Multiple Exciter Placement for Ground Vibration Test of X-59 Aircraft using Topology Optimization

Prepared By:
Chan-gi Pak, Ph.D.
Structural Dynamics Group, Aerostructures Branch (Code 560)
NASA Armstrong Flight Research Center

Model Correlation Requirements

- **References:** The military and NASA standards for **comparing the analytical and experimental modal data** are observed in
  - MIL-STD-1540C   Section 6.2.10
  - NASA-STD-5002   Section 4.2.6.d
  - AFFTC-TIH-90-001 (Structures Flight Test Handbook)

- **Frequency correlation**
  - Primary modes: within **5%** (NASA-STD) or **3%** (MIL-STD) of test frequencies

- **Mass orthogonality**
  - Use orthogonality matrix: $\mathbf{\Phi_G}^T \mathbf{M \Phi_G}$
    - $\mathbf{\Phi_G}$ = mode shape from GVT (**mass normalize** for measured modes)
    - $\mathbf{M}$ = **analytical** mass matrix ($\mathbf{M} = \mathbf{\Phi_{TMP}}^T \mathbf{\Phi_{TMP}}$; $\mathbf{\Phi_{TMP}} = \left(\mathbf{\Phi_A}^T \mathbf{\Phi_A}\right)^{-1} \mathbf{\Phi_A}^T$)
  - Primary modes: **off-diagonal terms** should be less than **10%** (Use orthonormalized matrix: 0.1 when diagonal is 1.0)

.1

Mode	7	8	12	14	15	16	17	18	20	22	26	30	31	34	35	36	37
7		.000	.000	.000	.000	-.001	-.002	-.002	-.001	-.004	.000	.000	.000	-.003	-.001	-.001	-.002
8	.000		.000	.000	.000	.000	.000	.000	.000	.002	.000	.000	.000	.000	.002	.000	.000
12	.000	.000		.000	.000	.000	.000	.000	.000	-.001	.000	.000	.000	.000	-.001	.000	.001
14	.000	.000	.000		.000	.000	.001	.001	.000	.001	.000	.000	.000	.001	.000	.000	-.001
15	.000	.000	.000	.000		.001	.004	.003	.001	.006	.000	.000	.000	.003	.001	.000	.001
16	-.001	.000	.000	.000	.001		-.005	-.001	-.004	-.006	.000	-.001	.000	-.006	.000	-.001	.000
17	-.002	.000	.000	.001	.004	-.005		**.098**	-.004	-.019	[illegible]	-.001	.001	-.012	-.003	-.002	-.004
18	-.002	.000	.000	.001	.003	-.001	**.098**		-.012	-.036	.000	-.003	.001	-.025	-.005	-.003	-.008
20	-.001	.000	.000	.000	.001	-.004	-.004	-.012		-.006	.000	.000	.000	-.002	-.001	.000	.000
22	-.004	.002	-.001	.001	.006	-.006	-.019	-.036	-.006		.000	-.007	.003	.013	.000	-.002	.004
26	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000		.000	.001	.001	.000	.000	.000
30	.000	.000	.000	.000	.000	-.001	-.001	-.003	.000	-.007	.000		.000	.025	.016	.001	-.006
31	.000	.000	.000	.000	.000	.000	.001	.001	.000	.003	.001	.000		-.002	-.001	-.001	.002
34	-.003	.000	.000	.001	.003	-.006	-.012	-.025	-.002	.013	.001	.025	-.002		-.007	-.012	.008
35	-.001	.002	-.001	.000	.001	.000	-.003	-.005	-.001	.000	.000	.016	-.001	-.007		-.002	.012
36	-.001	.000	.000	.000	.000	-.001	-.002	-.003	.000	-.002	.000	.001	-.001	-.012	-.002		.005
37	-.002	.000	.001	-.001	.001	.000	-.004	-.008	.000	.004	.000	-.006	.002	.008	.012	.005	

- **Mode shape correlation**
  - Use cross-orthogonality matrix: $\mathbf{\Phi_G}^T \mathbf{M \Phi_A}$
    - $\mathbf{\Phi_A}$ = mode shape from analysis (**mass normalize** for numerical modes: **use "MASS" option with NASTRAN**)
  - Primary modes:
    - **Diagonal terms** should be greater than **0.9** (NASA-STD) or **0.95** (MIL-STD)
    - **Off-diagonal terms** should be less than **10%** (0.1)

.9

Mode	7	8	12	14	15	16	17	18	20	22	26	30	31	34	35	36	37
7	1.00	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
8	.000	1.00	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
12	.000	.000	.977	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
14	.000	.000	.000	.943	.001	.000	.000	-.001	.000	.000	.000	.000	.000	.000	.000	.000	.000
15	.000	.000	.000	-.001	.941	.002	-.001	-.004	.000	.000	.000	.000	.000	.000	.000	.000	.000
16	-.001	.000	.000	.000	.003	-.948	.002	.011	.001	-.001	.000	-.001	.000	.000	.000	.000	.000
17	-.002	.000	[illegible]	.001	.005	.008	**.853**	**.146**	.003	-.003	.000	-.001	.001	-.001	.000	-.001	.000
18	-.002	.000	.000	.001	.006	.011	-.066	.921	.002	-.003	.000	-.002	.001	-.001	-.001	-.001	.000
20	-.001	.000	.000	.001	.001	.005	-.005	-.014	[illegible]	-.002	.000	.000	.000	.000	.000	.000	.000
22	-.005	.002	.000	.001	.006	.005	-.010	-.033	-.004	**-.812**	.000	-.011	.003	-.007	-.004	-.004	-.002
26	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.934	-.001	.001	.000	.000	.000	.000
30	.000	.000	.000	.000	.000	.000	.000	-.001	.000	.000	.000	**.815**	.000	-.002	.000	.000	.000
31	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	-.001	-.001	.904	.000	.000	.001	.000
34	-.003	.000	.000	.001	.003	.005	-.006	-.022	-.002	.013	.002	.032	-.004	.949	-.017	-.015	-.012
35	-.001	.002	-.001	.000	.000	.000	-.002	-.004	-.001	.002	.000	.018	-.001	.007	.935	-.005	-.003
36	-.001	.000	.000	.000	-.001	.001	.000	-.002	.000	.001	.000	.002	-.002	.002	.003	.921	-.002
37	-.002	.000	.001	-.001	.001	.000	-.002	-.007	.000	.004	.000	-.007	.002	.015	.013	.006	**.889**

.1

Create Reduced Order Model

1) Reduced Order Model is needed for a future structural model tuning.
2) Check the accuracy of the Reduced Order Model.

Full Order Model

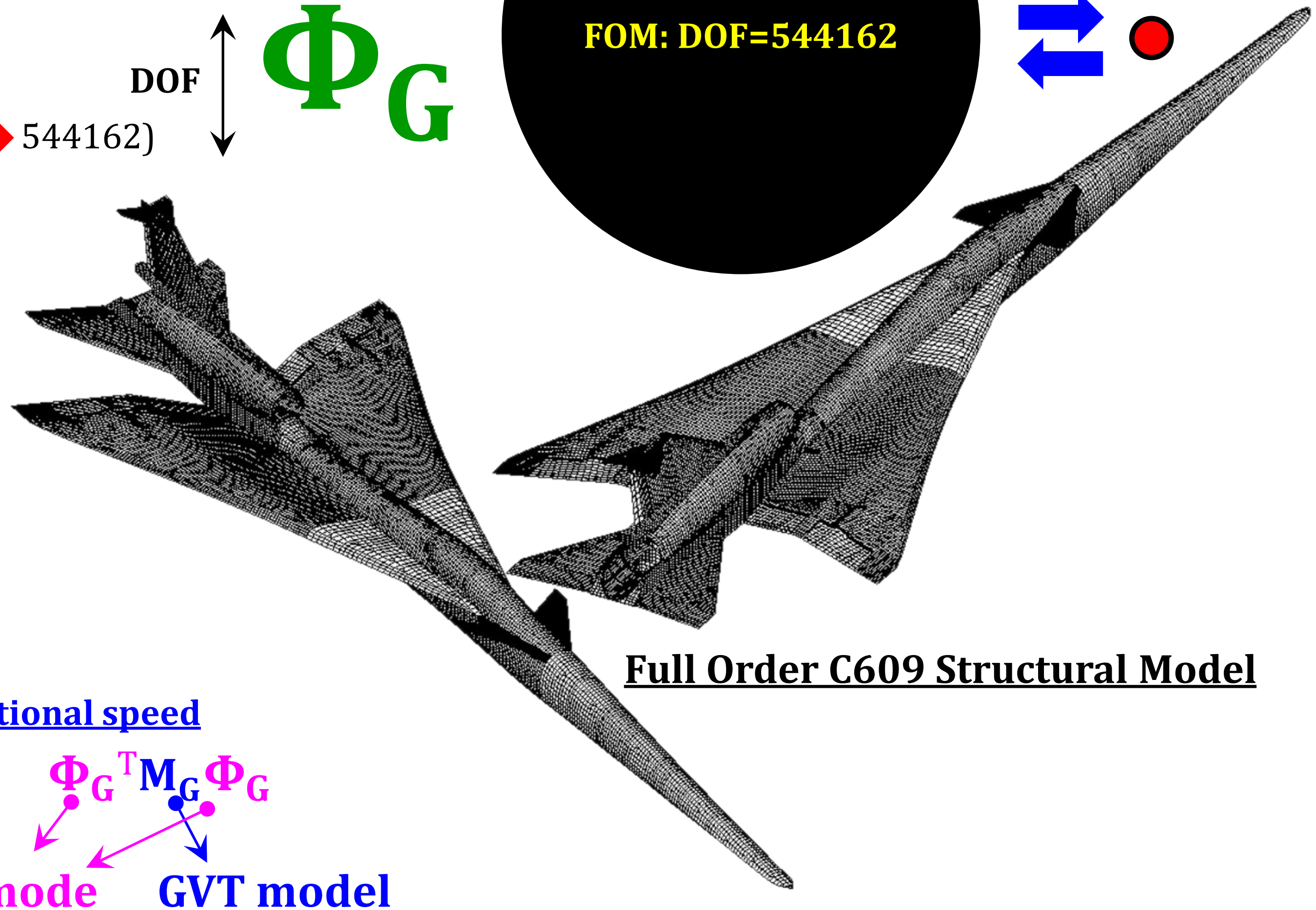
- Use Cruise Design Weight configuration: during methodology development using C609 Model
- Analytical Model: Full Order Model (FOM). (ex. X-59 C609 FEM; DOF = 544162)
  - **M**: 544162 × 544162
  - $\mathbf{\Phi_F}$: 544162 × *nmode*
- GVT Data: from accelerometers $\quad \Phi_G{}^T M \Phi_G \quad \Phi_G{}^T M \Phi_A$
  - $\mathbf{\Phi_G}$: 40 × *nmode*
- Expand measured mode shape $\Phi_G (= \Phi_M)$ to $\mathbf{\Phi_F}$ (ex. DOF = 40 ➡ 544162)

$$\Phi_{EF} = \begin{bmatrix} \Phi_M \left(\Phi_M{}^T \Phi_M\right)^{-1} \Phi_M{}^T \\ \Phi_S \left(\Phi_M{}^T \Phi_M\right)^{-1} \Phi_M{}^T \end{bmatrix} \Phi_G$$

$\Phi_{EF}{}^T M \Phi_{EF}$

Expanded mode $\quad$ **FOM**

$\Phi_{EF}{}^T M \Phi_F$



Full Order C609 Structural Model

  - Issues associated with **computer memory size** and **computational speed**
  - Matrix multiplication is **not** possible with FOM
- Or reduce the mass matrix **M** to $\mathbf{M_G}$ (ex. DOF = 544162 ➡ 40)
  - Over-simplified
  - **Accuracy issue**

$\Phi_G{}^T M_G \Phi_G$

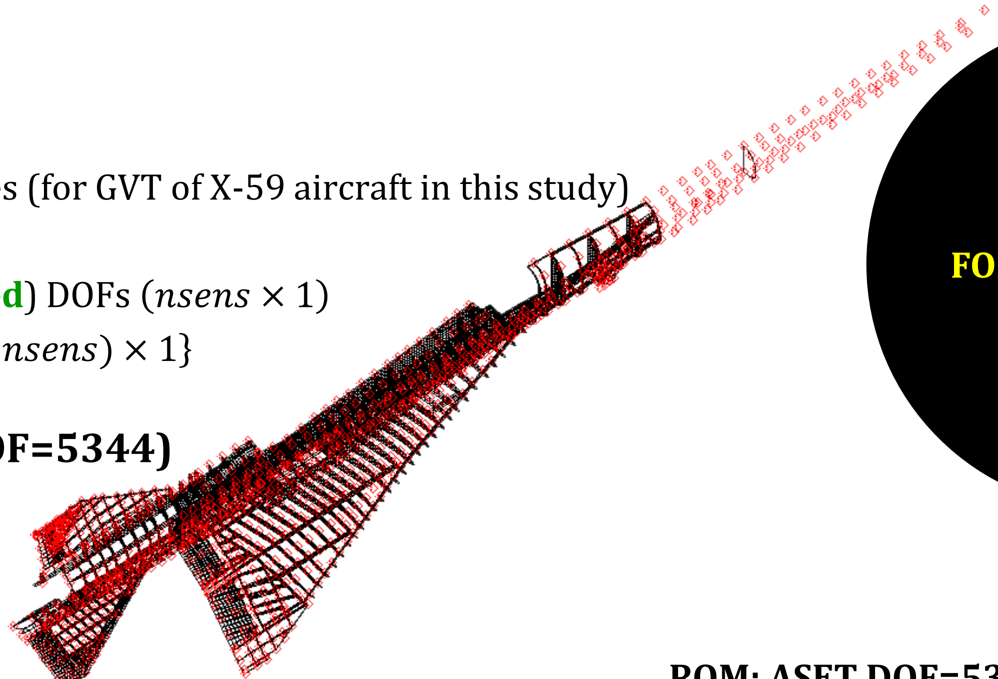
Measured mode $\quad$ **GVT model**

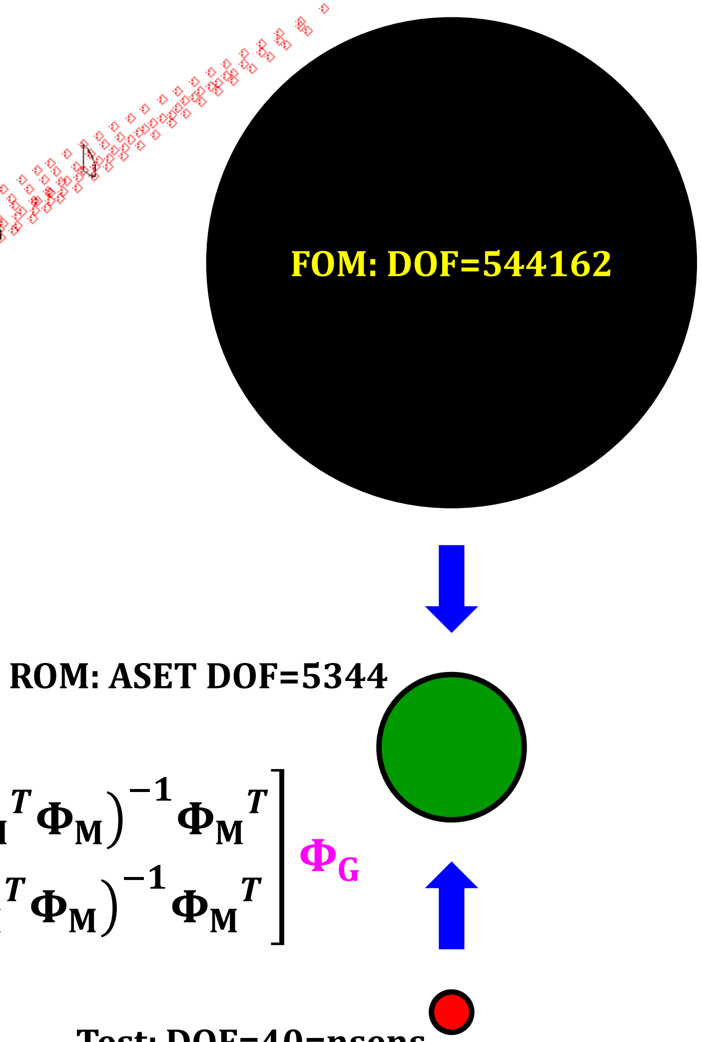
$\Phi_G{}^T M_G \Phi_{RG}$

Reduced Order Model

- Create mirror image of input grid (right-hand side) **except vertical tail**
  - Use MSC/PATRAN, create candidate grid points for an accelerometer placement study. (**candidate accelerometer locations**)
- Read in NASA's ASET1 cards, then keep centerline and right-hand side grid. (ex. DOF = 544162 → 5344)
- Need to create analytical Reduced Order Model (ROM)
- $\mathbf{\Phi}_\mathrm{R}$= mode shape from reduced order model (5344 × 1)
  - $\mathbf{\Phi}_\mathrm{R} = \begin{bmatrix} \mathbf{\Phi}_\mathrm{M} \\ \mathbf{\Phi}_\mathrm{S} \end{bmatrix}$
    - Use 40 modes for computations of correlation matrices (for GVT of X-59 aircraft in this study)
    - $nsens$: number of sensor (accelerometer)
    - $\mathbf{\Phi}_\mathrm{M}$: mode shapes correspond to **master** (or **measured**) DOFs ($nsens \times 1$)
    - $\mathbf{\Phi}_\mathrm{S}$: mode shapes correspond to **slave** DOFs $\{(5344 - nsens) \times 1\}$
    - $\mathbf{\Phi}_\mathrm{R}$:$5344 \times lmode$
- **M = mass matrix from reduced order model**
  - $\mathbf{M} = \mathbf{\Phi}_\mathrm{TMP}{}^T \mathbf{\Phi}_\mathrm{TMP}$
    - $\mathbf{\Phi}_\mathrm{TMP} = \left(\mathbf{\Phi}_\mathrm{R}{}^T \mathbf{\Phi}_\mathrm{R}\right)^{-1} \mathbf{\Phi}_\mathrm{R}{}^T$
    - $\mathbf{M}$: 5344 × 5344
- $\mathbf{\Phi}_\mathrm{G} = \mathbf{\Phi}_\mathrm{M}$ = measured mode shape ($nsens$ × 1) (mode shapes correspond to the measured master DOFs)
  - GVT Data: compute expanded GVT data (ex. DOF = 40 → 5344)
    - $\mathbf{\Phi}_\mathrm{E}$:$5344 \times lmode$
    - Matrix multiplication is possible.

ROM (DOF=5344)





$\mathbf{\Phi}_\mathrm{E}{}^\mathrm{T} \mathbf{M} \mathbf{\Phi}_\mathrm{E}$

Expanded GVT Data   Reduced Analytical Model

$\mathbf{\Phi}_\mathrm{E}{}^\mathrm{T} \mathbf{M} \mathbf{\Phi}_\mathrm{R}$

$$\mathbf{\Phi}_\mathrm{E} = \begin{bmatrix} \mathbf{\Phi}_\mathrm{M}\left(\mathbf{\Phi}_\mathrm{M}{}^T \mathbf{\Phi}_\mathrm{M}\right)^{-1} \mathbf{\Phi}_\mathrm{M}{}^T \\ \mathbf{\Phi}_\mathrm{S}\left(\mathbf{\Phi}_\mathrm{M}{}^T \mathbf{\Phi}_\mathrm{M}\right)^{-1} \mathbf{\Phi}_\mathrm{M}{}^T \end{bmatrix} \mathbf{\Phi}_\mathrm{G}$$

Primary Modes

- There's no big differences between mode shapes and numbers from zero fuel weight (ZFW) and cruise design weight (CDW) configurations.
- The primary modes are selected using flutter analyses and the basis functions for a jig shape optimization study of the X-59 LBFD aircraft.
  - Flutter analyses are based on ZFW and CDW weight configurations.
  - Jig shape optimization was based on CDW weight configuration.

17 Modes

Aerodynamic model

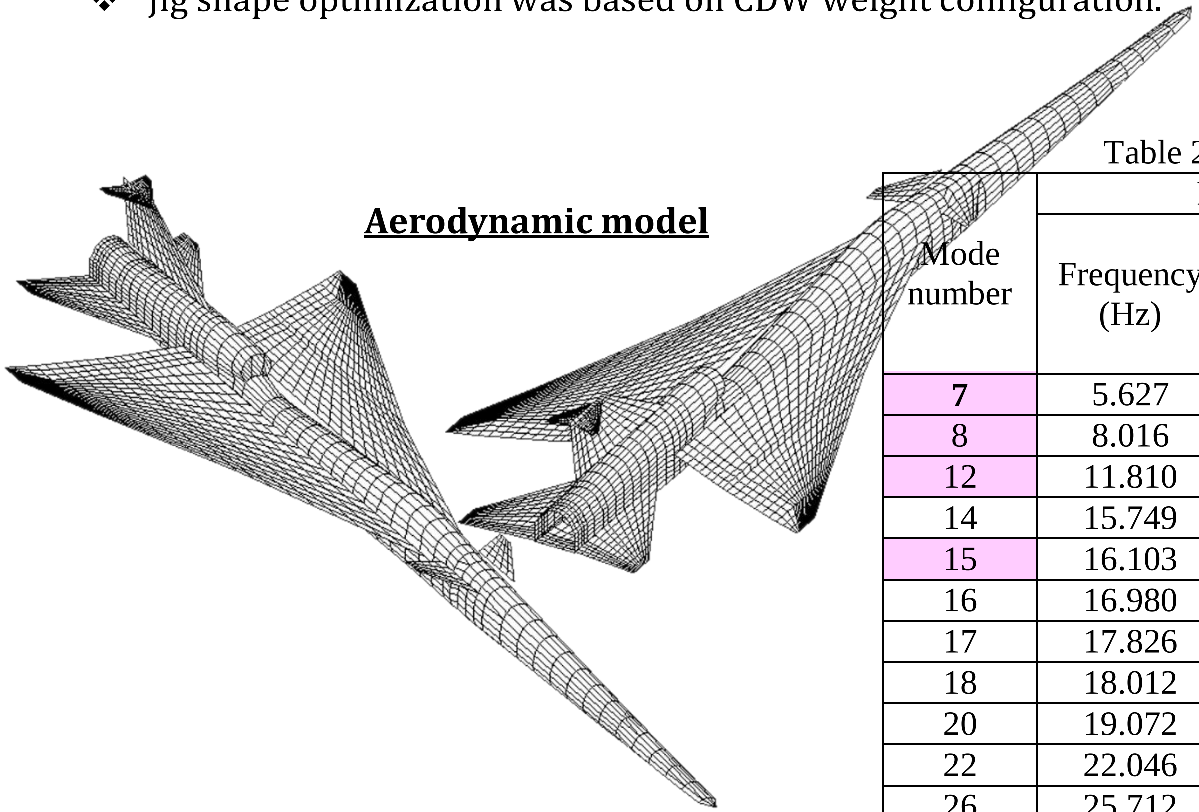


Table 2. Selection of primary modes from flutter analyses and jig shape optimization

Mode number	FOM using ZFW			FOM using CDW			
	Frequency (Hz)	Primary mode ranking	Modal participation factor	Frequency (Hz)	Primary mode ranking	Modal participation factor	Modal participation factor for basis functions used in jig shape optimization
7	5.627		0.0000	5.548		0.0000	0.1398
8	8.016		0.1033	7.793		0.0004	-1.4535
12	11.810		0.0703	11.468		0.1404	0.2762
14	15.749	2	34.25	15.742	1	37.46	
15	16.103		0.0009	16.092		0.0034	0.6809
16	16.980	10	1.259	16.802	6	1.057	
17	17.826	7	1.847	17.442		0.0091	
18	18.012		0.1195	17.726	5	1.270	
20	19.072	5	3.210	18.647		0.7743	-0.3346
22	22.046	1	34.97	21.686	2	37.36	
26	25.712	8	1.746	24.767		0.0831	0.1245
30	27.461	4	5.949	27.417		0.3513	-0.02954
31	28.261	9	1.434	27.699	4	1.543	
34	29.902	6	3.039	29.334		0.1395	
35	30.852		0.2033	30.600		0.1045	0.04405
36	31.855		0.6808	31.833		0.2679	-0.4546
37	34.799	3	7.116	33.229	3	8.142	

Frequency Comparisons: Full Model vs. Reduced Order Model

Table 4. Frequency comparison between results from FOM, LM's ROM, and NASA's ROM

FOM (544,162)		LM's ROM (1,879)			NASA's ROM (5,344)		
Mode number	Frequency (Hz)	Mode number	Frequency (Hz)	% error	Mode number	Frequency (Hz)	% error
7	5.548	**7**	5.552	0.07	7	5.550	0.03
8	7.793	8	7.797	0.05	8	7.795	0.02
12	11.175	12	11.477	0.08	12	11.470	0.03
14	15.742	14	15.753	0.07	14	15.750	0.05
15	16.092	15	16.105	0.08	15	16.101	0.06
16	16.802	16	16.824	0.13	16	16.814	0.07
17	17.442	17	17.531	0.51	17	17.498	**0.32**
18	17.726	18	17.744	0.10	18	17.737	0.06
20	18.647	20	18.680	0.17	20	18.659	0.06
22	21.686	22	21.801	0.53	22	21.741	0.26
26	24.767	25	24.855	0.36	25	24.792	0.10
30	27.417	29	27.438	0.08	29	27.429	0.04
31	27.699	30	27.810	0.40	30	27.774	0.27
34	29.334	32	29.584	**0.85**	32	29.349	0.05
35	30.600	33	30.710	0.36	33	30.640	0.13
36	31.833	34	31.888	0.17	34	31.866	0.11
37	33.229	35	33.457	0.69	35	33.282	0.16

Accuracy of Reduced Order Model: : FOM(544162) vs. NASA's ROM(5344)

- MAC matrix: $\mathbf{MAC}_{ij} = \dfrac{\left(\sum_{k=1}^{n_d} \Phi_E^T(k,i)\Phi_R(k,j)\right)^T}{\left(\sum_{k=1}^{n_d} \Phi_E^T(k,i)\Phi_E(k,i)\right)\left(\sum_{k=1}^{n_d} \Phi_R^T(k,j)\Phi_R(k,j)\right)}$
- Φ_R is the mode shape from ROM.
- Φ_E is the mode shape from FOM.
- Number of DOFs for a reduced order model should be increased to increase the accuracy of target matrices.
  - Number of DOF for a ROM is increased, then computation time as well as required computer memory size for NASTRAN run are also increased drastically. (memory size issue of the computer)
  - Automation of this is a time consuming procedure.

$\Phi_E^T \mathbf{M} \Phi_E$

$\Phi_E^T \mathbf{M} \Phi_R$

Cross-Orthogonality Matrix

Mode	7	8	12	14	15	16	17	18	20	22	26	30	31	34	35	36	37
7	1.00	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
8	.000	1.00	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
12	.000	.000	.977	.000	.000	.000	.000	.000	[illegible]	[illegible]	[illegible]	.000	.000	.000	.000	.000	.000
14	.000	.000	.000	.943	.001	.000	.000	-.001	[illegible]	[illegible]	[illegible]	.000	.000	.000	.000	.000	.000
15	.000	.000	.000	-.001	.941	.002	-.001	-.004	.000	.000	.000	.000	.000	.000	.000	.000	.000
16	-.001	.000	.000	.000	.003	-.948	.002	.011	.001	-.001	.000	[illegible]	[illegible]	[illegible]	.000	.000	.000
17	-.002	.000	.000	.001	.005	.008	.853	.146	.003	-.003	.000	[illegible]	[illegible]	[illegible]	.000	-.001	.000
18	-.002	.000	.000	.001	.006	.011	-.066	.921	.002	-.003	.000	-.002	.001	-.001	-.001	-.001	.000
20	-.001	.000	.000	.001	.001	.005	-.005	-.014	.936	-.002	.000	.000	.000	.000	.000	.000	.000
22	-.005	.002	.000	.001	.006	.005	-.010	-.033	-.004	-.812	.000	-.011	.003	-.007	-.004	-.004	-.002
26	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.934	-.001	.001	.000	.000	.000	.000
30	.000	.000	.000	.000	.000	.000	.000	-.001	.000	.000	.000	.815	.000	-.002	.000	.000	.000
31	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	-.001	-.001	.904	.000	.000	.001	.000
34	-.003	.000	.000	.001	.003	.005	-.006	-.022	-.002	.013	.002	.032	-.004	.949	-.017	-.015	-.012
35	-.001	.002	-.001	.000	.000	.000	-.002	-.004	-.001	.002	.000	.018	-.001	.007	.935	-.005	-.003
36	-.001	.000	.000	.000	-.001	.001	.000	-.002	.000	.001	.000	.002	-.002	.002	.003	.921	-.002
37	-.002	.000	.001	-.001	.001	.000	-.002	-.007	.000	.004	.000	-.007	.002	.015	.013	.006	.889

.146

-.812

Orthonormalized Mass Matrix

Mode	7	8	12	14	15	16	17	18	20	22	26	30	31	34	35	36	37
7	[redacted]	.000	.000	.000	.000	-.001	-.002	-.002	-.001	-.004	.000	.000	.000	-.003	-.001	-.001	-.002
8	.000	[redacted]	.000	.000	.000	.000	.000	.000	.000	.002	.000	.000	.000	.000	.002	.000	.000
12	.000	.000	[redacted]	.000	.000	.000	.000	.000	.000	-.001	.000	.000	.000	.000	-.001	.000	.001
14	.000	.000	.000	[redacted]	.000	.000	.001	.001	.000	.001	.000	.000	.000	.001	.000	.000	-.001
15	.000	.000	.000	.000	[redacted]	.001	.004	.003	.001	.006	.000	.000	.000	.003	.001	.000	.001
16	-.001	.000	.000	.000	.001	[redacted]	-.005	-.001	-.004	-.006	.000	-.001	.000	-.006	.000	-.001	.000
17	-.002	.000	.000	.001	.004	-.005	[redacted]	.098	-.004	-.019	.000	-.001	.001	-.012	-.003	-.002	-.004
18	-.002	.000	.000	.001	.003	-.001	.098	[redacted]	-.012	-.036	.000	-.003	.001	-.025	-.005	-.003	-.008
20	-.001	.000	[illegible]	[illegible]	[illegible]	-.004	-.004	-.012	[redacted]	-.006	.000	.000	.000	-.002	-.001	.000	.000
22	-.004	.002	[illegible]	[illegible]	[illegible]	-.006	-.019	-.036	-.006	[redacted]	.000	-.007	.003	.013	.000	-.002	.004
26	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	[redacted]	.000	.001	.001	.000	.000	.000
30	.000	.000	.000	.000	.000	-.001	-.001	-.003	.000	-.007	.000	[redacted]	.000	.025	.016	.001	-.006
31	.000	.000	.000	.000	.000	.000	.001	.001	.000	.003	.001	.000	[redacted]	-.002	-.001	-.001	.002
34	-.003	.000	.000	.001	.003	-.006	-.012	-.025	-.002	.013	.001	.025	-.002	[redacted]	-.007	-.012	.008
35	-.001	.002	-.001	.000	.001	.000	-.003	-.005	-.001	.000	.000	.016	-.001	-.007	[redacted]	-.002	.012
36	-.001	.000	.000	.000	.000	-.001	-.002	-.003	.000	-.002	.000	.001	-.001	-.012	-.002	[redacted]	.005
37	-.002	.000	.001	-.001	.001	.000	-.004	-.008	.000	.004	.000	-.006	.002	.008	.012	.005	[redacted]

.098

MAC Mass Matrix

Mode	7	8	12	14	15	16	17	18	20	22	26	30	31	34	35	36	37
7	1.000	0.000	0.000	0.000	0.018	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.009	0.002	0.019	0.000
8	0.000	1.000	0.042	0.000	0.000	0.000	0.002	0.000	0.003	0.026	0.000	0.000	0.000	0.000	0.005	0.000	0.002
12	0.000	0.042	1.000	0.003	0.000	0.002	0.017	0.002	0.047	0.166	0.000	0.003	0.000	0.002	0.027	0.002	0.046
14	0.000	0.000	0.003	1.000	0.000	0.000	0.001	0.000	0.003	0.000	0.000	0.000	0.000	0.002	0.008	0.000	0.000
15	0.018	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.015	0.000
16	0.000	0.000	0.002	0.000	0.000	1.000	0.085	0.012	0.164	0.001	[illegible]	[illegible]	[illegible]	0.000	0.001	0.000	0.008
17	0.000	0.002	0.018	0.001	0.000	0.099	0.985	0.001	0.040	0.007	[illegible]	[illegible]	[illegible]	0.000	0.004	0.000	0.039
18	0.000	0.000	0.001	0.000	0.000	0.003	0.040	0.988	0.002	0.000	0.000	0.000	0.000	0.010	0.000	0.000	0.001
20	0.000	0.003	0.047	0.003	0.000	0.159	0.032	0.006	1.000	0.009	0.000	0.000	0.000	0.000	0.000	0.000	0.000
22	0.000	0.019	0.142	0.000	0.001	0.001	0.011	0.001	0.008	0.795	0.000	0.012	0.001	0.000	0.018	0.001	0.036
26	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.001	0.000	0.000	0.000
30	0.000	0.000	0.004	0.000	0.000	0.006	0.009	0.001	0.000	0.013	0.000	0.969	0.000	0.007	0.017	0.000	0.002
31	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.999	0.008	0.000	0.001	0.000
34	0.011	0.000	0.002	0.001	0.001	0.000	0.000	0.012	0.000	0.017	0.001	0.007	0.009	0.993	0.058	0.006	0.004
35	0.002	0.005	0.027	0.008	0.000	0.001	0.004	0.000	0.000	0.016	0.000	0.018	0.000	0.073	1.000	0.000	0.049
36	0.019	0.000	0.002	0.000	0.015	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.006	0.000	1.000	0.000
37	0.000	0.002	0.046	0.000	0.000	0.009	0.036	0.003	0.000	0.043	0.000	0.004	0.000	0.007	0.043	0.000	0.998

.795

Select Accelerometers

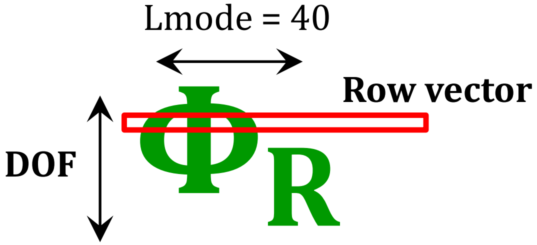
1) Effective Independence
2) Driving Point Acceleration Transfer Function

Accelerometer Selection using Effective Independence

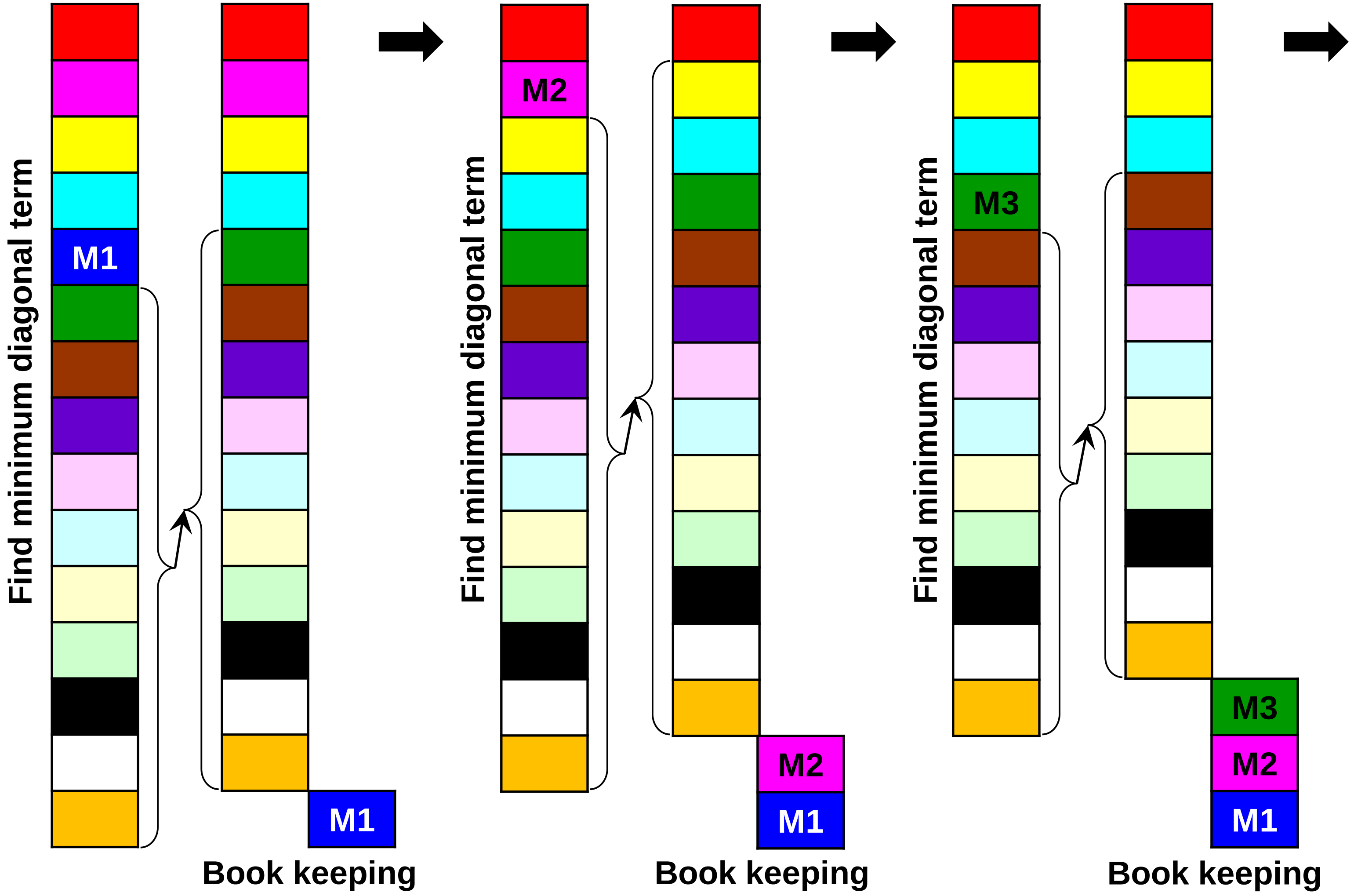
- Ranking DOFs based on "Effective Independence" method
  - Ranking DOFs based on "determinant of Fisher's matrix ($\Phi^T\Phi$)" gives identically same results.
- The objective of this method is to select measurement locations which make the mode shapes of interest **as linearly independent as possible**, while retaining **as much information as possible** about the selected modal responses in the measurement data.
- Compute the following matrix:

$$E = \Phi\left(\Phi^{T}\Phi\right)^{-1}\Phi^{T}$$ (Φ: mode shape matrix)

- Thus, terms on the diagonal of E represent the fractional contribution of each measurement location to the rank of E, and hence to the independence of the chosen modes. (max. = 1.0)



Starting DOF=5344 → **Ending DOF=40**



(a) First minimum (b) Second minimum (c) Third minimum

Driving Point Acceleration Transfer Function Analysis

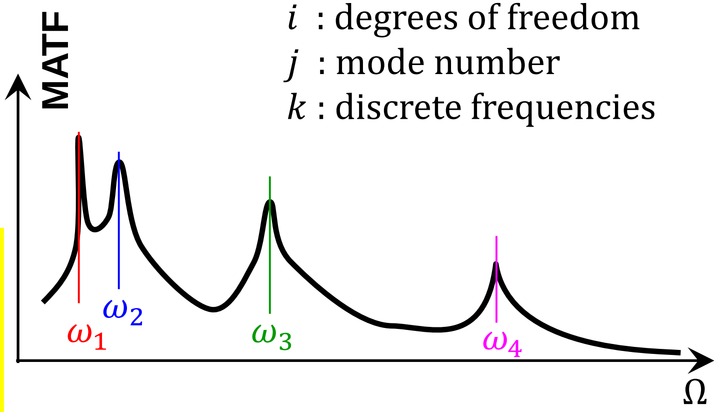
- Governing Equations of Motion: $[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = \{f\}$ (1)
- Use orthonormal coordinates: $\{q\} = [\Phi]\{\eta\}$ (2)
  - Substituting Eq.(2) into Eq.(1): $[M][\Phi]\{\ddot{\eta}\} + [C][\Phi]\{\dot{\eta}\} + [K][\Phi]\{\eta\} = \{f\}$ (3)
  - Pre-multiply $[\Phi]^T$ to Eq.(3): $[\Phi]^T[M][\Phi]\{\ddot{\eta}\} + [\Phi]^T[C][\Phi]\{\dot{\eta}\} + [\Phi]^T[K][\Phi]\{\eta\} = [\Phi]^T\{f\}$ (4)
- Orthonormal Governing Equations of Motion: $[I]\{\ddot{\eta}\} + [2\zeta\omega]\{\dot{\eta}\} + [\omega^2]\{\eta\} = \{N\}$ (5)
  - Where, $\{N\}=[\Phi]^T\{f\}$ (6) , ω (natural frequencies), and ζ (equivalent viscous damping)
  - In Laplace domain: $\left(s^2[I] + s[2\zeta\omega] + [\omega^2]\right)\{\bar{\eta}\} = \{\bar{N}\} \xrightarrow{\text{yields}} \{\bar{\eta}\} = \left(s^2[I] + s[2\zeta\omega] + [\omega^2]\right)^{-1}\{\bar{N}\}$ (7)
    - Where, matrices $[I]$, $[2\zeta\omega]$, and $[\omega^2]$ are diagonal matrices.
- Derive Acceleration Transfer Function
  - From Eq.(2) in Laplace domain: $\{\ddot{q}\} = [\Phi]\{\ddot{\eta}\} \xrightarrow{\text{yields}}$ *Acceleration* $s^2\{\bar{q}\}=s^2[\Phi]\{\bar{\eta}\}$ (8)
  - Substituting Eqs.(6) & (7) into Eq.(8): $s^2\{\bar{q}\}=s^2[\Phi]\left(s^2[I] + s[2\zeta\omega] + [\omega^2]\right)^{-1}[\Phi]^T\{\bar{f}\}$ (9)

Driving Point Acceleration Transfer Function Analysis (continue)

- Acceleration Transfer Function (ATF): $\frac{s^2\{\bar{q}\}}{\{f\}}=[\Phi]s^2[I]\left(s^2[I]+s[2\zeta\omega]+[\omega^2]\right)^{-1}[\Phi]^T=[\Phi]\left[\frac{s^2}{s^2+2\zeta\omega s+\omega^2}\right][\Phi]^T$ (10)
  - Let $s=i\Omega$ (Ω: *driving frequency*) then Eq.(10) becomes
  - ATF= $[\Phi]\left[\frac{-\Omega^2}{\omega^2-\Omega^2+i2\zeta\omega\Omega}\right][\Phi]^T$
    - Where, the matrix $\left[\frac{-\Omega^2}{\omega^2-\Omega^2+i2\zeta\omega\Omega}\right]$ is a **diagonal matrix**.

$$\mathrm{DPATF}(i,k)=\sum_{j=1}^{n}\frac{-{\Omega_k}^2}{\omega_j^2-{\Omega_k}^2+i2\zeta\omega_j\Omega_k}\phi_{ij}^2$$

- Driving Point ATF
  - Let $[\Phi]=\begin{bmatrix}\lfloor\phi_{11} & \phi_{12} & \cdots & \phi_{1n}\rfloor\\ \lfloor\phi_{21} & \phi_{22} & \cdots & \phi_{2n}\rfloor\\ & \vdots & & \\ \lfloor\phi_{m1} & \phi_{m2} & \cdots & \phi_{mn}\rfloor\end{bmatrix}$, then $[\Phi]^T=\begin{bmatrix}\begin{Bmatrix}\phi_{11}\\ \phi_{12}\\ \vdots\\ \phi_{1n}\end{Bmatrix} & \begin{Bmatrix}\phi_{21}\\ \phi_{22}\\ \vdots\\ \phi_{2n}\end{Bmatrix} & \cdots & \begin{Bmatrix}\phi_{m1}\\ \phi_{m2}\\ \vdots\\ \phi_{mn}\end{Bmatrix}\end{bmatrix}$
  - ATF= $[\Phi]\left[\frac{-\Omega^2}{\omega^2-\Omega^2+i2\zeta\omega\Omega}\right][\Phi]^T=\begin{bmatrix}\lfloor\phi_{11} & \phi_{12} & \cdots & \phi_{1n}\rfloor\\ \lfloor\phi_{21} & \phi_{22} & \cdots & \phi_{2n}\rfloor\\ & \vdots & & \\ \lfloor\phi_{m1} & \phi_{m2} & \cdots & \phi_{mn}\rfloor\end{bmatrix}\left[\frac{-\Omega^2}{\omega^2-\Omega^2+i2\zeta\omega\Omega}\right]\begin{bmatrix}\begin{Bmatrix}\phi_{11}\\ \phi_{12}\\ \vdots\\ \phi_{1n}\end{Bmatrix} & \begin{Bmatrix}\phi_{21}\\ \phi_{22}\\ \vdots\\ \phi_{2n}\end{Bmatrix} & \cdots & \begin{Bmatrix}\phi_{m1}\\ \phi_{m2}\\ \vdots\\ \phi_{mn}\end{Bmatrix}\end{bmatrix}$



i : degrees of freedom
j : mode number
k : discrete frequencies

- Driving point ATF$(i,k)\equiv$ **ATF at i-th DOF sensor due to i-th DOF loading under frequency Ω_k**
  - $\mathrm{DPATF}(i,k)=\lfloor\phi_{i1}\quad\phi_{i2}\quad\cdots\quad\phi_{in}\rfloor\begin{bmatrix}\frac{-\Omega^2}{\omega_1^2-\Omega^2+i2\zeta\omega_1\Omega} & 0 & \cdots & 0\\ 0 & \frac{-\Omega^2}{\omega_2^2-\Omega^2+i2\zeta\omega_2\Omega} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \frac{-\Omega^2}{\omega_n^2-\Omega^2+i2\zeta\omega_n\Omega}\end{bmatrix}\begin{Bmatrix}\phi_{i1}\\ \phi_{i2}\\ \vdots\\ \phi_{in}\end{Bmatrix}=\sum_{j=1}^{n}\frac{-{\Omega_k}^2}{\omega_j^2-{\Omega_k}^2+i2\zeta\omega_j\Omega_k}\phi_{ij}^2$

Accelerometer Selection using Magnitude of Driving Point Acceleration Transfer Function

- Compute the magnitude of the driving point acceleration transfer function(MDPATF) matrix from real and imaginary part of the driving point ATF matrix in equation (18).

1) Go to a discrete frequency for a "primary mode"

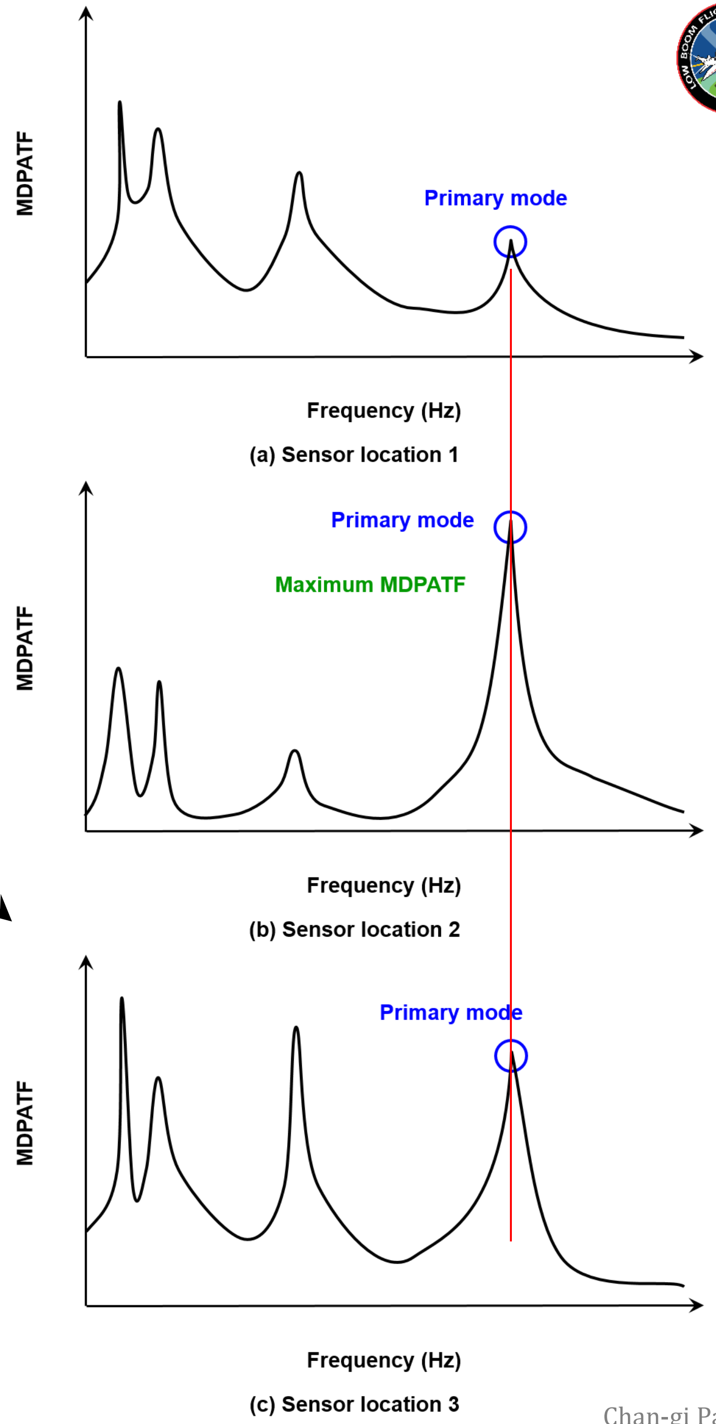
❖ $$DPATF\ matrix = \begin{bmatrix} DPATE(1,1) & DPATE(1,2) & \cdots & DPATE(1,n_k) \\ DPATE(2,1) & DPATE(2,2) & \cdots & DPATE(2,n_k) \\ \cdots & \cdots & \vdots & \cdots \\ DPATE(n_d,1) & DPATE(n_d,2) & \cdots & DPATE(n_d,n_k) \end{bmatrix} \quad \text{Eq.(18)}$$

2) Pick the accelerometer location where MDPATF is "maximum"

$$\mathrm{DPATF}(i,k) = \sum_{j=1}^{n} \frac{-{\Omega_k}^2}{\omega_j^2 - {\Omega_k}^2 + i2\zeta\omega_j\Omega_k} \phi_{ij}^2$$

i : degrees of freedom
j : mode number
k : discrete frequencies

1) Select k-th column of the DPATF matrix **based on the natural frequency of a primary mode**.
2) Using the k-th column vector, select the i-th DOF for an accelerometer location where the magnitude is a **maximum value**.



Forty accelerometers selected using MDPATF and EI methods

Ranking based on EI

Forty accelerometers selected using MDPATF and EI methods

Accelerometer number	Ranking in table 8	Grid number	DOF	X (inch)	Y (inch)	Z (inch)
1	11	200001	3	25.4649	1.78E-07	82.5134
2	12	200001	2	25.4649	1.78E-07	82.5134
3	47	2420290	2	1111.38	24	199.783
4	35	2441530	3	1148.39	85.3158	100.004
5	34	441530	3	1148.39	-85.3158	100.004
6	1	342501	3	505.348	-48.0879	115.501
7	6	2342501	3	505.348	48.0879	115.501
8	13	2281159	3	1018.51	177	100.811
9	32	440869	3	998.698	-20.5	94.9772
10	39	440869	2	998.698	-20.5	94.9772
11	17	420290	3	1111.38	-24	199.783
12	2	400123	3	979.803	-74.924	99.6877
13	14	2410127	3	1007.34	150.003	101.18
14	31	2440869	3	998.698	20.5	94.9772
15	3	410127	3	1007.34	-150.003	101.18
16	4	213094	2	540.267	0	132.724
17	5	280571	3	[illegible]	[illegible]	[illegible]
18	7	200424	2	[illegible]	[illegible]	[illegible]
19	8	281159	3	[illegible]	[illegible]	[illegible]
20	9	2400123	3	[illegible]	[illegible]	[illegible]
21	10	410066	3	[illegible]	[illegible]	[illegible]
22	15	254577	3	[illegible]	[illegible]	[illegible]
23	16	420001	3	[illegible]	[illegible]	[illegible]
24	18	200364	3	[illegible]	[illegible]	[illegible]
25	19	211559	2	[illegible]	[illegible]	[illegible]
26	20	2280571	3	[illegible]	[illegible]	[illegible]
27	21	200424	3	[illegible]	[illegible]	[illegible]
28	22	2420290	3	[illegible]	[illegible]	[illegible]
29	23	2410066	3	[illegible]	[illegible]	[illegible]
30	24	2430128	2	[illegible]	[illegible]	[illegible]
31	25	2430125	2	[illegible]	[illegible]	[illegible]
32	26	2254577	3	[illegible]	[illegible]	[illegible]
33	27	2420001	3	[illegible]	[illegible]	[illegible]
34	28	2350277	2	990.001	0.91178	163.961
35	29	253655	3	742.749	-79.3372	107.187
36	30	2253655	3	742.749	79.3372	107.187
37	33	2350730	3	1110.83	0.499999	199.903
38	36	441379	3	1134.49	-20.5	94.2836
39	37	2441379	3	1134.49	20.5	94.2836
40	38	2440869	2	998.698	20.5	94.9772

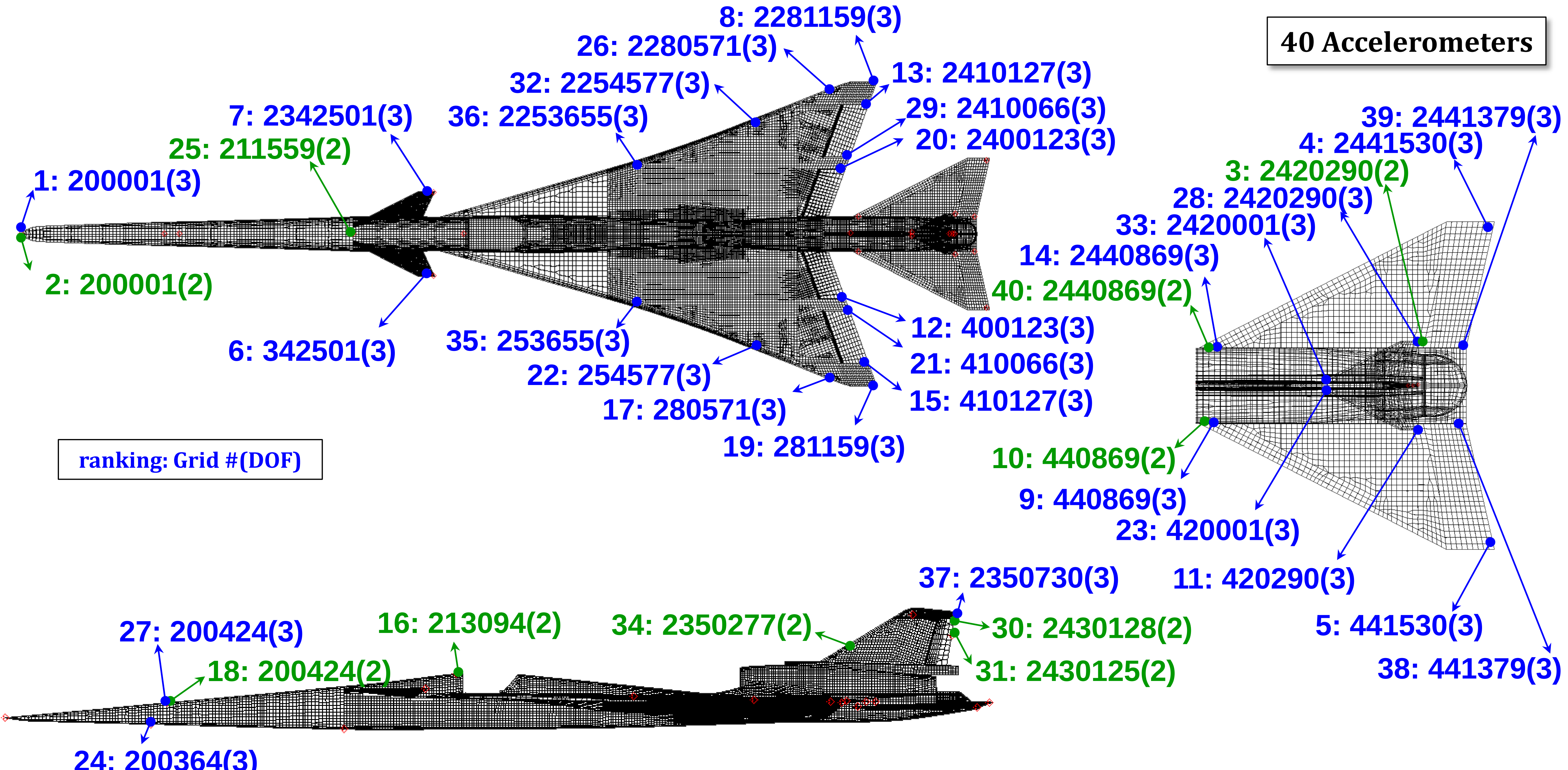
Use MDPATF (accelerometers 1–14)

Use EI (accelerometers 15–40)

Table 10. MDPATF at the first 14 accelerometer locations (M=Mode)

Accel. num.	Grid	DOF	MDPATF at frequencies of primary modes																
			M7	M8	M12	M14	M15	M16	M17	M18	M20	M22	M26	M30	M31	M34	M35	M36	M37
1	200001	3	500	1.8	9.1	0.0	4.2	0.2	2.6	6.6	1.0	285	1.3	2.6	6.2	475	27.7	11.1	5.9
2	200001	2	1.0	500	112	1.8	1.7	1.6	8.1	2.1	8.5	2.1	1.5	1.4	1.4	1.4	2.5	1.4	1.9
3	2420290	2	0.4	24.8	500	5.3	1.5	0.9	13.5	1.6	3.1	280	2.7	4.8	1.4	5.9	69.8	3.3	175
4	2441530	3	26.3	1.4	4.6	501	495	16.1	16.4	9.3	5.8	24.8	0.7	2.3	4.5	39.9	423	310	39.3
5	441530	3	26.4	2.3	14.0	497	502	16.5	10.9	9.1	5.9	14.0	0.8	1.8	3.7	39.5	215	500	16.1
6	342501	3	1.2	1.1	34.5	12.3	15.7	500	99.4	501	323	87.5	6.1	5.6	5.0	5.9	4.4	4.0	66.1
7	2342501	3	1.2	1.4	62.2	13.6	17.2	491	503	331	294	9.6	6.2	5.8	5.3	5.7	4.7	4.3	65.3
8	2281159	3	0.4	1.0	335	2.7	3.6	49.3	17.1	60.7	500	379	5.1	3.4	7.9	4.3	4.7	0.7	500
9	440869	3	0.9	0.7	2.4	301	383	10.3	13.2	6.0	4.3	500	3.2	0.3	2.7	63.0	247	469	13.8
10	440869	2	0.1	0.1	0.3	2.0	1.7	1.5	10.3	0.3	1.2	12.8	500	8.5	4.8	32.2	38.9	8.7	3.0
11	420290	3	16.0	1.4	75.2	0.9	0.8	0.8	15.0	0.6	1.3	15.6	7.6	500	14.3	108	24.9	7.5	86.2
12	400123	3	0.1	0.2	0.5	1.2	1.3	4.3	2.4	3.8	38.2	12.6	9.3	84.7	500	344	21.4	10.7	216
13	2410127	3	0.2	0.2	77.7	0.2	0.3	0.5	0.7	0.8	16.5	30.4	3.0	8.1	62.2	500	17.0	9.4	4.5
14	2440869	3	0.9	0.7	2.2	305	378	10.4	22.7	6.1	4.3	315	1.7	2.0	4.6	43.0	500	284	23.2

Forty Accelerometer Locations (continue)



Minimum Number of Accelerometers for GVT

- Select the first 14 accelerometers based on MDPATF analysis
- Select the next 26 accelerometers using "Effective Independence" method
- Φ_R is the mode shape from ROM.
- Φ_E is the expanded mode shape from Φ_M .
  - Φ_M are obtained from FOM during pre-test analysis.
  - $\Phi_M = \Phi_G$ during GVT.

$\Phi_E{}^T M \Phi_E$

$\Phi_E{}^T M \Phi_R$

$$\Phi_E = \begin{bmatrix} \Phi_M(\Phi_M{}^T\Phi_M)^{-1}\Phi_M{}^T \\ \Phi_S(\Phi_M{}^T\Phi_M)^{-1}\Phi_M{}^T \end{bmatrix} \Phi_M$$

40 Accelerometers

Cross-Orthogonality Matrix

Mode	7	8	12	14	15	16	17	18	20	22	26	30	31	34	35	36	37
7	1.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
8	.000	1.001	.000	.000	.000	-.001	-.001	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
12	.000	.000	.977	.000	.000	.000	.001	.000	[illegible]	[illegible]	[illegible]	.000	.000	.000	.000	.000	.000
14	.000	.000	.000	.943	.001	.000	.000	.000	[illegible]	[illegible]	[illegible]	.000	.000	.000	.000	.000	.000
15	.001	.000	.000	-.001	.941	.002	-.001	-.003	.000	.000	.000	.000	.000	.000	.000	.000	.000
16	-.001	.000	.000	.000	.004	-.948	.003	.010	.001	.000	.000	[illegible]	[illegible]	[illegible]	.000	.000	.000
17	-.003	.000	-.001	.001	.006	.008	.853	.145	.003	-.002	.000	[illegible]	[illegible]	[illegible]	.000	.000	.000
18	-.003	.000	-.001	.001	.007	.012	-.065	.919	.002	-.003	.000	-.001	.001	.001	.000	.000	.000
20	-.001	-.001	.000	.000	.002	.007	-.003	-.015	.936	-.001	.000	.000	.000	.000	.000	.000	.000
22	-.009	-.004	-.001	-.002	.009	.017	.002	-.041	-.003	-.808	-.001	-.010	.003	-.003	-.001	-.001	-.001
26	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.934	-.001	.001	.000	.000	.000	.000
30	.000	.001	.000	.001	.000	-.002	-.003	.000	-.001	.000	.000	.815	.000	-.001	.000	.000	.000
31	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	-.001	-.001	.904	.000	.000	.001	.000
34	-.009	-.001	-.001	.001	.008	.010	-.004	-.030	-.003	.016	.002	.035	-.007	.952	-.014	-.008	-.012
35	-.002	-.001	.000	-.003	.002	.015	.016	-.012	.000	.005	.000	.020	-.002	.008	.936	-.004	-.002
36	-.001	-.001	.000	.000	.000	.002	.000	-.003	.000	.002	.000	.002	-.002	.003	.003	.922	-.002
37	-.004	.001	-.001	.001	.003	-.005	-.011	-.008	-.003	.006	.000	-.007	.000	.017	.013	.009	.893

.145

-.808

Orthonormalized Mass Matrix

Mode	7	8	12	14	15	16	17	18	20	22	26	30	31	34	35	36	37
7		.000	.000	.000	.001	-.001	-.003	-.004	-.001	-.008	.000	.000	.000	-.009	-.002	-.001	-.005
8	.000		.000	.000	.000	.000	-.001	.000	-.001	-.003	.000	.001	.000	-.002	-.002	-.001	.001
12	.000	.000		.000	.000	-.001	.000	-.001	.000	-.001	.000	.000	.000	-.001	.001	.000	-.001
14	.000	.000	.000		.000	.000	.000	.000	.000	-.002	.000	.001	.000	.001	-.003	.000	.001
15	.001	.000	.000	.000		.002	.006	.004	.002	.009	.000	.001	.000	.008	.002	.000	.004
16	-.001	[illegible]	[illegible]	[illegible]	[illegible]		-.005	-.003	-.006	-.017	.000	.002	.000	-.011	-.015	-.002	.005
17	-.003	[illegible]	[illegible]	[illegible]	[illegible]	-.005		.098	-.003	-.009	.000	-.004	.001	-.010	.015	.000	-.015
18	-.004	.000	-.001	.000	.004	-.003	.098		-.014	-.042	.000	-.001	.002	-.032	-.014	-.003	-.009
20	-.001	-.001	.000	.000	.002	-.006	-.003	-.014		-.005	.000	-.001	.000	-.003	.000	.000	-.003
22	-.008	-.003	-.001	-.002	.009	-.017	-.009	-.042	-.005		-.001	-.005	.003	.022	.003	.001	.008
26	.000	.000	.000	.000	.000	.000	.000	.000	.000	-.001		.000	.001	.002	.000	.000	.000
30	.000	.001	.000	.001	.001	.002	-.004	-.001	-.001	-.005	.000		.000	.028	.018	.002	-.006
31	.000	.000	.000	.000	.000	.000	.001	.002	.000	.003	.001	.000		-.005	-.001	-.002	.000
34	-.009	-.002	-.001	.001	.008	-.011	-.010	-.032	-.003	.022	.002	.028	-.005		-.001	-.004	.010
35	-.002	-.002	.001	-.003	.002	-.015	.015	-.014	.000	.003	.000	.018	-.001	-.001		.000	.013
36	-.001	-.001	.000	.000	.000	-.002	.000	-.003	.000	.001	.000	.002	-.002	-.004	.000		.008
37	-.005	.001	-.001	.001	.004	.005	-.015	-.009	-.003	.008	.000	-.006	.000	.010	.013	.008	

.098

MAC Mass Matrix

Mode	7	8	12	14	15	16	17	18	20	22	26	30	31	34	35	36	37
7	1.000	0.000	0.000	0.000	0.018	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.008	0.002	0.019	0.000
8	0.000	1.000	0.042	0.000	0.000	0.000	0.002	0.000	0.003	0.026	0.000	0.000	0.000	0.000	0.005	0.000	0.002
12	0.000	0.042	1.000	0.003	0.000	0.002	0.017	0.002	0.047	0.165	0.000	0.003	0.000	0.002	0.027	0.002	0.046
14	0.000	0.000	0.003	1.000	0.000	0.000	0.001	0.000	0.003	0.000	0.000	0.000	0.000	0.002	0.008	0.000	0.000
15	0.018	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.015	0.000
16	0.000	0.000	0.002	0.000	0.000	1.000	0.085	0.012	0.164	0.001	[illegible]	[illegible]	[illegible]	0.000	0.001	0.000	0.008
17	0.000	0.002	0.018	0.001	0.000	0.099	0.985	0.001	0.040	0.007	[illegible]	[illegible]	[illegible]	0.000	0.005	0.000	0.038
18	0.000	0.000	0.001	0.000	0.000	0.003	0.040	0.988	0.002	0.000	0.000	0.000	0.000	0.010	0.000	0.000	0.001
20	0.000	0.003	0.047	0.003	0.000	0.159	0.032	0.006	1.000	0.009	0.000	0.000	0.000	0.000	0.000	0.000	0.000
22	0.000	0.019	0.142	0.000	0.001	0.001	0.011	0.001	0.008	0.792	0.000	0.012	0.001	0.000	0.018	0.001	0.036
26	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.001	0.000	0.000	0.000
30	0.000	0.000	0.004	0.000	0.000	0.006	0.009	0.001	0.000	0.013	0.000	0.969	0.000	0.007	0.017	0.000	0.002
31	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.999	0.008	0.000	0.001	0.000
34	0.011	0.000	0.002	0.001	0.001	0.000	0.000	0.013	0.000	0.017	0.001	0.007	0.009	0.992	0.058	0.006	0.004
35	0.002	0.005	0.027	0.008	0.000	0.001	0.004	0.000	0.000	0.016	0.000	0.018	0.000	0.072	1.000	0.000	0.049
36	0.019	0.000	0.002	0.000	0.015	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.006	0.000	1.000	0.000
37	0.000	0.002	0.046	0.000	0.000	0.009	0.036	0.003	0.000	0.043	0.000	0.004	0.000	0.007	0.043	0.000	0.998

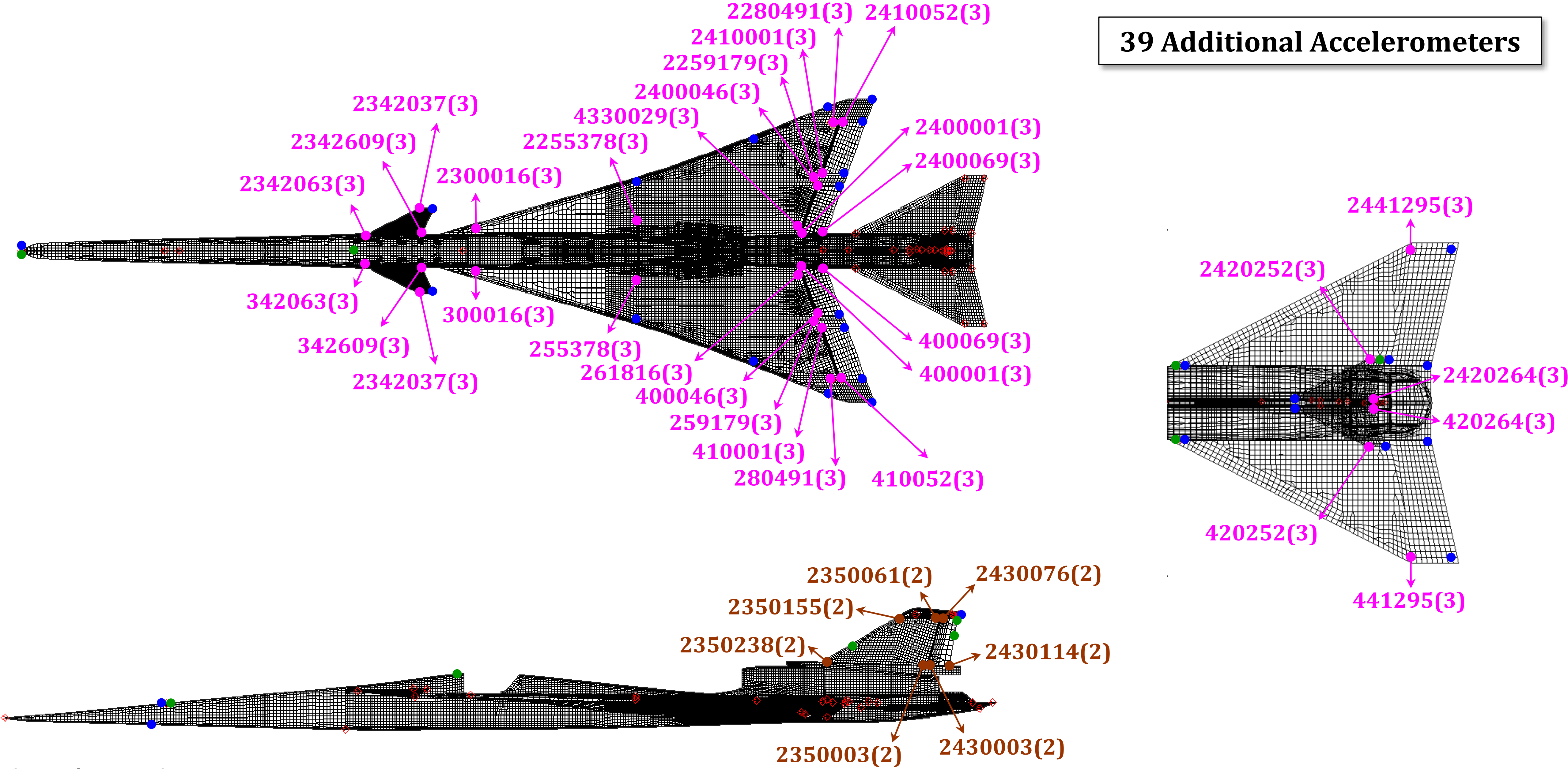
0.792

Additional 39 Accelerometers for Mode Visualization during GVT

Accelerometer number	Grid number	DOF	X (inch)	Y (inch)	Z (inch)
41	2350003	2	1070.76	1.8208	142.838
42	2350061	2	1086.01	0.993923	196
43	2350155	2	1043.33	0.722021	196.002
44	2350238	2	960.455	0.957993	146.208
45	2430076	2	1090.74	0.864463	195.5
46	2430114	2	1099.76	4.50E-02	142.452
47	2255378	3	742.749	34.6499	103.934
48	2300016	3	554.999	25.2244	109.364
49	2342037	3	489.38	48.4112	116.246
50	2342063	3	426.387	16.2727	113.456
51	2342609	3	492.294	20.3561	106.404
52	2400001	3	935.567	19.7273	87.0531
53	2400046	3	955.36	74.6253	101.369
54	2400069	3	960.783	21.4446	84.5237
55	2410001	3	960.591	90.0195	103.353
56	2410052	3	982.223	150.017	102.578
57	2420252	3	1103.07	24	199.929
58	2420264	3	1104.95	2.25	199.783
59	2430003	3	1075.76	1.57729	143.186
60	2441295	3	1125.66	85.3158	100.459
61	4330029	3	931.297	28.9999	89.1548
62	255378	3	742.7485	-34.6499	103.9337
63	300016	3	554.9989	-25.2244	109.3644
64	342037	3	489.3803	-48.4112	116.2455
65	342063	3	426.3869	-16.2727	113.4559
66	342609	3	492.294	-20.3561	106.4036
67	400001	3	935.5669	-19.7273	87.05307
68	400046	3	955.36	-74.6253	101.3687
69	400069	3	960.7825	-21.4445	84.52368
70	410001	3	960.5913	-90.0195	103.3525
71	410052	3	982.2228	-150.017	102.5775
72	420252	3	1103.066	-24	199.9287
73	420264	3	1104.946	-2.25	199.7829
74	441295	3	1125.656	-85.3158	100.459
75	261816	3	931.2971	-28.9999	89.15475
76	2259179	3	950.406	81.9998	103.292
77	259179	3	950.4058	-81.9998	103.292
78	2280491	3	972.435	150.47	102.842
79	280491	3	972.4346	-150.47	102.8418

Additional Accelerometers for Mode Shape Visualization during GVT

39 Additional Accelerometers



Additional Accelerometers for Mode Shape Visualization during GVT

- Select the first 14 accelerometers based on DPATF analysis
- Select 26 accelerometers using “Effective Independence” method
- Select 39 additional accelerometers for **mode visualization during GVT**

79 Accelerometers

Cross-Orthogonality Matrix

Mode	7	8	12	14	15	16	17	18	20	22	26	30	31	34	35	36	37
7	1.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
8	.000	1.001	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
12	.000	.000	.977	.000	.000	.000	.001	.000	[illegible]	[illegible]	[illegible]	.000	.000	.000	.000	.000	.000
14	.000	.000	.000	.943	.001	.000	.000	-.001	[illegible]	[illegible]	[illegible]	.000	.000	.000	.000	.000	.000
15	.000	.000	.000	-.001	.941	.002	-.001	-.004	.000	.000	.000	.000	.000	.000	.000	.000	.000
16	.000	.000	.000	.000	.003	-.948	.003	.011	.001	-.001	.000	[illegible]	[illegible]	[illegible]	.000	.000	.000
17	-.001	.000	.000	.001	.005	.008	.853	.146	.003	-.003	.000	[illegible]	[illegible]	[illegible]	-.001	-.001	.000
18	-.001	.000	.000	.001	.006	.012	-.065	.921	.002	-.003	.000	-.002	.001	-.002	-.001	-.001	-.001
20	.000	.000	.000	.000	.001	.006	-.004	-.015	.936	-.002	.000	.000	.000	-.001	.000	.000	.000
22	-.002	-.002	.000	-.001	.005	.013	-.001	-.035	-.004	-.811	.000	-.010	.003	-.009	-.004	-.004	-.002
26	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.934	-.001	.001	.000	.000	.000	.000
30	.000	.000	.000	.000	.000	-.001	-.002	.000	.000	.000	.000	.815	.000	-.002	-.001	.000	.000
31	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	-.001	-.001	.903	.000	.000	.001	.000
34	.001	-.001	-.001	.001	.001	.005	-.008	-.022	-.003	.014	.002	.033	-.007	.944	-.018	-.013	-.012
35	.000	.000	.000	-.002	.000	.009	.009	-.008	-.001	.004	.001	.019	-.002	.007	.935	-.005	-.002
36	.000	.000	.000	.000	-.001	.001	-.001	-.002	.000	.001	.000	.002	-.002	.002	.003	.921	-.002
37	.000	.000	.001	.000	.000	-.001	-.005	-.007	-.001	.005	.000	-.006	.000	.012	.012	.007	.890

.146

-.811

Orthonormalized Mass Matrix

Mode	7	8	12	14	15	16	17	18	20	22	26	30	31	34	35	36	37
7	[redacted]	.000	.000	.000	.000	.000	-.001	-.001	.000	-.002	.000	.000	.000	.001	.000	.000	.000
8	.000	[redacted]	.000	.000	.000	.000	-.001	.000	.000	-.002	.000	.000	.000	-.001	.000	.000	.000
12	.000	.000	[redacted]	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	-.001	.000	.000	.001
14	.000	.000	.000	[redacted]	.000	.000	.001	.000	.000	-.001	.000	.001	.000	.001	-.002	.000	.000
15	.000	.000	.000	.000	[redacted]	.001	.004	.002	.001	.005	.000	.000	.000	.001	.000	.000	.000
16	.000	.000	.000	.000	.001	[redacted]	-.005	-.002	-.005	-.014	.000	.001	.000	-.007	-.010	-.001	.000
17	-.001	-.001	.000	.001	.004	-.005	[redacted]	.100	-.004	-.011	.000	-.004	.001	-.015	.007	-.002	-.007
18	-.001	.000	.000	.000	.002	-.002	.100	[redacted]	-.013	-.038	.000	-.001	.001	-.026	-.011	-.003	-.008
20	.000	.000	[illegible]	[illegible]	[illegible]	-.005	-.004	-.013	[redacted]	-.006	.000	-.001	.000	-.004	-.001	-.001	-.001
22	-.002	-.002	[illegible]	[illegible]	[illegible]	-.014	-.011	-.038	-.006	[redacted]	.000	-.006	.002	.011	-.001	-.002	.004
26	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	[redacted]	.000	.001	.002	.000	.000	.000
30	.000	.000	.000	.001	.000	.001	-.004	-.001	-.001	-.006	.000	[redacted]	.000	.025	.017	.001	-.006
31	.000	.000	.000	.000	.000	.000	.001	.001	.000	.002	.001	.000	[redacted]	-.005	-.001	-.002	.000
34	.001	-.001	-.001	.001	.001	-.007	-.015	-.026	-.004	.011	.002	.025	-.005	[redacted]	-.009	-.011	.004
35	.000	.000	.000	-.002	.000	-.010	.007	-.011	-.001	-.001	.000	.017	-.001	-.009	[redacted]	-.002	.013
36	.000	.000	.000	.000	.000	-.001	-.002	-.003	-.001	-.002	.000	.001	-.002	-.011	-.002	[redacted]	.006
37	.000	.000	.001	.000	.000	.000	-.007	-.008	-.001	.004	.000	-.006	.000	.004	.013	.006	[redacted]

.100

MAC Mass Matrix

Mode	7	8	12	14	15	16	17	18	20	22	26	30	31	34	35	36	37
7	1.000	0.000	0.000	0.000	0.018	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.010	0.002	0.019	0.000
8	0.000	1.000	0.042	0.000	0.000	0.000	0.002	0.000	0.003	0.026	0.000	0.000	0.000	0.000	0.005	0.000	0.002
12	0.000	0.042	1.000	0.003	0.000	0.002	0.017	0.002	0.047	0.166	0.000	0.003	0.000	0.002	0.027	0.002	0.046
14	0.000	0.000	0.003	1.000	0.000	0.000	0.001	0.000	0.003	0.000	0.000	0.000	0.000	0.002	0.008	0.000	0.000
15	0.018	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.015	0.000
16	0.000	0.000	0.002	0.000	0.000	1.000	0.085	0.012	0.164	0.001	[illegible]	[illegible]	[illegible]	0.000	0.001	0.000	0.008
17	0.000	0.002	0.018	0.001	0.000	0.099	0.985	0.001	0.040	0.007	[illegible]	[illegible]	[illegible]	0.000	0.005	0.000	0.038
18	0.000	0.000	0.001	0.000	0.000	0.003	0.040	0.988	0.002	0.000	0.000	0.000	0.000	0.010	0.000	0.000	0.001
20	0.000	0.003	0.047	0.003	0.000	0.159	0.032	0.006	1.000	0.009	0.000	0.000	0.000	0.000	0.000	0.000	0.000
22	0.000	0.019	0.142	0.000	0.001	0.001	0.011	0.001	0.008	0.794	0.000	0.012	0.001	0.000	0.018	0.001	0.036
26	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.001	0.000	0.000	0.000
30	0.000	0.000	0.004	0.000	0.000	0.006	0.009	0.001	0.000	0.013	0.000	0.969	0.000	0.007	0.017	0.000	0.002
31	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.999	0.008	0.000	0.001	0.000
34	0.011	0.000	0.002	0.001	0.001	0.000	0.000	0.012	0.000	0.017	0.001	0.007	0.009	0.993	0.058	0.006	0.004
35	0.002	0.005	0.027	0.008	0.000	0.001	0.004	0.000	0.000	0.016	0.000	0.018	0.000	0.073	1.000	0.000	0.049
36	0.019	0.000	0.002	0.000	0.015	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.006	0.000	1.000	0.000
37	0.000	0.002	0.046	0.000	0.000	0.009	0.036	0.003	0.000	0.043	0.000	0.004	0.000	0.007	0.043	0.000	0.998

0.794

Comparison between 40 and 79 accelerometers

Orthonormalized mass matrix

40 accelerometers

.098

Mode	7	8	12	14	15	16	17	18	20	22	26	30	31	34	35	36	37
7		.000	.000	.000	.001	-.001	-.003	-.004	-.001	-.008	.000	.000	.000	-.009	-.002	-.001	-.005
8	.000		.000	.000	.000	.000	-.001	.000	-.001	-.003	.000	.001	.000	-.002	-.002	-.001	.001
12	.000	.000		.000	.000	-.001	.000	-.001	.000	-.001	.000	.000	.000	-.001	.001	.000	-.001
14	.000	.000	.000		.000	.000	.000	.000	.000	-.002	.000	.001	.000	.001	-.003	.000	.001
15	.001	.000	.000	.000		.002	.006	.004	.002	.009	.000	.001	.000	.008	.002	.000	.004
16	-.001	[illegible]	[illegible]	[illegible]	[illegible]		-.005	-.003	-.006	-.017	.000	.002	.000	-.011	-.015	-.002	.005
17	-.003	[illegible]	[illegible]	[illegible]	[illegible]	-.005		.098	-.003	-.009	.000	-.004	.001	-.010	.015	.000	-.015
18	-.004	.000	-.001	.000	.004	-.003	.098		-.014	-.042	.000	-.001	.002	-.032	-.014	-.003	-.009
20	-.001	-.001	.000	.000	.002	-.006	-.003	-.014		-.005	.000	-.001	.000	-.003	.000	.000	-.003
22	-.008	-.003	-.001	-.002	.009	-.017	-.009	-.042	-.005		-.001	-.005	.003	.022	.003	.001	.008
26	.000	.000	.000	.000	.000	.000	.000	.000	.000	-.001		.000	.001	.002	.000	.000	.000
30	.000	.001	.000	.001	.001	.002	-.004	-.001	-.001	-.005	.000		.000	.028	.018	.002	-.006
31	.000	.000	.000	.000	.000	.000	.001	.002	.000	.003	.001	.000		-.005	-.001	-.002	.000
34	-.009	-.002	-.001	.001	.008	-.011	-.010	-.032	-.003	.022	.002	.028	-.005		-.001	-.004	.010
35	-.002	-.002	.001	-.003	.002	-.015	.015	-.014	.000	.003	.000	.018	-.001	-.001		.000	.013
36	-.001	-.001	.000	.000	.000	-.002	.000	-.003	.000	.001	.000	.002	-.002	-.004	.000		.008
37	-.005	.001	-.001	.001	.004	.005	-.015	-.009	-.003	.008	.000	-.006	.000	.010	.013	.008	

Target

.098

Mode	7	8	12	14	15	16	17	18	20	22	26	30	31	34	35	36	37
7		.000	.000	.000	.000	-.001	-.002	-.002	-.001	-.004	.000	.000	.000	-.003	-.001	-.001	-.002
8	.000		.000	.000	.000	.000	.000	.000	.000	.002	.000	.000	.000	.000	.002	.000	.000
12	.000	.000		.000	.000	.000	.000	.000	.000	-.001	.000	.000	.000	.000	-.001	.000	.001
14	.000	.000	.000		.000	.000	.001	.001	.000	.001	.000	.000	.000	.001	.000	.000	-.001
15	.000	.000	.000	.000		.001	.004	.003	.001	.006	.000	.000	.000	.003	.001	.000	.001
16	-.001	.000	.000	.000	.001		-.005	-.001	-.004	-.006	.000	-.001	.000	-.006	.000	-.001	.000
17	-.002	.000	.000	.001	.004	-.005		.098	-.004	-.019	.000	-.001	.001	-.012	-.003	-.002	-.004
18	-.002	.000	.000	.001	.003	-.001	.098		-.012	-.036	.000	-.003	.001	-.025	-.005	-.003	-.008
20	-.001	.000	[illegible]	[illegible]	[illegible]	-.004	-.004	-.012		-.006	.000	.000	.000	-.002	-.001	.000	.000
22	-.004	.002	[illegible]	[illegible]	[illegible]	-.006	-.019	-.036	-.006		.000	-.007	.003	.013	.000	-.002	.004
26	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000		.000	.001	.001	.000	.000	.000
30	.000	.000	.000	.000	.000	-.001	-.001	-.003	.000	-.007	.000		.000	.025	.016	.001	-.006
31	.000	.000	.000	.000	.000	.000	.001	.001	.000	.003	.001	.000		-.002	-.001	-.001	.002
34	-.003	.000	.000	.001	.003	-.006	-.012	-.025	-.002	.013	.001	.025	-.002		-.007	-.012	.008
35	-.001	.002	-.001	.000	.001	.000	-.003	-.005	-.001	.000	.000	.016	-.001	-.007		-.002	.012
36	-.001	.000	.000	.000	.000	-.001	-.002	-.003	.000	-.002	.000	.001	-.001	-.012	-.002		.005
37	-.002	.000	.001	-.001	.001	.000	-.004	-.008	.000	.004	.000	-.006	.002	.008	.012	.005	

79 accelerometers

.100

Mode	7	8	12	14	15	16	17	18	20	22	26	30	31	34	35	36	37
7		.000	.000	.000	.000	.000	-.001	-.001	.000	-.002	.000	.000	.000	.001	.000	.000	.000
8	.000		.000	.000	.000	.000	-.001	.000	.000	-.002	.000	.000	.000	-.001	.000	.000	.000
12	.000	.000		.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	-.001	.000	.000	.001
14	.000	.000	.000		.000	.000	.001	.000	.000	-.001	.000	.001	.000	.001	-.002	.000	.000
15	.000	.000	.000	.000		.001	.004	.002	.001	.005	.000	.000	.000	.001	.000	.000	.000
16	.000	.000	.000	.000	.001		-.005	-.002	-.005	-.014	.000	.001	.000	-.007	-.010	-.001	.000
17	-.001	-.001	.000	.001	.004	-.005		.100	-.004	-.011	.000	-.004	.001	-.015	.007	-.002	-.007
18	-.001	.000	.000	.000	.002	-.002	.100		-.013	-.038	.000	-.001	.001	-.026	-.011	-.003	-.008
20	.000	.000	[illegible]	[illegible]	[illegible]	-.005	-.004	-.013		-.006	.000	-.001	.000	-.004	-.001	-.001	-.001
22	-.002	-.002	[illegible]	[illegible]	[illegible]	-.014	-.011	-.038	-.006		.000	-.006	.002	.011	-.001	-.002	.004
26	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000		.000	.001	.002	.000	.000	.000
30	.000	.000	.000	.001	.000	.001	-.004	-.001	-.001	-.006	.000		.000	.025	.017	.001	-.006
31	.000	.000	.000	.000	.000	.000	.001	.001	.000	.002	.001	.000		-.005	-.001	-.002	.000
34	.001	-.001	-.001	.001	.001	-.007	-.015	-.026	-.004	.011	.002	.025	-.005		-.009	-.011	.004
35	.000	.000	.000	-.002	.000	-.010	.007	-.011	-.001	-.001	.000	.017	-.001	-.009		-.002	.013
36	.000	.000	.000	.000	.000	-.001	-.002	-.003	-.001	-.002	.000	.001	-.002	-.011	-.002		.006
37	.000	.000	.001	.000	.000	.000	-.007	-.008	-.001	.004	.000	-.006	.000	.004	.013	.006	

Comparison between 40 and 79 accelerometers (continued)

Cross-orthogonality matrix

40 accelerometers

Mode	7	8	12	14	15	16	17	18	20	22	26	30	31	34	35	36	37
7	1.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
8	.000	1.001	.000	.000	.000	-.001	-.001	.000	[illegible]	[illegible]	[illegible]	.000	.000	.000	.000	.000	.000
12	.000	.000	.977	.000	.000	.000	.001	.000	[illegible]	[illegible]	[illegible]	.000	.000	.000	.000	.000	.000
14	.000	.000	.000	.943	.001	.000	.000	.000	[illegible]	[illegible]	[illegible]	.000	.000	.000	.000	.000	.000
15	.001	.000	.000	-.001	.941	.002	-.001	-.003	.000	.000	.000	[illegible]	[illegible]	[illegible]	.000	.000	.000
16	-.001	.000	.000	.000	.004	-.948	.003	.010	.001	.000	.000	[illegible]	[illegible]	[illegible]	.000	.000	.000
17	-.003	.000	-.001	.001	.006	.008	**.853**	**.145**	.003	-.002	.000	[illegible]	[illegible]	[illegible]	.000	.000	.000
18	-.003	.000	-.001	.001	.007	.012	-.065	.919	.002	-.003	[illegible]	-.001	.001	.001	.000	.000	.000
20	-.001	-.001	.000	.000	.002	.007	-.003	-.015	.936	-.001	.000	.000	.000	.000	.000	.000	.000
22	-.009	-.004	-.001	-.002	.009	.017	.002	-.041	-.003	**-.808**	-.001	-.010	.003	-.003	-.001	-.001	-.001
26	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.934	-.001	.001	.000	.000	.000	.000
30	.000	.001	.000	.001	.000	-.002	-.003	.000	-.001	.000	.000	**.815**	.000	-.001	.000	.000	.000
31	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	-.001	-.001	.904	.000	.000	.001	.000
34	-.009	-.001	-.001	.001	.008	.010	-.004	-.030	-.003	.016	.002	.035	-.007	.952	-.014	-.008	-.012
35	-.002	-.001	.000	-.003	.002	.015	.016	-.012	.000	.005	.000	.020	-.002	.008	.936	-.004	-.002
36	-.001	-.001	.000	.000	.000	.002	.000	-.003	.000	.002	.000	.002	-.002	.003	.003	.922	-.002
37	-.004	.001	-.001	.001	.003	-.005	-.011	-.008	-.003	.006	.000	-.007	.000	.017	.013	.009	**.893**

.145

-.808

Target

Mode	7	8	12	14	15	16	17	18	20	22	26	30	31	34	35	36	37
7	1.00	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
8	.000	1.00	.000	.000	.000	.000	.000	.000	[illegible]	[illegible]	[illegible]	.000	.000	.000	.000	.000	.000
12	.000	.000	.977	.000	.000	.000	.000	.000	[illegible]	[illegible]	[illegible]	.000	.000	.000	.000	.000	.000
14	.000	.000	.000	.943	.001	.000	.000	-.001	[illegible]	[illegible]	[illegible]	.000	.000	.000	.000	.000	.000
15	.000	.000	.000	-.001	.941	.002	-.001	-.004	.000	.000	.000	[illegible]	[illegible]	[illegible]	.000	.000	.000
16	-.001	.000	.000	.000	.003	-.948	.002	.011	.001	-.001	.000	[illegible]	[illegible]	[illegible]	.000	.000	.000
17	-.002	.000	.000	.001	.005	.008	**.853**	**.146**	.003	-.003	.000	[illegible]	[illegible]	[illegible]	.000	-.001	.000
18	-.002	.000	.000	.001	.006	.011	-.066	.921	.002	-.003	[illegible]	-.002	.001	-.001	-.001	-.001	.000
20	-.001	.000	.000	.001	.001	.005	-.005	-.014	.936	-.002	.000	.000	.000	.000	.000	.000	.000
22	-.005	.002	.000	.001	.006	.005	-.010	-.033	-.004	**-.812**	.000	-.011	.003	-.007	-.004	-.004	-.002
26	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.934	-.001	.001	.000	.000	.000	.000
30	.000	.000	.000	.000	.000	.000	.000	-.001	.000	.000	.000	**.815**	.000	-.002	.000	.000	.000
31	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	-.001	-.001	.904	.000	.000	.001	.000
34	-.003	.000	.000	.001	.003	.005	-.006	-.022	-.002	.013	.002	.032	-.004	.949	-.017	-.015	-.012
35	-.001	.002	-.001	.000	.000	.000	-.002	-.004	-.001	.002	.000	.018	-.001	.007	.935	-.005	-.003
36	-.001	.000	.000	.000	-.001	.001	.000	-.002	.000	.001	.000	.002	-.002	.002	.003	.921	-.002
37	-.002	.000	.001	-.001	.001	.000	-.002	-.007	.000	.004	.000	-.007	.002	.015	.013	.006	**.889**

.146

-.812

79 accelerometers

Mode	7	8	12	14	15	16	17	18	20	22	26	30	31	34	35	36	37
7	1.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
8	.000	1.001	.000	.000	.000	.000	.000	.000	[illegible]	[illegible]	[illegible]	.000	.000	.000	.000	.000	.000
12	.000	.000	.977	.000	.000	.000	.001	.000	[illegible]	[illegible]	[illegible]	.000	.000	.000	.000	.000	.000
14	.000	.000	.000	.943	.001	.000	.000	-.001	[illegible]	[illegible]	[illegible]	.000	.000	.000	.000	.000	.000
15	.000	.000	.000	-.001	.941	.002	-.001	-.004	.000	.000	.000	[illegible]	[illegible]	[illegible]	.000	.000	.000
16	.000	.000	.000	.000	.003	-.948	.003	.011	.001	-.001	.000	[illegible]	[illegible]	[illegible]	.000	.000	.000
17	-.001	.000	.000	.001	.005	.008	**.853**	**.146**	.003	-.003	.000	[illegible]	[illegible]	[illegible]	-.001	-.001	.000
18	-.001	.000	.000	.001	.006	.012	-.065	.921	.002	-.003	[illegible]	-.002	.001	-.002	-.001	-.001	-.001
20	.000	.000	.000	.000	.001	.006	-.004	-.015	.936	-.002	.000	.000	.000	-.001	.000	.000	.000
22	-.002	-.002	.000	-.001	.005	.013	-.001	-.035	-.004	**-.811**	.000	-.010	.003	-.009	-.004	-.004	-.002
26	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.934	-.001	.001	.000	.000	.000	.000
30	.000	.000	.000	.000	.000	-.001	-.002	.000	.000	.000	.000	**.815**	.000	-.002	-.001	.000	.000
31	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	-.001	-.001	.903	.000	.000	.001	.000
34	.001	-.001	-.001	.001	.001	.005	-.008	-.022	-.003	.014	.002	.033	-.007	.944	-.018	-.013	-.012
35	.000	.000	.000	-.002	.000	.009	.009	-.008	-.001	.004	.001	.019	-.002	.007	.935	-.005	-.002
36	.000	.000	.000	.000	-.001	.001	-.001	-.002	.000	.001	.000	.002	-.002	.002	.003	.921	-.002
37	.000	.000	.001	.000	.000	-.001	-.005	-.007	-.001	.005	.000	-.006	.000	.012	.012	.007	**.890**

.146

-.811

Comparison between 40 and 79 accelerometers (continued)

MAC matrix

40 accelerometers

Mode	7	8	12	14	15	16	17	18	20	22	26	30	31	34	35	36	37
7	1.000	0.000	0.000	0.000	0.018	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.008	0.002	0.019	0.000
8	0.000	1.000	0.042	0.000	0.000	0.000	0.002	0.000	0.003	0.026	0.000	0.000	0.000	0.000	0.005	0.000	0.002
12	0.000	0.042	1.000	0.003	0.000	0.002	0.017	0.002	0.047	0.165	0.000	0.003	0.000	0.002	0.027	0.002	0.046
14	0.000	0.000	0.003	1.000	0.000	0.000	0.001	0.000	0.003	0.000	0.000	0.000	0.000	0.002	0.008	0.000	0.000
15	0.018	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.015	0.000
16	0.000	0.000	0.002	0.000	0.000	1.000	0.085	0.012	0.164	0.001	[illegible]	[illegible]	[illegible]	0.000	0.001	0.000	0.008
17	0.000	0.002	0.018	0.001	0.000	0.099	0.985	0.001	0.040	0.007	[illegible]	[illegible]	[illegible]	0.000	0.005	0.000	0.038
18	0.000	0.000	0.001	0.000	0.000	0.003	0.040	0.988	0.002	0.00[illegible]	0.000	0.000	0.000	0.010	0.000	0.000	0.001
20	0.000	0.003	0.047	0.003	0.000	0.159	0.032	0.006	1.000	0.009	0.000	0.000	0.000	0.000	0.000	0.000	0.000
22	0.000	0.019	0.142	0.000	0.001	0.001	0.011	0.001	0.008	0.792	0.000	0.012	0.001	0.000	0.018	0.001	0.036
26	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.001	0.000	0.000	0.000
30	0.000	0.000	0.004	0.000	0.000	0.006	0.009	0.001	0.000	0.013	0.000	0.969	0.000	0.007	0.017	0.000	0.002
31	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.999	0.008	0.000	0.001	0.000
34	0.011	0.000	0.002	0.001	0.001	0.000	0.000	0.013	0.000	0.017	0.001	0.007	0.009	0.992	0.058	0.006	0.004
35	0.002	0.005	0.027	0.008	0.000	0.001	0.004	0.000	0.000	0.016	0.000	0.018	0.000	0.072	1.000	0.000	0.049
36	0.019	0.000	0.002	0.000	0.015	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.006	0.000	1.000	0.000
37	0.000	0.002	0.046	0.000	0.000	0.009	0.036	0.003	0.000	0.043	0.000	0.004	0.000	0.007	0.043	0.000	0.998

0.792

Target

Mode	7	8	12	14	15	16	17	18	20	22	26	30	31	34	35	36	37
7	1.000	0.000	0.000	0.000	0.018	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.009	0.002	0.019	0.000
8	0.000	1.000	0.042	0.000	0.000	0.000	0.002	0.000	0.003	0.026	0.000	0.000	0.000	0.000	0.005	0.000	0.002
12	0.000	0.042	1.000	0.003	0.000	0.002	0.017	0.002	0.047	0.166	0.000	0.003	0.000	0.002	0.027	0.002	0.046
14	0.000	0.000	0.003	1.000	0.000	0.000	0.001	0.000	0.003	0.000	0.000	0.000	0.000	0.002	0.008	0.000	0.000
15	0.018	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.015	0.000
16	0.000	0.000	0.002	0.000	0.000	1.000	0.085	0.012	0.164	0.001	[illegible]	[illegible]	[illegible]	0.000	0.001	0.000	0.008
17	0.000	0.002	0.018	0.001	0.000	0.099	0.985	0.001	0.040	0.007	[illegible]	[illegible]	[illegible]	0.000	0.004	0.000	0.039
18	0.000	0.000	0.001	0.000	0.000	0.003	0.040	0.988	0.002	0.00[illegible]	0.000	0.000	0.000	0.010	0.000	0.000	0.001
20	0.000	0.003	0.047	0.003	0.000	0.159	0.032	0.006	1.000	0.009	0.000	0.000	0.000	0.000	0.000	0.000	0.000
22	0.000	0.019	0.142	0.000	0.001	0.001	0.011	0.001	0.008	0.795	0.000	0.012	0.001	0.000	0.018	0.001	0.036
26	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.001	0.000	0.000	0.000
30	0.000	0.000	0.004	0.000	0.000	0.006	0.009	0.001	0.000	0.013	0.000	0.969	0.000	0.007	0.017	0.000	0.002
31	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.999	0.008	0.000	0.001	0.000
34	0.011	0.000	0.002	0.001	0.001	0.000	0.000	0.012	0.000	0.017	0.001	0.007	0.009	0.993	0.058	0.006	0.004
35	0.002	0.005	0.027	0.008	0.000	0.001	0.004	0.000	0.000	0.016	0.000	0.018	0.000	0.073	1.000	0.000	0.049
36	0.019	0.000	0.002	0.000	0.015	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.006	0.000	1.000	0.000
37	0.000	0.002	0.046	0.000	0.000	0.009	0.036	0.003	0.000	0.043	0.000	0.004	0.000	0.007	0.043	0.000	0.998

.795

79 accelerometers

Mode	7	8	12	14	15	16	17	18	20	22	26	30	31	34	35	36	37
7	1.000	0.000	0.000	0.000	0.018	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.010	0.002	0.019	0.000
8	0.000	1.000	0.042	0.000	0.000	0.000	0.002	0.000	0.003	0.026	0.000	0.000	0.000	0.000	0.005	0.000	0.002
12	0.000	0.042	1.000	0.003	0.000	0.002	0.017	0.002	0.047	0.166	0.000	0.003	0.000	0.002	0.027	0.002	0.046
14	0.000	0.000	0.003	1.000	0.000	0.000	0.001	0.000	0.003	0.000	0.000	0.000	0.000	0.002	0.008	0.000	0.000
15	0.018	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.015	0.000
16	0.000	0.000	0.002	0.000	0.000	1.000	0.085	0.012	0.164	0.001	[illegible]	[illegible]	[illegible]	0.000	0.001	0.000	0.008
17	0.000	0.002	0.018	0.001	0.000	0.099	0.985	0.001	0.040	0.007	[illegible]	[illegible]	[illegible]	0.000	0.005	0.000	0.038
18	0.000	0.000	0.001	0.000	0.000	0.003	0.040	0.988	0.002	0.00[illegible]	0.000	0.000	0.000	0.010	0.000	0.000	0.001
20	0.000	0.003	0.047	0.003	0.000	0.159	0.032	0.006	1.000	0.009	0.000	0.000	0.000	0.000	0.000	0.000	0.000
22	0.000	0.019	0.142	0.000	0.001	0.001	0.011	0.001	0.008	0.794	0.000	0.012	0.001	0.000	0.018	0.001	0.036
26	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.001	0.000	0.000	0.000
30	0.000	0.000	0.004	0.000	0.000	0.006	0.009	0.001	0.000	0.013	0.000	0.969	0.000	0.007	0.017	0.000	0.002
31	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.999	0.008	0.000	0.001	0.000
34	0.011	0.000	0.002	0.001	0.001	0.000	0.000	0.012	0.000	0.017	0.001	0.007	0.009	0.993	0.058	0.006	0.004
35	0.002	0.005	0.027	0.008	0.000	0.001	0.004	0.000	0.000	0.016	0.000	0.018	0.000	0.073	1.000	0.000	0.049
36	0.019	0.000	0.002	0.000	0.015	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.006	0.000	1.000	0.000
37	0.000	0.002	0.046	0.000	0.000	0.009	0.036	0.003	0.000	0.043	0.000	0.004	0.000	0.007	0.043	0.000	0.998

0.794

Select a Single Shaker

Shaker Selection

- Select shaker position where the "standard deviation" of the pick driving point acceleration transfer function (MDPATF) is small.
- Compute the magnitude of the MDPATF matrix from real and imaginary part of the driving point ATF matrix in equation (18).

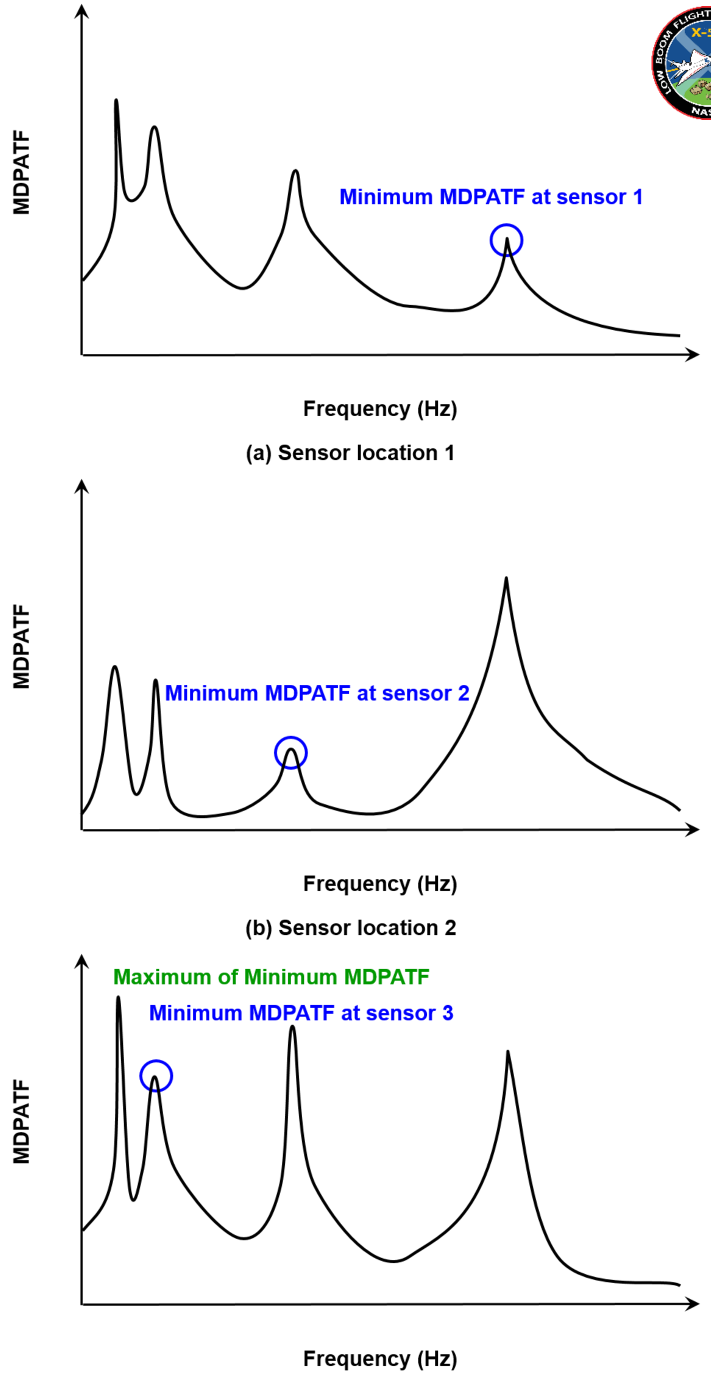
1) Go to an accelerometer location

❖ $$DPATF\ matrix = \begin{bmatrix} DPATE(1,1) & DPATE(1,2) & \cdots & DPATE(1,n_k) \\ DPATE(2,1) & DPATE(2,2) & \cdots & DPATE(2,n_k) \\ \cdots & \cdots & \vdots & \cdots \\ DPATE(n_d,1) & DPATE(n_d,2) & \cdots & DPATE(n_d,n_k) \end{bmatrix} \quad \text{Eq.(18)}$$

2) Make the minimum MDPATF vector

3) Select maximum

$$\begin{Bmatrix} select\ minimum\ MDPATF\ at\ accelerometer\ \#1 \\ select\ minimum\ MDPATF\ at\ accelerometer\ \#2 \\ \vdots \\ select\ minimum\ MDPATF\ at\ accelerometer\ \#na \end{Bmatrix}$$



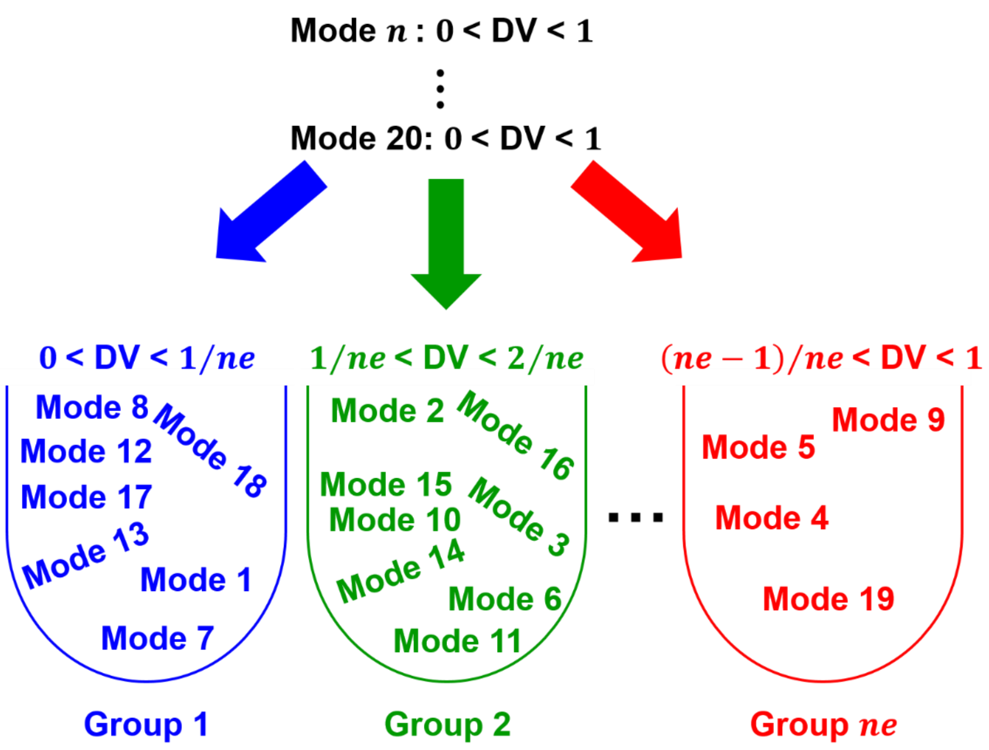
(c) Sensor location 3

1) Select minimum MDPATF at each accelerometer location.
2) Make the minimum MDPATF vector
3) Compare the element of the minimum MDPATF vector and select the maximum accelerometer location as an shaker location

Select Multiple Shakers

Select Multiple Shakers using Topology Optimization

- Random grouping of modes based on random design variable
  - Number of modes in each group is also randomly decided
  - Number of DV = number of modes (40 in this study)
  - Grouping is performed using integerization table.



***ne*: number of group**

Use four shakers

Mode number	Group 1 modes	Group 2 modes	Group 3 modes	Group 4 modes
1	3	1	2	5
2	4	6	15	12
3	7	8	16	18
4	10	9	17	20
5	22	11	19	27
6	23	13	21	35
7	24	14	25	37
8	26	36	28	
9	29	40	30	
10	31			
11	32			
12	33			
13	34			
14	38			
15	39			

Cared by Shaker #1

Cared by Shaker #2

Cared by Shaker #3

Cared by Shaker #4

Select Shaker for each Group

Accelerometer number	Group 1 modes	Group 2 modes	Group 3 modes	Group 4 modes
1	0.34	0.78	1.70	**6.88(12)***
2	1.95	1.40	1.40	2.86
3	0.26	0.70	0.27	1.70
4	2.56	3.84	0.48	0.78
5	1.68	**5.86(14)***	0.36	0.43
6	1.20	1.25	1.49	5.21
7	1.15	1.24	1.68	5.86
8	5.66	1.08	0.42	5.13
9	0.84	0.87	0.83	5.61
10	0.32	0.15	0.33	0.80
11	3.24	2.31	1.99	3.00
12	1.14	0.28	1.56	1.63
13	1.70	0.84	2.21	2.56
14	0.62	0.86	0.81	1.48
15	1.36	0.66	**4.41(28)***	2.14
16	0.38	0.03	0.22	0.02
17	0.86	0.43	0.35	0.49
18	0.35	0.13	0.08	0.17
19	**6.69(23)***	0.91	0.47	2.45

Minimum MDPATF for each group

- ❑ Choose multiple shakers using the single shaker selection technique

Mode number

Select maximum

Four shaker locations & exciting DOFs

Shaker number	Accelerometer number	Grid number	DOF	X (inch)	Y (inch)	Z (inch)
1	19	281159	3	1018.51	-177.00	100.81
2	5	441530	3	1148.39	-85.32	100.00
3	15	410127	3	1007.34	-150.00	101.18
4	1	200001	3	25.46	0.00	82.51

Objective Function for Topology Optimization

Accelerometer number	Group 1 modes	Group 2 modes	Group 3 modes	Group 4 modes	All modes
1	2.85	4.89	7.66	24.24	2.85
2	3.44	1.45			
3	5.00	0.48			
4	4.47	9.84			
5	5.45	**16.89(13)***			
6	8.76	3.33			
7	9.96	2.26			
8	6.49	5.06			
9	3.88	8.59			
10	2.76	0.71	1.92	4.33	0.71
11	4.25	2.72	15.09	8.02	2.72
12	9.20	7.85	7.06	9.28	7.06
13	2.12	2.31	3.54	3.07	2.12
14	3.89	1.33	4.30	14.08	1.33
15	6.22	5.72	14.74	9.61	5.72
16	3.45	0.28	0.39	0.53	0.28
17	5.85	4.32	4.71	7.50	4.32
18	4.17	0.45	2.04	0.32	0.32
19	**14.20(23)***	11.23	11.06	**24.71(35)***	**11.06(19)***
20	3.85	5.51			
21	2.35	6.82			
22	4.32	3.56			
23	0.77	1.09			
24	2.23	2.22			
25	1.95	0.18			
26	4.30	1.57			
27	1.66	1.53			
28	3.97	4.50	**15.78(2)***	3.94	3.94

Minimum MATF for each group

Table 23. The maximum of minimum MATF for all the modes of interest using different numbers of shakers.

Number of shakers	Maximum of minimum MATF value (= objective function value at the global optimum point)	Increase of maximum of minimum MATF value based on number of shaker increase (percent)
1	1.987	N/A
2	5.031	153.1
3	7.5522	50.1
4	11.06	46.5

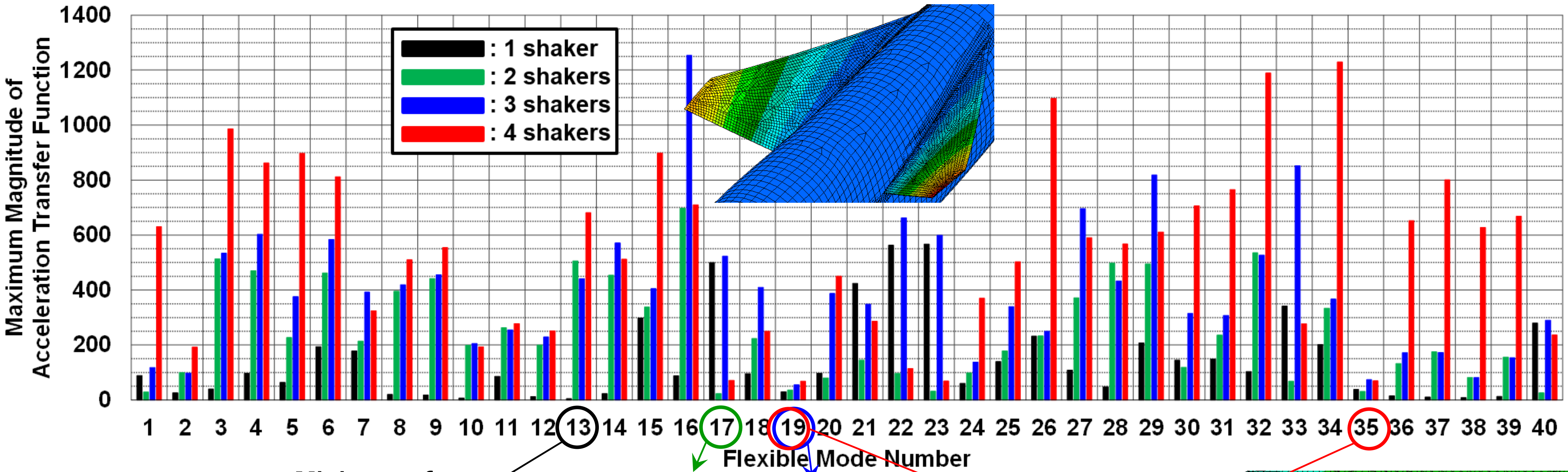
Objective function value for topology optimization

Maximize the objective function using genetic algorithm

Comparisons: MDPATF vs. MATF

Shaker number	Maximum of minimum MDPATF for shaker selection (summary of Table 19)	Maximum of minimum MATF (summary of Table 21)	MATF at same accelerometer location with same mode in the second column
1	6.69(23)*	14.20(23)*	14.20(23)*
2	5.86(14)*	**16.89(13)***	**17.31(14)*+-**
3	4.41(28)	**15.78(2)***	**33.26(28)*+**
4	6.88(12)*	**24.71(35)***	**29.51(12)*++**

Maximum MATF for each mode with different number of shakers



Minimum of maximum MATF for 1 shaker

Minimum of maximum MATF for 2 shakers

Minimum of maximum MATF for 3 & 4 shakers

Difficult to shake compare to other modes

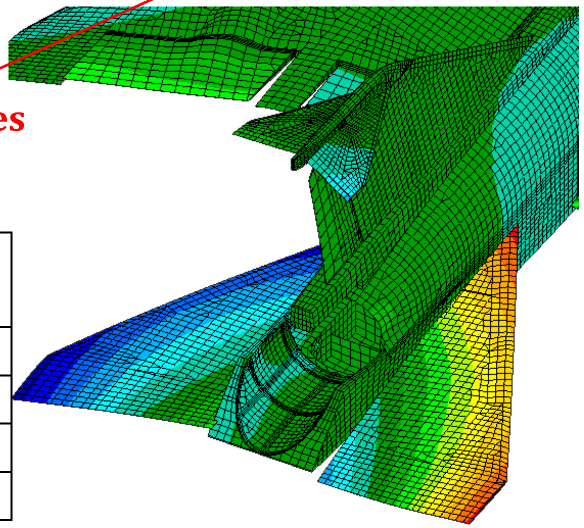


Table 25. The minimum of maximum MATF for all the modes of interest using different numbers of shakers.

Number of shakers	Minimum of maximum MATF value	Increase of minimum of maximum MATF value based on number of shaker increase (percent)
1	4.95 **(13)**	N/A
2	23.19 **(17)**	368.5
3	55.11 **(19)**	137.6
4	67.82 **(19)**	23.1

Summaries

- The tool developed in this study has been tested using the **C609 FE model** of the X-59 LBFD aircraft.
  - Final configuration is **C612 FE model**.
- Total number of **79** accelerometers are selected for the GVT.
  - Driving point acceleration transfer function method: **14** accelerometers (number of primary modes = 17)
  - Effective independence method: **26** accelerometers
  - mode visualization during the ground vibration test: **39** accelerometers
- **Four** shakers are selected using the **topology optimization** technique.
  - **Genetic algorithm** is used to maximize the objective function value.
    - Compute the magnitude of the acceleration transfer function (MATF) matrix.
      - ✓ Row vectors: MATFs at each accelerometer location
      - ✓ Column vectors: MATF at each natural frequency
    - Select the minimum MATF at each accelerometer location and construct a vector.
    - The maximum value of this vector is selected as an objective function value.