

Shape Validation and RF Performance of Inflatable Antennas

Bryan Welch

NASA Glenn Research Center

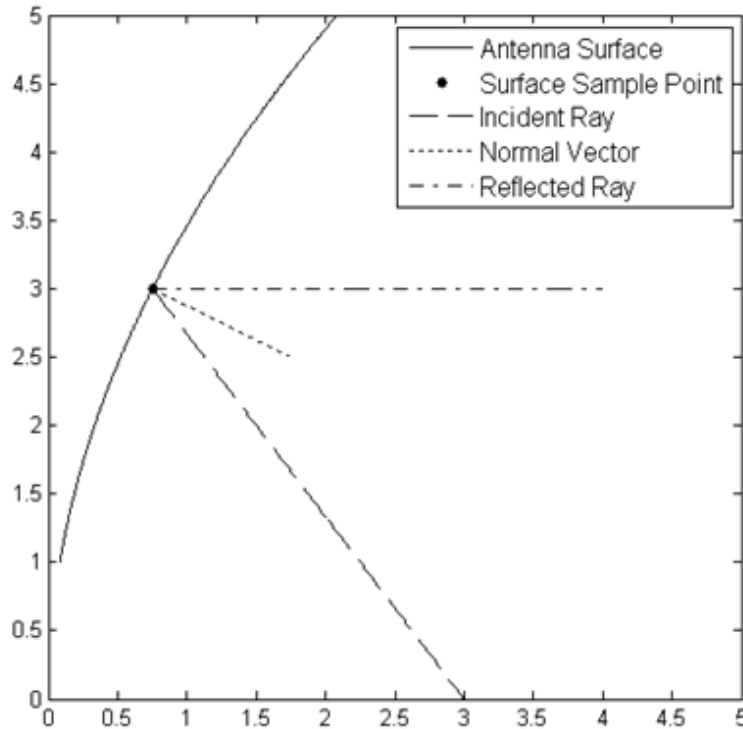
Doctoral Dissertation Defense

February 6, 2020

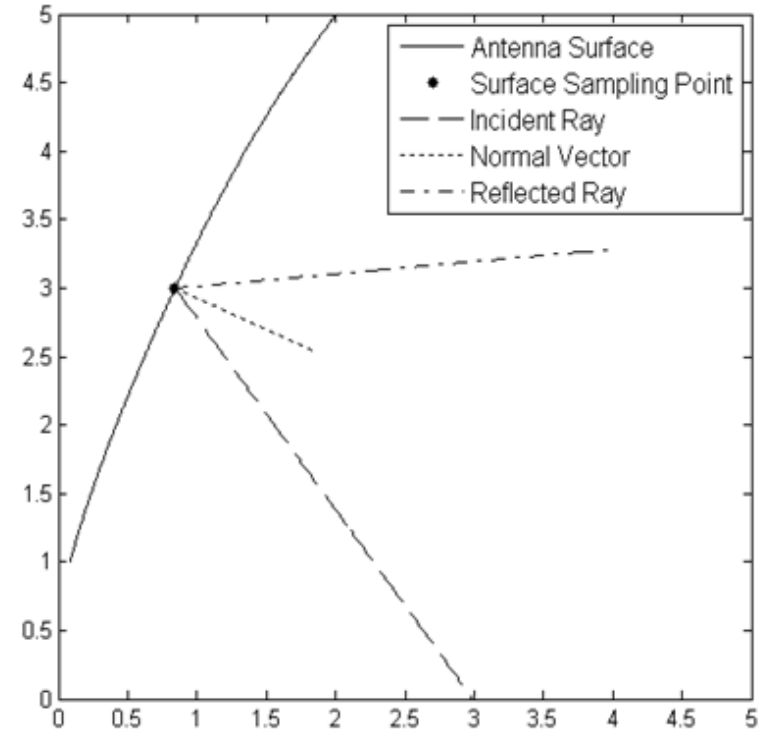
- ◆ **Introduction**
- ◆ **Inflatable Aperture Antenna Surface Shape Profile Derivation**
- ◆ **Inflatable Aperture Antenna RF Performance Model Derivation**
- ◆ **RF Performance Model Comparison of Inflatable Aperture Antenna with Parabolic Aperture Antenna**
- ◆ **Conclusions**

INTRODUCTION

Snell's Law Illustrations



Ideal Parabolic Antenna Surface Reflection



Non-Ideal Parabolic Antenna Surface Reflection

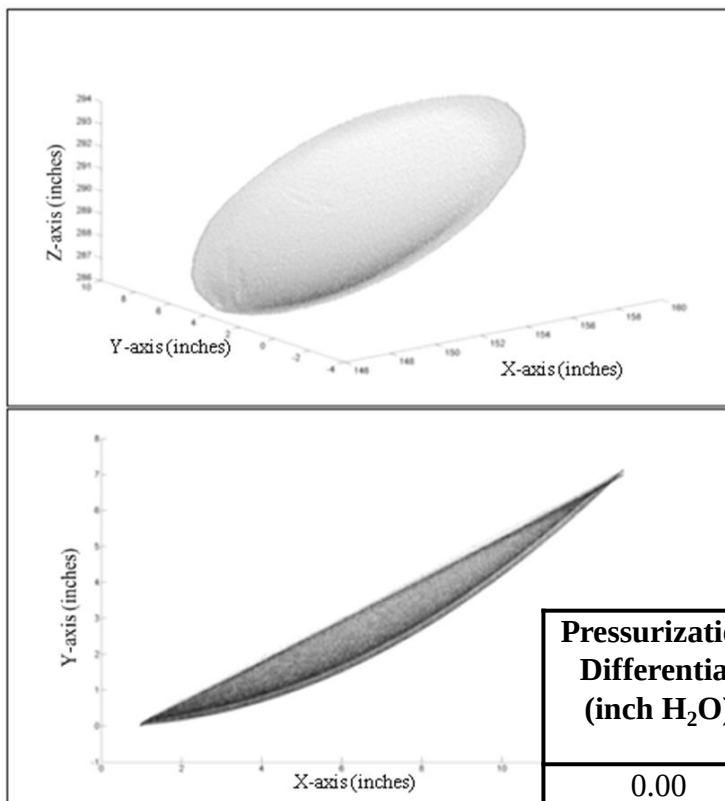
Ruze Equation

$$L_R = e^{-\left(\frac{4\pi\epsilon}{\lambda_A}\right)^2}$$

Ruze Equation assumes errors are random in nature, uniformly distributed in fixed circular correlation regions much smaller than the antenna diameter, and have a Gaussian spatial phase correlation.

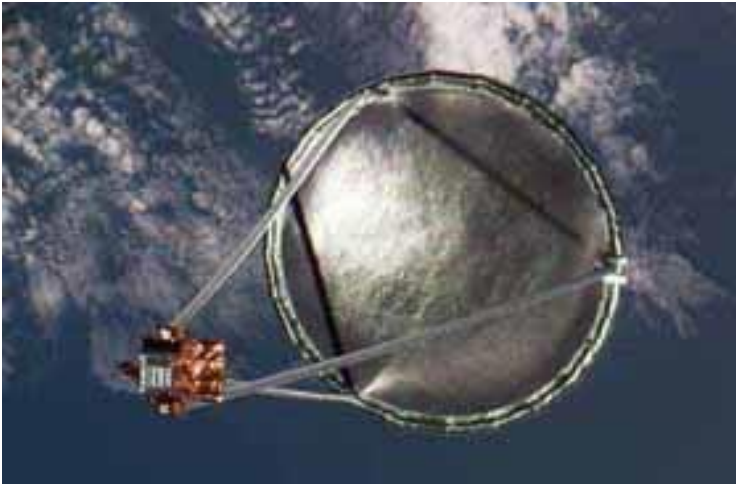
Application of Antenna Surface Theory to Inflatable Aperture Antenna

- Master's Thesis proved that Ruze Equation didn't apply to Inflatable Aperture Antennas



Pressurization Differential (inch H ₂ O)	RF Gain (dBi)	RF-Derived Gain Degradation (dB)	RMS Surface Error (inch)	Ruze-Derived Gain Degradation (dB)	Difference (dB)
0.00	14.42	-12.33	1.20	-402.02	389.69
0.03	23.14	-3.61	0.34	-32.21	28.60
0.04	24.37	-2.38	0.26	-19.05	16.67
0.05	24.52	-2.23	0.23	-15.12	12.89
0.06	24.47	-2.28	0.19	-9.86	7.58
0.07	24.45	-2.30	0.26	-19.47	17.17

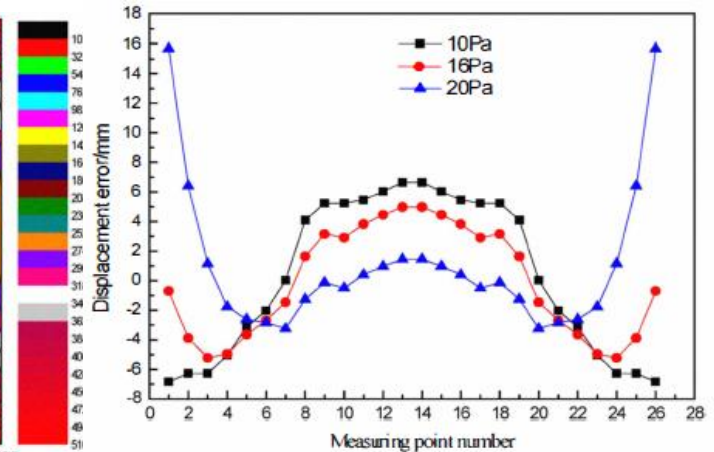
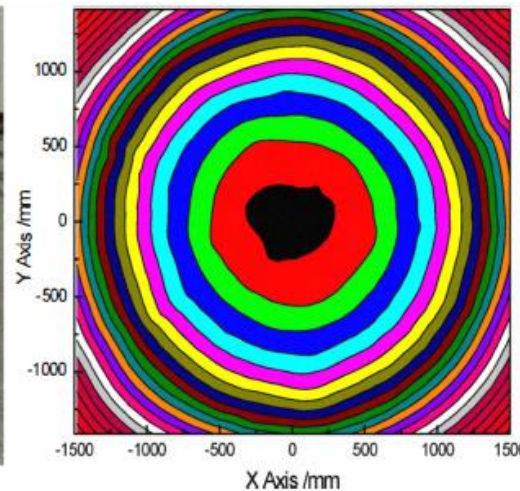
Inflatable Antenna Experiment



NASA SBIR Characterization



Harbin Institute of Technology Photogrammetry Analysis





Dissertation Motivation and Contributions

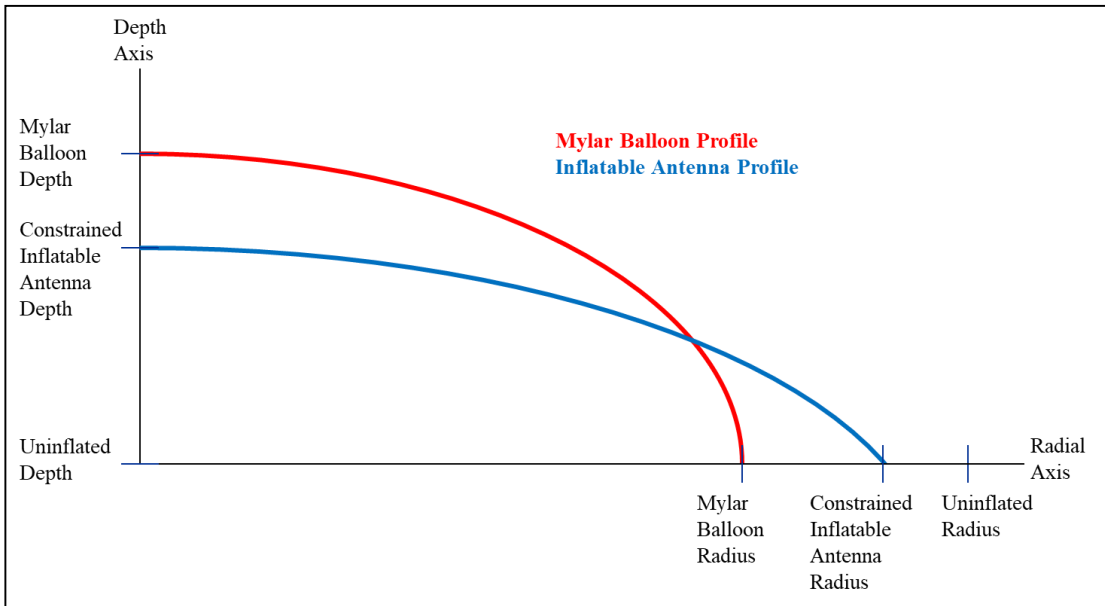


- ◆ **Derived a mathematical model of the inflatable antenna surface.**
- ◆ **Validated mathematical model of the inflatable aperture antenna.**
- ◆ **Derived physical optics model of the inflatable aperture antenna.**
- ◆ **Performed a degradation analysis of the inflatable aperture antenna RF performance model against standard parabolic antenna RF performance model using common diameter, depth, and arc-length.**

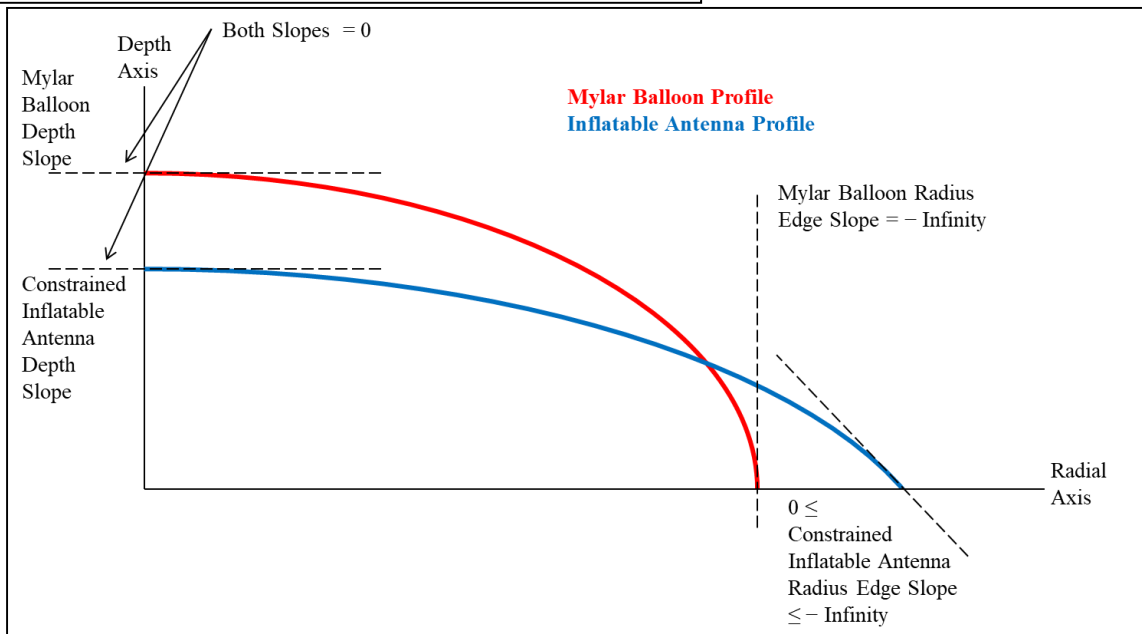
INFLATABLE APERTURE ANTENNA SURFACE SHAPE PROFILE DERIVATION

Comparison of Surface Profiles

Surface Shapes



Surface Slopes



Generic Calculus of Variations Extremal Function

$$J = \int_{t_0}^{t_1} f(t, x(t), x'(t)) dt$$

Euler-Lagrange Equation

$$\frac{\partial f}{\partial x} - \frac{d}{dt} \left(\frac{\partial f}{\partial x'} \right) = 0$$

Calculus of Variations Endpoint-Curve Shape Equation

$$\int_0^{r_B} \sqrt{1 + z'(x)^2} dx = a$$

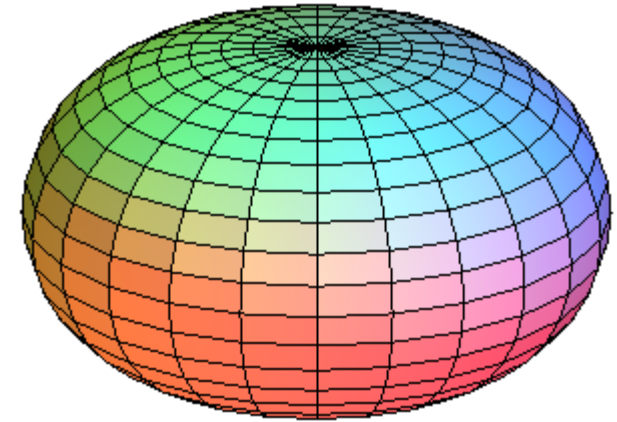
Balloon Volume Equation

$$V = 4\pi \int_0^{r_B} xz(x) dx$$

Constraints

$$z'(x = 0) = 0$$

$$\lim_{x \rightarrow r_B^-} z' = -\infty$$



Parameterized Shape Model

$$x(u, v) = r_B \operatorname{cn} \left(u, \frac{1}{\sqrt{2}} \right) \cos(v)$$

$$y(u, v) = r_B \operatorname{cn} \left(u, \frac{1}{\sqrt{2}} \right) \sin(v)$$

$$z(u) = r_B \sqrt{2} \left[E \left(\operatorname{sn} \left(u, \frac{1}{\sqrt{2}} \right), \frac{1}{\sqrt{2}} \right) - \frac{1}{2} F \left(\operatorname{sn} \left(u, \frac{1}{\sqrt{2}} \right), \frac{1}{\sqrt{2}} \right) \right]$$

Derivation of Inflatable Aperture Antenna Surface Shape Model

Alternate Slope Constraint

$$z'(x = r_A) = \frac{-(x = r_A)^2}{\sqrt{m^4 - (x = r_A)^4}} = -A$$

Inflatable Antenna Slope Equation

$$z'(x) = \frac{-x^2}{\sqrt{\frac{r_A^4(1 + A^2)}{A^2} - x^4}}$$

Inflation Ratio Equation

$$I = \frac{a - r_A}{a - r_B}$$

Shape Equation of Inflatable Aperture Antenna needs to integrate the Slope Equation, but the specific slope of the inflated radius needs to be known a priori.

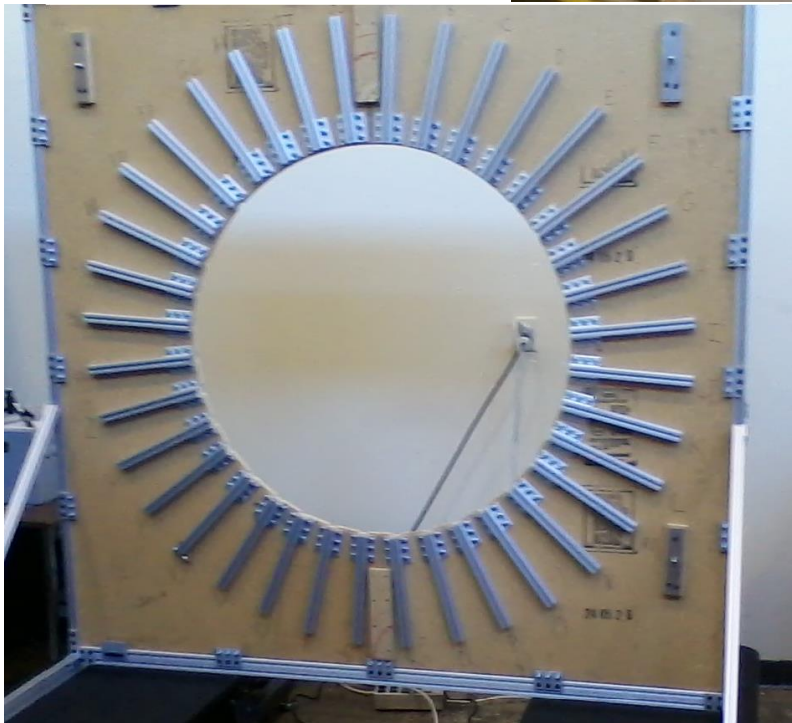
A Slope Equation over the entire Inflation Ratio range needed to be developed.

Inflatable Antenna Edge Slope Derivation

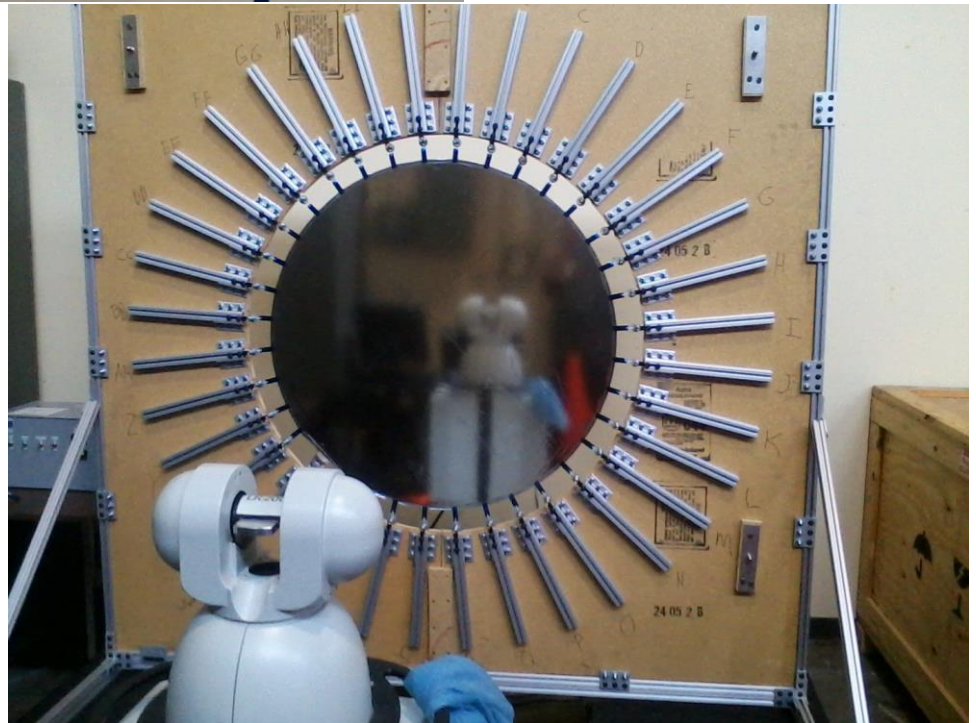
Pre-Mounted
Test Article



Test
Structure

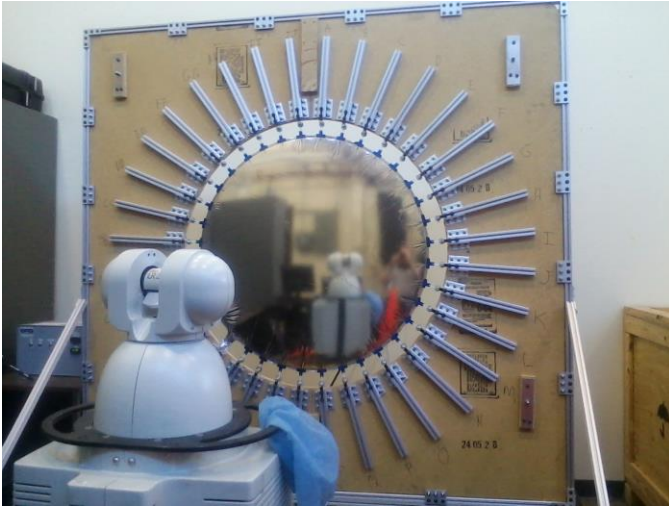


Post-Mounted
Test Article



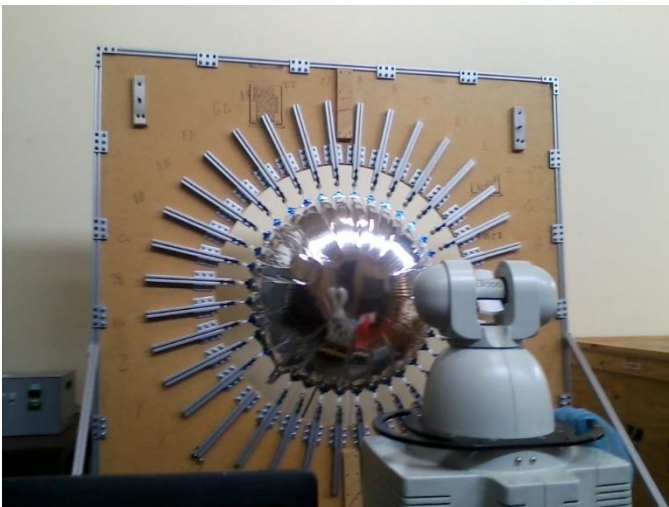
Inflatable Antenna Edge Slope Derivation

Test Article at Diameter of 34.375 Inches



Test Article at Diameter of 32.0 Inches

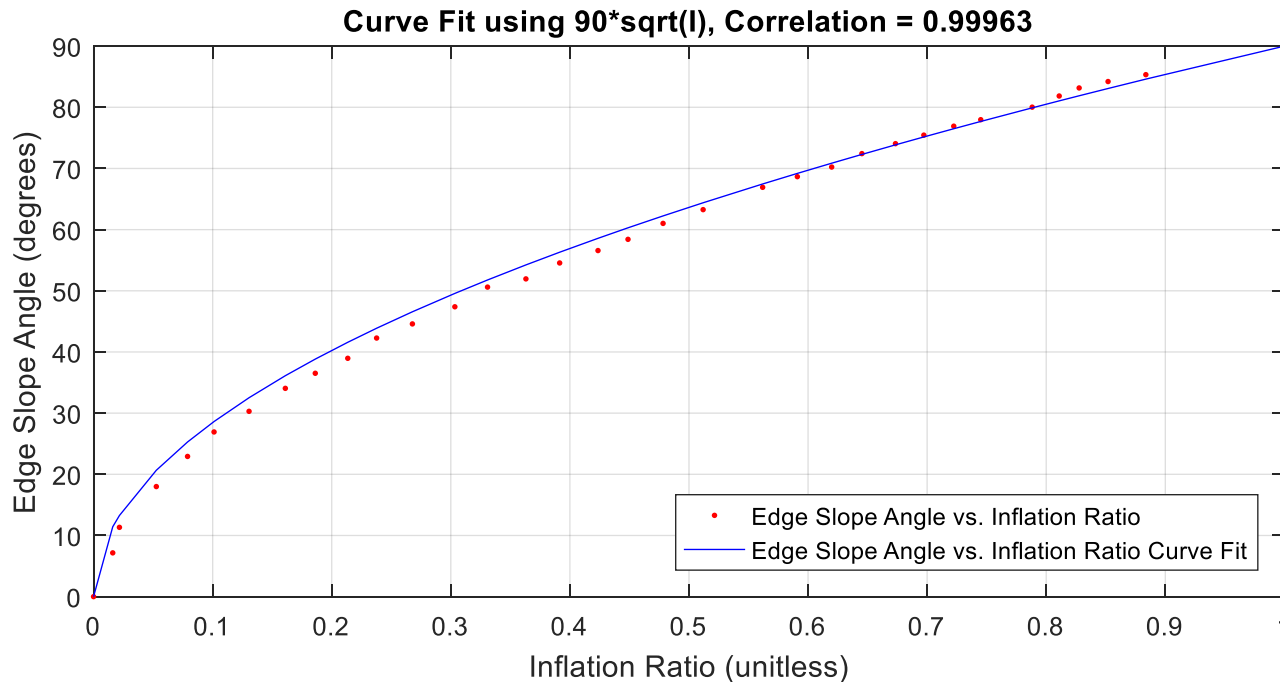
Test Article at Diameter of 29.75 Inches



Test Article at Diameter of 26.75 Inches

Inflatable Antenna Edge Slope Derivation

Optimal Slope Equation Curve Fit



Curve Fit Model Description	Correlation with Test Article Data
Square Root	0.99963
Sine	0.99487
Arc-Tangent	0.99248
Cubic Root	0.99513
Polynomial	0.99592

Inflatable Antenna Slope Equation

$$A = \tan \left(\frac{\pi}{2} \sqrt{\frac{a - r_A}{a - r_B}} \right)$$

Inflatable Aperture Antenna Surface Profile Derivation

Parameterized Shape Model

$$x(u) = r_c \operatorname{cn}\left(u, \frac{1}{\sqrt{2}}\right)$$

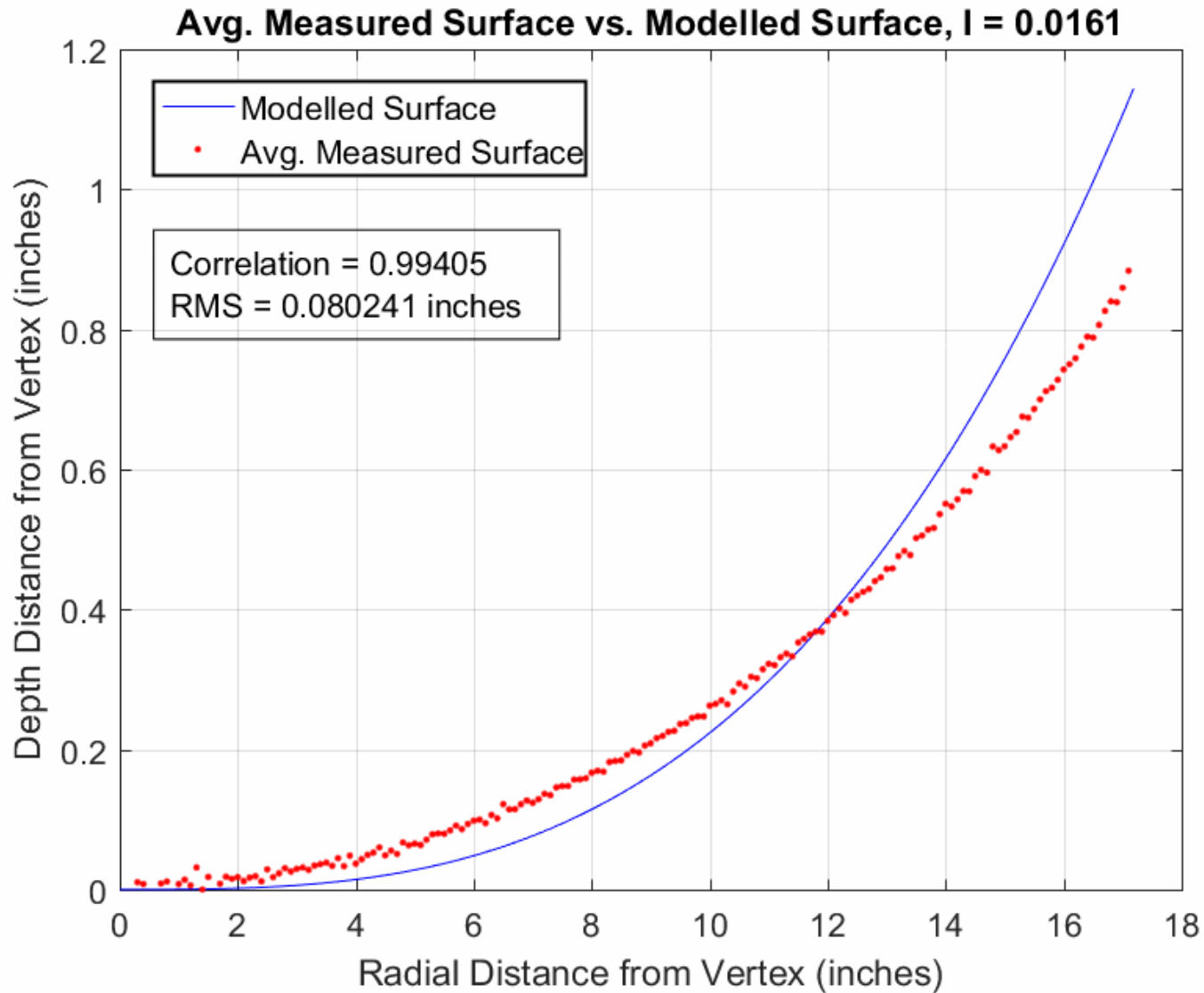
$$z(u) = r_c \sqrt{2} \left[E\left(\operatorname{sn}\left(u, \frac{1}{\sqrt{2}}\right), \frac{1}{\sqrt{2}}\right) - \frac{1}{2} F\left(\operatorname{sn}\left(u, \frac{1}{\sqrt{2}}\right), \frac{1}{\sqrt{2}}\right) \right]$$

$$r_c = r_A \sqrt[4]{1 + \cot^2\left(\frac{\pi}{2} \sqrt{\frac{a - r_A}{a - r_B}}\right)}$$

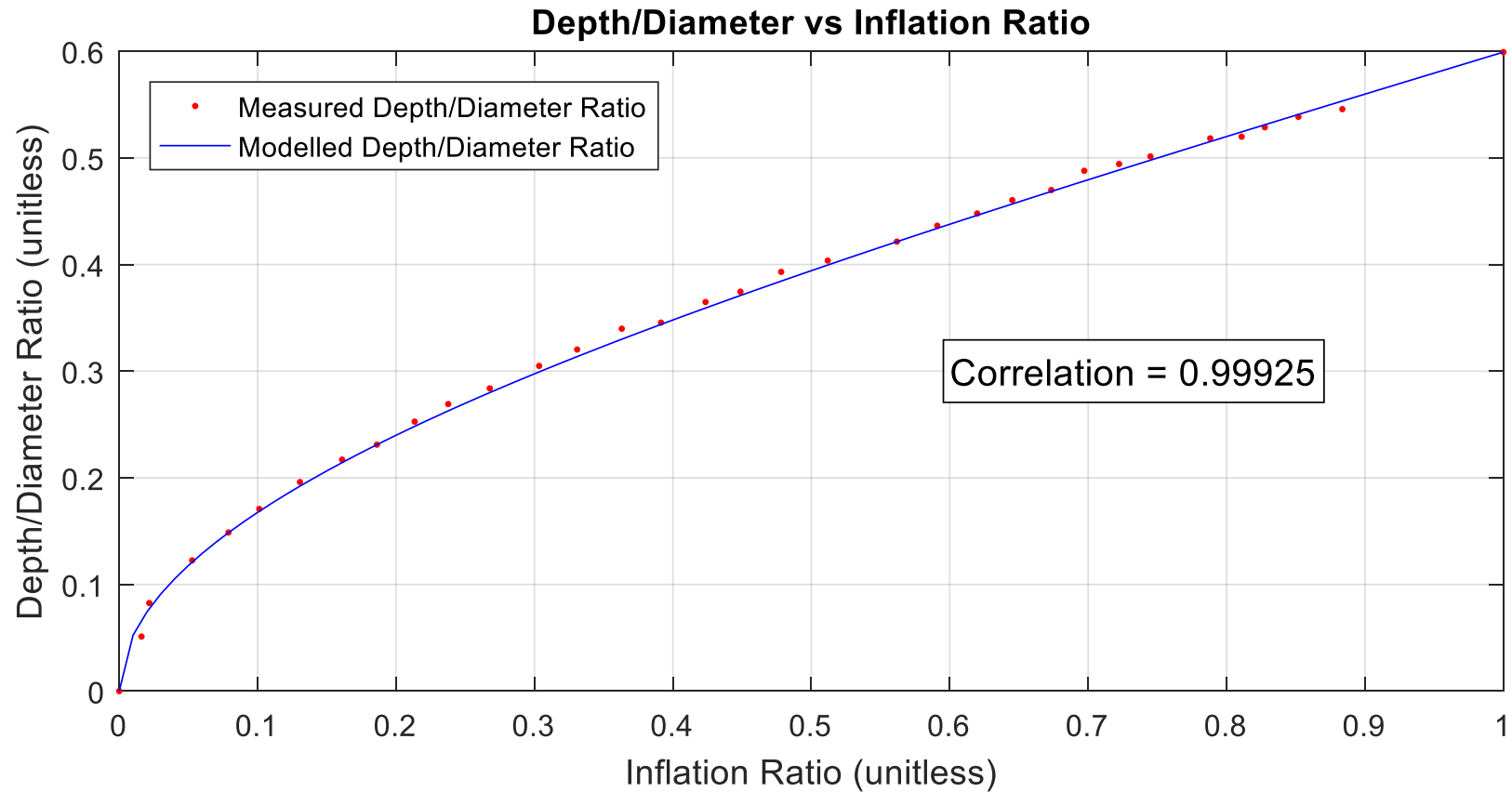
Non-Parameterized Shape Model

$$z(x, y) = r_c \sqrt{2} \left[E\left(\operatorname{sn}\left(\operatorname{arccn}\left(\frac{\sqrt{x^2 + y^2}}{r_c}, \frac{1}{\sqrt{2}}\right), \frac{1}{\sqrt{2}}\right), \frac{1}{\sqrt{2}}\right) - \frac{1}{2} F\left(\operatorname{sn}\left(\operatorname{arccn}\left(\frac{\sqrt{x^2 + y^2}}{r_c}, \frac{1}{\sqrt{2}}\right), \frac{1}{\sqrt{2}}\right), \frac{1}{\sqrt{2}}\right) \right] \\ - r_c \sqrt{2} \left[E\left(\operatorname{sn}\left(u_0, \frac{1}{\sqrt{2}}\right), \frac{1}{\sqrt{2}}\right) - \frac{1}{2} F\left(\operatorname{sn}\left(u_0, \frac{1}{\sqrt{2}}\right), \frac{1}{\sqrt{2}}\right) \right]$$

Inflatable Aperture Antenna Surface Profile Validation

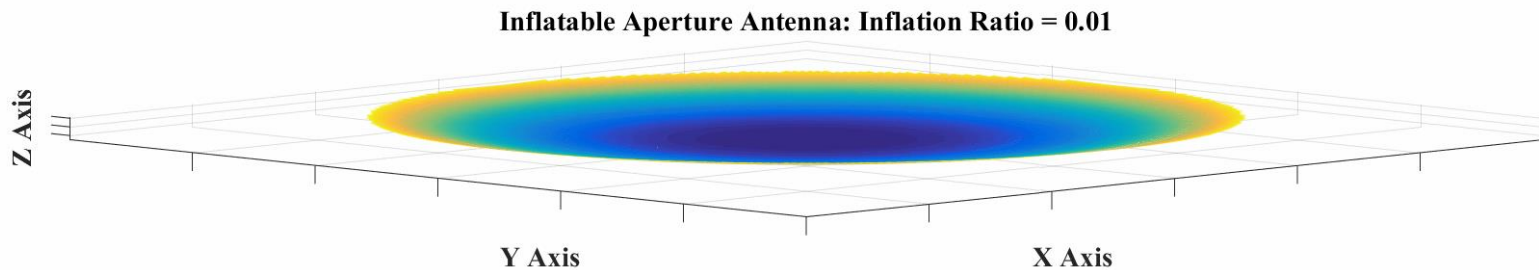


Inflatable Aperture Antenna Surface Profile Validation



Inflatable Aperture Antenna Surface Profile Validation

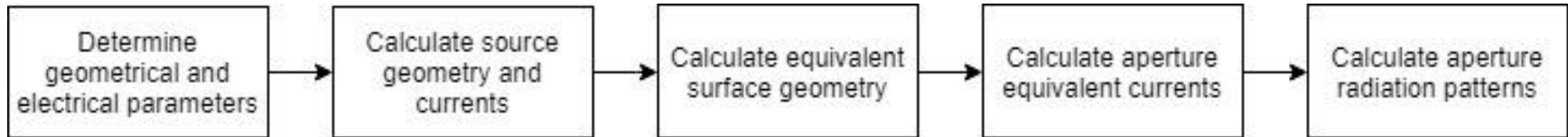
Inflatable Aperture Antenna Surface Shape Visualization



- ◆ **Derived a mathematical model of the inflatable antenna surface using Calculus of Variations methodology.**
- ◆ **Derived a mathematical model of the inflatable antenna slope based on laser radar measurements.**
- ◆ **Validated mathematical model of the inflatable aperture slope against measured test data.**
- ◆ **Validated mathematical model of the inflatable aperture surface against measured test data.**

INFLATABLE APERTURE ANTENNA RF PERFORMANCE MODEL DERIVATION

Physical Optics Calculation Workflow



Standard Parabolic Antenna Shape

$$z = \frac{(x^2 + y^2)}{4F} - F$$

Cosinusoidal Feed Taper

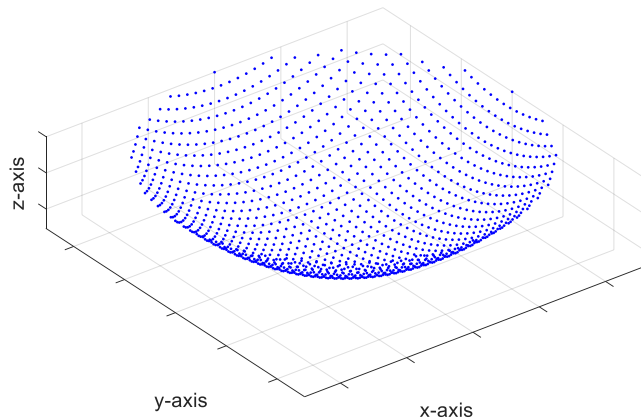
$$\theta_{subtended} = 2 \tan^{-1} \left(\frac{D}{4F} \right)$$

$$q_E = \frac{1}{2} \frac{T}{10 \log_{10}(\cos(\theta_{subtended}))}$$

$$q_H = \frac{1}{2} \frac{T}{10 \log_{10}(\cos(\theta_{subtended}))}$$

Sampled Parabolic Antenna Surface

$$f(r_{s,x}, r_{s,y}, r_{s,z}) = r_{s,z} - \frac{(r_{s,x}^2 + r_{s,y}^2)}{4F} + F$$



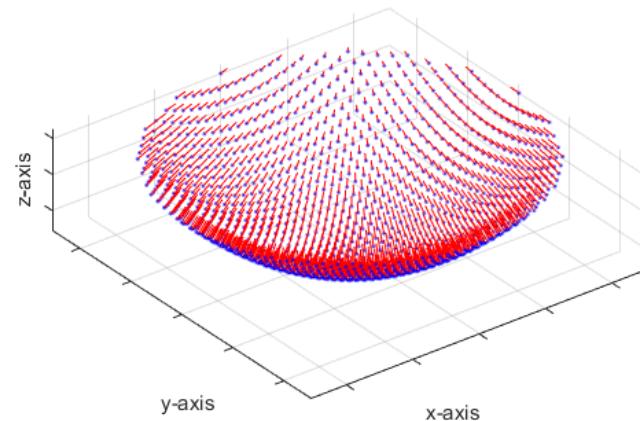
Normalized Normal Vectors

$$n'_x = \frac{-\frac{r_{s,x}}{2F}}{\sqrt{\left(-\frac{r_{s,x}}{2F}\right)^2 + \left(-\frac{r_{s,y}}{2F}\right)^2 + (1)^2}}$$

$$n'_y = \frac{-\frac{r_{s,y}}{2F}}{\sqrt{\left(-\frac{r_{s,x}}{2F}\right)^2 + \left(-\frac{r_{s,y}}{2F}\right)^2 + (1)^2}}$$

$$n'_z = \frac{1}{\sqrt{\left(-\frac{r_{s,x}}{2F}\right)^2 + \left(-\frac{r_{s,y}}{2F}\right)^2 + (1)^2}}$$

Sampled Parabolic Antenna Surface With F/D of 0.25



Surface Area Equations

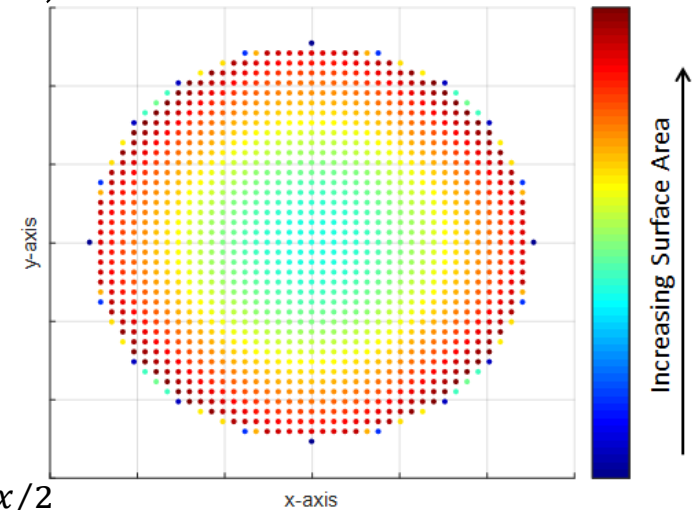
$$\Delta S = \int_{x-\Delta x/2}^{x+\Delta x/2} \int_{y-\Delta y/2}^{y+\Delta y/2} \sqrt{1 + \left(\frac{\partial z(x, y)}{\partial x}\right)^2} \sqrt{1 + \left(\frac{\partial z(x, y)}{\partial y}\right)^2} dS$$

$$dS = \Delta x \Delta y \sqrt{1 + \left(-\frac{x}{2F}\right)^2} \sqrt{1 + \left(-\frac{y}{2F}\right)^2}$$

$$\Delta S = dS_x dS_y$$

$$dS_x = F \left[\ln \left(\frac{x}{2F} + \sqrt{\left(\frac{x}{2F}\right)^2 + 1} \right) + \frac{x}{2F} \sqrt{\left(\frac{x}{2F}\right)^2 + 1} \right]_{x-\Delta x/2}^{x+\Delta x/2}$$

$$dS_y = F \left[\ln \left(\frac{y}{2F} + \sqrt{\left(\frac{y}{2F}\right)^2 + 1} \right) + \frac{y}{2F} \sqrt{\left(\frac{y}{2F}\right)^2 + 1} \right]_{y-\Delta y/2}^{y+\Delta y/2}$$



Sampled Reflector
Surface Area

Incident Currents

$$\mathbf{J}_{inc}(\mathbf{r}_s) = \mathbf{n}(\mathbf{r}_s) \times \mathbf{H}_{inc}(\mathbf{r}_s)$$

$$\mathbf{M}_{inc}(\mathbf{r}_s) = -\mathbf{n}(\mathbf{r}_s) \times \mathbf{E}_{inc}(\mathbf{r}_s)$$

Reflected Incident Currents

$$\mathbf{J}_{Refl} = -\mathbf{J}_{Inc}$$

$$\mathbf{M}_{Refl} = \mathbf{M}_{Inc}$$

Magnetic and Electric Far-Field Patterns

$$\mathbf{H}_{FF}(\boldsymbol{\theta}, \boldsymbol{\varphi}) = -\frac{jk}{4\pi} \hat{\mathbf{r}} \int \mathbf{M}_{refl}(\mathbf{r}_s) e^{-jk\hat{\mathbf{r}}R_{FF}} dS_{FF}$$

$$\mathbf{E}_{FF}(\boldsymbol{\theta}, \boldsymbol{\varphi}) = \frac{jk}{4\pi} \hat{\mathbf{r}} \int \mathbf{J}_{refl}(\mathbf{r}_s) e^{-jk\hat{\mathbf{r}}R_{FF}} dS_{FF}$$

Theta Polarized Far-Field Pattern

$$\mathbf{F}_{\theta}(\boldsymbol{\theta}, \boldsymbol{\varphi}) = (\mathbf{E}_{FF}(\boldsymbol{\theta}, \boldsymbol{\varphi}) + Z_0 \mathbf{H}_{FF}(\boldsymbol{\theta}, \boldsymbol{\varphi})) \hat{\boldsymbol{\theta}}$$

Phi Polarized Far-Field Pattern

$$\mathbf{F}_{\varphi}(\boldsymbol{\theta}, \boldsymbol{\varphi}) = (\mathbf{E}_{FF}(\boldsymbol{\theta}, \boldsymbol{\varphi}) + Z_0 \mathbf{H}_{FF}(\boldsymbol{\theta}, \boldsymbol{\varphi})) \hat{\boldsymbol{\varphi}}$$

Inflatable Antenna Surface Shape Equation

$$z(x,y) = r_c \sqrt{2} \left[E \left(\operatorname{sn} \left(\operatorname{arccn} \left(\frac{\sqrt{x^2 + y^2}}{r_c}, \frac{1}{\sqrt{2}} \right), \frac{1}{\sqrt{2}} \right), \frac{1}{\sqrt{2}} \right) - \frac{1}{2} F \left(\operatorname{sn} \left(\operatorname{arccn} \left(\frac{\sqrt{x^2 + y^2}}{r_c}, \frac{1}{\sqrt{2}} \right), \frac{1}{\sqrt{2}} \right), \frac{1}{\sqrt{2}} \right) \right] \\ - r_c \sqrt{2} \left(E \left(1, \frac{1}{\sqrt{2}} \right) - \frac{1}{2} F \left(1, \frac{1}{\sqrt{2}} \right) \right) - F$$

Identities Normal Vector Calculations

$$\frac{\partial}{\partial u} E(u, k) = \sqrt{1 - k^2 \sin^2(u)}$$

$$\frac{\partial}{\partial u} F(u, k) = \frac{1}{\sqrt{1 - k^2 \sin^2(u)}}$$

$$\frac{\partial}{\partial u} \operatorname{sn}(u, k) = \operatorname{cn}(u, k) \operatorname{dn}(u, k)$$

$$\frac{\partial}{\partial u} \operatorname{cn}(u, k) = -\operatorname{sn}(u, k) \operatorname{dn}(u, k)$$

Quadruple Function Chain Rule

$$P(x) = f \left(g \left(h(p(x)) \right) \right)$$

$$\frac{\partial}{\partial x} P(x) = \frac{\partial}{\partial x} \left(f \left(g \left(h(p(x)) \right) \right) \right) \frac{\partial}{\partial x} \left(g \left(h(p(x)) \right) \right) \frac{\partial}{\partial x} \left(h(p(x)) \right) \frac{\partial}{\partial x} (p(x))$$

Inflatable Antenna Surface Normalized Normal Vector

$$n'_x = \frac{\frac{-x\sqrt{x^2 + y^2}}{\sqrt{r_c^4 - (x^2 + y^2)^2}}}{\sqrt{\left(\frac{-x\sqrt{x^2 + y^2}}{\sqrt{r_c^4 - (x^2 + y^2)^2}}\right)^2 + \left(\frac{-y\sqrt{x^2 + y^2}}{\sqrt{r_c^4 - (x^2 + y^2)^2}}\right)^2 + (1)^2}}$$

$$n'_y = \frac{\frac{-y\sqrt{x^2 + y^2}}{\sqrt{r_c^4 - (x^2 + y^2)^2}}}{\sqrt{\left(\frac{-x\sqrt{x^2 + y^2}}{\sqrt{r_c^4 - (x^2 + y^2)^2}}\right)^2 + \left(\frac{-y\sqrt{x^2 + y^2}}{\sqrt{r_c^4 - (x^2 + y^2)^2}}\right)^2 + (1)^2}}$$

$$n'_z = \frac{1}{\sqrt{\left(\frac{-x\sqrt{x^2 + y^2}}{\sqrt{r_c^4 - (x^2 + y^2)^2}}\right)^2 + \left(\frac{-y\sqrt{x^2 + y^2}}{\sqrt{r_c^4 - (x^2 + y^2)^2}}\right)^2 + (1)^2}}$$

Inflatable Antenna Surface Area

$$\Delta S = \int_{x-\Delta x/2}^{x+\Delta x/2} \int_{y-\Delta y/2}^{y+\Delta y/2} \sqrt{1 + \left(\frac{-x\sqrt{x^2 + y^2}}{\sqrt{r_c^4 - (x^2 + y^2)^2}} \right)^2} \sqrt{1 + \left(\frac{-y\sqrt{x^2 + y^2}}{\sqrt{r_c^4 - (x^2 + y^2)^2}} \right)^2} dS$$

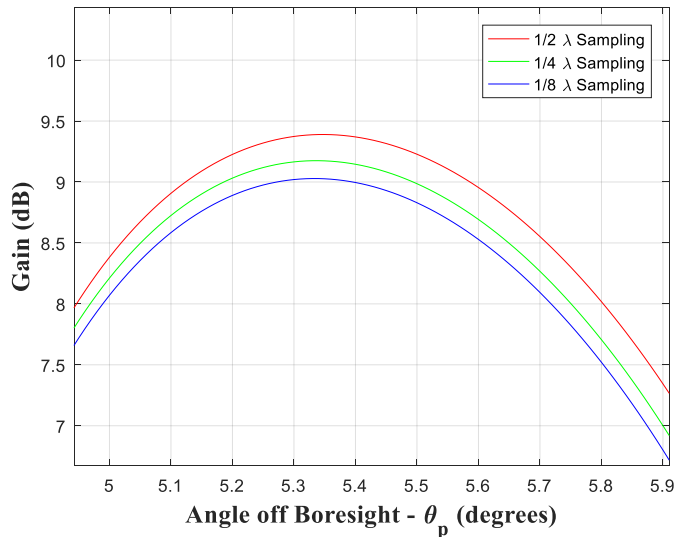
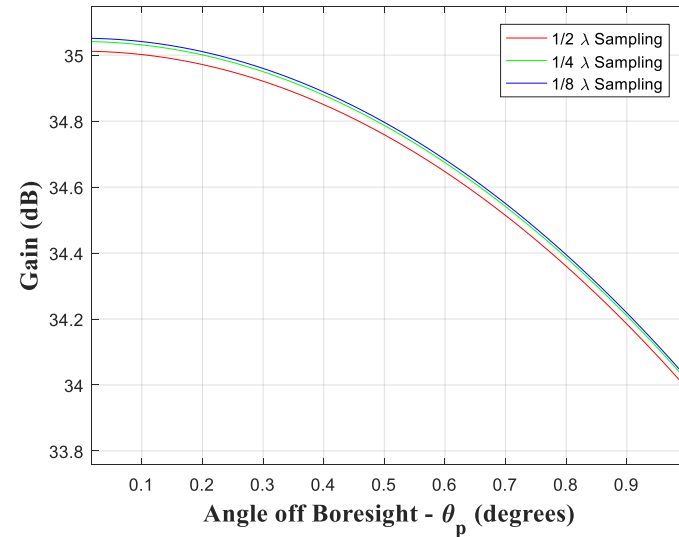
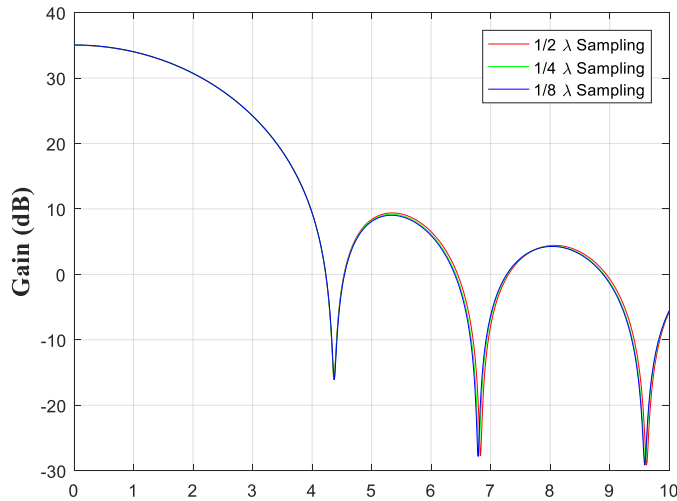
$$\Delta S = \int_{x-\Delta x/2}^{x+\Delta x/2} \int_{y-\Delta y/2}^{y+\Delta y/2} \frac{\sqrt{(r_c^4 - x^2y^2 - y^4)(r_c^4 - x^2y^2 - x^4)}}{r_c^4 - (x^2 + y^2)^2} dS$$

Unlike for the standard parabolic antenna, the surface area equation of the inflatable antenna is not separable into X- and Y- axis components. This is due to the partial derivatives being a function of both axis components.

- ◆ **Provided the physical optics model methodology for a standard parabolic and inflatable aperture reflector antenna.**
- ◆ **Developed the inflatable antenna surface shape model into the necessary format for physical optics modeling.**
- ◆ **Developed the inflatable antenna surface normalized normal vector model into the necessary format for physical optics modeling.**
- ◆ **Developed the inflatable antenna surface area model into the necessary format for physical optics modeling.**

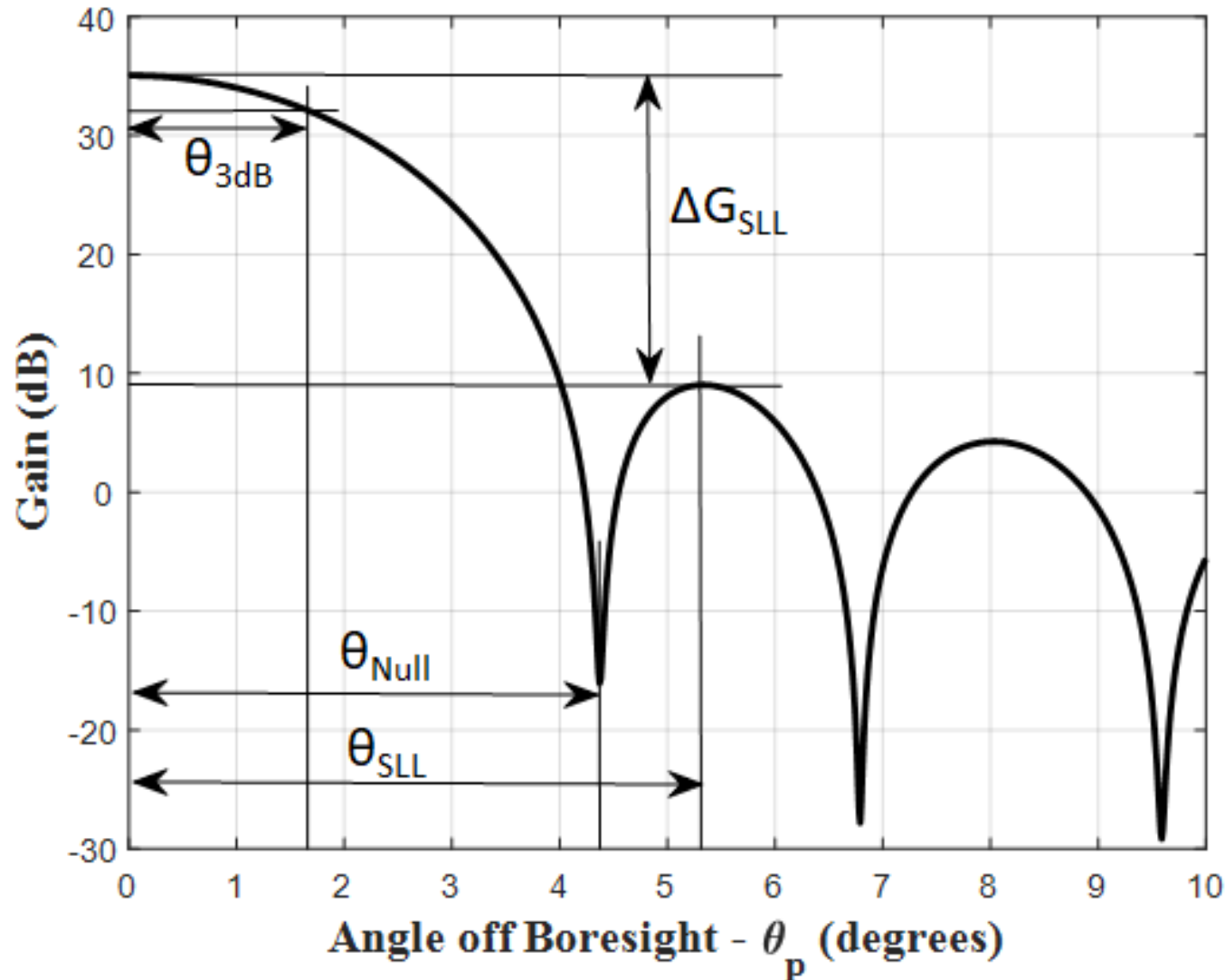
RF PERFORMANCE MODEL COMPARISON OF INFLATABLE APERTURE ANTENNA WITH PARABOLIC APERTURE ANTENNA

Parabolic Reflector Patterns as a function of Sample Spacing

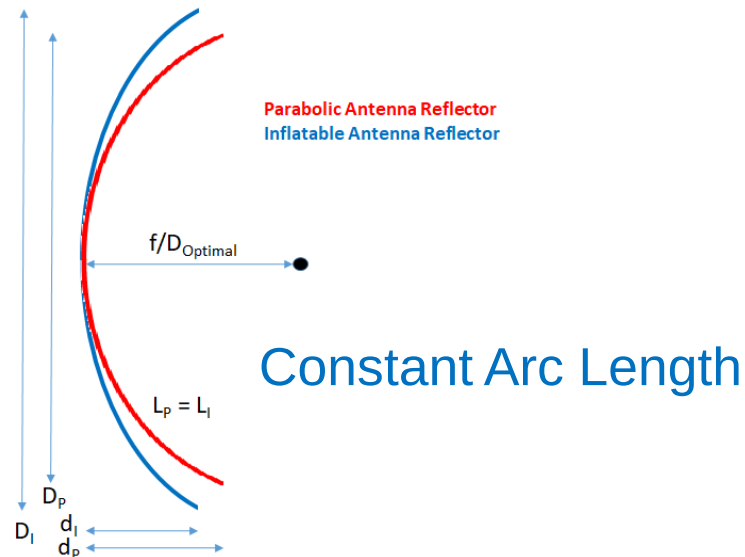
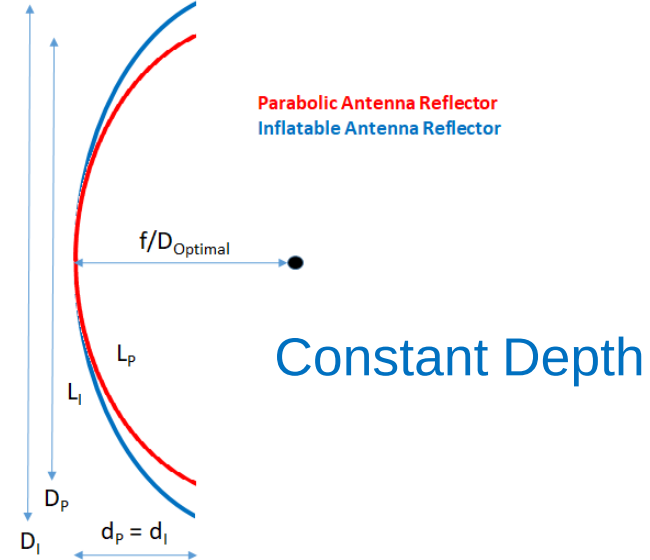
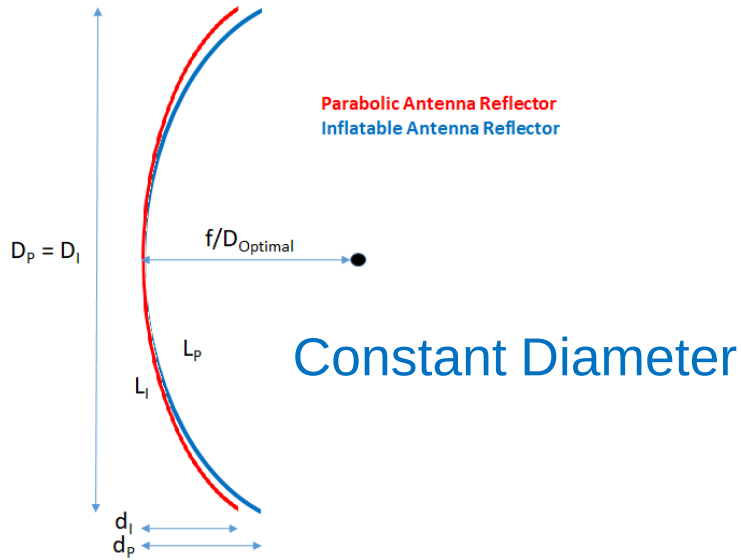


Utilized Nyquist Sampling Points

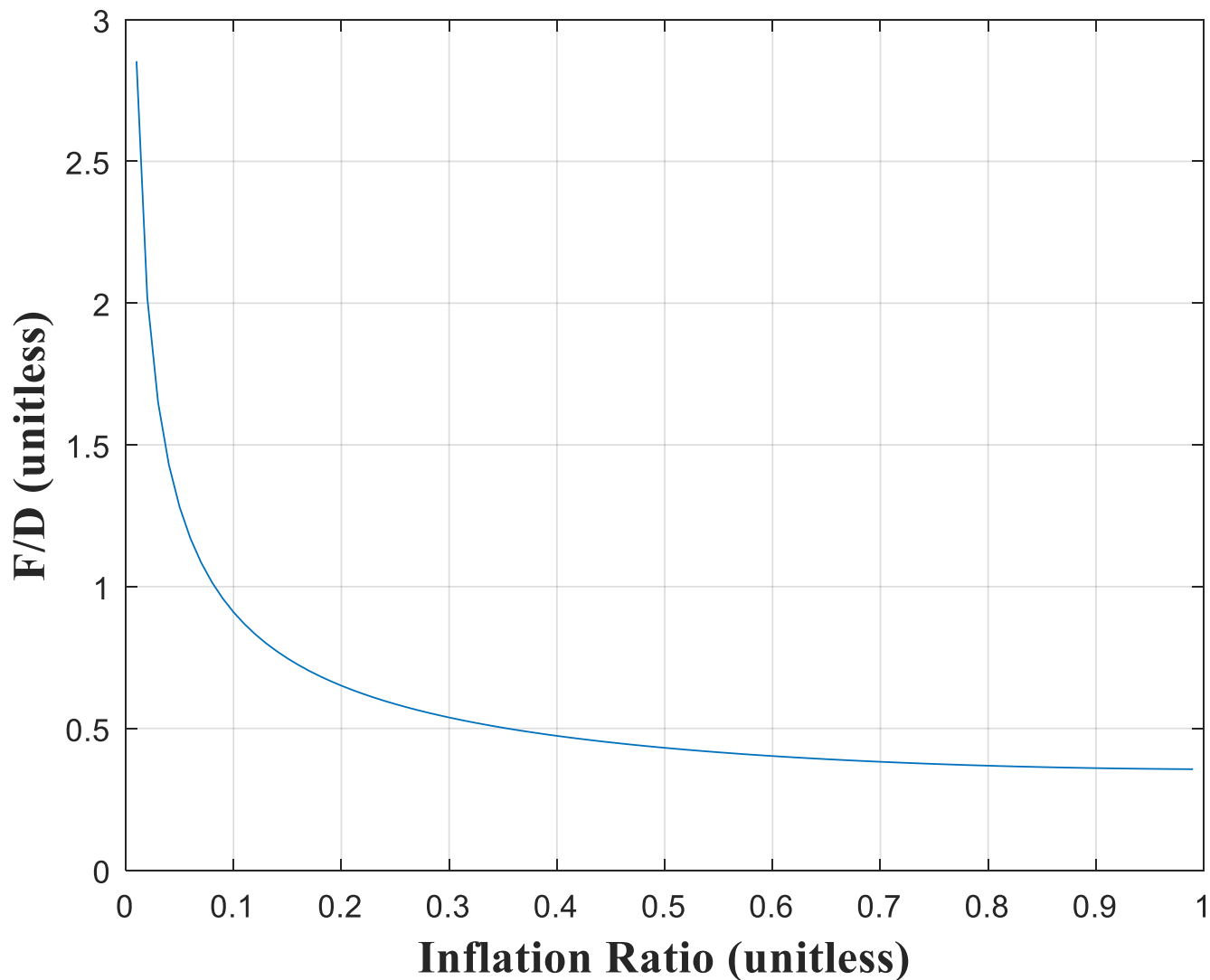
$$N = \text{floor}\left(\frac{D}{8\lambda}\right) + 1$$



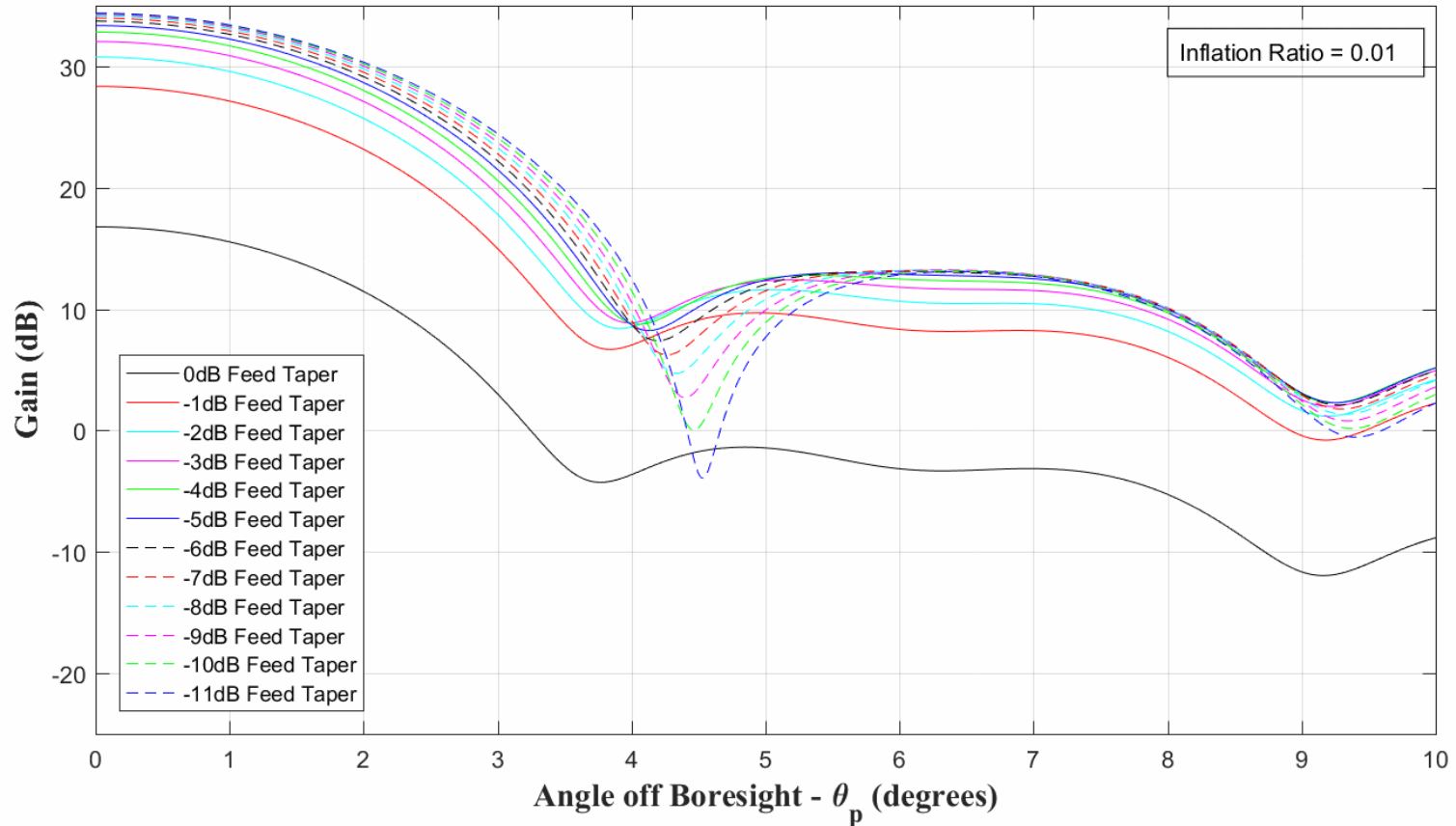
Methodologies of RF Performance Model Comparisons



RF Performance Assessment of Inflatable Aperture Antenna

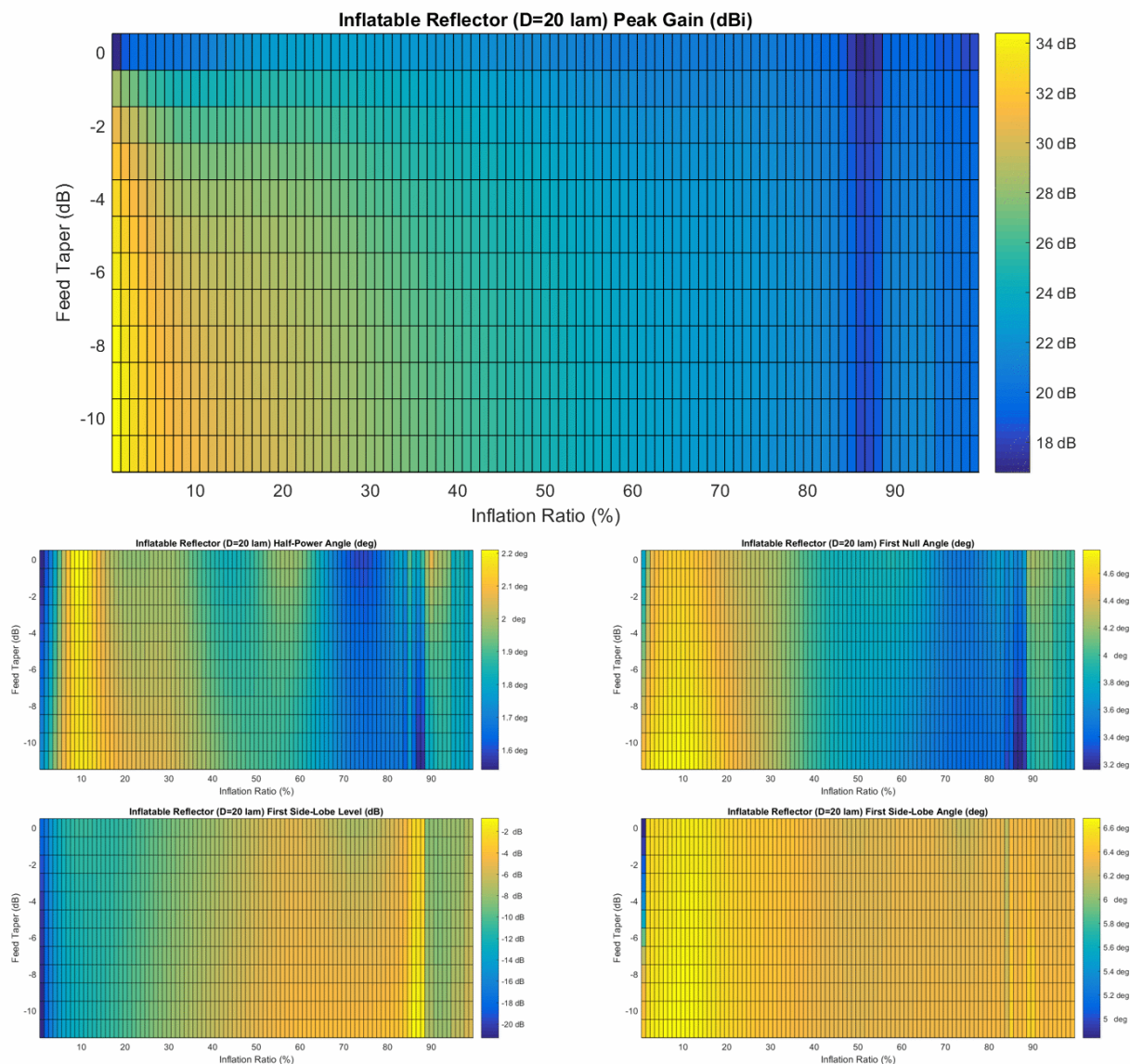


RF Performance against Inflation Ratio and Feed Taper



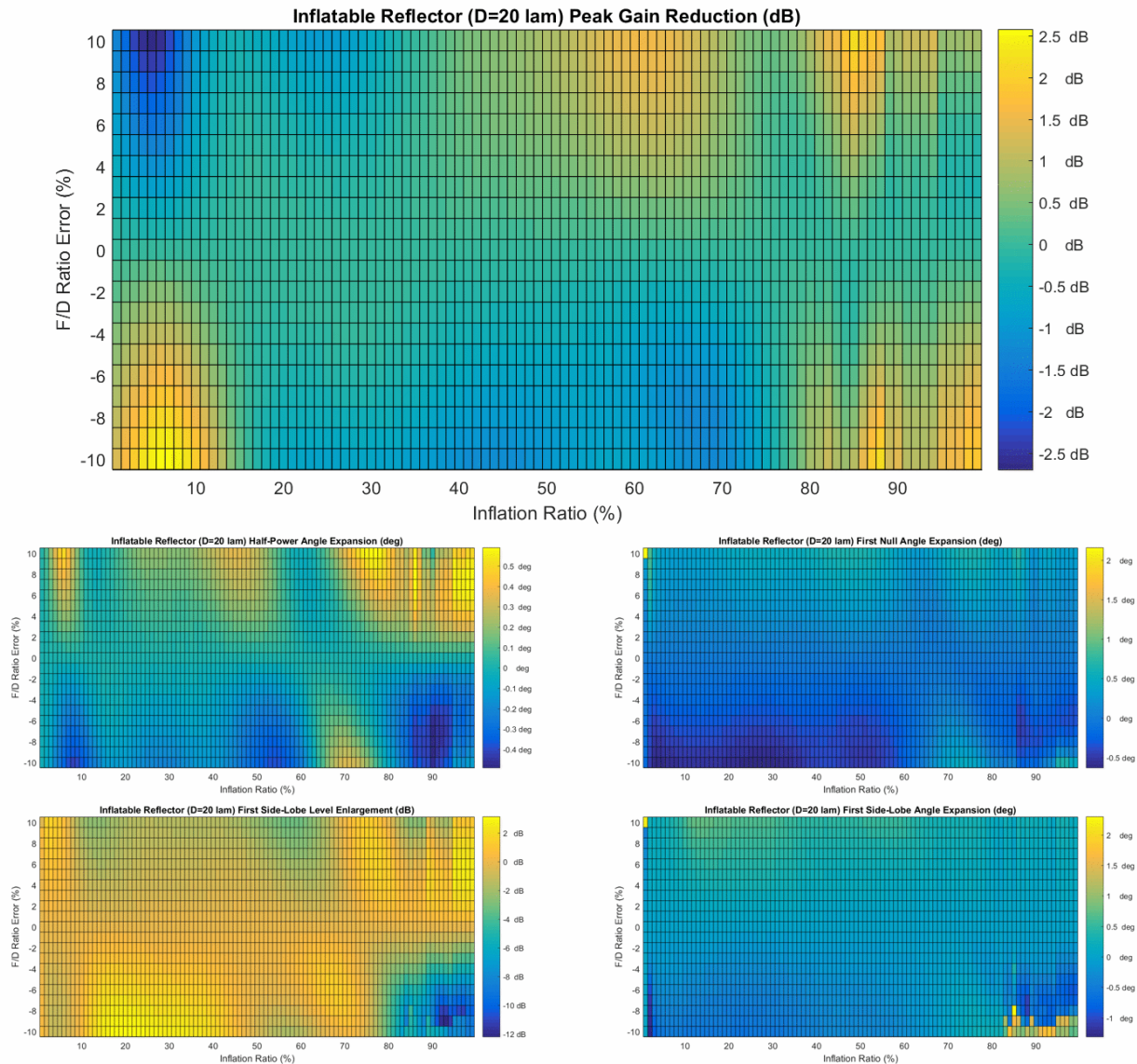
RF Performance Assessment of Inflatable Aperture Antenna

RF
Performance
against
Diameter

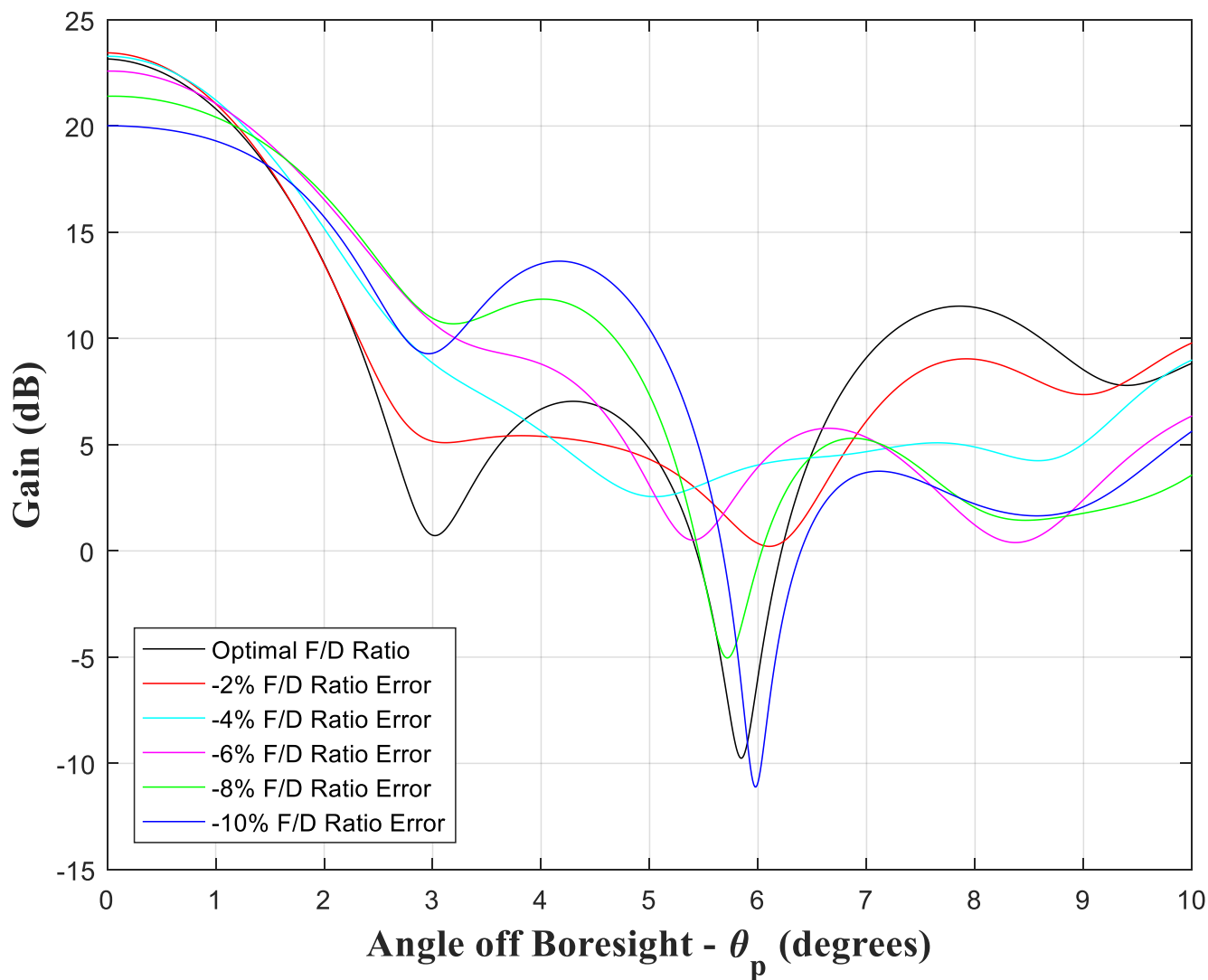


RF Performance Assessment of Inflatable Aperture Antenna

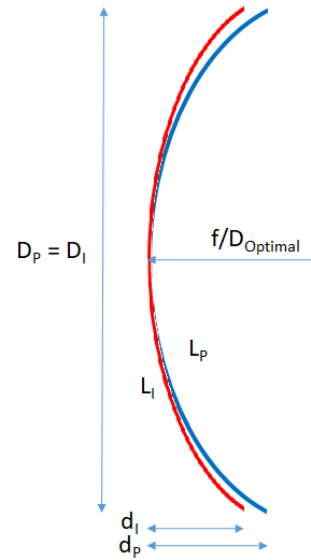
RF
Performance
Comparison
against f/D
Error



Specific Focal Length Variations Comparison at 30 Wavelengths

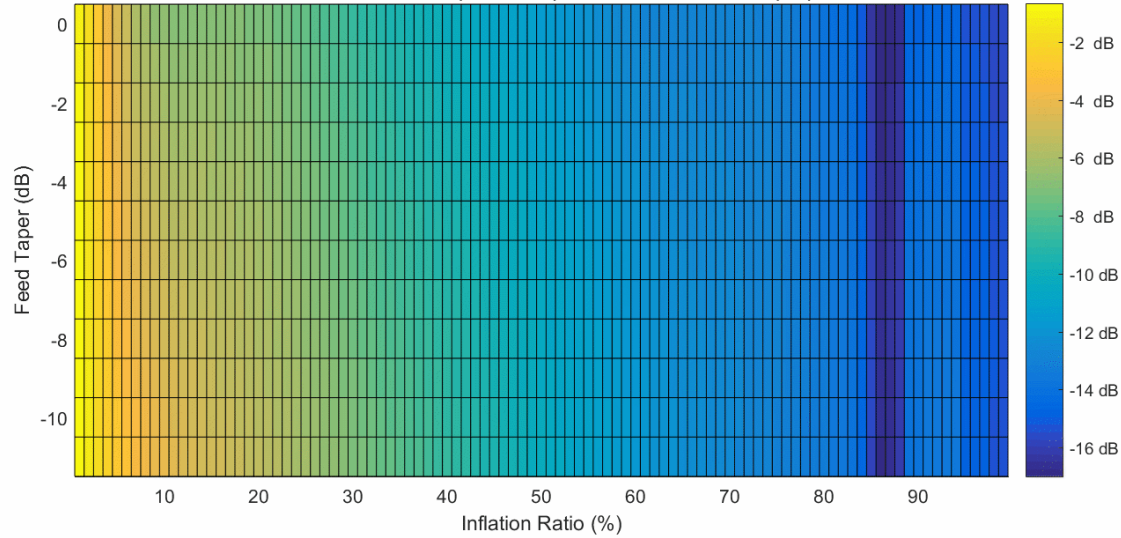


RF Performance Comparison using Common Diameter

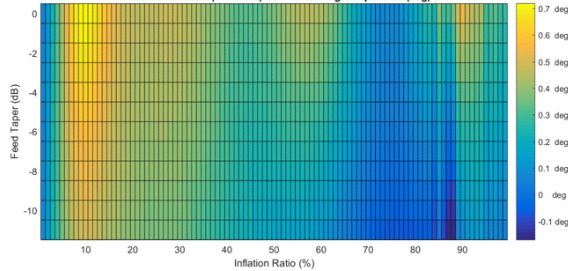


Common
Diameter

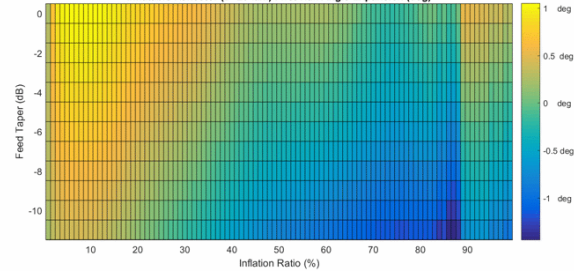
Inflatable Reflector (D=20 lam) Peak Gain Reduction (dB)



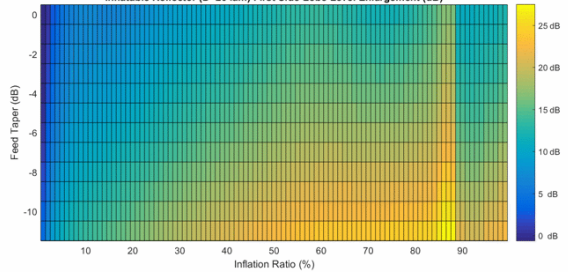
Inflatable Reflector (D=20 lam) Half-Power Angle Expansion (deg)



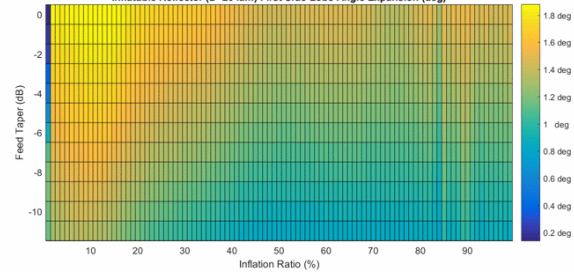
Inflatable Reflector (D=20 lam) First Null Angle Expansion (deg)



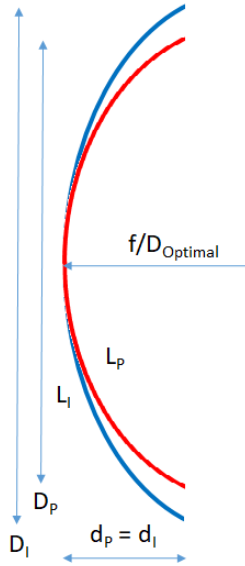
Inflatable Reflector (D=20 lam) First Side-Lobe Level Enlargement (dB)



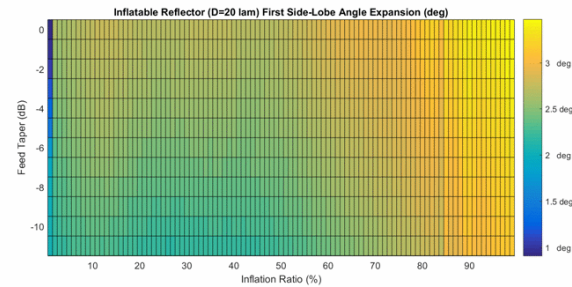
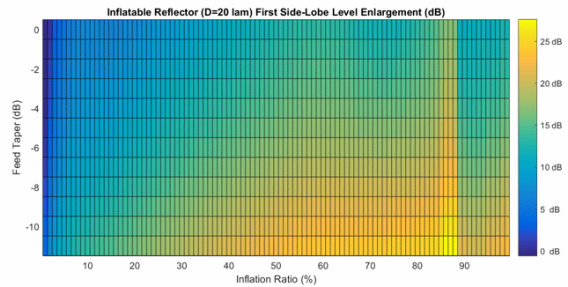
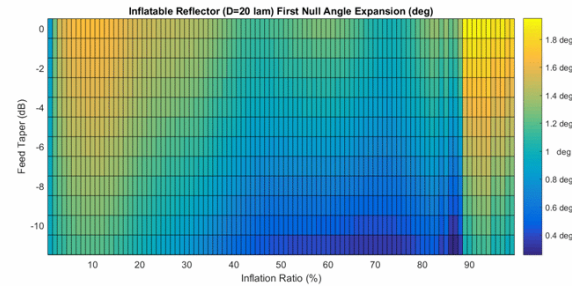
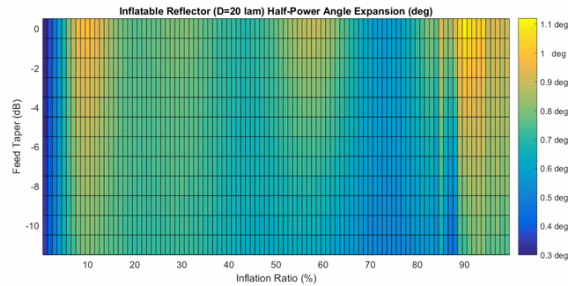
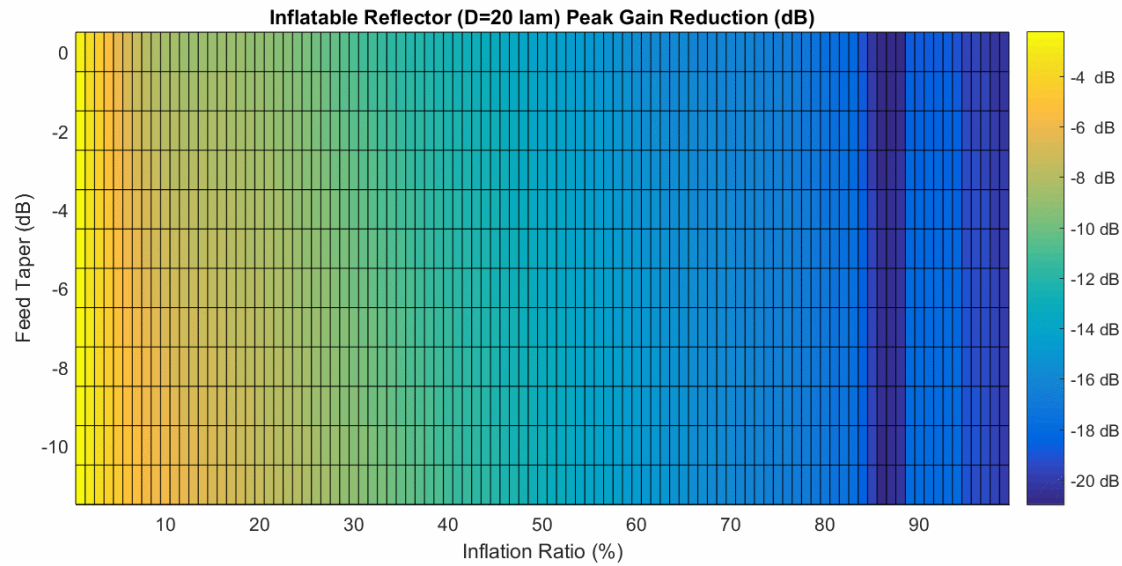
Inflatable Reflector (D=20 lam) First Side-Lobe Angle Expansion (deg)



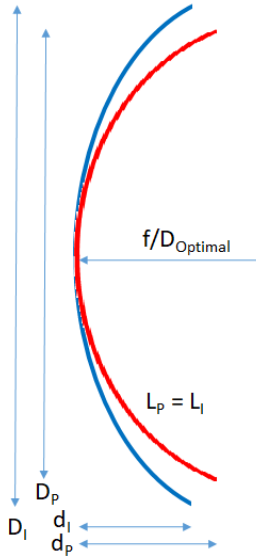
RF Performance Comparison using Common Depth



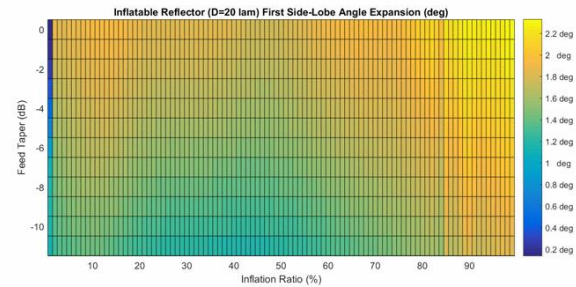
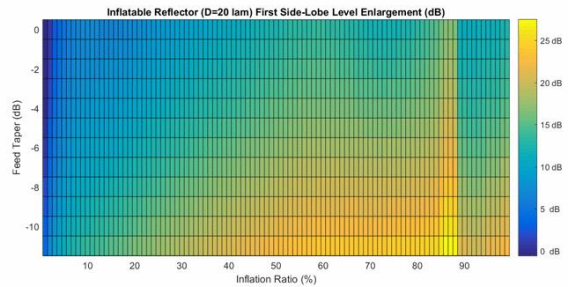
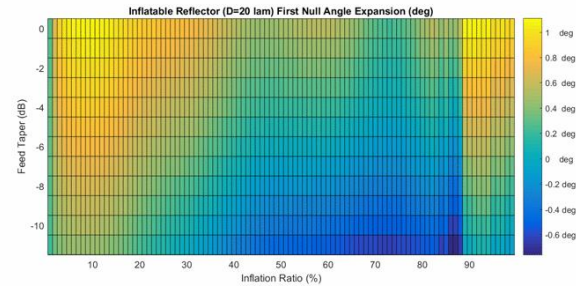
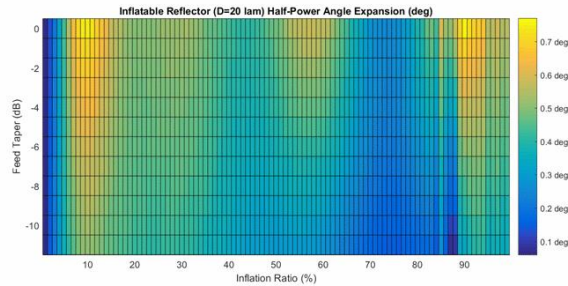
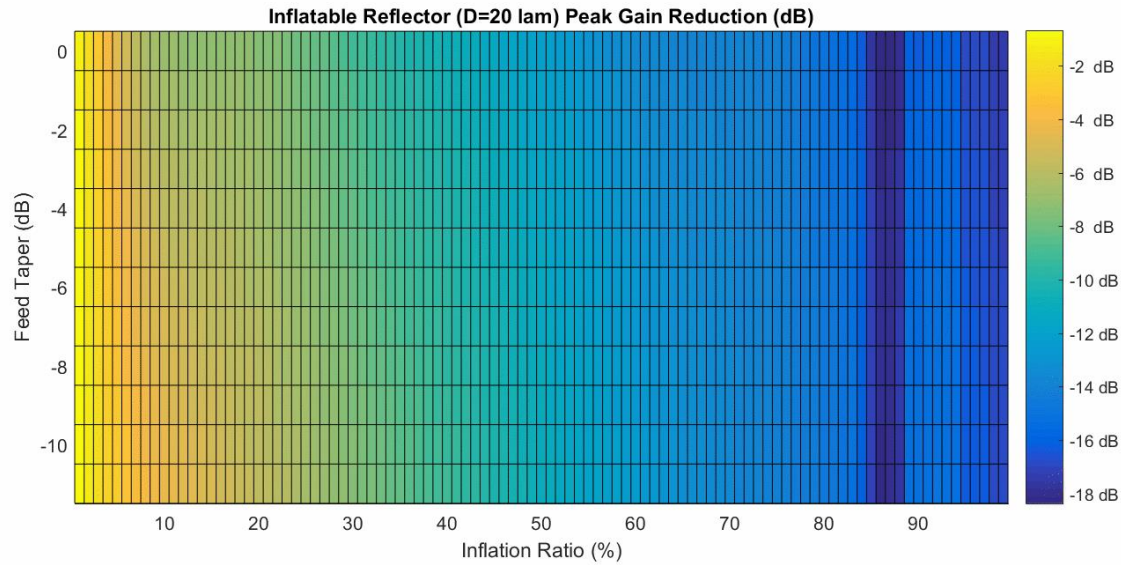
Common
Depth



RF Performance Comparison using Common Arc Length



Common
Arc
Length



- ◆ **Implemented physical optics modeling of inflatable aperture antennas.**
- ◆ **Evaluated performance metrics of inflatable aperture antenna gain patterns against best-fit f/D ratio errors as a function of feed taper and inflation ratio.**
- ◆ **Compared performance metrics of inflatable aperture antenna gain patterns against performance metrics of a standard parabolic antenna gain pattern using a common diameter, depth, or arc length as a function of feed taper and inflation ratio.**
- ◆ **No single configuration provides optimized inflatable aperture antenna performance, but does show improved performance of the inflatable aperture antenna at lower inflation ratios.**

CONCLUSIONS

- ◆ **Derived the mathematical model of the inflatable aperture antenna surface profile shape from the Calculus of Variations technique.**
- ◆ **Validated the mathematical model of the inflatable aperture antenna surface profile shape using laser radar metrology.**
- ◆ **Derived the RF performance model of the inflatable aperture antenna from the Physical Optics technique.**
- ◆ **Simulated the RF performance (gain, half-power beamwidth, first-null beamwidth, first side-lobe beamwidth, and first side-lobe-level degradation) comparisons of the inflatable aperture antenna against a standard parabolic antenna using a common diameter, depth, and arc-length.**
 - NOTE: Ruze method only provides gain degradation

- ◆ **Construction of an inflatable aperture antenna from a shaped-mandrel, designed from a desired inflation ratio and diameter.**
- ◆ **RF performance testing, in a near-field anechoic chamber, of the inflatable aperture antenna constructed from the shaped mandrel using different feed taper profiles.**
- ◆ **Extension of RF performance model of inflatable aperture antenna to include a phased array antenna feed, to aid in compensation of non-parabolic shape, or to negate regions of the antenna near the rim. If model extension looks promising, extend testing RF performance testing to include a phased array antenna feed.**

QUESTIONS?