Negative Mass in Contemporary Physics

and its Application to Propulsion

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What do we mean by Mass?

Physicists mean several different things when we refer to “mass”

- Inertial mass $F=m_i a$
- Active (source) gravitational mass $g=GM_g/r^2$

*(more complicated in General Relativity)

- Passive gravitational mass $F_g=m_g g$
- Schrödinger mass $ihd\psi/dt=-(h^2/2m_s)d^2\psi/dx^2+V$

*(or the corresponding term in Dirac or relativistic Schrödinger equation)

- Conserved mass/energy equivalent of mass: $m_o=E/c^2$

*(relativistic 4-vector)

- Quantum number m (*e.g.,* $m_{e^-}= 511\ MeV/c^2$)

These terms are related by constitutive relations

Inertial mass=gravitational mass

Equivalence principle

Active gravitational mass = Passive gravitational mass:

Newton's third law

Schrödinger mass= inertial mass:

correspondence principle

Energy equivalent mass= inertial mass:

special relativity

Quantum number

quantum indistinguishability

Can mass ever be negative?

- Einstein put “energy condition” into general relativity that energy must be positive.
  - $E=Mc^2$; so positive energy = positive mass
  - Condition added to avoid “absurd” solutions to field equations
  - Many possible formulations: null, weak, strong, dominant energy condition; averaged null, weak, strong, dominant energy condition
- Bondi pointed out that negative mass is not contradictory to GR (*Rev. Mod. Phys.*, 1957)
  - However, he pointed out strange consequences!
  - Not the same as Dirac negative energy (mass) states
  - Forward elaborated consequences 1990
  - Landis commented on Forward 1990

Negative mass reacts oppositely to all forces!

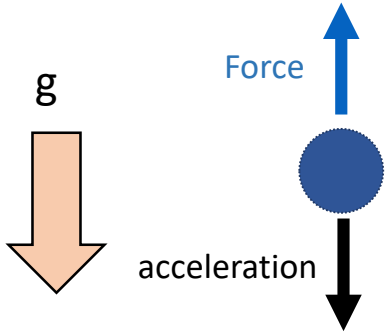
- Force and acceleration are opposite
- F = ma
  - When you push on negative mass it moves toward you
  - When you pull on negative mass it moves away from you

Negative mass still falls “down” in a gravitational field

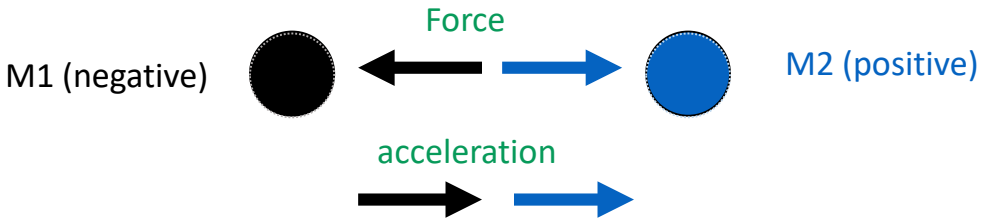
- Force of gravity is opposite:

$$F = GmM/r^2$$

  - so when m is negative, gravity pushes
- but
  - When you **push** on negative mass it moves **toward** you
  - So negative mass still is attracted downward (toward positive mass)



Negative mass pushes positive mass *away* gravitationally

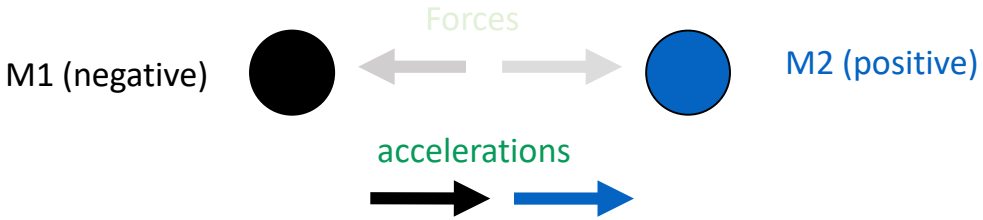


- $F = GM_1M_2/r^2$; so if M_1 is negative, gravity pushes instead of pulls
- Positive mass moves away from negative mass
- Negative mass moves toward positive mass
- Negative and positive mass chase each other!

Motion with no external force!

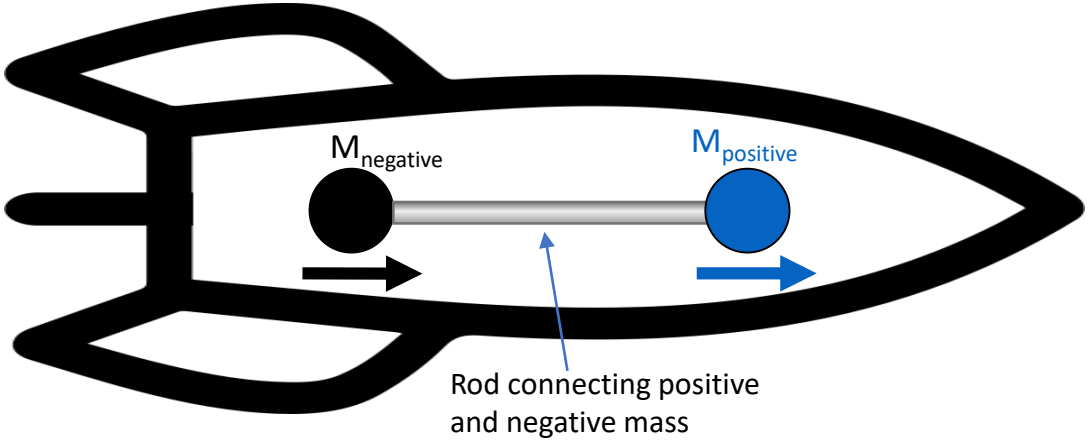
Forward pointed out this result applies to all forces between a negative and positive mass, not just gravity, but springs, rods, electric fields, etc.

Negative mass chasing positive mass



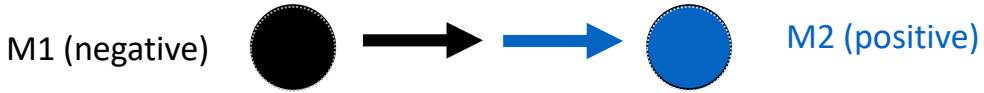
- **But… momentum and energy are both still conserved:**
  - negative mass has negative momentum ($p = mV$)
  - and negative energy ($E = \frac{1}{2} mV^2$)
  - So the energy and momentum cancel out

Negative mass and positive mass for propulsion (as proposed by Forward)



- Push on the rod and the positive mass moves to the right… and so does the negative mass
- Spaceship moves to the right

masses stay at constant distance only masses are equal in magnitude

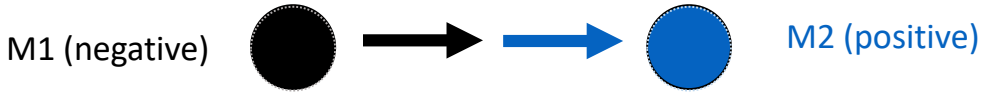


- **Equal mass:**
  - Acceleration of negative mass equals acceleration of positive

- **Negative mass larger:**
  - Acceleration of positive mass is higher (a = F/m): masses move apart

- **positive mass larger:**
  - Acceleration of negative mass equals acceleration of positive: masses catch up

masses stay at constant distance only masses are equal in magnitude



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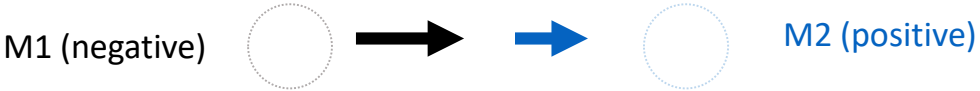

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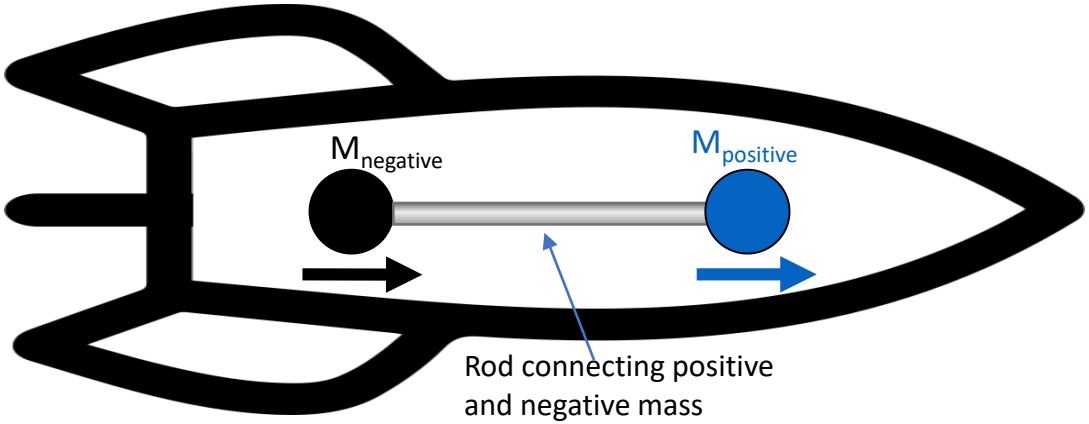
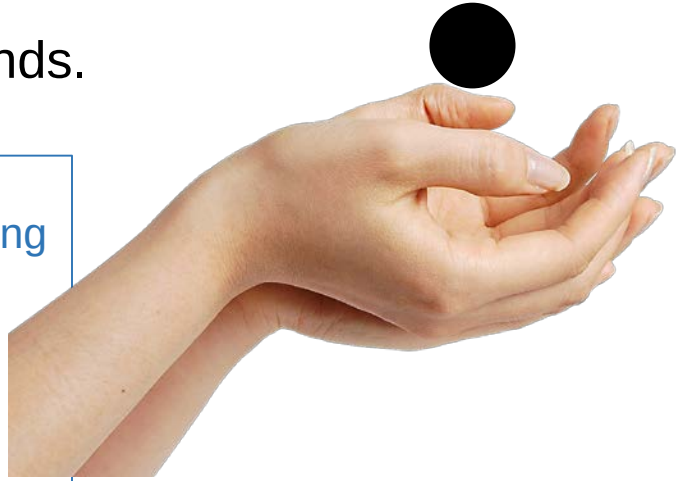


- The propulsion system only works if total of negative & positive mass = zero
- Spaceship with zero mass can move without external force
- This also means its response to external force is nearly infinite
  - Will be buffeted around by every impact of dust or cosmic blackbody photon

What would happen if you held a piece of negative mass in your hands?

- Nothing-- it would fall right through your hands.



- The reason solid matter is “solid” is that the **Pauli exclusion principle** prevents electrons from passing through other electrons in the same energy state
- Whatever negative matter is… it’s *not* electrons, protons, and neutrons in the same state as your hand
  - no exclusion principle, no way for you to hold it
- Negative matter could still be manipulated by fields

What would happen to bulk negative matter in the early universe?

- Uncharged negative matter would repel itself gravitationally.
- Charged negative matter would attract itself strongly
  - For positive matter, like charges push each other apart, so ordinary matter is bulk neutral
  - For negative matter, like charges push each other apart, so they move toward each other: **negative matter is not bulk neutral**
  - A universe of negative matter would look like a universe of positive matter if gravity and electrostatic forces switched: **gravity repels and electrostatic forces attract**
  - But… the electrostatic force is 10^{36} times stronger than gravity!
  - Negative matter would accumulate into huge charged balls… which push away balls of opposite charge
    - And also gravitationally push away normal matter
    - Speculation: could this be the “voids” we see in the universe?

Cosmic voids

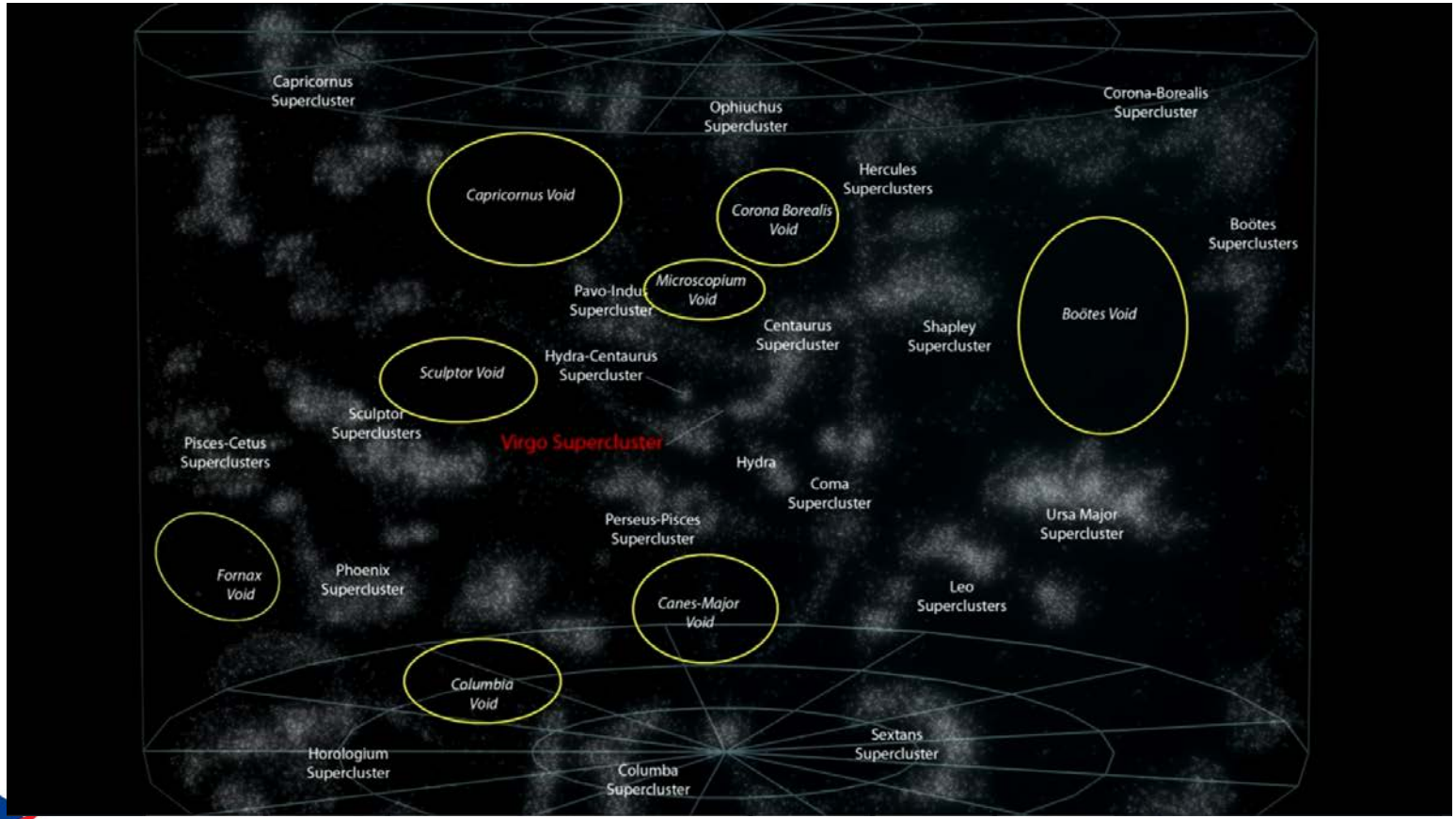
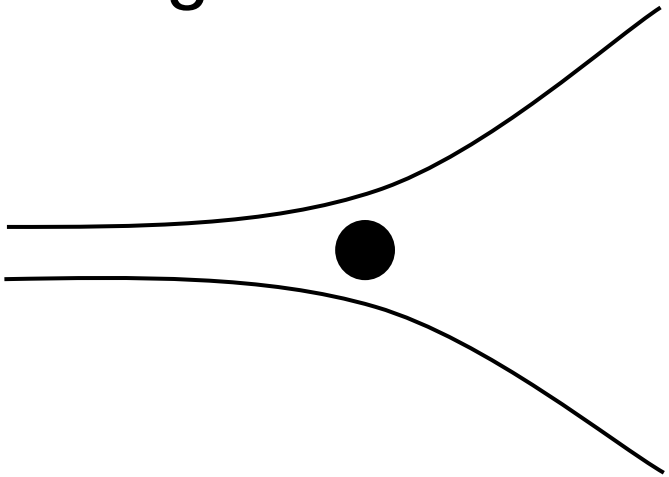


Image courtesy Wikipedia

In GR, negative mass means parallel geodesics diverge

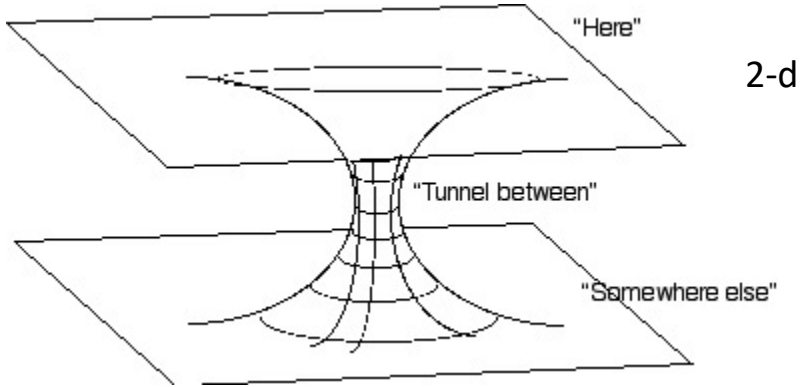


Equation of geodesic deviation: Second derivative of the distance between geodesics is the mass density* (in the co-moving frame)

*more specifically, a function of the stress-energy tensor, which reduces to the mass density in a properly chosen co-moving frame

Diverging geodesics are a geometrical result of any wormhole:

Negative mass* is required to make a wormhole!



2-d embedding

*or the equivalent

Follow the parallel lines-- they diverge if they go through the wormhole.

This is a necessary result of the geometry

Theory suggests that negative mass is required for any FTL travel

Existence of stable negative-mass matter is hard to reconcile with the existence of vacuum
(in quantum field theory)

Consider the reaction

Vacuum → m(positive) + m(negative)

- (Conservation of momentum indicates this must be at least a three-particle reaction)

- It costs nothing to produce negative matter
  - In fact, the energy cost is less that zero: making negative mass *generates* energy
- What stops this from happening spontaneously?
  - Thermodynamics says the universe will increase the number of states
  - If this reaction can happen, it should happen all the time, everywhere
  - The universe ought to disappear into a burst of energy plus negative energy.

Since the universe doesn't spontaneously generate showers of positive and negative-mass matter, we will assume that there is a conservation of some (possibly still unknown) quantum number that prevents this

Since vacuum doesn't spontaneously generate negative mass, we also know that negative mass doesn't react with positive mass on contact to produce nothing ("nullification," as proposed by Forward).

- this would be the reverse of spontaneous generation, so since that doesn't occur, nullification doesn't either.

Some types of negative mass/energy is now conventional thinking in modern physics

- “Negative energy” is the force behind cosmic inflation, which is now our **standard theory** of the early moments of the big bang
- cosmological constant ("quintessence,” “dark energy”) seems to exist
- “negative energy” is pushing the universe apart

Negative mass/energy in classical physics

- **“bare” mass of elementary particles**
  - Electromagnetic fields have energy,
    - so particle mass = the bare mass of the particle, plus integral of the field mass density (E^2+B^2)
    - But E^2+B^2 diverges
  - Bare mass must be negative (aka “renormalization of mass”)
  - But… how do you strip away the field?

Things that act like negative mass/energy in General Relativity

- Charge acts negative to mass in Einstein equation
  - In fact, the extended solution of a charged black hole contains a wormhole

Reissner-Nordstöm solution:

$$g_{\mu\nu} = -(1-\phi_{gravitational}+\phi_{electric}^{2})c^2dt^2$$

$$+(1+\phi_{gravitational}-\phi_{electric}^{2})\,dr^2$$

$$+r^2(d\theta^2+\sin^2\theta d\varphi^2)$$

Where

$\Phi_{gravitational}=M/r$

$\Phi_{electric}=q/r$

Things that act like negative mass/energy in modern physics

- **mass fluctuations of vacuum**
  - Time average energy E = zero… but due to uncertainty, this fluctuates.
- **"bare" mass of vacuum, with no virtual particles**
  - Empty space = true vacuum plus virtual particles
  - If virtual particles have positive mass, “bare” empty space must have negative mass
  - but how do you strip the virtual particles?
  - Casimir vacuum
    - If you put conducting parallel plates close together, these plates can forbid some modes of the virtual photons of the quantum vacuum
    - The Casimir vacuum between the plates, stripped of these virtual photons, has energy less than zero
    - This is the Casimir effect, which can be measured experimentally

Things that act like negative mass/energy in quantum gravity

- Hawking/Hartle vacuum (virtual particles surrounding a black hole)
- Unruh vacuum (virtual particles surrounding an accelerated reference frame)

Quick and dirty visualization: space is full of transiently existing virtual particle pairs, which have zero net energy. In a gravitational field, the particles relax to lower energy. Since they started at zero energy, lower energy is less than zero.

This is a side effect of the celebrated “Hawking radiation”

Things that act like negative mass/energy in speculative quantum physics

- Intrinsically negative rest mass particles
  - Elementary particles exist with a wide range of non-negative masses. It is not ruled out that not-yet-discovered particles exist with negative rest mass.
  - Why haven't we seen them?
- Negative mass states of virtual particles "off the mass shell"
  - May exist transiently in the quantum foam
- Particles in the negative-energy Dirac Sea
  - Note that "the Dirac sea" is a mostly-obsolete way of looking at the Dirac equation. Not actually discredited, but no longer used because it's not very useful
  - There's no reasonable approach to make that consistent with General Relativity (you can't simply renormalize and ignore background infinity!)

Things that act like negative mass/energy in cosmology

- Dark energy
  - The energy that causes the Hubble expansion to accelerate
  - This now the standard model of cosmology
- Cosmic Inflation during the Big Bang
  - During the earliest stage of the big bang, “inflation” caused the universe to expand faster than the speed of light
  - Inflation required negative energy
  - This now the standard model of cosmology

Some more speculative forms of negative mass/energy

- Negative mass cosmic strings.
  - A cosmic string is a geometrical flaw in the structure of space; in principle a cosmic string could have either positive or negative mass
  - Could exist from the early universe (however, if inflation theory is correct, expected number density = one per universe)
  - However, positive mass strings are under tension, while negative mass strings would have negative tension: how do you make a string with negative tension stable?
- Negative mass wormhole mouths
  - Gravitational field lines coming out of a wormhole mouth can look like negative mass
  - An effective negative mass can be created from a wormhole mouth from which mass emerges
  - Requires a wormhole

Conclusions

- Negative mass is allowed by the general theory of relativity
- Negative mass doesn't fall upwards… but in other ways its properties would be extremely strange
- Since the inertial of negative mass subtracts from that of positive mass, a collection of negative and positive mass could have arbitrarily low, or even zero, inertia.
  - Could accelerate to arbitrary velocity with little or no expenditure of energy!
- Nevertheless, some objects with the properties of negative mass are now being accepted in mainstream physics
  - But it's not "matter as we know it"

Things that act like negative mass/energy in General Relativity

- **“mass” of gravitational field.**

Gravity is a source of gravity, but formally gravity doesn’t have mass in General Relativity: the “mass” of gravity is accounted in the nonlinearity of the theory.

Gravitational waves have positive mass

Static gravitational field has apparent **negative** mass

–Take two positive mass particles, M(initial)=m_1+m_2

–Allow them to move together, and energy is released (and by conservation, total mass decreases)

–The total mass can be assigned as the mass of the two individual particles, plus the gravitational field,

M(final)=m_1+m_2+m(gravity)<M(initial)