

# A Frequency Diversity Algorithm for Extending the Radar Doppler Velocity Nyquist Interval

V. Venkatesh\*, L. Li<sup>†</sup>, M. McLinden<sup>†</sup>, M. Coon<sup>†</sup>, G. M. Heymsfield<sup>†</sup>

\* Jet Propulsion Lab <sup>†</sup> NASA/Goddard Space Flight Center

## Abstract

Compact millimeter wavelength radars have been widely used for applications such as remote sensing of clouds, guidance avionics and recently, automotive navigation. However, the short wavelength of these radars limit their maximum unambiguous Doppler velocity. A common solution to this problem is to subsequently unfold the Doppler velocity estimate with the staggered pulse repetition time (PRT) algorithm, which requires two different PRTs to be employed in sequence. This work investigates a novel and potentially more rapid method to extend the Doppler velocity Nyquist interval. The investigated algorithm uses a pair of frequency diverse pulses separated by a short time lag for Doppler velocity estimation while the unambiguous range still corresponds to the pulse repetition time employed. During the first PRT, two pulses with center frequencies  $f_1$  followed by  $f_2$  separated by a lag  $\tau$  are transmitted. During the next PRT, the pulses transmitted are in the order  $f_2$  followed by  $f_1$ . Doppler velocity is then estimated using the sum of the Doppler phases derived from  $f_1/f_2$  and  $f_2/f_1$  sequences. Based on Monte-Carlo simulations and data collected with the NASA Goddard Space Flight center (GSFC) Cloud Radar System (CRS), the algorithm is demonstrated on surface echoes. Projected aircraft speeds and traditional pulse-pair estimates were employed as a reference. The novel algorithm was found to perform adequately for a low spectrum width of the order of 0.5 m/s and SNR better than 20 dB. A simple-point target based model is used to interpret results, and simulations are used as a baseline to transition from point-target to surface scattering scenarios.

## Index Terms

Corresponding author email : vijay.venkatesh@jpl.nasa.gov .

25 Millimeter wave radar, algorithm design, data analysis, signal processing, Doppler velocity. Fre-  
26 quency diversity, pulse-pair.

## 27 I. INTRODUCTION

28 Millimeter wave radars have been used for applications such as atmospheric remote sensing  
29 [1], [2], [3], target tracking [4], [5], guidance and navigation [6], [7]. In part, the popularity of  
30 these millimeter wave radars is because short wavelengths ( $\lambda$ ) allow fine angular resolution to  
31 be realized while still allowing for physically small and lightweight designs. The choice of a  
32 short wavelength however exacerbates the range-Doppler ambiguity dilemma in pulse trains. For  
33 a given pulse repetition time ( $T$ ), the unambiguous Doppler velocity ( $v_u$ ) is given by  $v_u = \frac{\lambda}{4T}$   
34 while the unambiguous range ( $R_u$ ) is given by  $R_u = \frac{cT}{2}$ . Since pulse repetition time is directly  
35 proportional to  $R_u$  but inversely proportional to  $v_u$ , a “see-saw” trade results. This trade is  
36 exacerbated at short wavelengths, since  $v_u$  is proportional to  $\lambda$ . Millimeter wavelength radars  
37 that are required to have long range coverage often have Doppler Nyquist intervals that are much  
38 smaller than the relative speeds of target with respect to the platform.

39 Techniques to mitigate the Doppler-Range ambiguity dilemma have been developed. For  
40 example, the staggered PRT [8] and Polarization Diversity Pulse-Pair (PDPP) [9] techniques  
41 decouple the maximum unambiguous range from the maximum unambiguous Doppler velocity.  
42 The staggered PRT is a multi-rate sampling method that unfolds the Doppler velocity based  
43 on differential aliasing in Doppler velocities measured using 2 unique Pulse Repetition Times  
44 (PRT). The idea is that as the true velocity increases, it aliases onto different values within the two  
45 Doppler Nyquist intervals. Depending on the difference in measured Doppler velocities from the  
46 2 PRTs, a correction is computed. A limitation of the staggered PRT method is that the dwell time  
47 approaches double that of single PRT operation. This is because of the need to estimate Doppler  
48 velocity with sufficient precision sequentially at two lags. The Polarization Diversity Pulse-Pair  
49 method provides more rapid measurements than staggered PRTs by using a significantly shorter  
50 lag  $\tau$  between the pulses used for Doppler velocity estimation and eliminating the need for a  
51 dwell with a second PRT. Here, two pulses with orthogonal polarizations are placed within a  
52 PRT and Doppler velocity is estimated by cross-correlating voltages from these two pulses with  
53 orthogonal polarizations. The unambiguous Doppler velocity now corresponds to the lag between  
54 the pair of pulses with orthogonal polarizations, while the unambiguous range still corresponds to

55 the PRT. For non-depolarizing scatterers, the cross-polarization isolation of the antenna provides  
56 a mechanism to isolate returns from each polarization. A limitation of this method is the need  
57 for additional microwave hardware such as an orthomode transducer and dual-polarized feeds  
58 for the antenna. Spillage due to finite cross-polarization isolation may further limit the extent  
59 of Doppler retrievals within profiles that have high SNR regions [10]. This is often the case for  
60 downward looking radars because the surface echo is atleast one high SNR region with a profile.

61 This work investigates a novel approach that uses frequency diverse waveforms for Doppler  
62 velocity estimation [11]-[12]. Relative to the polarization diversity, isolation is traded for co-  
63 herence. For applications where this loss in coherence is not precipitous, digital filters provide  
64 isolation that is several orders of magnitude better than antenna cross-polarization isolation at  
65 millimeter wavelengths. Similar methods have been formulated for use in automotive radars  
66 [13], and the novel part herein is the adaptation and subsequent demonstration to a beam-  
67 filled scenario with non-trivial spectrum width. Numerical simulations and analysis of real  
68 data are used to demonstrate this novel algorithm. The following section outlines a conceptual  
69 formulation of the Frequency Diversity Pulse-Pair algorithm (FDPP) using a point-target model.  
70 Subsequently, numerical simulations are employed as a baseline to transition from point-target  
71 to surface scattering scenarios. Finally, surface echo data is used to demonstrate the algorithm  
72 using projected aircraft speeds and traditional pulse-pair measurements as references.

## 73 II. ALGORITHM FORMULATION

74 The algorithm proposed herein decouples unambiguous velocity from unambiguous range by  
75 employing two frequency shifted waveforms within a pulse repetition period. As shown in Fig.  
76 1, the two transmitted pulses at center frequencies of  $f_1$  and  $f_2$  are separated by a time-lag  $\tau$ .  
77 The order of transmitted frequencies is reversed after every transmit/receive cycle while retaining  
78 the same lag  $\tau$ . The choice of  $f_1$  and  $f_2$  is such that the return signals can be separated by filters  
79 implemented in the digital receiver. Cross-covariances from the two sequences are individually  
80 accumulated and stored (denote as  $c_{12}$  and  $c_{21}$  respectively). Doppler velocity is then estimated  
81 from the sum of the cross-covariance estimates from each pair.

82 Herein, a simple point target based analysis of the phase estimate produced by the FDPP  
83 algorithm is presented. A more detailed analytical treatise may be found in [11], and we present  
84 a simplified model herein. Denote the received voltage at the antenna at frequency  $f_1$  due to a

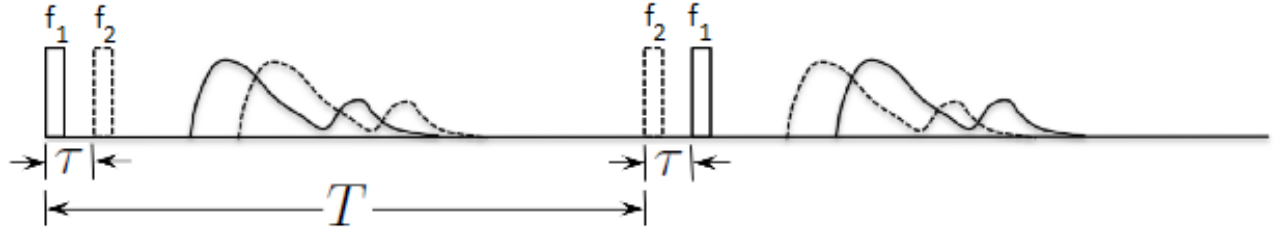


Fig. 1: Illustration of the Frequency Diverse Pulse-Pair (FDPP) algorithm concept. The first sequence transmitted has two frequencies  $f_1$  followed by  $f_2$  and separated by lag  $\tau$ . After one elapsed pulse-repetition time  $T$ , the next transmitted sequence has  $f_2$  followed by  $f_1$  separated by the same lag  $\tau$ . The concept is that errors and range dependent phase terms between the  $f_1/f_2$  and  $f_2/f_1$  pairs cancel out.

85 point-target at range  $R$  as [14]

$$V_1(t) = S_1 \exp\{-2jk_1 R\}. \quad (1)$$

86 For the same range-bin of interest, the received voltage at frequency 2 after a time-lag  $\tau$ , can  
87 be written as

$$V_2(t + \tau) = S_2 \exp\{-2jk_2(R + v_r \tau)\}, \quad (2)$$

88 where  $v_r$  is the radial velocity of the scatterer towards or away from the radar,  $S_1$  and  $S_2$  are  
89 radar system dependent constants. Since this component of velocity causes the most phase shifts,  
90 it suffices to consider this component of scatterer motion alone. Assuming the random process  
91 to be wide-sense stationary, we may omit the dependence on  $t$  and express the covariance of  
92 the  $f_1/f_2$  pair as

$$\langle c_{12}(\tau) \rangle = \langle V_1^* V_2(\tau) \rangle \approx S_1 S_2 \exp\{-[2j(k_2 - k_1)R + (2k_2 v_r \tau)]\}, \quad (3)$$

93 During the next pulse repetition interval, the covariance of the  $f_2/f_1$  pair is

$$\langle c_{21}(\tau) \rangle = \langle V_2^* V_1(\tau) \rangle \approx S_1 S_2 \exp\{-[2j(k_1 - k_2)R + (2k_1 v_r \tau)]\}. \quad (4)$$

94 Now denote the phase of the covariances as

$$\langle \Delta\phi_{12} \rangle = \angle \langle c_{12}(\tau) \rangle, \quad (5)$$

$$\langle \Delta\phi_{21} \rangle = \angle \langle c_{21}(\tau) \rangle. \quad (6)$$

95 The sum of the phases of the two covariances can be written as

$$\langle \Delta\Phi \rangle = \langle \Delta\phi_{12} \rangle + \langle \Delta\phi_{21} \rangle. \quad (7)$$

96 Using (3) and (4) in (7), the phase of the covariance sum simplifies as

$$\langle \Delta\Phi \rangle = 2(k_1 + k_2)\tau v_r. \quad (8)$$

97 The implementation we consider herein typically has frequencies that are separated by several  
98 MHz for a millimeter wave radar, such as a 94 GHz W-band radar system. Therefore, we  
99 approximate  $k_1 + k_2 \approx 2k_1$ . The above equation simplifies to

$$v_r = \frac{1}{4k_1\tau} \langle \Delta\Phi \rangle. \quad (9)$$

100 Since all values other than  $\Delta\Phi$  are solely system dependent, the radial component of target  
101 mean radial velocity  $v_r$  can be obtained from the ensemble-averaged  $\Delta\Phi$ . Two comments about  
102 the Nyquist velocity of the frequency diversity pulse-pair estimates are in order. First, for a  
103 given lag  $\tau$  the Nyquist interval is diminished by a factor of 2 compared to a traditional pulse-  
104 pair or a polarization diversity pulse-pair. This is because the frequency diversity pulse-pair  
105 algorithm accumulates two phase estimates  $\Delta\phi_{12}$  and  $\Delta\phi_{21}$  and each of these variables that form  
106 the resultant sum have a range of variation from  $[-\pi, \pi]$ . This is in contrast to the traditional  
107 pulse-pair algorithm which directly uses a phase estimate ranging from  $[-\pi, \pi]$ . Second, the  
108 frequency diversity pulse-pair algorithm developed herein allows an extension of the Nyquist  
109 interval as compared to a traditional pulse-pair which necessarily needs long lags to accommodate  
110 sufficiently large unambiguous ranges. For example, if the FDPP estimate is obtained with a lag  
111  $\tau$  and the traditional pulse-pair estimate is obtained with a lag  $T$ , then the Nyquist interval of  
112 the FDPP algorithm is improved by a factor  $\frac{T}{2\tau}$ .

113 A brief analysis of the variance of  $\Delta\Phi$  reveals a mechanism that partially alleviates the loss of  
114 pulse-pair coherence due to Rayleigh fading. Let  $Var$  denote the variance operator,  $Cov$  denote  
115 the covariance operator and  $\rho$  denote the correlation operator for the random variables  $\Delta\Phi_{12}$   
116 and  $\Delta\Phi_{21}$ . From (7), we know that

$$Var(\Delta\Phi) = Var(\Delta\Phi_{12}) + Var(\Delta\Phi_{21}) + 2Cov(\Delta\Phi_{12}, \Delta\Phi_{21}) \quad (10)$$

117 Now, the covariance term can be conveniently decomposed as

$$Cov(\Delta\Phi_{12}, \Delta\Phi_{21}) = \rho(\Delta\Phi_{12}, \Delta\Phi_{21}) \cdot \sqrt{Var(\Delta\Phi_{12}) \cdot Var(\Delta\Phi_{21})} \quad (11)$$

118 Since the same time-lag  $\tau$  is used for the  $f_1/f_2$  pair and the  $f_2/f_1$  pair,  $Var(\Delta\Phi_{12}) =$   
 119  $Var(\Delta\Phi_{21})$ ,

$$Cov(\Delta\Phi_{12}, \Delta\Phi_{21}) = \rho(\Delta\Phi_{12}, \Delta\Phi_{21}) \cdot Var(\Delta\Phi_{12}) \quad (12)$$

120 From the above relationships,

$$Var(\Delta\Phi) = 2Var(\Delta\Phi_{12}) + 2\rho(\Delta\Phi_{12}, \Delta\Phi_{21}) \cdot \sigma(\Delta\Phi_{12}) \quad (13)$$

121 Rearranging terms in (13),

$$Var(\Delta\Phi) = 2Var(\Delta\Phi_{12})[1 + \rho(\Delta\Phi_{12}, \Delta\Phi_{21})] \quad (14)$$

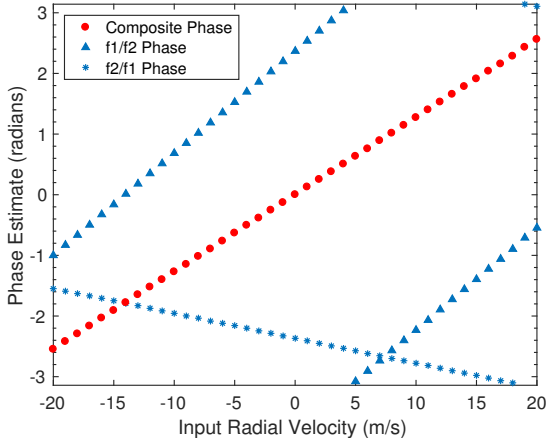
122 From (14), we see that as  $\rho(\Delta\Phi_{12}, \Delta\Phi_{21})$  tends to -1,  $Var(\Delta\Phi)$  tends to 0. In other words,  
 123 the variances of the  $f_1/f_2$  and  $f_2/f_1$  phase estimates can be large since there is little correlation  
 124 between the  $f_1$  and  $f_2$  pulses. However, if the  $f_1/f_2$  and  $f_2/f_1$  phase estimates are highly anti-  
 125 correlated, the sum of the two phase estimates has a much smaller variance. For sufficiently small  
 126 Doppler velocities in (3) and (4), it can be shown that  $\rho(\Delta\Phi_{12}, \Delta\Phi_{21}) = -\rho(\Delta\Phi_{12}, \Delta\Phi_{12})$ . For  
 127 low velocity turbulence and short lags, it appears reasonable to approximate the auto-correlation  
 128  $\rho(\Delta\Phi_{12}) \approx 1$ . This in turn results in  $\rho(\Delta\Phi_{12}, \Delta\Phi_{21}) \approx -1$ .

129 Note that a similar identity was first exploited on contiguous pulse-pairs [15]. The difference  
 130 here is that alternating  $f_1/f_2$  and  $f_2/f_1$  pairs brings about an anti-correlation, as opposed to  
 131 the ‘‘common pulse’’ in contiguous pulse-pairs. The algorithm herein has more similarities  
 132 to the alternating polarization sequence used for ‘‘Differential propagation phase shift’’ ( $\Phi_{DP}$ )  
 133 measurements in [16]. In that case, it was shown that the correlation between phase estimates  
 134 from an alternately reversing sequence approaches -1 for zero spectrum width and  $SNR > 20$   
 135 dB (equation 6.57 in [17]).

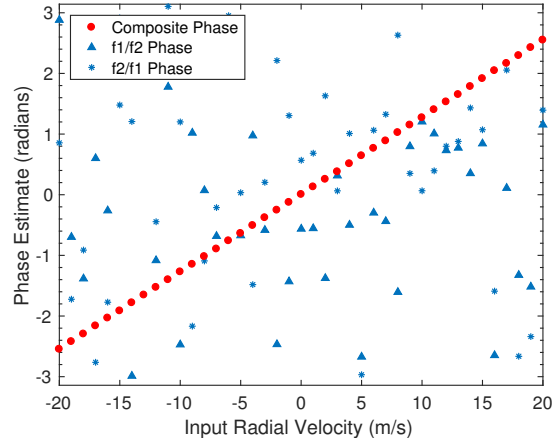
### 136 III. PERFORMANCE EVALUATION

#### 137 A. Simulations

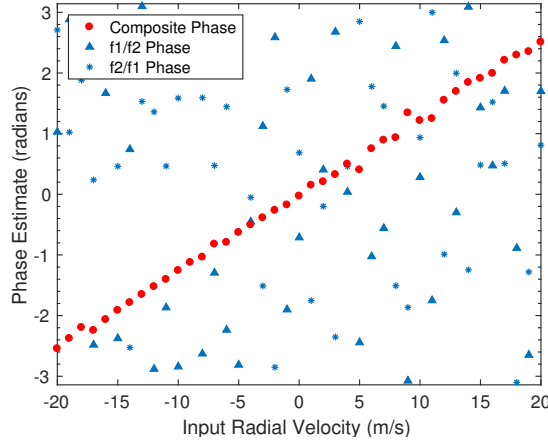
138 The point-target based model in the earlier section was solely meant to aid interpretation.  
 139 Numerical simulations are used as a baseline to go from point-target to beam-filled scattering.  
 140 The approach to simulations adopted herein is to compute backscattered voltages using a Monte-  
 141 Carlo like simulator [18] and estimate the phases at lag  $\tau$  on the simulated voltages. The voltage  
 142 at the radar receiver due to a collection of scatterers is given as



(a) Spectrum Width = 0 m/s  
Number of Scatterers = 1



(b) Spectrum Width = 0 m/s  
Number of Scatterers = 1000



(c) Spectrum Width = 1 m/s  
Number of Scatterers = 1000

Fig. 2: Monte-Carlo simulations showing the frequency diversity Doppler estimation algorithm concept. As the number of scatterers increase, the f1/f2 and f2/f1 pair phases become increasingly “noisy”. Nonetheless, the composite phase (sum of f1/f2 and f2/f1 phase) produces a good estimate of Doppler velocity for trivial spectrum width.

$$V(t) = W \sum_{n=1}^{N_s} A_n R_n \exp\{-2jkr_n\}. \quad (15)$$

143 Here,  $W$  is the radar constant for beam-filled surface scattering. Further,  $n$  denotes the  $n^{\text{th}}$   
144 scatterer,  $A_n$  denotes the antenna weighting function and  $R_n$  denotes the range weighting function

145 evaluated for the  $n^{th}$  scatterer. The antenna weighting function is defined below

$$A(X_n, Y_n) = \exp \left\{ -\frac{(X_0 - X_n)^2}{2r_0^2\sigma_\phi^2} - \frac{(Y_0 - Y_n)^2}{2r_0^2\sigma_\theta^2} \right\} \quad (16)$$

146 where the antenna phase center is denoted by  $[X_0, Y_0]$  and is the origin for the co-ordinate  
 147 system adopted herein. The scatterer locations are indicated by  $[X_n, Y_n]$  and  $\sigma_\phi$  and  $\sigma_\theta$  are the  
 148 second central moments of the effective two way antenna patterns in the azimuthal and elevation  
 149 planes respectively. These are related to antenna 3-dB beamwidths by the relationship

$$\sigma_\phi = \frac{\theta_{3dB}}{\sqrt{(8 * \ln 2)}} \quad (17)$$

150 where,  $\theta_{3dB}$  denotes the antenna 3 dB beamwidth. The range weighting function is given by

$$R(Z_n) = \exp \left\{ -\frac{(Z_0 - Z_n)^2}{2\sigma_R^2} \right\} \quad (18)$$

151 where,  $Z_0$  is the range to the center of the resolution volume and  $Z_n$  indicates the scatterer  
 152 location [18]. The location of each scatterer is updated at every pulse repetition interval  $\tau$  using  
 153 the relationship  $\vec{P}_n = \vec{P}_n + \vec{v}_n\tau$ . Here,  $\vec{P}_n$  denotes the three-dimensional position of the scatterer  
 154 and  $\vec{v}_n$  denotes the velocity of each scatterer. Each component of three-dimensional velocity  
 155 is in turn decomposed into deterministic and random velocity components. Gaussian distributed  
 156 zero-mean random variables are used to synthesize the turbulent velocity fields. As the scatterers  
 157 evolve, the voltages at the frequencies of interest are obtained in a manner consistent with Fig.  
 158 1. After scaling to the desired SNR, thermal noise is added to the synthesized signals at the  
 159 respective frequencies before the phase estimates are produced, after scaling for the desired SNR.  
 160 Phase estimates at the lag  $\tau$  are then obtained for the sequences  $f_1/f_2$  and  $f_2/f_1$ . These phase  
 161 estimates are then added and subsequently scaled using equation (9) to obtain Doppler velocity.

162 Fig. 2 shows simulated estimates of Doppler velocity using the frequency diverse algorithm  
 163 for a dwell time of 100 ms. Here, ‘‘composite phase’’ is the sum of the  $f_1/f_2$  and  $f_2/f_1$  phases  
 164 after wrapping into the interval  $[-\pi, \pi]$ . As is evident in Fig. 2a and Fig. 2c, the algorithm works  
 165 well for a point scatterer. As the number of scatterers alone is increased, the estimator still holds  
 166 up well. But the combination of Rayleigh fading and spectrum width degrades the precision of  
 167 the Doppler estimate. Consequently, the application of the algorithm is limited to beam-filled  
 168 scenarios with low spectrum width.

## 169 *B. Data analysis*

170 To evaluate performance, the FDPP technique was implemented on the NASA Goddard Space  
171 Flight Center (GSFC) W-band Cloud Radar System (CRS) during the 2015 NASA Olympic  
172 Mountain Experiment (OLYMPEX) and the 2017 NOAA Geostationary Operational Environ-  
173 mental Satellite R (GOES-R) series calibration campaign. During these field campaigns, CRS  
174 was installed on the NASA ER-2 aircraft along with the other sensors for cloud and precipi-  
175 tation observation. Fig. 3 shows qualitative comparisons of FDPP retrievals on isolated surface  
176 echoes with traditional pulse-pair measurements. Simulations are used as a baseline to interpret  
177 differences between traditional pulse-pair and FDPP Doppler estimates. CRS was mounted on  
178 an ER-2 aircraft with the antenna pointed near nadir. Pointing angle deviations away from nadir  
179 are introduced because of antenna mounting errors and aircraft attitude. In turn, this induces  
180 a non-zero surface velocity measurement due the motion of the aircraft. In Fig. 3, “Measured  
181 pulse-pair velocities” shown in Fig. 3a and the error sources shown in Fig. 3b were used as  
182 inputs to the simulation. The simulation output is shown by the red dotted line in Fig. 3a and  
183 aids interpretation of the FDPP algorithm performance. Two comments are in order. First, FDPP  
184 estimates of Doppler velocity are in reasonable agreement with traditional pulse-pair Doppler  
185 velocity estimates in regions of high SNR (i.e. of the order of 20 dB). This constitutes a qualitative  
186 demonstration of the algorithm concept developed herein. Second, both FDPP measurements  
187 from CRS and simulations show large deviations from the pulse-pair Doppler velocity estimate  
188 in regions of low SNR (i.e. comparable to 0 dB). This is because the error cancellation mechanism  
189 of the FDPP algorithm relies on high anti-correlation between the  $f_1/f_2$  pair and the  $f_2/f_1$  pair.  
190 Recall that thermal noise is merely uncorrelated.

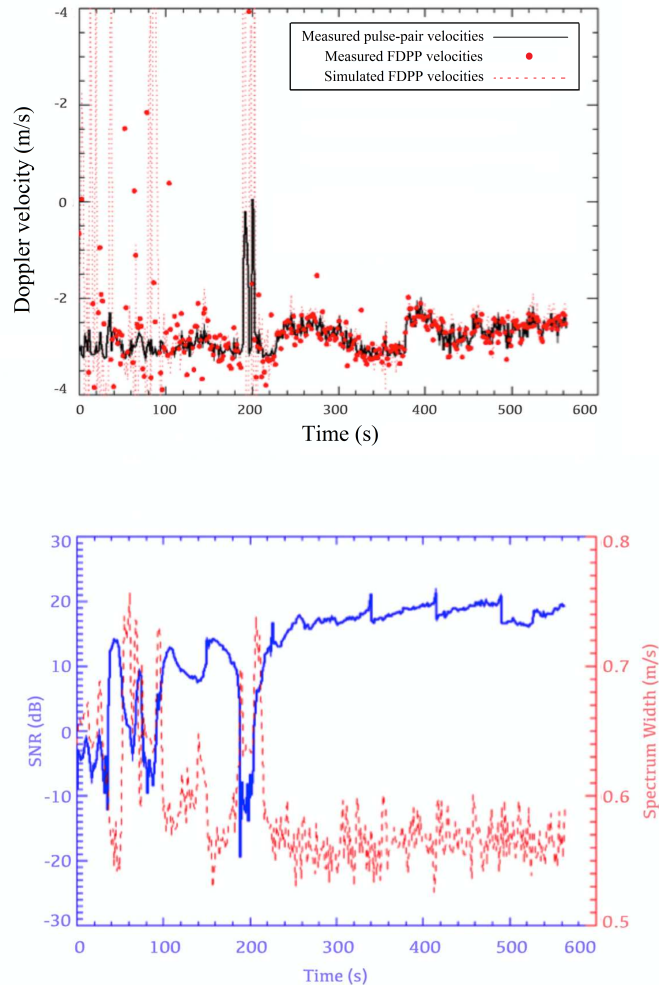


Fig. 3: Top - Evaluation of frequency diversity Doppler velocity measurements using traditional pulse-pair measurements as a reference. Simulations are employed as a baseline for interpretation. While the FDPP algorithm performs well for SNR of the order of 20 dB, both measurements and simulations show that the algorithm has poor immunity to thermal noise. Bottom - SNR and spectrum width corresponding to the duration over which the measurements were made. The “blue” line corresponds to SNR in dB and the “red” line corresponds to Spectrum width in m/s.

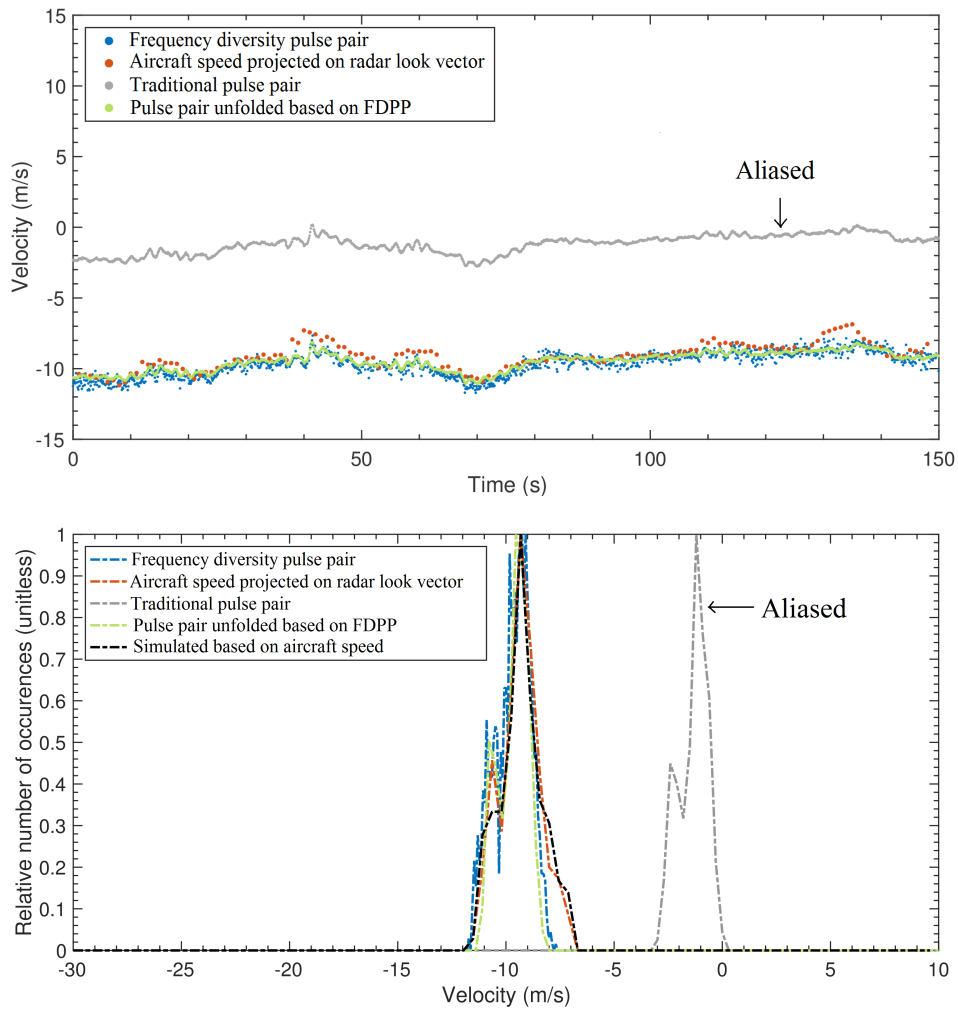


Fig. 4: Comparison of FDPP and pulse-pair measurements of Doppler velocity using projected aircraft speed as a reference. Top - While traditional pulse-pair estimates are aliased, FDPP estimates of Doppler velocity agree with aircraft speed projections on the radar look vector. However, unfolded pulse-pair velocities using the pulse-pair as reference show that the FDPP has a higher standard deviation relative to the traditional pulse-pair estimator. Bottom - The probability density functions of simulated FDPP Doppler estimates are in qualitative agreement with measurements. The simulations herein used projected aircraft speeds as input.

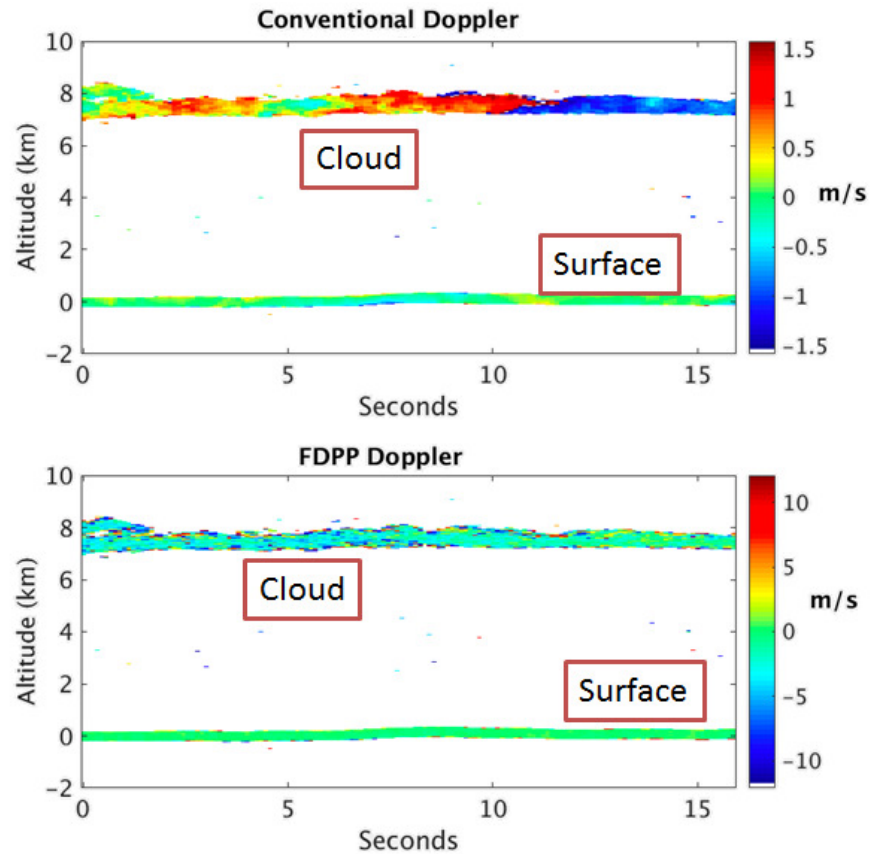


Fig. 5: Cloud radar System (CRS) Doppler velocity measurements from the NOAA GOES-R validation campaign. The dataset was collected on May 17th, 2017 at 03:53 UTC with the antenna was pointing “nearly” at nadir. Top - Traditional pulse-pair measurements in clouds show significant folding over the flight segment. Bottom - Frequency Diverse Pulse-pair measurements (FDPP) in clouds show little folding over the same flight segment. Surface Doppler measurements show little folding in both cases and are better suited for subsequent quantitative comparisons.

191 During one of the GOES-R flights, the ER-2 aircraft pilot performed high pitch maneuvers. For  
 192 one of these high pitched measurements, Fig. 4 shows FDPP measurements of Doppler velocity  
 193 compared to conventional pulse-pair velocities using projected aircraft speed as a reference. The  
 194 aircraft attitude was recorded using an Inertial Motion Unit and measured velocity, roll, pitch,  
 195 and heading. The geometric projection of the aircraft motion vector onto the radar look vector  
 196 gives the radial velocity that is employed as a reference herein. To compute this projection, two  
 197 coordinate system are defined; track relative and aircraft relative. The track relative coordinate

198 system is assumed to be static and representative of aircraft motion. The aircraft relative coordi-  
 199 nate system express the orientation of the aircraft for a given roll, pitch, and drift. The rotation  
 200 of the aircraft relative coordinate system can be mapped to track relative coordinate system using  
 201 3 rotation matrices [19]

$$M_P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(P) & -\sin(P) \\ 0 & \sin(P) & -\cos(P) \end{bmatrix},$$

$$M_D = \begin{bmatrix} \cos(D) & -\sin(D) & 0 \\ \sin(D) & \cos(D) & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad (19)$$

$$M_R = \begin{bmatrix} \cos(R) & 0 & \sin(R) \\ 0 & 1 & 0 \\ -\sin(R) & 0 & \cos(R) \end{bmatrix}.$$

202 The initial look vector of the antenna is given by

$$X_a = \begin{bmatrix} x_a \\ y_a \\ z_a \end{bmatrix} = \begin{bmatrix} \cos(\tau_a)\sin(\theta_a) \\ \sin(\tau_a) \\ \cos(\theta_a)\cos(\tau_a) \end{bmatrix}. \quad (20)$$

203

204 The off-nadir pitch offset of 2.8 degrees in Fig. 4 corresponds to  $\theta_a = \pi$  and  $\tau_a = 2\pi - 2.8^\circ$  in  
 205 equation (21). Finally, the rotation order is given by:

$$X_t = M_d M_P M_R X_a \quad (21)$$

206 where  $X_t$  is the track relative coordinate and  $X_a$  is the aircraft relative coordinate system.  
 207 Finally, the dot-product of  $X_t$  with the Cartesian aircraft motion vector gives the projected radial  
 208 velocity. Using the projected radial velocity as a reference, we note that the FDPP measurements  
 209 of Doppler velocity are in qualitative agreement in the expected value sense, while traditional  
 210 pulse pair velocities are aliased. The pulse-pair velocities shown herein correspond to a pulse  
 211 repetition time of 247 micro-seconds. However, the conventional pulse-pair measurements have  
 212 a lower standard deviation than the FDPP estimates of Doppler velocity. This is because of the

213 inevitable loss of coherence due to frequency swinging. Since pulse-pair estimates are already  
214 available in the time-series, we find that the best estimator of Doppler velocity is to use the  
215 FDPP as a reference to unfold traditional pulse-pair estimates. In this manner, the higher Doppler  
216 Nyquist interval of the FDPP for sufficiently high SNR may be combined with the lower standard  
217 deviation of the pulse-pair estimates. Lastly, the simulated statistics using the aircraft speed  
218 projections as input are in qualitative agreement with measurements for dwell times of 0.5  
219 seconds. This validates the simulation methodology and the deductions made therein.

220 Fig. 5 shows Doppler velocity estimates made with the traditional pulse-pair algorithm and  
221 the frequency diverse method investigated herein. Here, the surface is at an altitude that is nearly  
222 0 km while a thin layer of cirrus cloud is at an altitude of approximately 7.5 km. We see that  
223 there is little folding in the FDPP and traditional pulse-pair measurements from surface echoes  
224 with little motion along the radial direction. But the upward motion of the cirrus cloud makes  
225 traditional pulse-pair estimates of Doppler velocity fold more than the FDPP Doppler estimate.  
226 Here, traditional pulse-pair Doppler velocity estimates were made with a PRT of 504 micro-  
227 seconds. We also note that the FDPP algorithm yields noisier estimates on clouds as compared to  
228 a nearly static surface. This can be attributed to the loss in coherence in clouds relative to surface  
229 echoes. For the dataset shown here, the mean coherence in the surface echoes was 0.92 while the  
230 mean coherence in the cloud layer was 0.06. The reason we get some coherent Doppler velocity  
231 structure in the clouds is because significant phase anti-correlation brings about a cancellation of  
232 the variances of the  $f_1/f_2$  and the  $f_2/f_1$  pairs. Notwithstanding, we see that there is significantly  
233 less folding in the FDPP Doppler estimate in the cloud layer compared to traditional pulse-pair  
234 Doppler measurements. This shows that the frequency diversity algorithm investigated herein  
235 has a wider Nyquist interval for the Doppler velocity estimate relative to traditional pulse-pair  
236 measurements. However, it appears based on data that the FDPP Doppler estimates from surface  
237 echoes have significantly better precision than Doppler retrievals in clouds. This may potentially  
238 be attributed to differences in scattering mechanisms and are not well understood at this juncture.  
239 Further study is necessary in order to benchmark and potentially optimize the performance of  
240 the FDPP algorithm across these scattering mechanisms.

## IV. SUMMARY

Waveforms with frequency diversity have been employed for mitigating the range-Doppler ambiguity dilemma in automotive radars [13]. The novel part in this work was the random phase error and range dependent term self-cancelling algorithm design and the demonstration in a beam-filled scenario. With modern waveform generation, digital receiver and solid-state power amplifier technologies, the frequency diversity algorithm can be implemented without additional hardware such as a second millimeter wave receive chain. The transmit waveform sequence was multiplexed in time through the transmitter and already available electronics in the radar system. On receive, digital filters isolated returns from each of the frequencies. During the first PRT, the radar system transmitted a sequence of two pulses modulated by frequencies  $f_1$  and  $f_2$  in that order. During the next PRT, the order of  $f_2$  and  $f_1$  were reversed. The sum of the phase estimates from the  $f_1/f_2$  and  $f_2/f_1$  pairs yielded a composite phase which is linearly proportional to Doppler velocity.

The algorithm concept was investigated using Monte-Carlo simulations and data collected with the NASA GSFC's Cloud Radar System. A simple point-target based analytical formulation enabled interpretation of results. Simulations were employed to go between point-target and surface scattering. On data collected with the NASA Cloud Radar System, the surface echo Doppler estimates made with the FDPP algorithm were in reasonable agreement with traditional pulse-pair velocities for a low antenna pitch offset. On a flight where aircraft maneuvers were performed to result in high antenna pitch angles, the pulse-pair Doppler velocities from the surface were aliased while the FDPP Doppler estimate was in agreement with projected aircraft speed. The algorithm was found to perform well for a spectrum width of about 0.5 m/s and SNR greater than 20 dB. There were two mechanisms for error reduction. First, the phases due to the difference frequency from the  $f_1/f_2$  and  $f_2/f_1$  pairs canceled out in the expected value sense. Second, the sum of the two phase estimates has a much smaller variance relative to the individual phase estimates since the  $f_1/f_2$  and  $f_2/f_1$  phase estimates are highly anti-correlated.

FDPP Doppler estimates in clouds were found to have lower precision than surface echoes. However, the lack of sufficient SNR in clouds made it difficult to attribute differences in precision due to the scattering mechanism alone. Future work may focus on using the frequency diverse pulse-pair as a reference to optimally unfold Doppler estimates from a single PRT. In addition

271 to being used for correcting air-borne antenna pointing errors using the surface echo, this FDPP  
272 algorithm may also be investigated for use in terminal descent sensors that provide spacecraft  
273 guidance and navigation signals [20].

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