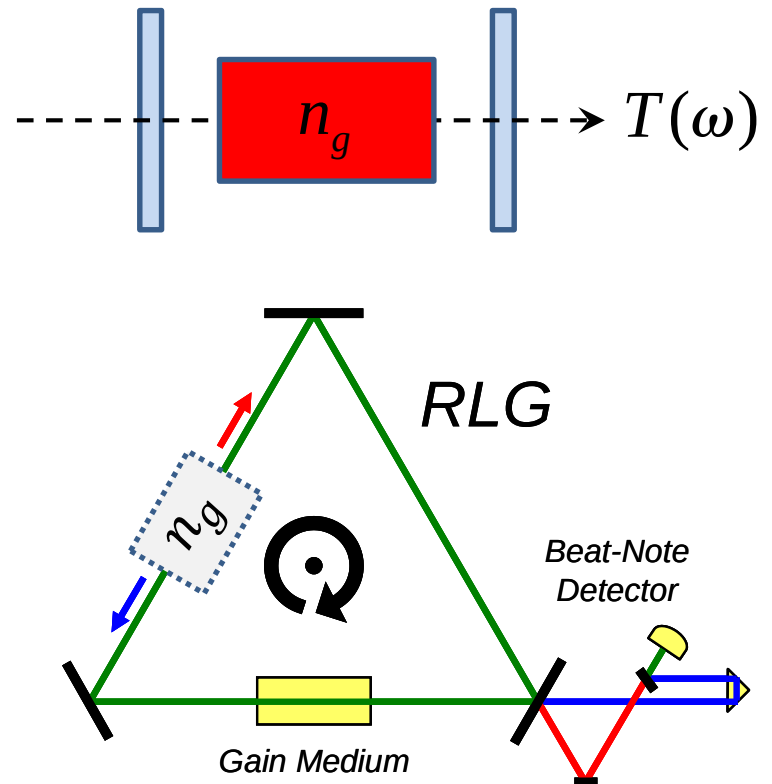
Interferometers with Dispersion

Idea: Put dispersive (slow or fast light) material into interferometer.

**Two-beam Interferometer
with dispersion**



**Multi-beam Interferometers
(Cavities) with Dispersion**



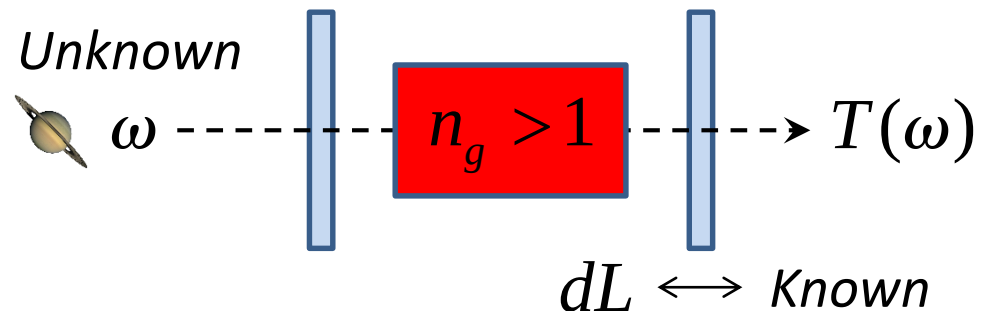
Slow or Fast Light?

Q: Is it better to use slow light or fast light?

A: It depends on what you're trying to measure.

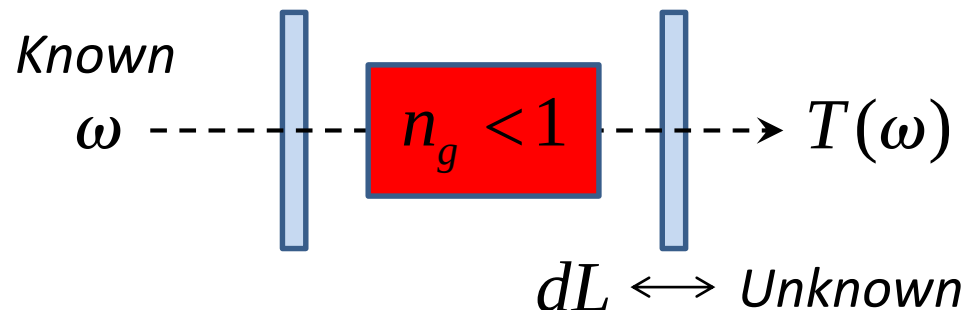
Spectroscopy:

⇒ use **SLOW** Light
Narrows cavity
resonances



Detection of OPL change:

⇒ use **FAST** Light
Narrows input
spectrum (in a
relative sense)



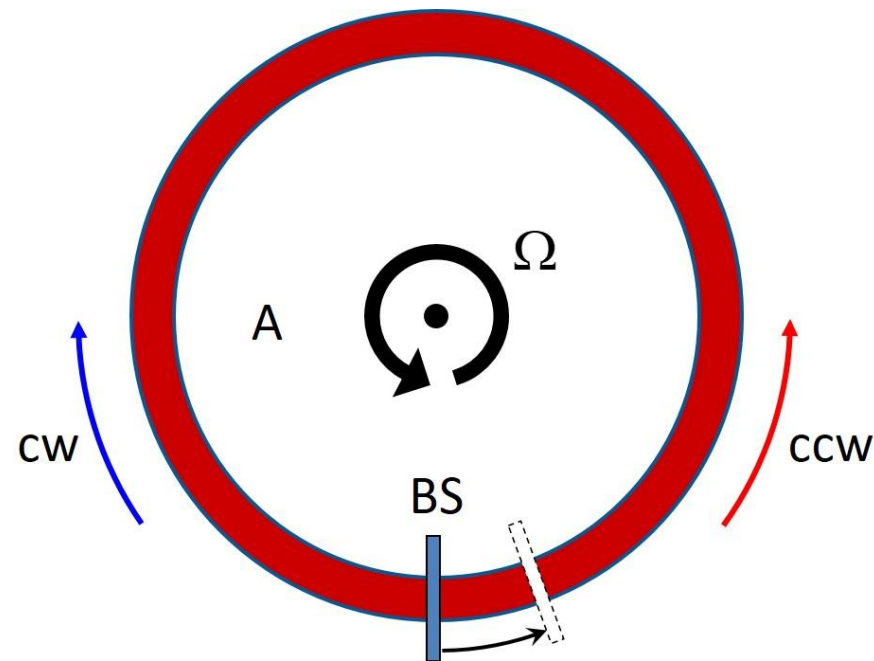
Sagnac Effect

A phase difference between cw / ccw beams due to different travel times around moving ring.

Time-difference around loop:

$$\Delta t = t_{cw} - t_{ccw} = \frac{4\Omega \cdot A}{c^2(1-\beta^2)} \approx \frac{4\Omega \cdot A}{c^2}$$

$$\beta = v / c \ll 1$$

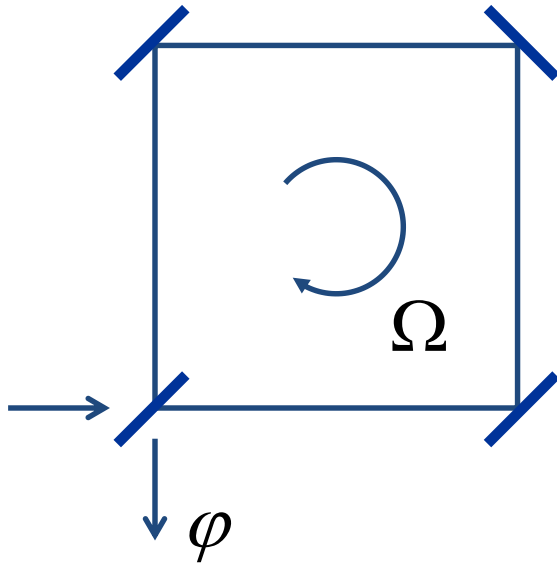


$$\Rightarrow \phi_s = \omega \Delta t = \frac{8\pi\omega}{c^2} \Omega \cdot A$$

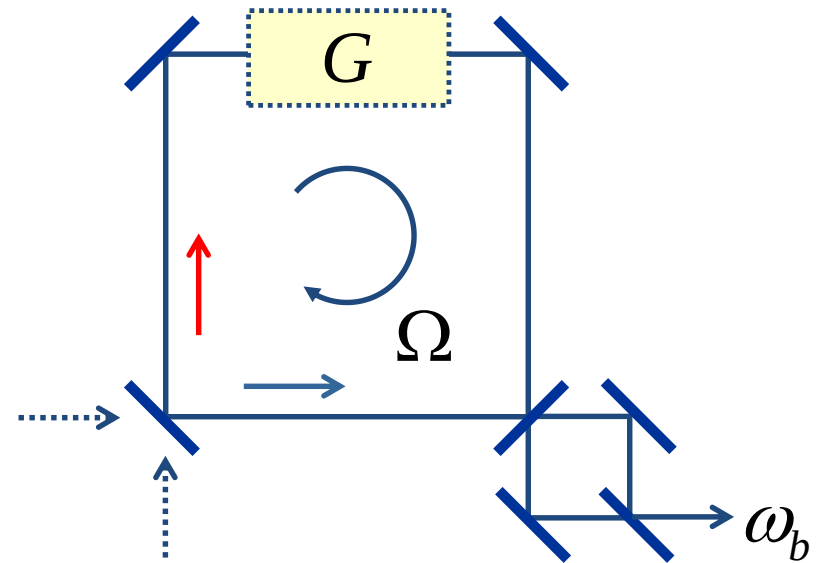
Sagnac Phase Shift!

Types of Optical Gyros

Interferometric:
(IFOG)



Ring Cavity (active & passive):
(PCG & RLG)



Sagnac phase shift independent of refractive index.

$\Rightarrow$ Only the cavity types can be enhanced!



Solution: Fast-Light (FL) Gyros



Scale Factors:

$$s^{(e)} = \frac{d\omega_b^{(e)}}{d\Omega} \propto \text{Area}$$

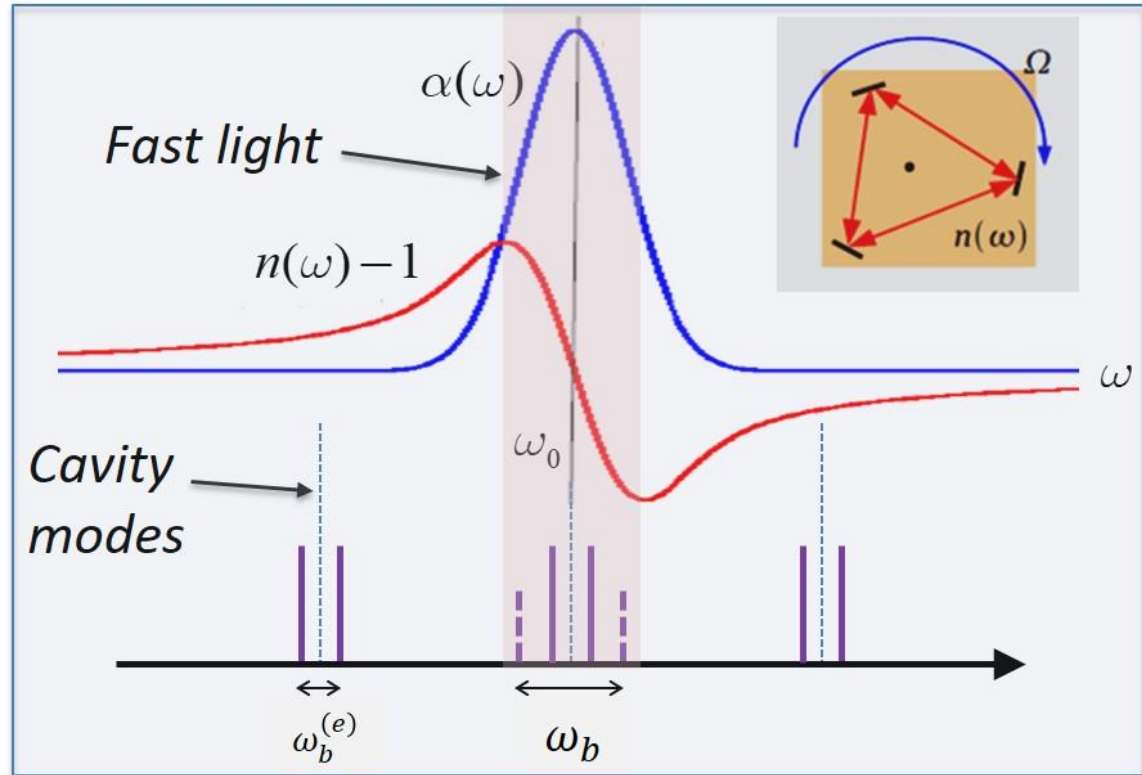
$$s = \frac{d\omega_b}{d\Omega}$$

Scale-Factor
Enhancement:

$$S = \frac{s}{s^{(e)}} \approx \frac{1}{n_g(\omega_0)} > 1 \text{ when } n_g < 1 \text{ (Fast Light!)}$$

Increase in Error: $\varepsilon = \frac{\sigma}{\sigma^{(e)}} \Rightarrow \text{Enhancement in Precision when:}$

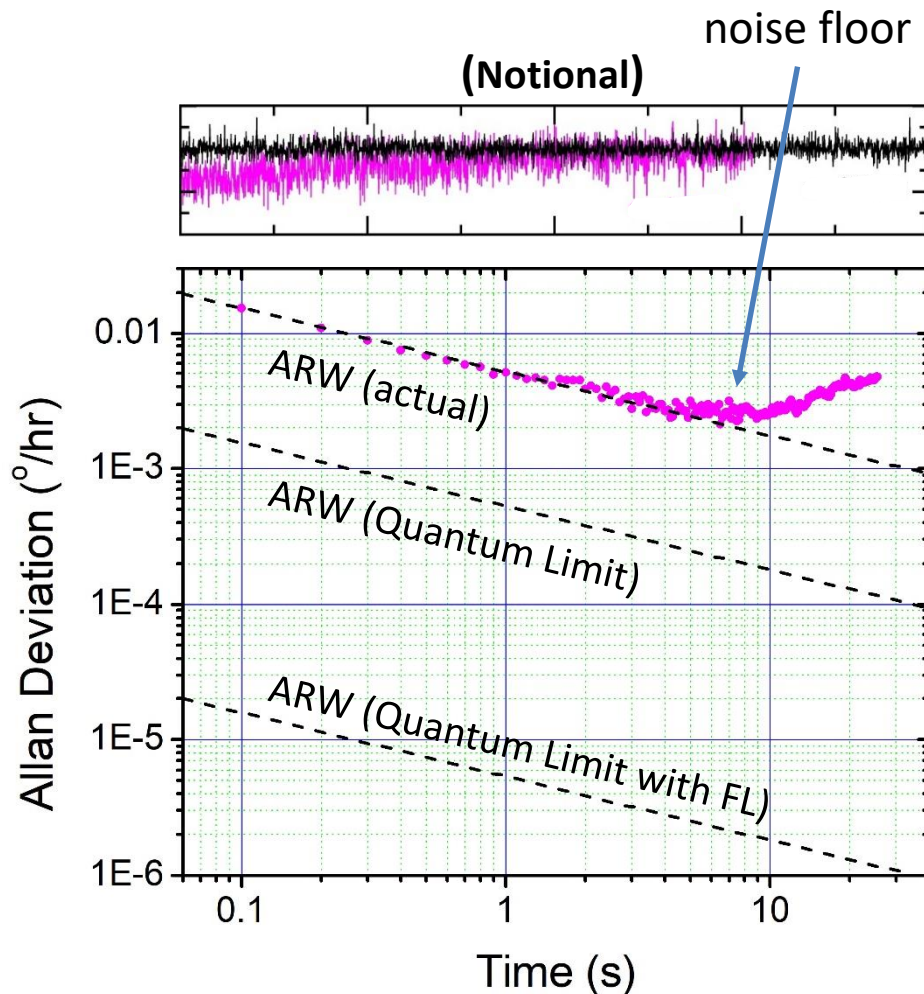
$$\zeta = \frac{S}{\varepsilon} > 1$$



Where would FL have greatest benefit?

$$\zeta = \frac{S}{\varepsilon} = \frac{ARW^{(e)}}{ARW} \quad ARW = \frac{1}{\sqrt{\tau}} \int_0^{\tau} \Omega_{\min} dt \quad \Omega_{\min} = \frac{\sigma}{s} \quad \sigma_{QNL} = \frac{\sqrt{2}\gamma}{SNR}$$

ARW decreases when s increases more than σ



- At short integration times where ARW dominates
 - Rapid accelerations
 - Tight controls
- Increased precision can be traded off for faster measurements or smaller gyroscopes
 - Where precision and accuracy are important
 - Where size & weight are important
 - Where high sampling is needed

Potential Applications

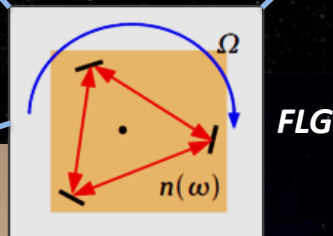


Kinetic Impactors

Sample Return

Precision EDL

Formation Flying

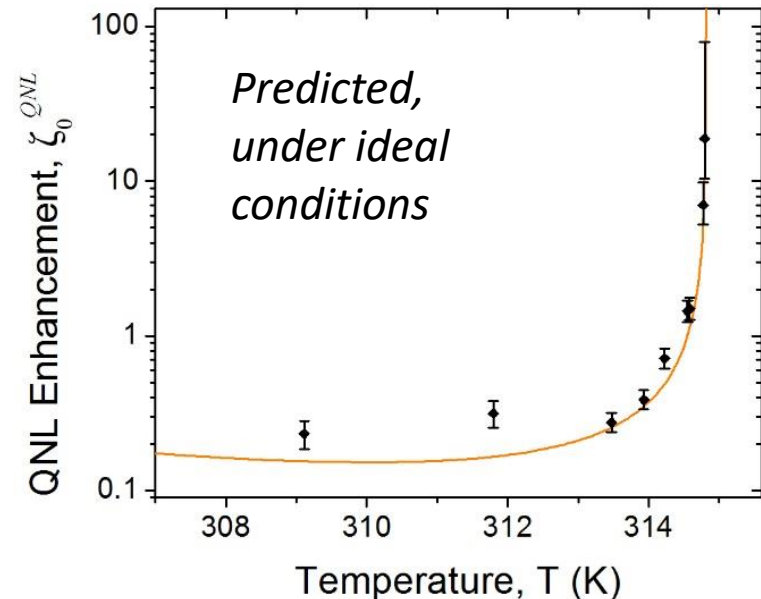
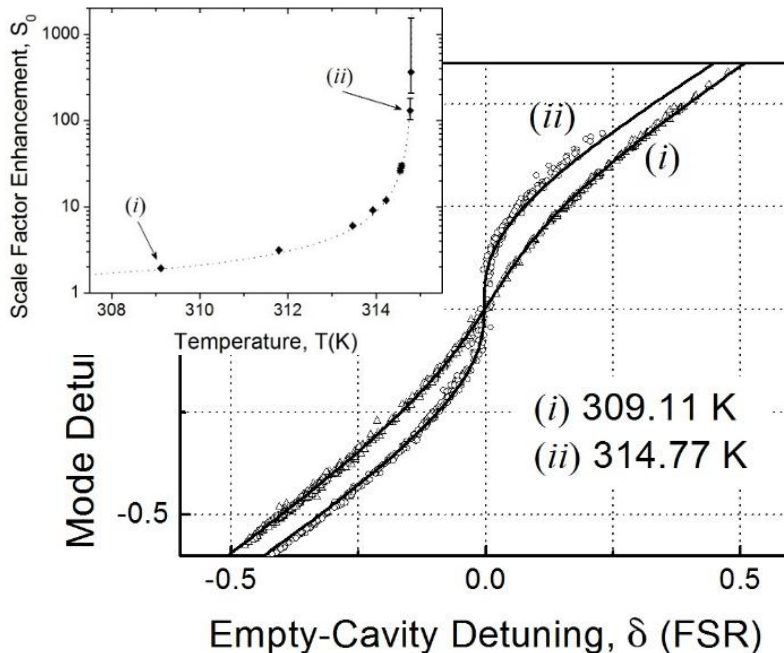
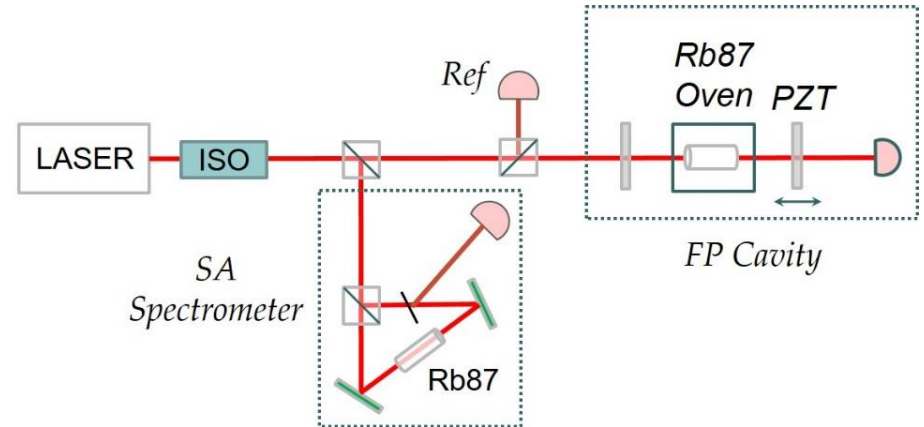


Passive Fast-Light Gyros (FLGs)

Passive FL Cavity



- First, largest, and most direct observation of enhanced scale-factor sensitivity ($S = 363$).
- Tuning of S by temperature (slow) and by optical pumping (fast).





Limitations

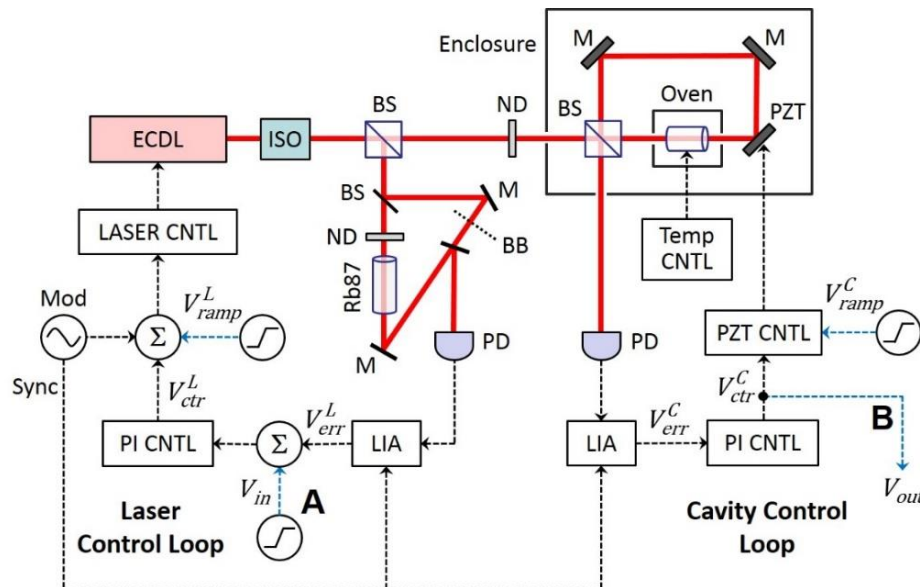
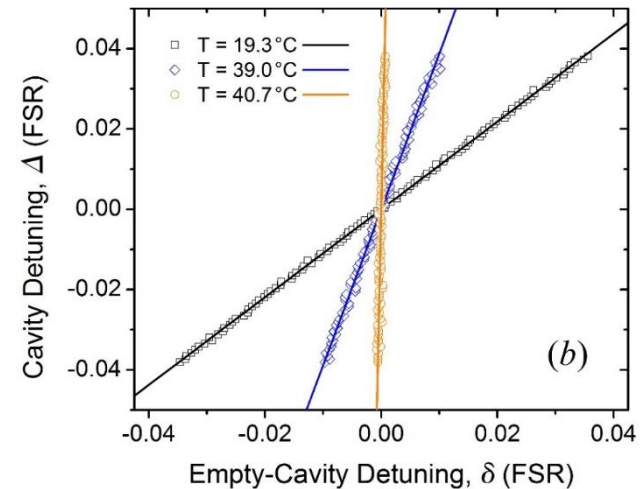


- Cavity detunings not measured directly in real time in an operating device. Instead S and ε are deduced, after the fact, from the spectra
 - Slow (5 mins vs. <1 sec. for closed-loop). Large amount of unnecessary data recorded.
 - No real experimental evidence of enhancement. ζ inferred for ideal (QNL & high-SNR) conditions only.
- Instability due to mode pushing → data scarce near resonance. Large uncertainty in S . Stabilization needed.

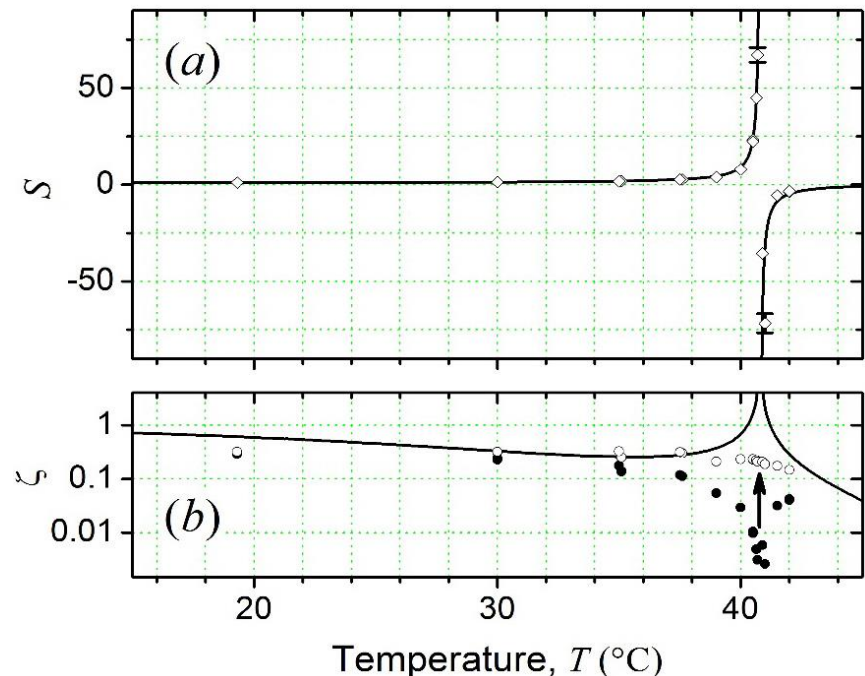
Closed-Loop Passive FL Cavity



- First direct measurement of boost in S in a **closed-loop device**. $\Rightarrow$ Paves way for passive FL gyro.
- Cavity stays locked through critical temperature. Both positive and negative values of S observed.
- Increase in ζ not yet observed due to classical noise. $\Rightarrow$ Gyro geometry needed.



D. D. Smith et al., *Opt. Expr.* 26, 14905, (2018).



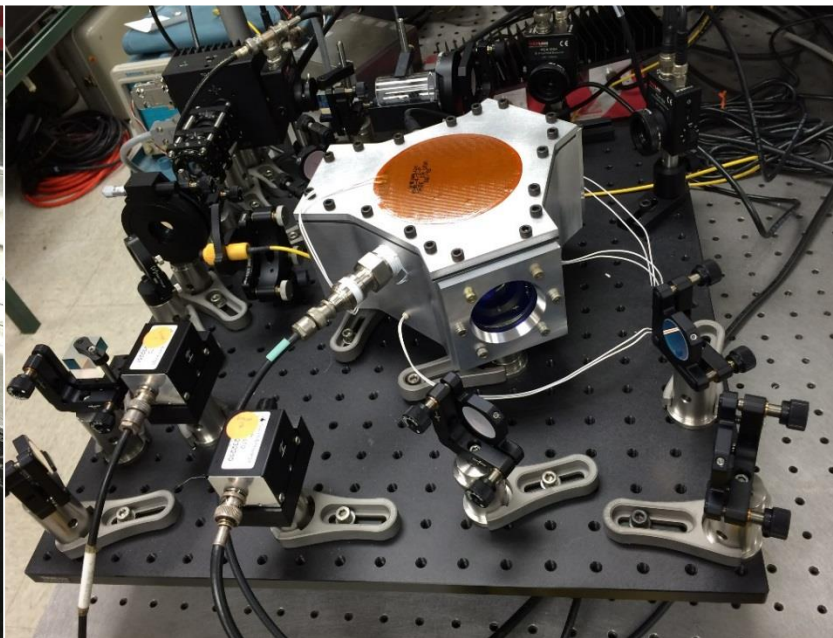
Passive FL Gyro (Atomic Vapor)



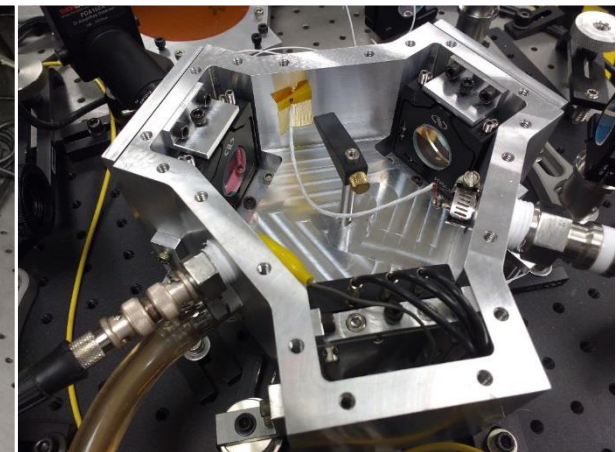
- Objective: Scale-factor enhancement for rotation has never been demonstrated in any experiment.



Table Shaker



Laser and Cavity on Rotation Stage



Passive Cavity

- Monolithic, vacuum-enclosed, magnetically-shielded, and temperature-stabilized to reduce noise.



Passive FLG: Advantages & Limitations



Advantages

- Simpler and less costly to obtain preliminary data
- Avoids nonlinear dynamics from gain medium
- Only systems so far to have shown boost in S to OPL changes.

Limitations

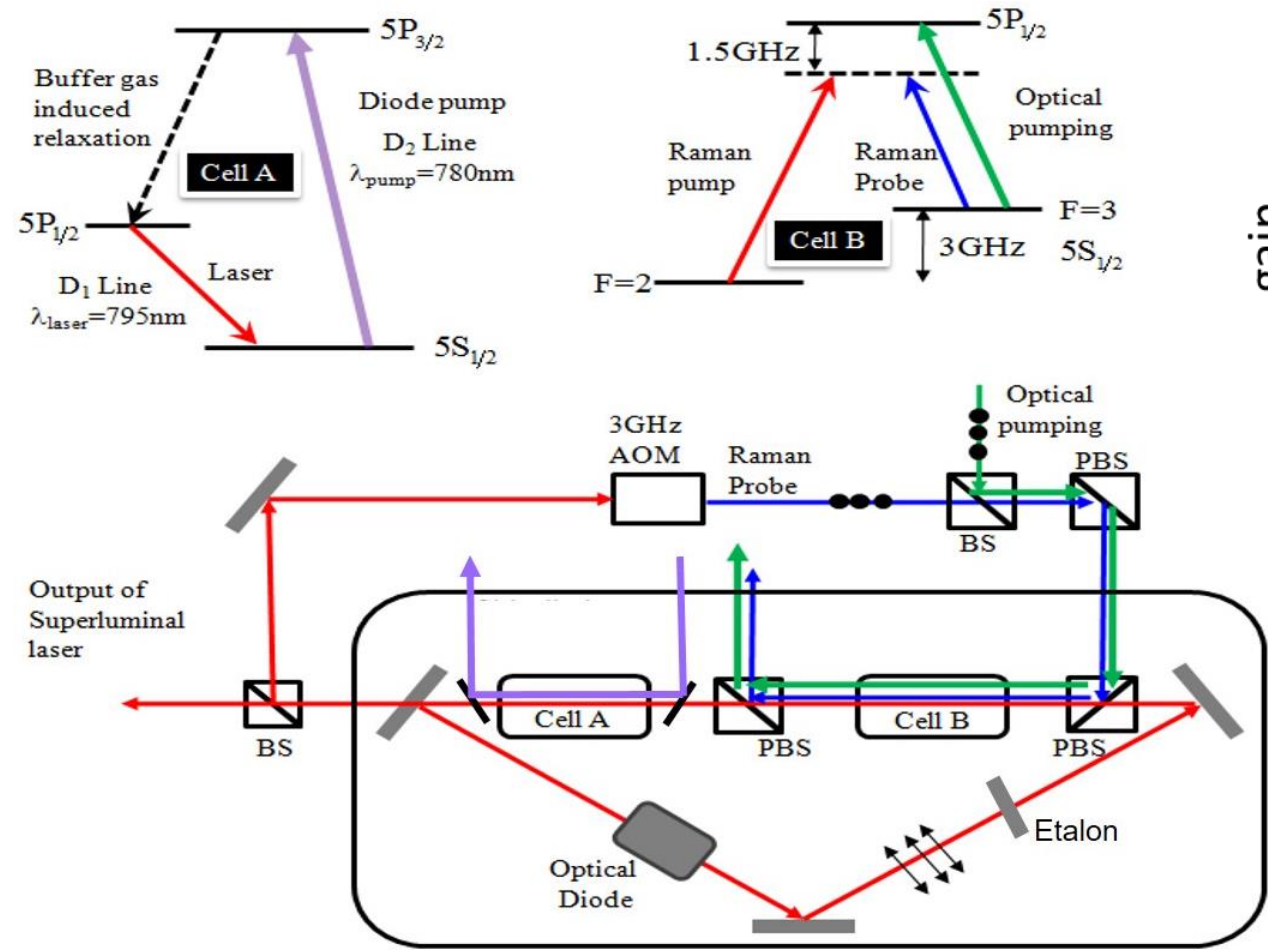
- Cavity modes attenuated by absorption. Reduced signal to noise.
- Saturation alters lineshape, couples counterpropagating beams, and limits achievable signal to noise.
- Require complicated closed-loop locking schemes
- Cavity linewidth also broadens. May not be true for active cavities. **Active FL gyros may achieve higher enhancement in precision!**

Active Fast-Light Gyros

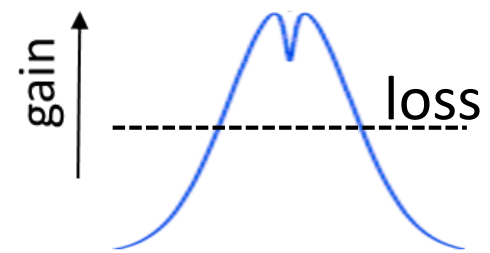


Active FLG: DPAL with SRS

SRS = Stimulated Raman Scattering



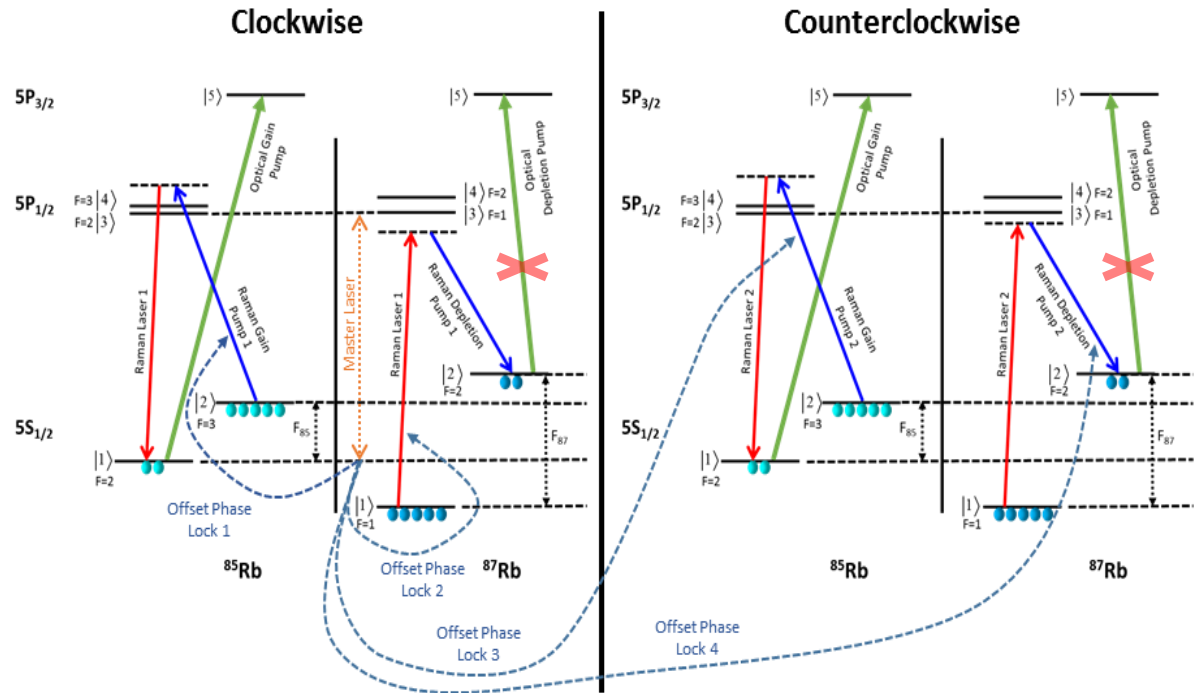
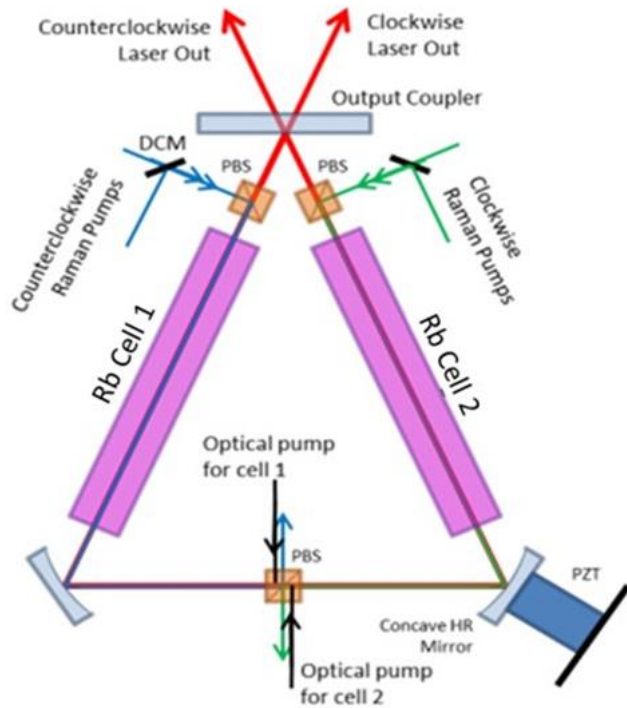
Dip in gain produced by SRS in second Rb cell



Diode-pumped Alkali laser (DPAL)

$S = 190$ (Inferred)

Active FLG: Bidirectional SRS



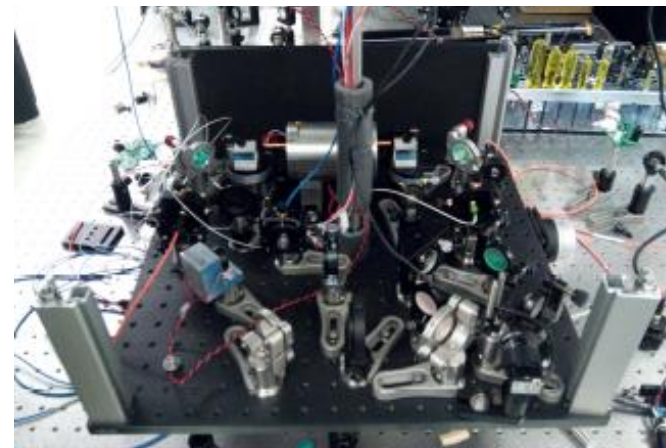
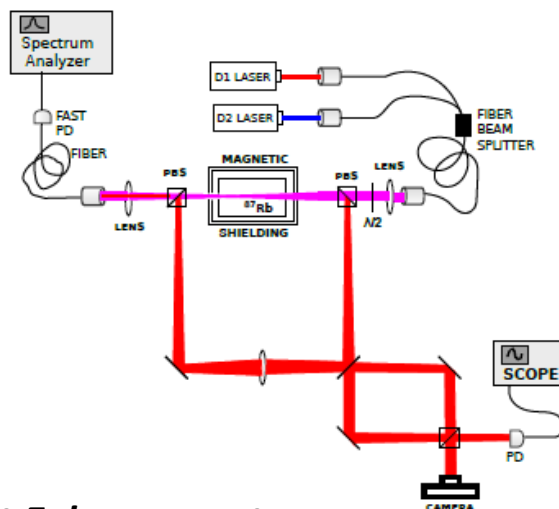
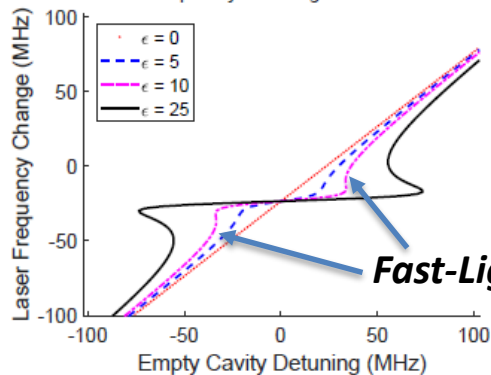
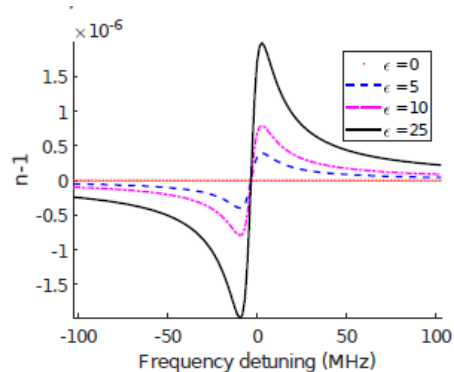
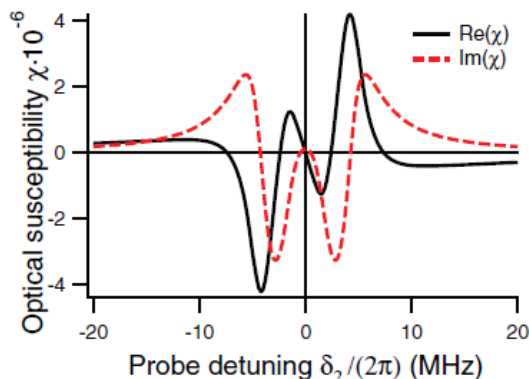
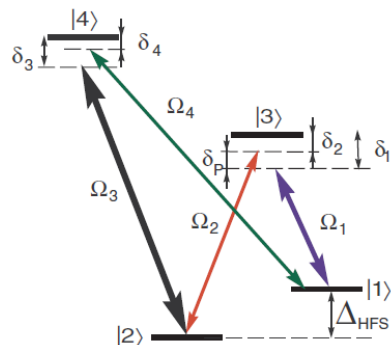
- Two Raman lasers, each one using dual-isotope ($^{85}\text{Rb} + ^{87}\text{Rb}$) vapor cells. Use different optical and Raman pumps for each isotope to produce a narrow dip in the gain. Apply Raman pumps for the two cells at different detunings resulting in a bias.
- Common path, bidirectional, self-biased FL gyro $\Rightarrow$ Bias eliminates dead-band and direction ambiguity. Improved common-mode rejection. Lower power.
- $S \approx 1000$ (Inferred, in unidirectional system only)



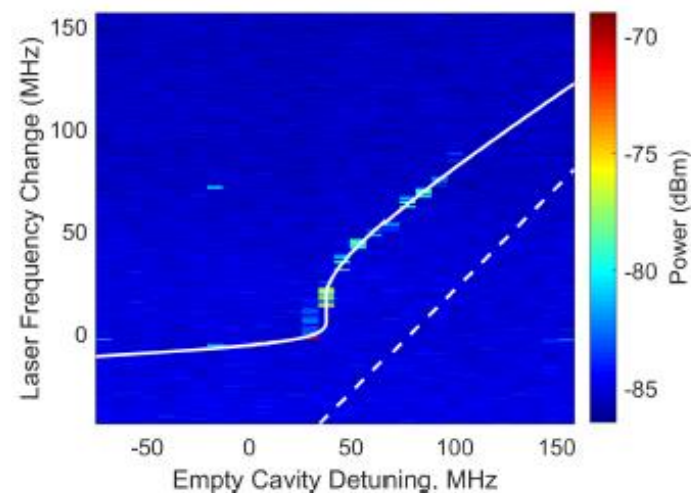
Active FLG: FWM Gyro



FWM = Four-Wave Mixing



First Enhancement to OPL in Active System!

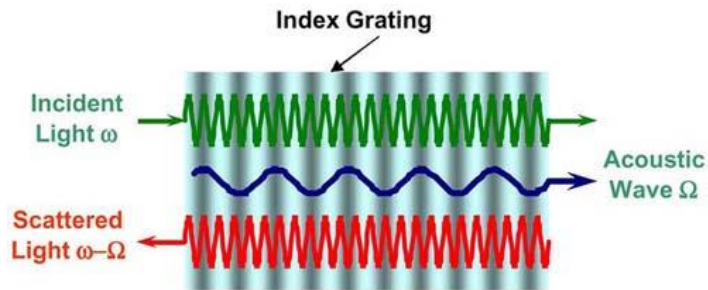


S. L. Cuozzo et al., PRA 100, 023846 (2019).

Active FLG: SBS Gyro



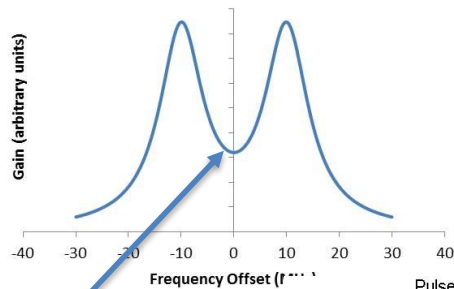
SBS = Stimulated-Brillouin Scattering



Problem: SBS gain produces slow light

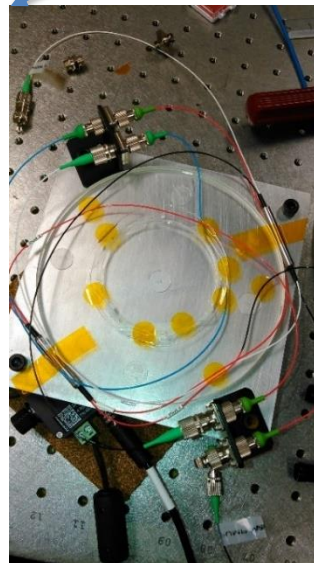
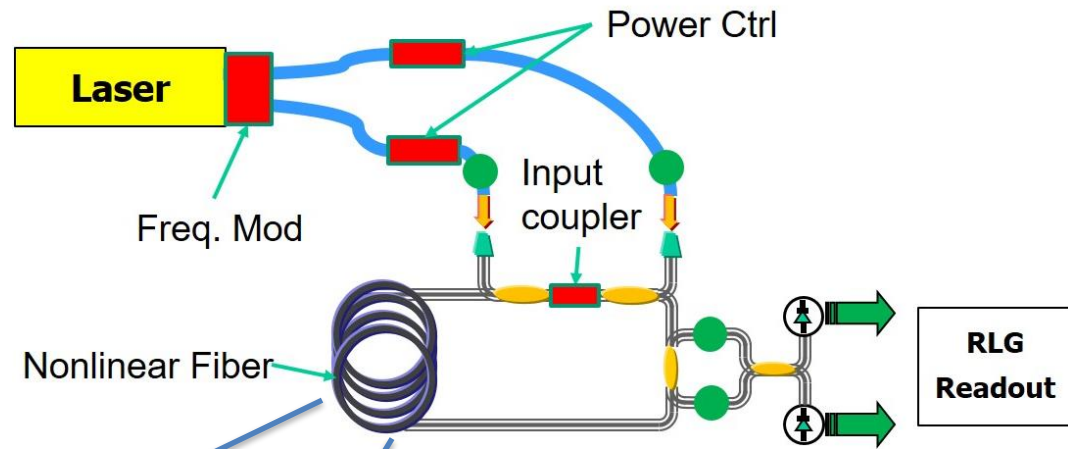
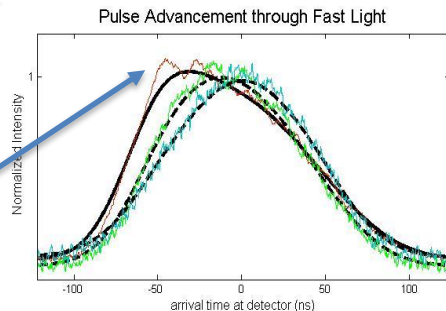
Solution: **Dual (two-color) pumping!**

Two gain features



Fast Light!

Pulse Advancement!



- Compact Design. First solid-state gyro. No gain competition!
- Lock-in effect eliminated by bias at FSR. No deadband!
- Dual pumping produces FL.
- Lasing at dip, and bidirectional with dual pump demonstrated.

Christensen et al., Proc. SPIE 10934, 109340S (2019).



Challenges for Active FL Gyros



- Gain competition $\Rightarrow$ unidirectional lasing
- Lock-in $\Rightarrow$ dead-band
- Dynamics of gain medium cancels FL enhancement to some degree.
- Often rely on NLO processes generated by additional pump beams
 - $\rightarrow$ Significantly more complex. **Difficult to miniaturize.**
 - $\rightarrow$ Sophisticated control schemes required
 - $\rightarrow$ Added sensitivity to environmental effects
- Enhancement in sensitivity, S , still not demonstrated directly to OPL changes, only inferred.



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Limitations of Atomic-Vapor FL Gyros



- Difficult to miniaturize
- Sophisticated control schemes
- Rely on discrete material transitions
 - Temperature dependent - requires SOA stabilization techniques.
 - Limited operation wavelength - inhibits wide adoption, manufacturers want to stick with He-Ne wavelengths.

Coupled-Resonator Gyros



Advantages of CR Gyros



- Easy to miniaturize via microfabrication
- Entirely linear effect, no saturation $\Rightarrow$ higher signal-to-noise
- Eliminates temperature dependence of atomic absorption $\Rightarrow$ better scale-factor stability.
- Not limited to operation at atomic resonance frequencies. **Any lasing wavelength possible, including He-Ne.**



Limitations of CRs



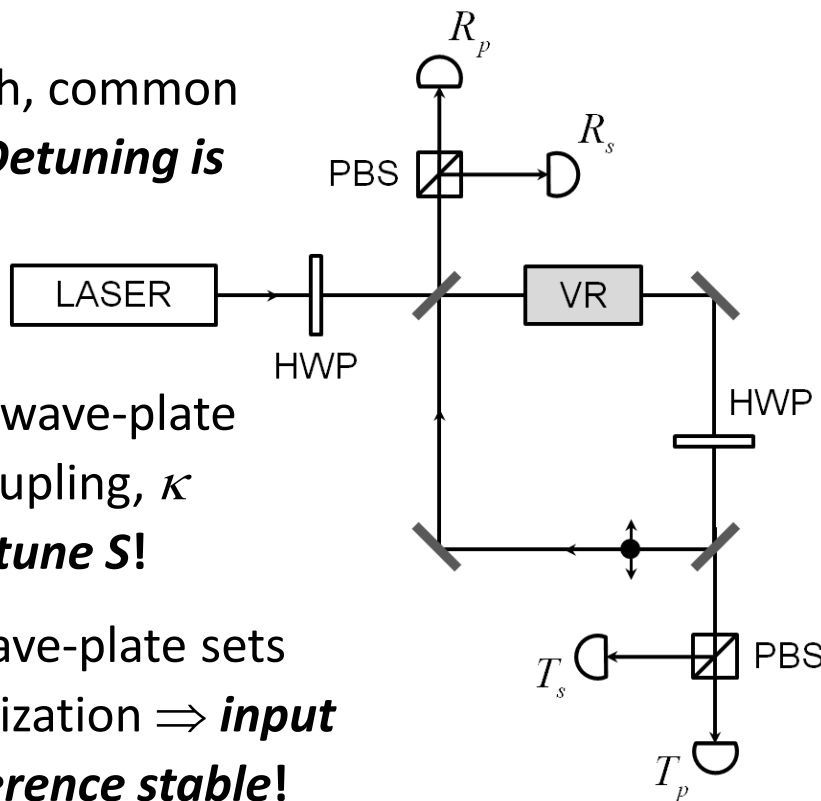
- **Not common path.** Resonators suffer from independent amounts of noise and drift.
⇒ Detuning between resonators not stable
- **Coupling not easily controlled**
⇒ S not easily tunable
- **Coherent control of S requires two input beams with a stable phase difference** (an interferometer)
⇒ Random phase between input beams.

Passive CRs: Polarization Mode Coupling

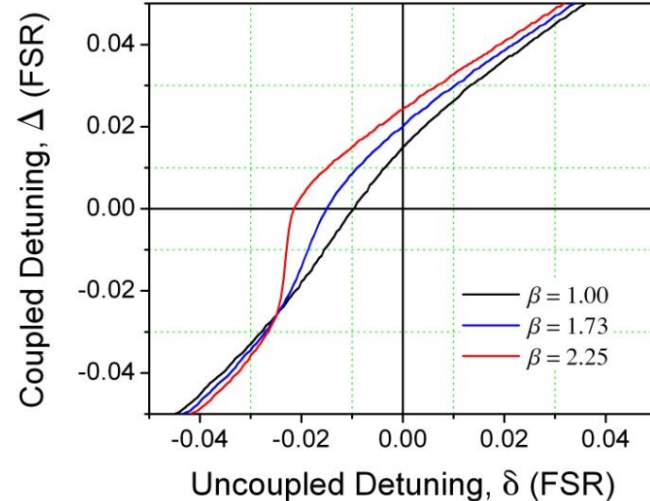


Soln: Polarization Coupling in a Single Cavity

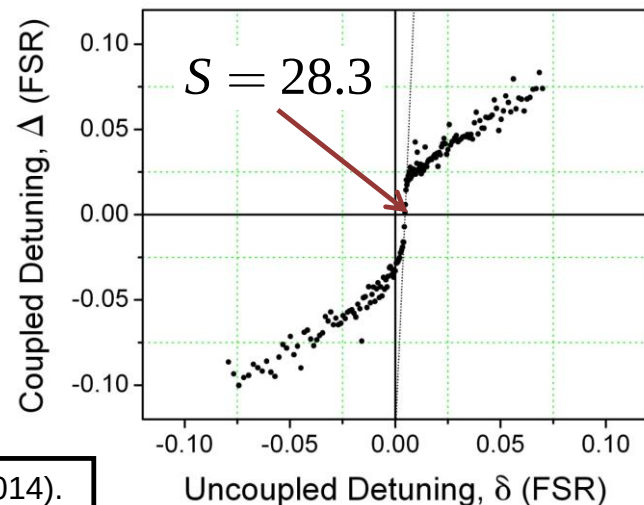
- Shared path, common mode. $\Rightarrow$ **Detuning is stable!**
 - Intracavity wave-plate controls coupling, κ
 $\Rightarrow$ **Easy to tune S!**
 - External wave-plate sets input polarization $\Rightarrow$ **input phase difference stable!**
- $\rightarrow$ Allows fast coherent control of S,
without changing anything inside cavity.



Coherent Control of S

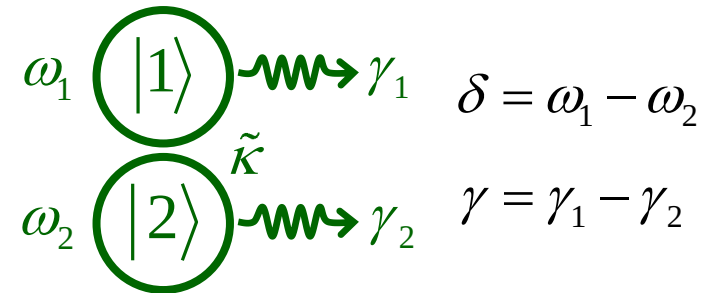


Critical FL Condition



Coupled Oscillators = Two Level Atom

$$|\psi(t)\rangle = E_1(t)|1\rangle + E_2(t)|2\rangle \quad \text{Coherent Superposition}$$



Coupled Modes = Schrod. Eqn. in RWA:

$$i\hbar |\dot{\psi}(t)\rangle = -\frac{\hbar}{2} \begin{pmatrix} -2\tilde{\omega}_1 & \tilde{\kappa} \\ \tilde{\kappa}^* & -2\tilde{\omega}_2 \end{pmatrix} |\psi(t)\rangle$$

$$= \tilde{H} |\psi(t)\rangle$$

$$\tilde{\omega}_{1,2} = \omega_{1,2} - i\gamma_{1,2}/2$$

$$\tilde{\delta} = \tilde{\omega}_1 - \tilde{\omega}_2 = \delta - i\frac{\gamma}{2}$$

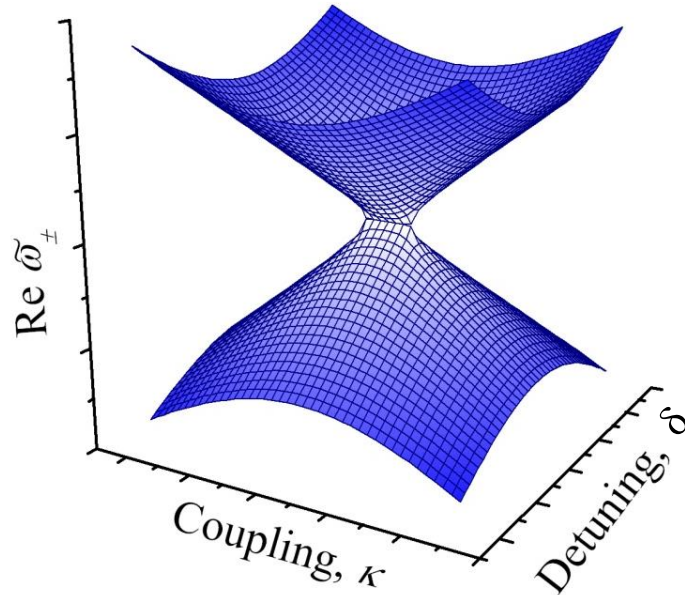
Complex Eigenvalues: $\tilde{\omega}_{\pm} = \tilde{\omega}_{avg} \pm \frac{\tilde{\Omega}}{2}$ $\tilde{\Omega} = [\tilde{\delta}^2 + \kappa^2]^{1/2}$ Generalized Rabi Freq.

Equivalent to Damped Rabi Problem $\rightarrow$ Expect Same Dynamics!

Complex Eigenvalues

$$\tilde{\omega}_{\pm} = \tilde{\omega}_{avg} \pm \frac{\tilde{\Omega}}{2} = \omega_{\pm} - i\gamma_{\pm}/2$$

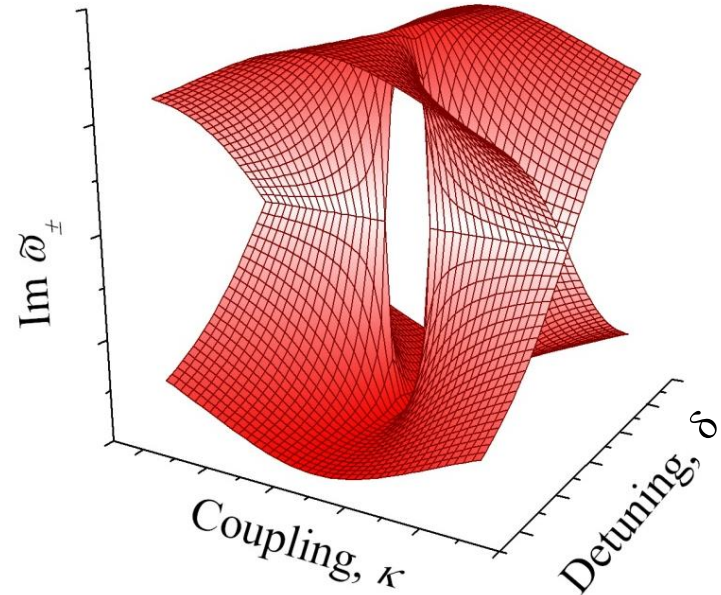
$\swarrow$ Freq. $\swarrow$ Width



Sub-exceptional

$$\kappa < |\gamma / 2|$$

Frequency Crossing
& Width Anti-Crossing



Exceptional Point (EP)

$$\kappa = |\gamma / 2|$$

Frequency
& Width Crossing

Super-exceptional

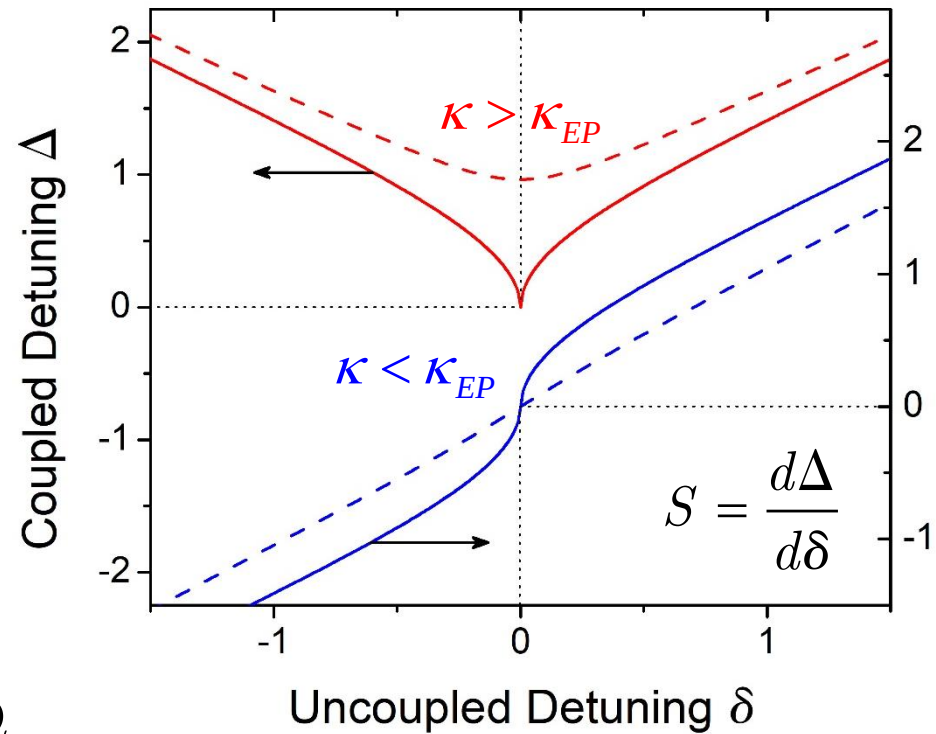
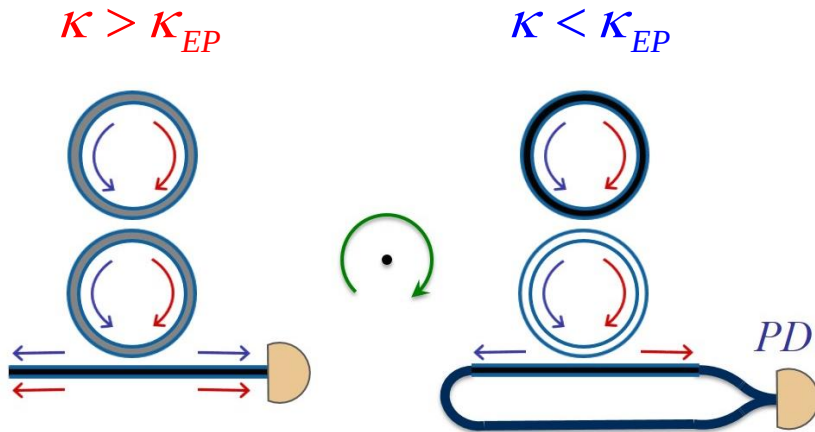
$$\kappa > |\gamma / 2|$$

Frequency Anti-Crossing
& Width Crossing

Coupled-Resonator (CR) Gyros



Two Possibilities:



Beat Frequency: $\Delta = \omega_+ - \omega_- = \text{Re } \tilde{\Omega}$

at EP: $\Delta \sim \sqrt{\delta} \Rightarrow S(0) \rightarrow \infty$

Exceptional Point (EP) = Fast Light (FL) Enhancement!



Maybe you
shouldn't have
done that roll.

You mean
Exceptional-
Point gyros,
right?

The AI has
tumbled.

Activate Fast-
Light Gyros.

Sure

Parity-Time Symmetry

$$H = -\frac{\hbar}{2} \begin{pmatrix} -2\tilde{\omega}_1 & \tilde{\kappa}_1 \\ \tilde{\kappa}_2 & -2\tilde{\omega}_2 \end{pmatrix}$$

P switches indices $1 \leftrightarrow 2$

$T \rightarrow$ complex conjugation

PT -Symmetric when: $[PT, H] = 0$

$$PTH = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -2\tilde{\omega}_1^* & \tilde{\kappa}_1^* \\ \tilde{\kappa}_2^* & -2\tilde{\omega}_2^* \end{pmatrix} = \begin{pmatrix} \tilde{\kappa}_2^* & -2\tilde{\omega}_2^* \\ -2\tilde{\omega}_1^* & \tilde{\kappa}_1^* \end{pmatrix}$$

$$HPT = \begin{pmatrix} -2\tilde{\omega}_1 & \tilde{\kappa}_1 \\ \tilde{\kappa}_2 & -2\tilde{\omega}_2 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} \tilde{\kappa}_1 & -2\tilde{\omega}_1 \\ -2\tilde{\omega}_2 & \tilde{\kappa}_2 \end{pmatrix}$$

PT -Symmetry when: $\tilde{\kappa}_1 = \tilde{\kappa}_2^*$ $\tilde{\omega}_1 = \tilde{\omega}_2^* \Rightarrow \delta = 0$ AND $\gamma_1 = -\gamma_2$

Conservative Coupling, No Detuning, and Balanced Gain/Loss!



Threshold Conditions (set $\gamma_{\pm} = 0$)



$$\tilde{\omega}_{\pm} = \omega_{avg} - i \frac{\gamma_{avg}}{2} \pm \frac{\tilde{\Omega}}{2}$$

Case I: $\tilde{\Omega}$ real ($\delta = 0$ & $\kappa \geq \kappa_{EP}$)

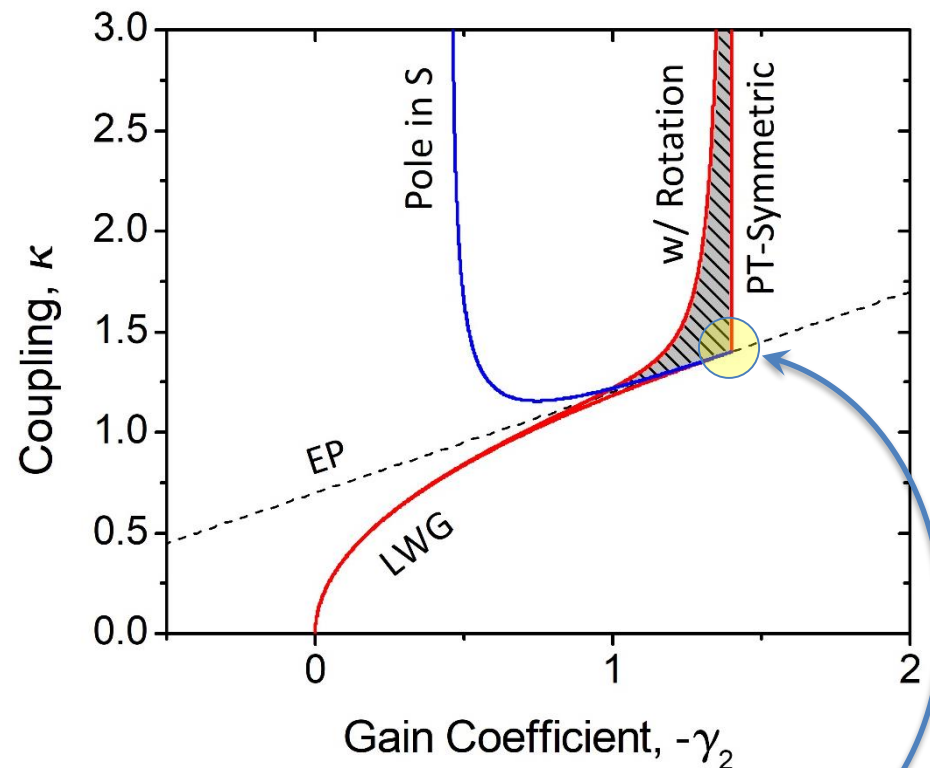
$$\gamma_2 = -\gamma_1 \Rightarrow PT\text{-Symmetric}$$

$$\gamma_{TOT} = \gamma_1 + \gamma_2 = 0 \quad \text{Usual lasing condition}$$

Case II: $\tilde{\Omega}$ complex ($\delta \neq 0$ or $\kappa < \kappa_{EP}$)

$$\kappa^2 = -\gamma_1 \gamma_2 + \delta^2 \left(\kappa_{EP}^2 / \gamma_{avg}^2 - 1 \right)$$

$$\gamma_{TOT} < 0 \quad \text{Lasing when net gain negative!}$$

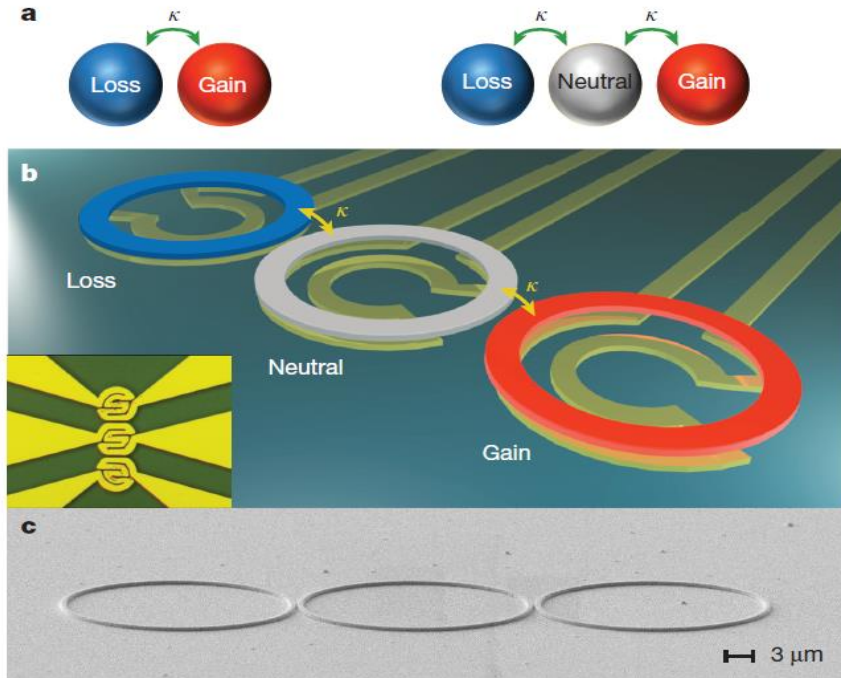


PT-symmetry breaking phase transition (EP, pole in S, laser threshold all coincide)

Broken PT-symmetry = Lasing Without Gain (LWG)



Active CRs: PT-Symmetric Systems



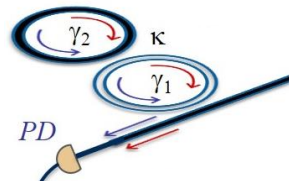
second-order EPs $\Rightarrow$ square-root
third-order EPs $\Rightarrow$ cube-root

H. Hodaei et al., *Nature* **548**, 187 (2017).

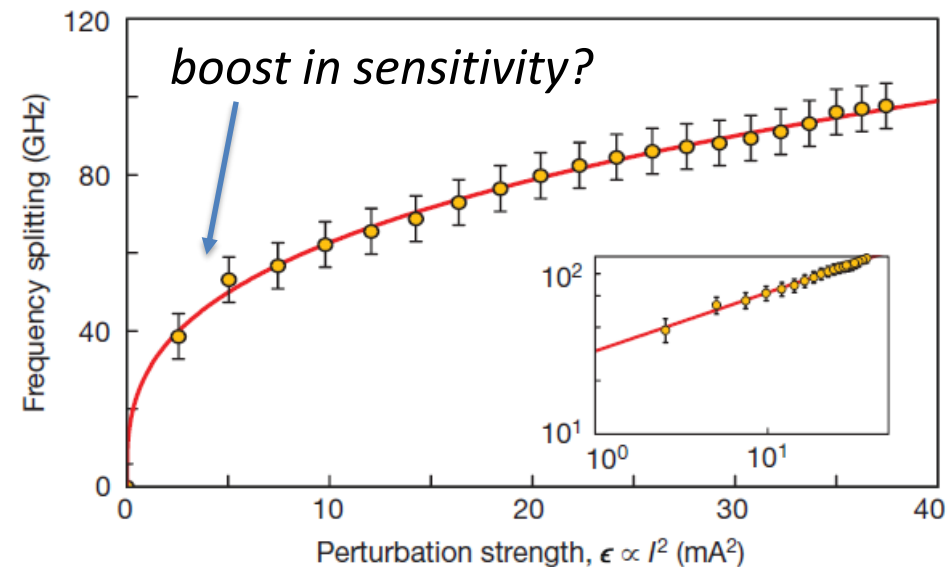
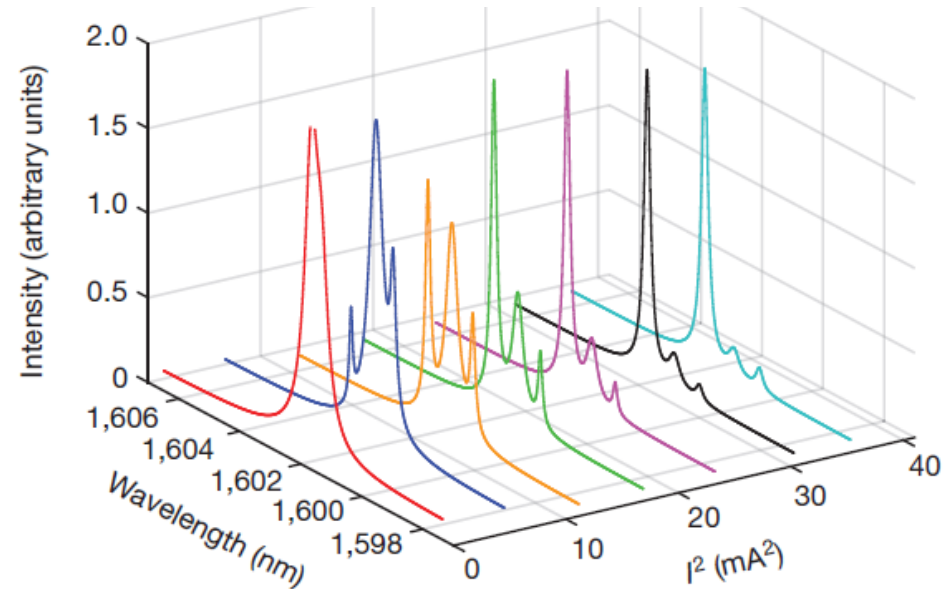
PT-Symmetric gyros:

Balanced gain/loss $\gamma_2 = -\gamma_1$

Unidirectional (no dead-band)



J. Ren et al., *Opt. Lett.* **42**, 1556 (2017).

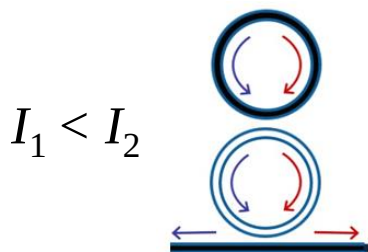


Photon Dynamics: Rabi oscillations

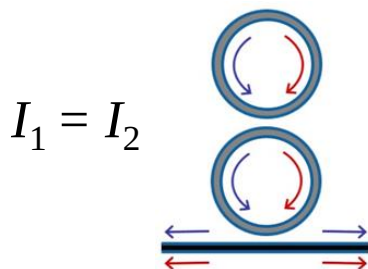
General Solution: $\psi(t) = \begin{bmatrix} \cos \frac{\tilde{\Omega}t}{2} - i \frac{\tilde{\delta}}{\tilde{\Omega}} \sin \frac{\tilde{\Omega}t}{2} & i \frac{\tilde{\kappa}}{\tilde{\Omega}} \sin \frac{\tilde{\Omega}t}{2} \\ i \frac{\tilde{\kappa}^*}{\tilde{\Omega}} \sin \frac{\tilde{\Omega}t}{2} & \cos \frac{\tilde{\Omega}t}{2} + i \frac{\tilde{\delta}}{\tilde{\Omega}} \sin \frac{\tilde{\Omega}t}{2} \end{bmatrix} \psi(0) e^{-i\omega_{\text{avg}}t} e^{-\gamma_{\text{avg}}t/2}$

Undamped for PT-symmetric

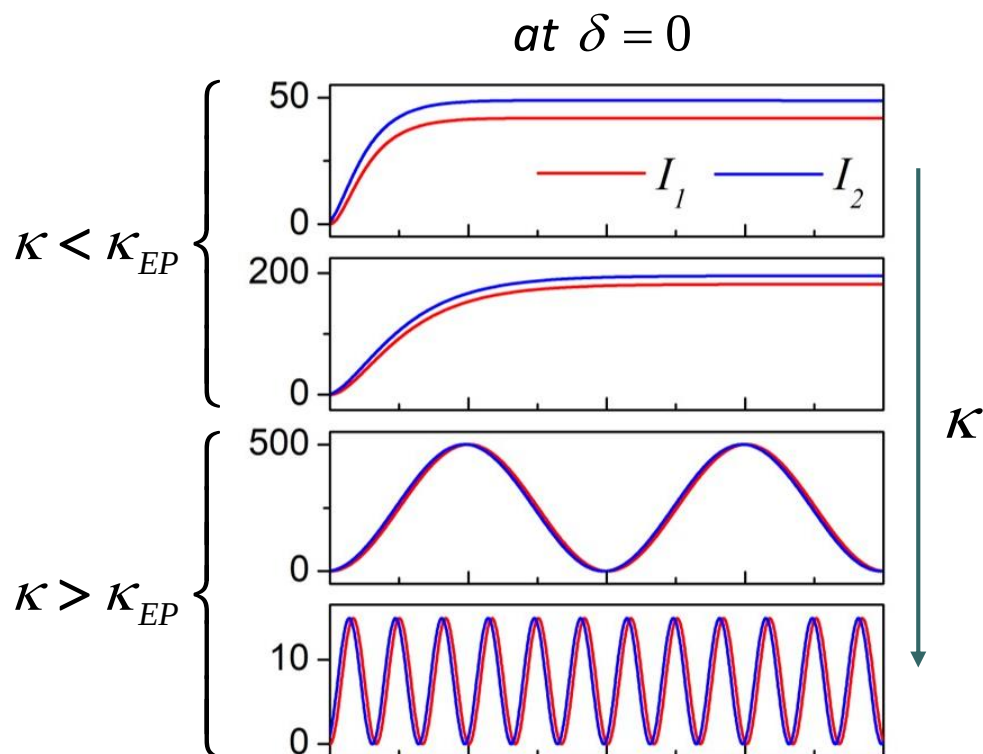
Photons
Localized!



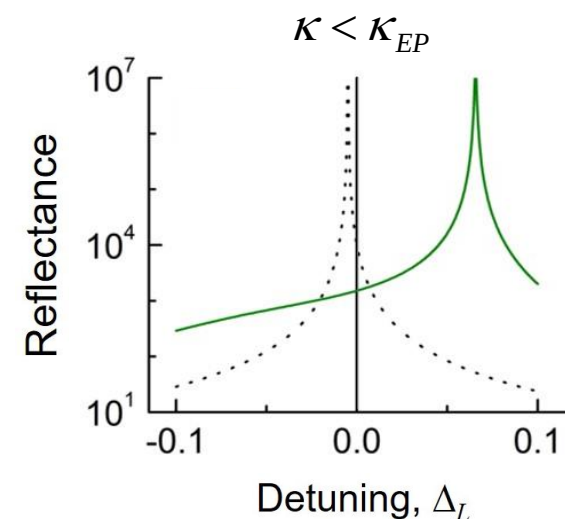
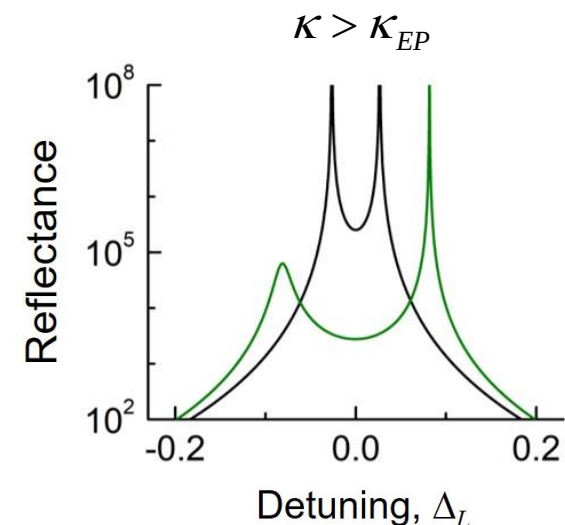
Overdamped!
(PM beat)



Undamped
(pure AM beat)



Spectra (from single output direction)



PT-Symmetric Gyros:

- Directional ambiguity
 - For small rotation rates, enhancement obtained only very close to EP. Otherwise sensitivity is reduced. (min value of $S = 0$)
 - One frequency disappears (subthreshold) if:
 - gain is homogeneously broadened
 - pumping / gain insufficient
 - rotation rate too large.
- Bidirectional operation required!***
- Pure AM beat signal $\Rightarrow$ strong undamped power oscillations $\rightarrow$ false rotation signal via Kerr effect

LWG Gyros:

- No ambiguity
- Enhancement even far from EP (min value of $S = 1$).
- PM beat signal $\Rightarrow$ No power oscillations



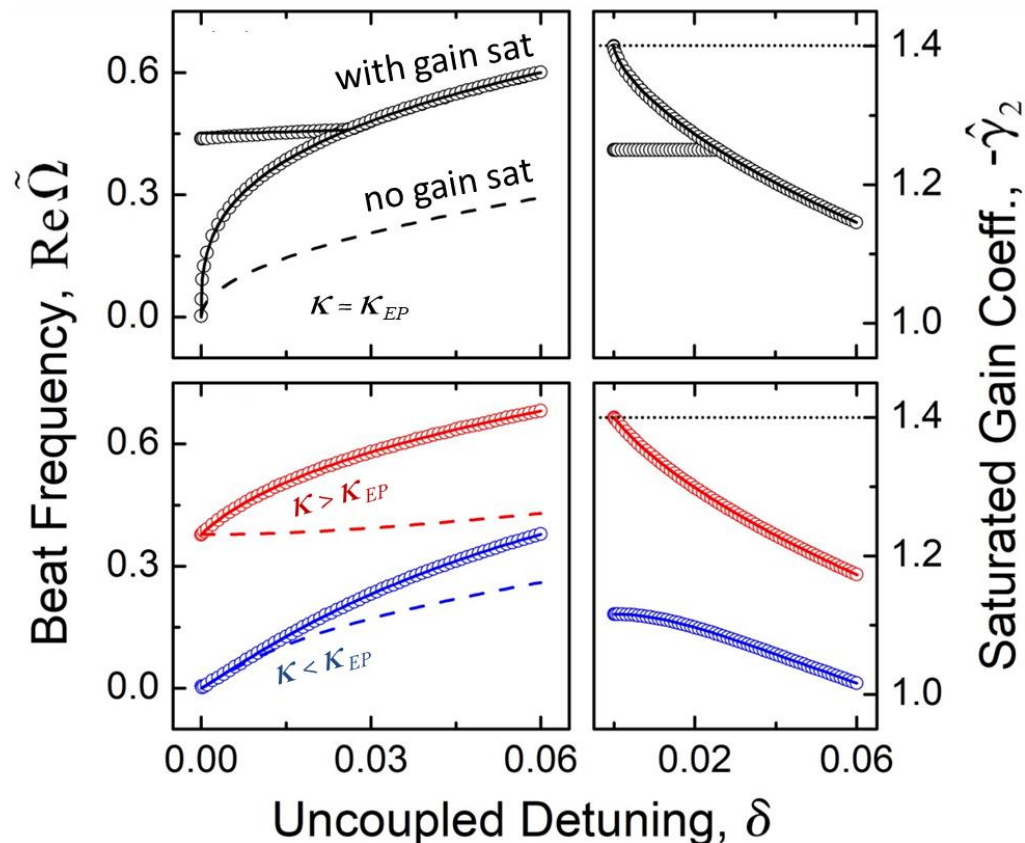
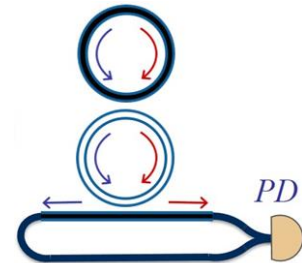
Effects of Gain Saturation



Previously assumed independent variables in $\tilde{\Omega}(\delta, \gamma, \kappa)$, but $\gamma(\delta)$.

$\Rightarrow$ scale factor $d(\text{Re}\tilde{\Omega})/d\delta$ will be altered by gain saturation.

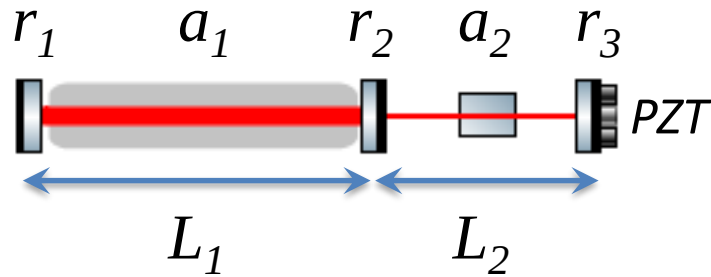
- Solve CM equations numerically, assuming homogeneous gain saturation
- Assume no backscattering within resonators (no dead-band).



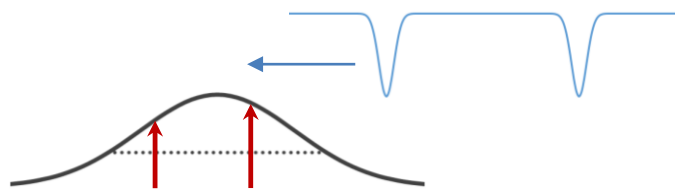
- Eliminates power oscillations
- Prevents one of the output frequencies from lasing
- Removes directional ambiguity
- **Boosts sensitivity enhancement (even larger than expected)**
- **Enlarges parameter space around EP over which enhancement occurs**



EP-Enhancement in He-Ne



- L_1 such that only two modes oscillate $\rightarrow \omega_b$ observed on RF spectrum analyzer.
- Scan passive cavity with PZT $\rightarrow$ pushes on the lasing modes.
- Sensitivity increases as EP approached.
- Chaos observed when too much feedback. Used absorber, a_2 , to reduce.

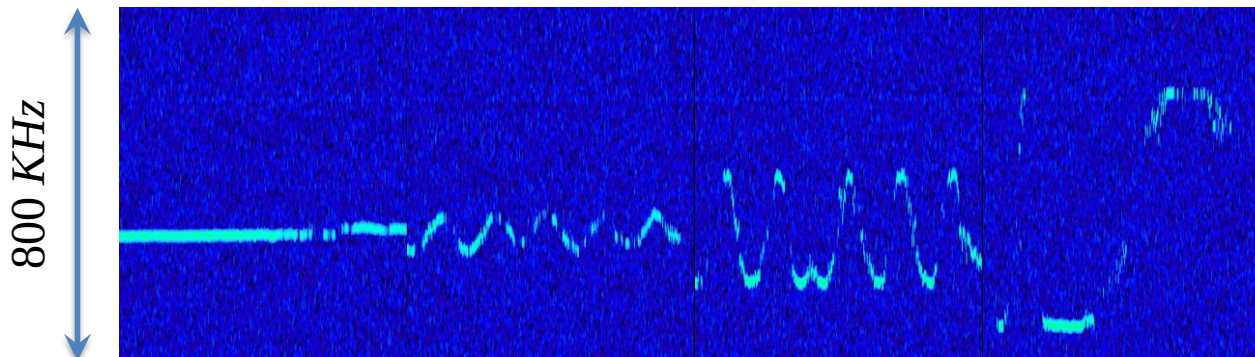


$a_2 = 0$

0.03

0.1

0.2



Oscillations in ω_b increase, but **one mode stops lasing**.

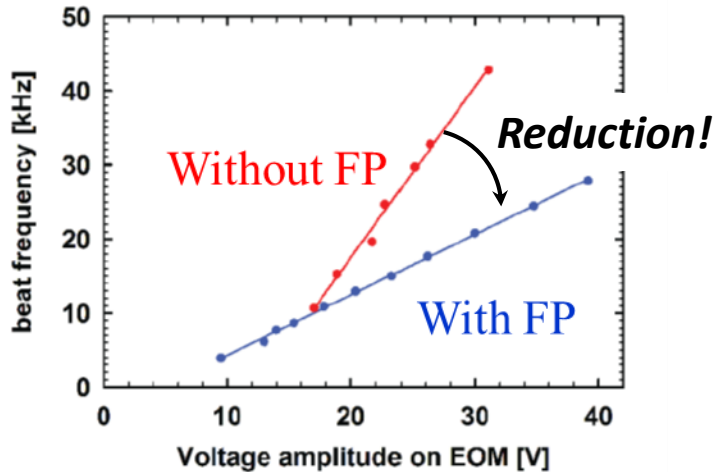
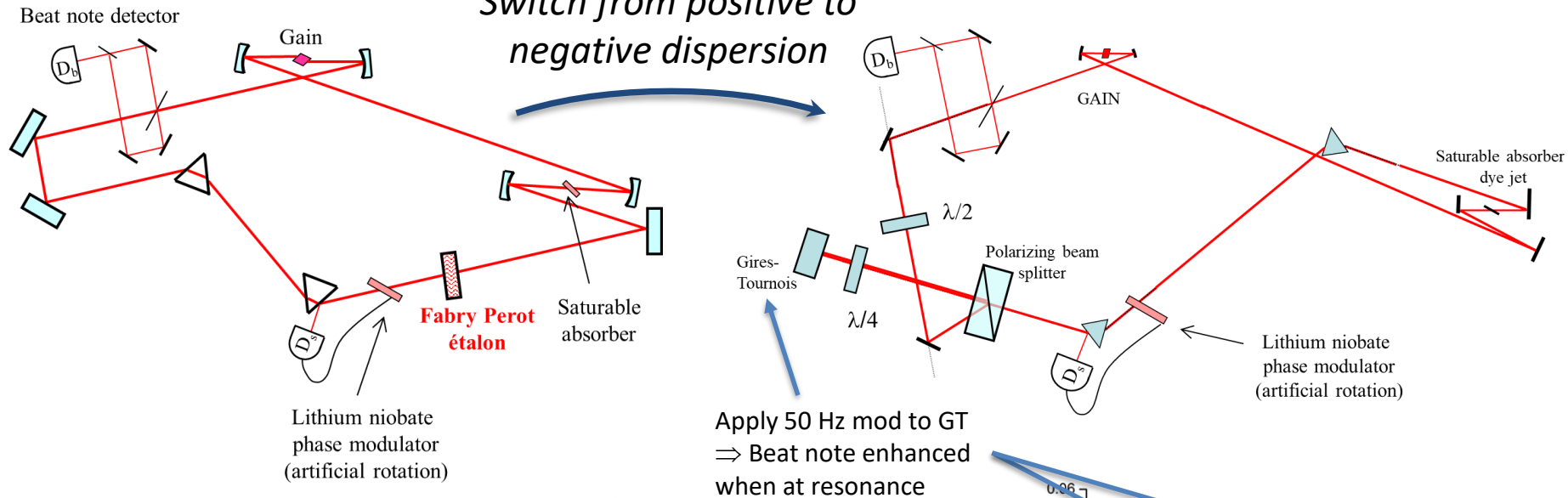
$\Rightarrow$ Need higher gain a_1 to get to EP.



EP-Enhancement in Pulsed Laser

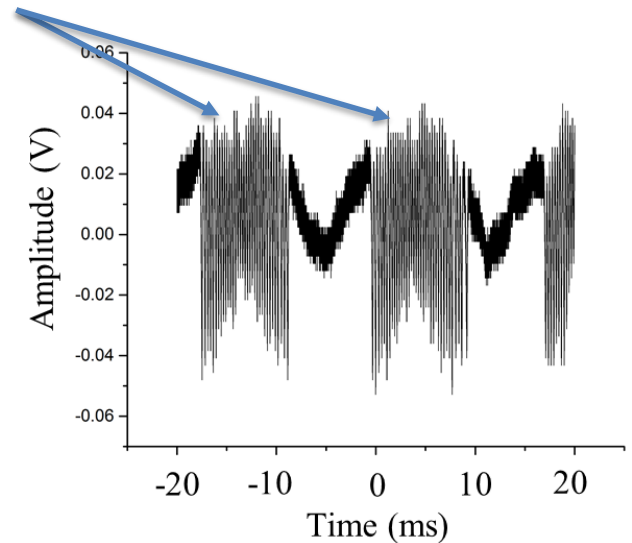


Switch from positive to negative dispersion



J. Hendrie et al., *Opt. Exp.* **24**, 30402 (2016).

Increase!



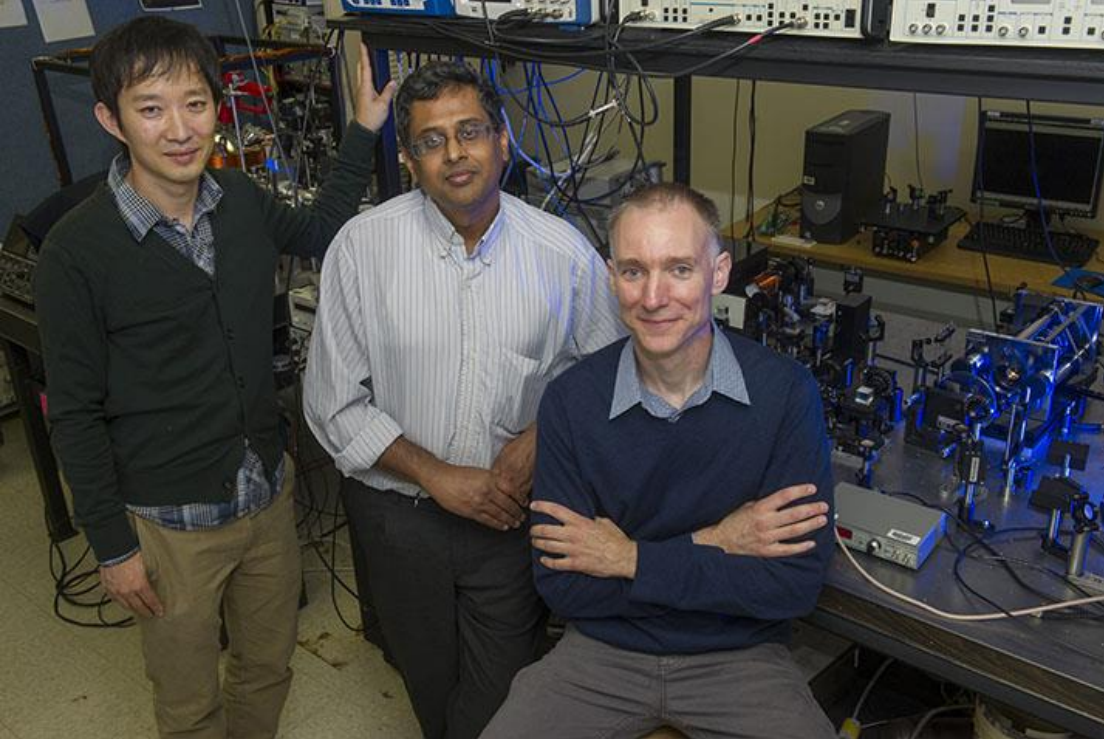
J.C. Diels et al., SPIE Phot. West (2019).



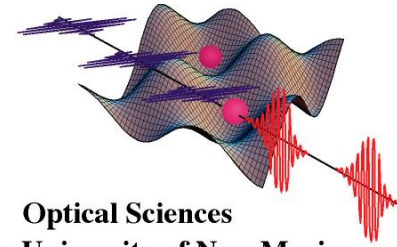
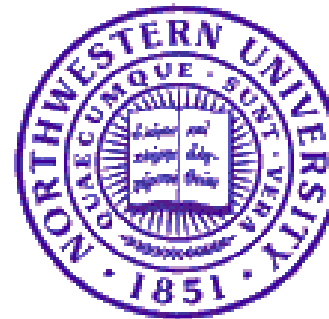
Conclusions



- PT-symmetric gyros will not work as originally-envisioned under a variety of conditions. Gyros with broken symmetry have advantages.
- Gain saturation helps by boosting sensitivity and allowing operation farther from EP.
- Experiments are getting better at demonstrating enhanced sensitivity.
- Claim: a FL laser is simply one that operates at an EP.
 - ⇒ It can't be simultaneously true that: (1) linewidth is unaffected by FL, and (2) the Petermann factor diverges near an EP.
 - ⇒ Need measurements of intrinsic linewidth to determine if precision can be enhanced.



MagiQ
Technologies



Optical Sciences
University of New Mexico

Thank You!



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