Negative Mass in Contemporary Physics, and its Application to Propulsion

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As first analyzed by Hermann Bondi in 1957, matter with negative mass is consistent with the structure of Einstein's general theory of Relativity. Although initially the concept was considered just a theoretical curiosity, negative mass, or "exotic matter," is now incorporated into the body of mainstream physics in a number of forms. Negative mass has a number of rather non-intuitive properties, which, as first noted by Bondi, and then later commented on by Forward (1990), Landis (1991), and others, results in possible applications for propulsion requiring little, or possibly no, expenditure of fuel. As pointed out by Morris and Thorne (1988) and others, negative mass (or, more strictly, a violation of the null energy condition) is also a requirement for any proposed faster than light travel. This paper presents the basic theory of negative mass, the ways by which it can manifest in contemporary physical theory, and the counterintuitive properties that result, including possible uses for interstellar propulsion. Although negative mass has moved from a theoretical curiosity to a concept fundamental to the contemporary understanding of physics, it is still not clear whether bulk negative mass can be manufactured, or if it is limited to only appearing at the cosmological scale (e.g., "dark energy:) or in the quantum (e.g., Casimir vacuum) limit. If it can be manufactured, the propulsion applications would be significant.

I. Introduction

IN everyday life, one of the most commonplace aspects of mechanics is the fact that if you push an object away from you, it moved away from you, and if you pull an object toward you, it moves toward you. Objects move in the direction

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of the force applied: this is such an unremarkable fact that we do not even notice it. This is an aspect of inertia that we do not even think about: due to Newton's law of acceleration

$$\vec{F} = m\vec{a} \qquad (1)$$

the acceleration is in the same direction as the force. But in making a conclusion that acceleration is in the same direction as the force, we are making the implicit assumption that mass can never be less than zero.

Einstein put an "energy condition" into the general theory of relativity, stating that energy cannot be negative. Since $E=mc^2$, the condition stating that energy cannot be negative is the same as specifying that mass cannot be negative. Later researchers have elucidated many different formulations of the energy condition, including the null, weak, strong, and dominant energy condition; and as well as the averaged null, weak, and strong energy condition; these are catalogued by Visser [1]. The non-negative energy condition (of whichever form), however, is not integral to the formulation of relativity; but was added to avoid what Einstein considered to be "absurd" solutions to field equations. Matter which violates the null energy condition is referred to in physics as "exotic matter". In this discussion, however, I limit the discussion to the properties of bulk matter of negative mass; more general cases of exotic matter are of interest, but not discussed here.

In 1957 Hermann Bondi pointed out that, despite Einstein's offhand rejection of the possibility, matter of negative mass is not inconsistent with the structure of Einstein's general theory of Relativity [2]. He also pointed out that such negative-mass matter would have odd properties; these odd properties were elaborated by Forward [3] and further implications pointed by Landis [4]. Visser [1] then continued on to list several areas in which contemporary theoretical physics incorporates forms of negative energy.

Physicists use the phrase "mass" to mean several different things, trusting in general for the right meaning to be clear in context. Here (following Bondi) I use the word mass in its classical physical meaning.[†] In this formulation, negative mass means both negative gravitational and negative inertial mass. Likewise, from Einstein's equation

$$E = mc^2 \qquad (2)$$

negative mass is identical to negative energy (and in this discussion, I will use the words negative mass and negative energy interchangeably, and not distinguish between them).

[†] in particular, when I refer to negative energy here, I am not referring to the negative energy states that can be found as solutions to the Dirac equation. This is an interesting topic in itself, but not related to the classical negative mass discussed here.

- Negative mass reacts oppositely to all forces. Force and acceleration are opposite. F = ma (eqn. 1) continues to apply, and thus
    - When you push on negative mass it moves toward you.
    - When you pull on negative mass it moves away from you.

The force of gravity on a negative is opposite: in a gravitational field g:

$$\vec{F} = m\vec{g} \tag{3}$$

so when m is negative, gravity pushes (instead of pulls). But the inertial mass also negative. So, negative mass still falls "down" in a gravitational field, and moves toward positive mass. Hence Einstein's equivalence principle still holds: all matter moves the same way in a gravitational field, regardless of mass, including negative matter.

II. Reactionless Propulsion with Negative Mass

As Bondi pointed out in 1957, this leads to an odd consequence. If an identical positive and negative mass objects were placed next to each other, the gravitational attraction between the two would lead to the positive mass being repelled from the negative mass… and the negative mass, also repelled, chasing after it (figure 1).

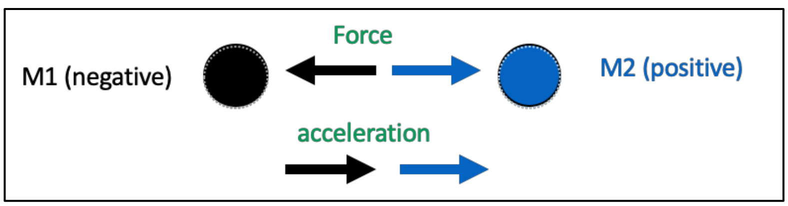

Figure 1: Negative mass chasing a positive mass [2]. The gravitational force repels the two masses, but the negative mass responds in reverse to the repulsion, and moves toward the positive mass.

The entire system of positive and negative mass spontaneously accelerates, but momentum is conserved, since the positive momentum of the positive mass is exactly cancelled by the negative momentum of the negative mass. Likewise, energy is also conserved.

Forward pointed out in 1990 that gravitational attraction is irrelevant to this motion. As long as the negative mass adds up exactly equal to the magnitude of the positive mass, any force between the two results in motion. A rod could be stretched between the two, and any desired acceleration could be achieved by pushing (or pulling) on the rod (figure 2).

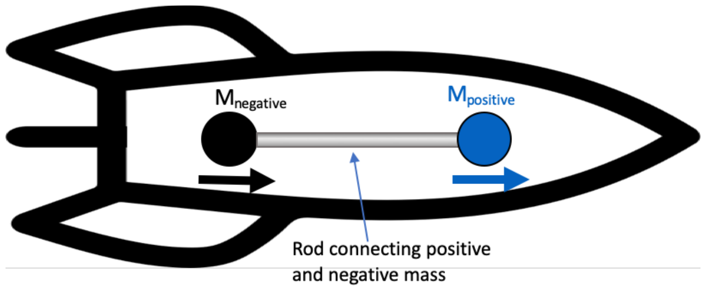

Figure 2: Spaceship drive using negative mass, according to Robert Forward [3]. By pushing on the rod, the mass on the right moves to the right, and the reaction force on the negative mass also moves it to the right.

I will point out that such propulsion only works if the net mass of the spaceship totals to exactly zero. If the positive mass is heavier than (the magnitude of) the mass of the negative mass, then the positive mass accelerates less rapidly than the negative, and the two masses move together. If the negative mass is more negative than the magnitude of the positive mass, then the negative mass accelerates less than the positive mass, and the two masses separate. Only if the two masses cancel is momentum conserved. A spaceship with zero mass can move without external force. This also means its response to external force is nearly infinite: the zero-mass spacecraft will be buffeted around by every impact of dust or cosmic blackbody photon.

III. Other Implications of Negative Mass

What would happen if you held a piece of negative mass matter in your hands?

The answer is, nothing. The negative mass matter would fall right through your hands. The reason solid matter is "solid" is that the Pauli exclusion principle prevents electrons from passing through other electrons in the same energy state. We do not know what negative matter might be: but we know what it is not. It will not consist of electrons, protons, and neutrons. Without Pauli exclusion, there would be no way for you to hold it. The "rod" in the spaceship example could not be a physical structure.

However, negative matter could still be manipulated by fields.

Uncharged negative matter would repel itself gravitationally. Since both the gravitational force and the inertial mass are negative, two negative masses would gravitationally accelerate away from each other. In the case of electric charge, however, two pieces of charged negative matter with like charge would attract itself strongly

For positive matter, like charges push each other apart, making bulk matter tending to be electrically uncharged. This is not the case for negative mass matter: negative mass matter does *not* self-neutralize, and hence bulk negative matter is not necessarily electrically uncharged.

A universe of negative matter would look like a universe of positive matter in which gravity and electrostatic forces switched: gravity is a repulsive force, and electrostatic forces attract.

But this has significant consequences. The electrostatic force is 10^{36} times stronger than gravity. Bulk negative matter, if it carried electrical charge, would accumulate into huge charged balls which push away balls of opposite charge, and also gravitationally push away normal matter.

It is tempting to speculate that if negative matter existed in the early universe, this could account for the "voids" we see in some places in the distant universe.

The existence of stable bulk negative-mass matter, however, is hard to reconcile with the existence of vacuum in quantum field theory. Consider the reaction where vacuum $\rightarrow m_{(positive)}+m_{(negative)}$

(Conservation of momentum indicates this must be at least a three-particle reaction)

It costs nothing to produce negative matter: in fact, the energy cost is less than zero: making negative mass generates energy. This leads to the question, what stops this from happening spontaneously? Thermodynamics tells us the universe will increase the number of states, so if this reaction can happen, it should happen all the time, everywhere, and the universe ought to disappear into a burst of (energy plus negative energy). Since the universe does not spontaneously generate showers of positive and negative-mass matter, we will assume that there is a conservation of some (possibly still unknown) quantum number that prevents this. Also, the fact that vacuum doesn't spontaneously decay into positive and negative mass tells us that negative mass doesn't react with positive mass on contact to produce nothing ("nullification," as proposed by Forward). This would be the reverse of spontaneous generation, so since that doesn't occur, nullification doesn't either.

As first pointed out by Morris and Thorne [5] and others [1, 5], a violation of the null energy condition is also a requirement for a "wormhole," and indeed, for any proposed faster than light travel [1]. Strictly, what is required is that in the region of the wormhole throat, the total tension (τ) is greater than the energy density (mc^2), and hence in at least some reference frame, the energy density (and hence the mass density) is less than zero. It has since been shown that this condition is general: any faster than light travel implies that at least some parallel light beams must diverge, and the law of geodesic deviation indicates that this must mean negative mass density (in at least some reference frame.)

Although negative mass was initially considered to be a theoretical curiosity unrelated to the real world, in modern physical theory, negative mass appears in many places.

IV. Negative Energy in Contemporary Physics

Although negative energy density was rejected by Einstein in the formulation of the General Theory of Relativity, effects that imply negative energy density ("exotic matter" or "exotic energy") are now commonly accepted in theoretical physics.

The mass of the vacuum is, by definition, zero. In the quantum-mechanical vacuum, however, while the *time average* of the energy density is zero, the instantaneous value of the energy will fluctuate as pairs of virtual particles appear and disappear. Since empty space consists of the true vacuum plus these transient virtual particles, if virtual particles have positive mass, "bare" empty space must have negative mass density. One way of manifesting this negative mass density is the Casimir vacuum. In a Casimir cavity, conductive plates bound a region of vacuum. The boundary condition on these places excludes some of the electromagnetic modes of the virtual photons present in the vacuum, and hence the vacuum between the plates is of lower energy density. This manifests in the form of an attraction between the two plates, as the higher energy true vacuum outside presses in against the negative energy density vacuum; the "Casimir effect". This Casimir vacuum was proposed by Morris, Thorne, and Yurtsever [6] as something that could serve as the negative energy to stabilize a wormhole throat (although later researchers showed that any hole in the negative energy, required to allow an object to reverse the wormhole, would destroy the wormhole faster than the time required to traverse it.).

A similar effect means that the vacuum surrounding a black hole, the Hartle-Hawking vacuum, is also of negative energy density.

Negative energy density has since become ubiquitous in cosmology as well. The best current model of the early universe suggests that the initial era incorporated cosmological inflation, a period of time in which the size of the universe inflated at a rate much greater than the speed of light. This cosmological inflation inherently implies a negative energy density. Likewise, current measurements show an increase in the rate of expansion of the universe. This expansion, referred to as "dark energy," is also an example of negative energy density.

It is interesting to note that negative energy density exists in purely classical physics as well. It is well understood that electromagnetic fields contain energy (of energy density equal to the square of the field strength). From Einstein's relation (eq. 2), this energy must contribute to the mass of any charged elementary particle. But for a charged point particle, the volume integral of the energy in the electric field is infinite. Therefore, classically, since the total mass of a charged particle is finite, the particle itself must have a "bare" mass that is negative (and infinite).

This bare negative mass can be directly seen, for example, in the Reissner-Nordström solution of the Einstein equations. In general relativity, the line element for a (non-rotating) spherical charged mass is:

$$g_{\mu\nu} = - (1 -(M/r +(q/r)^2)c^2dt^2 + (1 +M/r -(q/r)^2)\, dr^2 + r^2(d\theta^2+\sin^2\theta d\varphi^2) \qquad (4)$$

One can directly see that the effect of mass, M, is of opposite sign to that of charge, q: charge acts oppositely to gravity. What happens is that as the radial coordinate r comes increasingly close to r=0, the energy density of the electric field outside that radius becomes larger, and hence the mass interior to that radius r is smaller by an amount equal to the square of the electrical potential. At a certain distance from a charged particle, the time and radial components of $g_{\mu\nu}$ swap from greater to unity to less than unity, and the gravity reverses to that of a negative mass. (For an electron, this occurs at a distance of about 10^{-34} cm; too small for the reversal of the (already small) gravitational force to be experimentally measured).

Other effects in theoretical physics also imply negative energy.

While negative energy has moved from a theoretical curiosity to a concept fundamental to the contemporary understanding of physics, it is still not clear whether bulk negative mass can be manufactured, or if it is limited to only appearing at the cosmological scale (*e.g.,* "dark energy:) and in the quantum (*e.g.,* Casimir vacuum) limit.

V. Conclusions

Although Einstein rejected it, matter of negative mass is not disallowed by the general theory of relativity. Negative mass doesn't fall upwards… but in other ways its properties would be extremely strange.

Since the inertial of negative mass subtracts from that of positive mass, a collection of negative and positive mass could have arbitrarily low, or even zero, inertia. Thus, it could accelerate to arbitrary velocity with little or no expenditure of energy.

Despite the odd properties, some theoretical objects with the properties of negative mass are now being accepted in mainstream theoretical physics, although it is not “matter as we know it”. Whether conventional matter of negative-mass could exist in bulk form is still a question open for debate.

References

[1] Visser, M., “12: Energy Conditions,” in *Lorentzian Wormholes,* American Institute of Physics Pess, Woodbury NY (1995), pp. 115-136. ISBN 1-56396-394-9

[2] Bondi, H., “Negative Mass in General Relativity,” *Review of Modern Physics, Vol. 29,* No. 3 (1957), pp. 423-428.

[3] Forward, R. L., “Negative Matter Propulsion,” *Journal of Propulsion and Power, Vol. 6,* No. 1 (1990), pp. 28-37.

[4] Landis, G. A., “ Comments on Negative Mass Propulsion,” *Journal of Propulsion and Power, Vol. 7,* No. 2 (1991), p. 304.

[5] Morris, M. S., and Thorne, K. S., “Wormholes in spacetime and their use for interstellar travel: A tool for teaching general relativity.” *American Journal of Physics 56,* No. 5 (1988), pp. 395-412.

[6] Morris, M. S., Thorne, K. S., and Yurtsever, U., “Wormholes, time machines, and the weak energy condition, “*Phys. Rev. Lett., 61* (1988), pp. 395-412.