# Beam propagation through atmospheric turbulence using an altitude-dependent structure profile with non-uniformly distributed phase screens

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## Summary

For space-to-ground optical communications, numerical wave optics simulations provide a useful technique for modeling turbulence-induced beam degradation when the analytical theory is insufficient for characterizing receiver optics. Motivated by such an application we use a split-step method modeling the turbulence as a series of random phase screens using the Hufnagel-Valley turbulence profile. We examine the irradiance and phase statistics for uniformly and non-uniformly located screens and find better agreement with theory using a non-uniform discretization minimizing the contribution of each screen to the total scintillation. We evaluate this method as a flexible alternative to other layered models used in imaging applications.

## Split-step method

Beam propagation in moderate turbulence is governed by the linear Schrodinger equation (or 'parabolic wave equation')

$$i\frac{\partial u}{\partial z} = -\frac{1}{2k}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)u - kn_1(x,y,z)u =: -(A+B)u$$

whose solution in the plane $z = L$ can be related to the solution in the plane $z = 0$ in via an evolution operator $U(L,0)$. The split-step beam propagation method is a numerical method used to solve this equation by splitting the evolution operator into multiple steps $U(L,0) = U(L,z_{n-1})U(z_{n-1},z_{n-2})\dots U(z_1,0)$ and approximating the evolution for each step using the "phase screen approximation"

$$U(z_{j+1},z_j) \simeq e^{i\int_{z_j}^{z_{j+1}}(A+B(z))dz} \simeq e^{i\alpha_j\Delta z_j A} e^{i\int_{z_j}^{z_{j+1}} B(z)dz} e^{i(1-\alpha_j)\Delta z_j A} \quad (1)$$

where the step sizes $\Delta z_j = z_{j+1} - z_j$ and phase screen locations $\alpha_j$ are freely chosen parameters which specify the discretization. The middle term represents a phase screen $e^{i\phi}$ where the phase

$$\phi(x,y) = k\int_{z_j}^{z_{j+1}} n_1(x,y,z)dz$$

describes variations in the optical path length along rays parallel to the propagation axis.

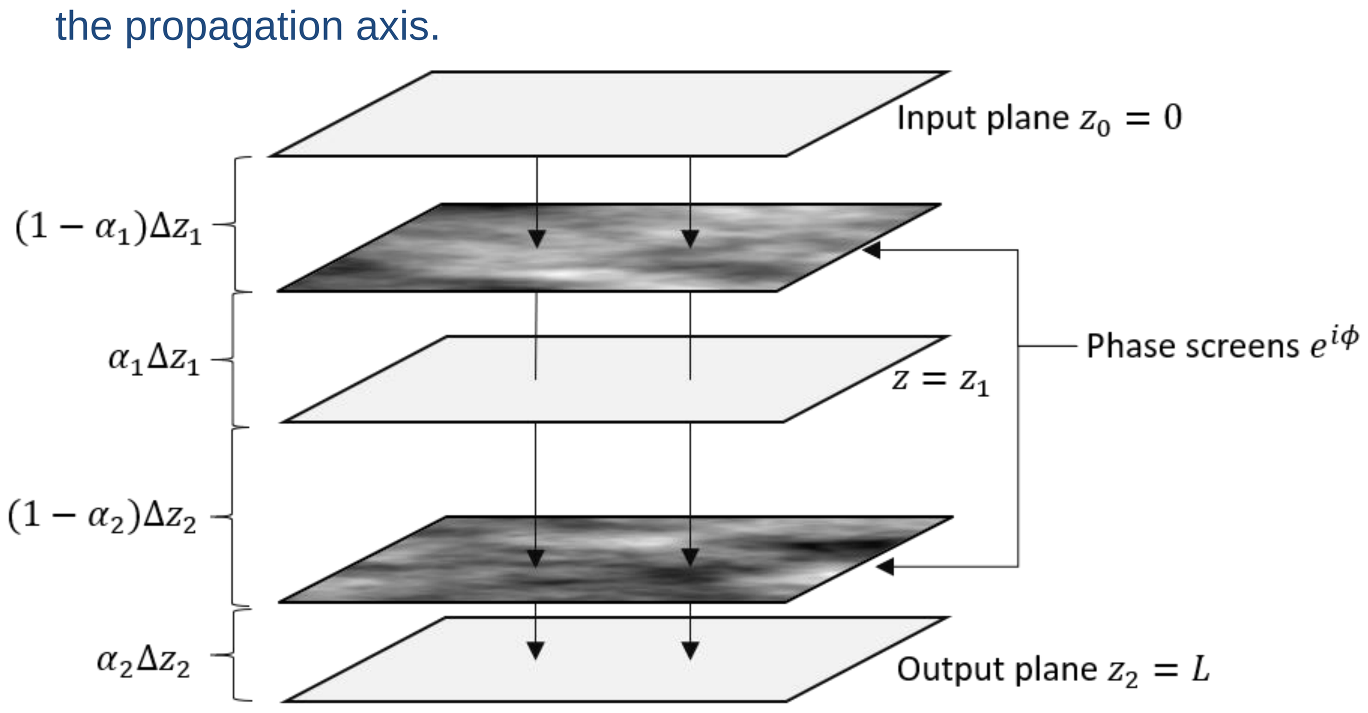


Figure 1. Schematic of split-step beam propagation method.

In this work, we study the error introduced by (1) for different discretizations $\{\Delta z_j, \alpha_j\}$ assuming that the refractive index fluctuations $n_1(x,y,z)$ satisfy modified von Karman refractive index statistics with path-dependent variance determined by the Hufnagel-Valley refractive index structure profile.

## Discretization of path

Several discretization algorithms to determine the location and strength of each phase screen (i.e. $\{\Delta z_j, \alpha_j\}$) were studied:

(PM) Minimize phase variance $\sigma_\phi^2$ for each segment, centered ($\alpha_j = 1/2$)

(DM) Minimize distance $\Delta z_j$ for each segment, centered ($\alpha_j = 1/2$)

(SM) Minimize scintillation $\sigma_\chi^2$ for each segment, centered ($\alpha_j = 1/2$)

(SM*) Minimize scintillation, center-of-mass located screens ($\alpha_j$ in Figure 2)

(MM) Moment-matched four-layer model of Troxel et al.[1] ($\Delta z_i, \alpha_i$ in Figure 2)

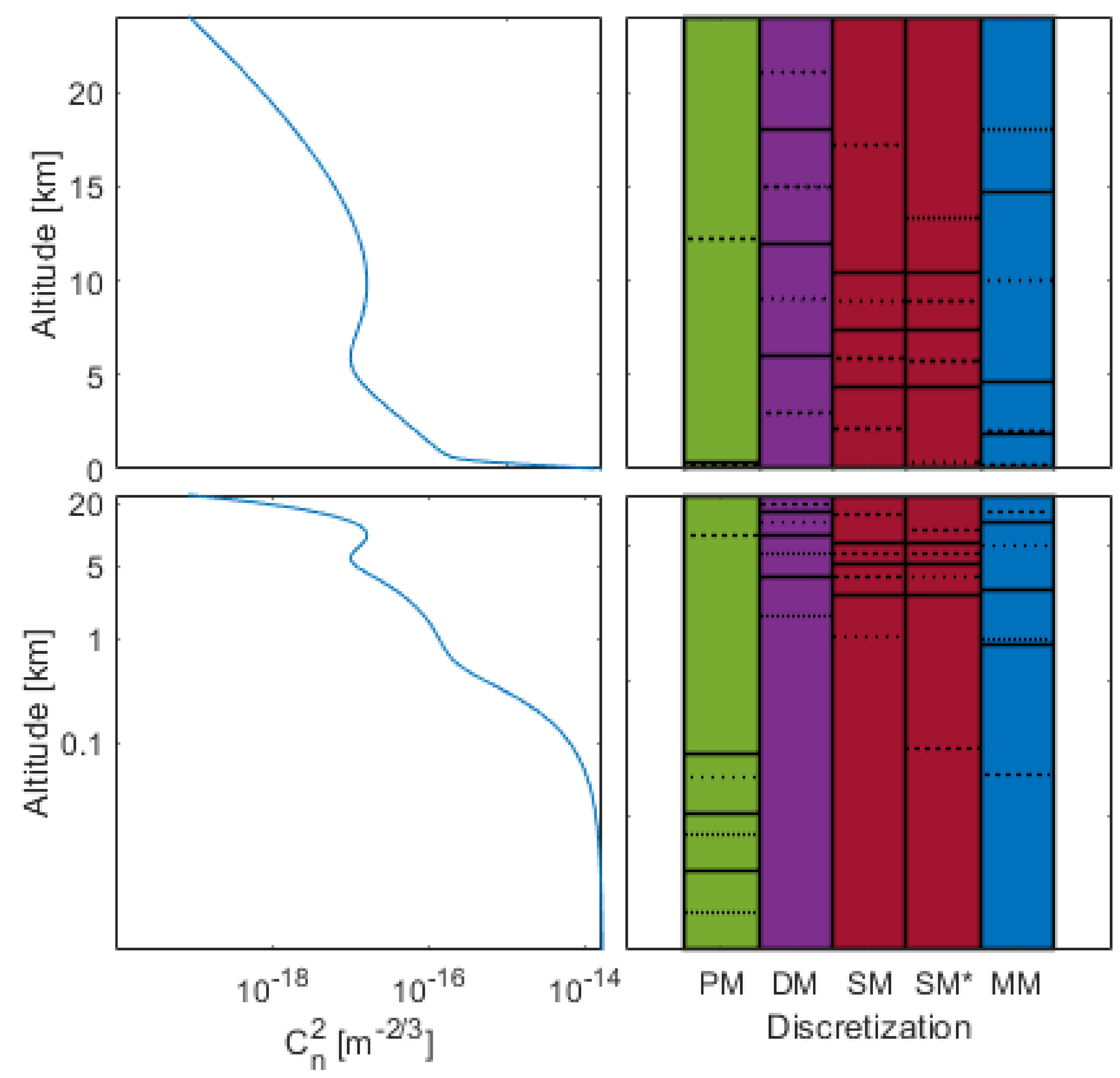


Figure 2. Discretization of the vertical Hufnagel-Valley 5/7 downlink propagation path. Solid lines are segment boundaries and dotted lines are phase screens.

We assume a 24 km vertical propagation path with the Hufnagel-Valley turbulence profile modeling a LEO-to-ground channel (turbulence above 24km is negligible). The path is divided into four segments and random "subharmonic" phase screens are generated for each segment via the spectral method studied by Frehlich.[2] The segmentation algorithms were evaluated by examining the statistics of the numerically propagated optical fields and comparing to those expected from the Rytov theory.

## Results

### Uplink Results

- The center-of-mass scintillation-minimized (SM*) discretization yielded the closest agreement to the wave structure function and normalized mean irradiance profile predicted by the Rytov theory.
- The moment-matched (MM) discretization gave similar results.
- The other centered discretizations all yielded results tending toward those expected from a similar propagation path with a path-averaged constant turbulence profile.

### Downlink Results

- The most significant differences were visible in the scintillation index
- The center-of-mass scintillation-minimized and moment-matched discretizations showed excellent agreement with the Rytov theory
- Other centered discretizations again tended toward the results expected from the path-averaged profile.

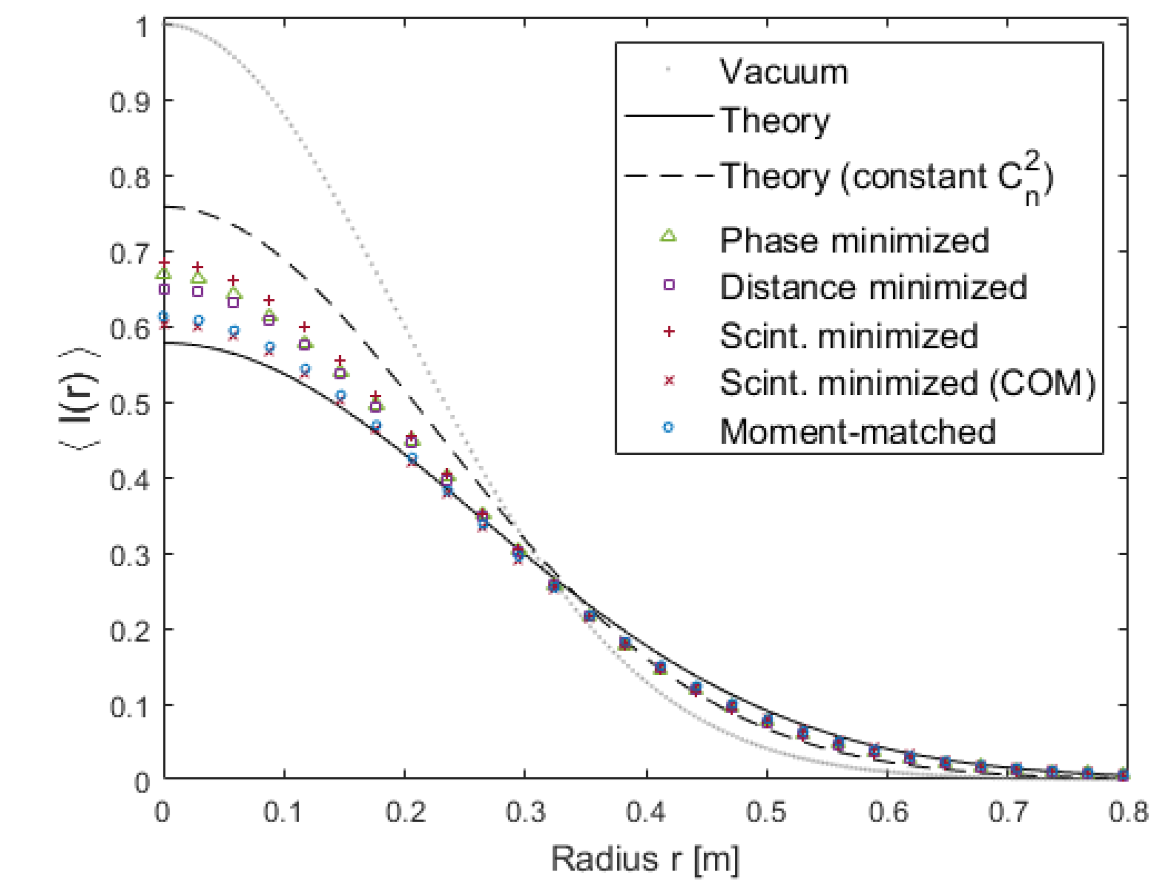


Figure 3. Mean irradiance for uplink HV-21 propagation path with ground level turbulence $C_n^2 = 6.8\cdot10^{-14}\ m^{-2/3}$ and a 6 cm input beam.

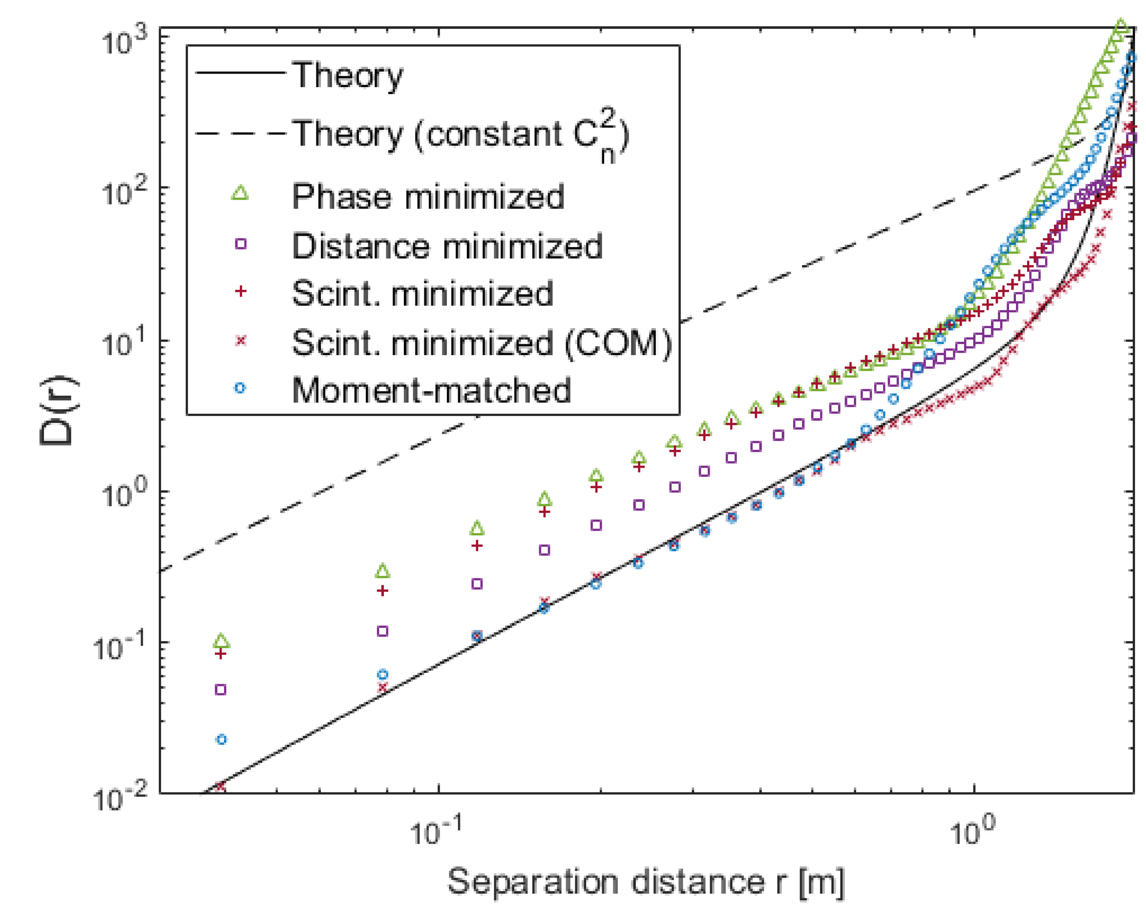


Figure 4. Wave structure function for uplink HV-21 propagation path with ground level turbulence $C_n^2 = 6.8\cdot10^{-14}\ m^{-2/3}$ and a 6 cm input beam.

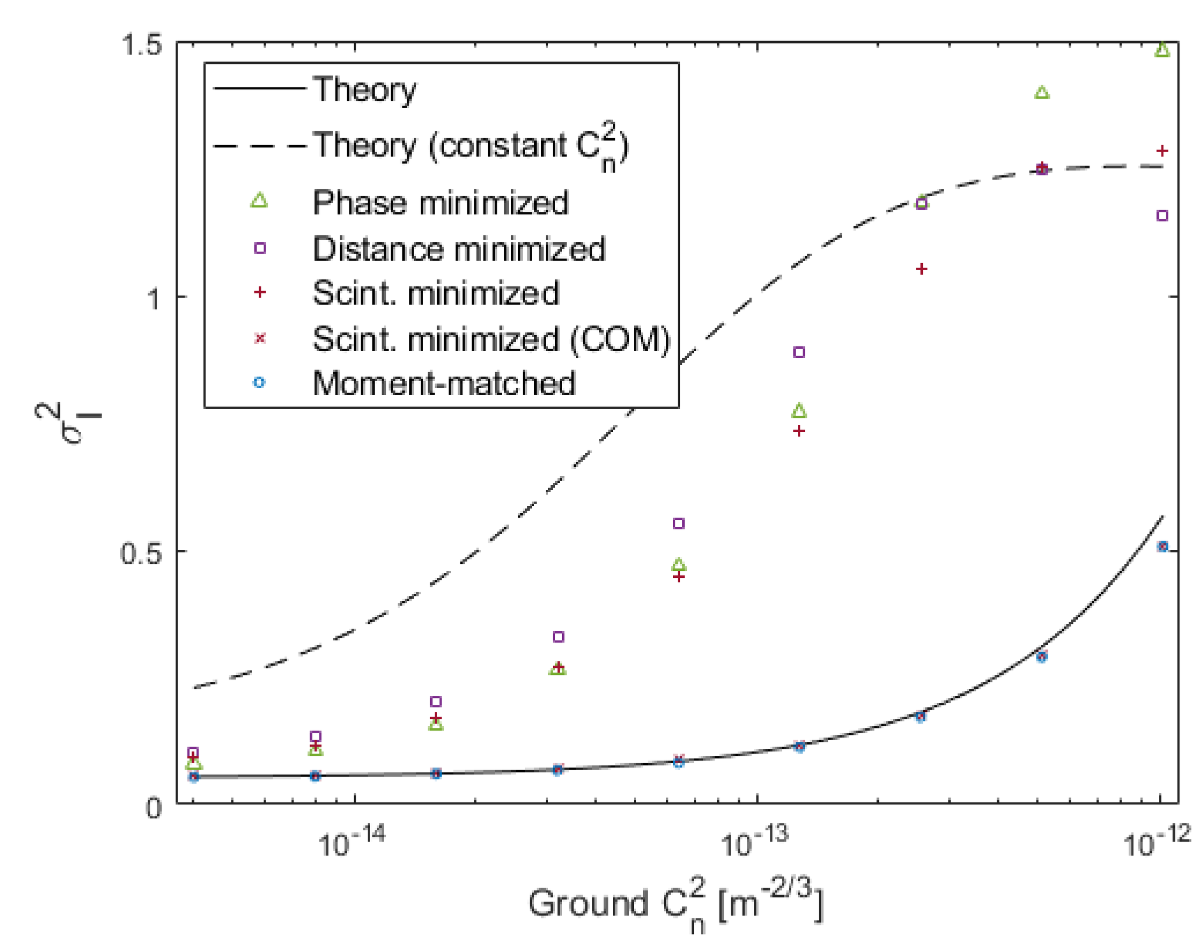


Figure 5. Scintillation index for downlink HV-21 propagation path at a range of ground turbulence levels for a 2 m beam. (Note: the scintillation-minimized (COM) and moment-matched results are almost identical.)

## REFERENCES

[1] S.E. Troxel, B.M. Welsh, M.C. Roggemann "Off-axis optical transfer function calculations in an adaptive-optics system by means of a diffraction calculation for weak index fluctuations.," *Journal of the Optical Society of America A* **11**, pp.2100-2111, Jul 1994.

[2] Frehlich, R., "Simulation of laser propagation in a turbulent atmosphere," Appl. Opt. 39(3), 393-397 (2000).