

Investigation of Transitional Shock-Wave/Boundary Layer Interactions Using Direct Numerical Simulations

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Interaction of a transitional boundary layer with a normal-shock is investigated using unstructured tetrahedral meshes under the numerical framework of the space-time conservation element, solution element (CESE) method. The computations mimic recent experimental efforts at the University of Tennessee Space Institute, where a Mach 2.0 flow interacts with a tall cylinder attached to a flat plate. The location of the cylinder with respect to the flat plate leading edge determines if the incoming boundary layer is laminar, transitional or fully turbulent. Four representative flow conditions exemplifying laminar and transitional boundary layers are analyzed by direct numerical simulations. Similar to what was observed in the experiments for the case of transitional interaction, the computations reveal an intermittent upstream influence (UI) shock that repeatedly travels upstream from the lambda-foot toward the leading edge before vanishing. Through detailed unsteady flow analysis obtained using Fourier analysis and dynamic mode decomposition techniques, the presence of disturbances with similar frequencies as those measured in experiments were identified in the flow along with locations that appear to influence the dynamics of the flow.

Nomenclature

d	= cylinder diameter
f	= frequency (Hz)
p	= pressure
Re_{unit}	= unit Reynolds number (1/m)
t	= time
x, y, z	= streamwise, wall-normal and spanwise coordinates
$\langle \rangle$	= time averaged mean quantity
Subscripts	
∞	= freestream conditions

I. Introduction

Shock/turbulence interaction is fundamental to many fluid-mechanics applications, especially in the area of high-speed flight where shock-wave/boundary-layer interaction (SWBLI) is a dominant feature. For this reason, various aspects of this problem have been the subject of many studies (e.g., Refs. [1] and [2]). In practical applications, unsteady thermal and acoustic loads due to SWBLI have significant impact on the vehicle performance. A majority of these studies involved fully turbulent boundary layers. From a flow physics standpoint, transitional SWBLI, in which the incoming boundary layer is in a transitional state (or the interaction makes the boundary layer transitional) exhibits more unsteady features than its fully turbulent counterpart due to the flow sensitivity to the disturbance environment, and strong nonlinear interaction between the instability waves. However, studies involving transitional boundary layers have been scant [3] perhaps due to a perceived lack of critical applications and the difficulties of performing experiments. Thus,

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our understanding of transitional SWBLI lags behind the fully turbulent SWBLI. In this regard, the studies by Korkegi [4] and Kaufman et al. [5] were amongst the earliest experimental efforts to focus on transitional SWBLI. This was then followed by experimental studies performed at the University of Texas at Austin by Dolling et al. [6] and Murphree et al. [7, 8], where the transitional interaction was generated through a tall cylinder mounted on a flat plate and the details about the interaction were obtained using a variety of techniques that included a kerosene-lampblack method, schlieren imaging and planar laser scattering. With an increased interest in high-speed vehicles and high-speed air breathing propulsion systems, transitional SWBLI has become important once again. The inlets of the high-speed air-breathing systems in particular feature complex shock systems that impinge up on the boundary layer at locations that span a wide range of Reynolds numbers. As a consequence, a significant spatial region around the inlet involves transitional SWBLI and this impacts the quality (in terms of uniformity/steadiness) of the flow entering the combustor. Similar aspects of flow interaction are also common in high-speed wind-tunnel testing. As a result, efforts to study transitional SWBLI have picked up over the last decade. The experimental efforts by Sandham et al. [9] that measured heat transfer for a reflected oblique shock impinging on a transitional boundary layer under hypersonic conditions and that by Davidson and Babinsky [10], where the focus was on normal shock impinging on laminar and transitional boundary layers under low supersonic speeds, being prime examples.

As a follow up to the earlier experimental work at the University of Texas at Austin [6–8], newer studies [11–14] are currently being pursued at the University of Tennessee Space Institute (UTSI) that focus on the interaction of a shock (for a Mach 2 freestream) induced by a vertical cylinder mounted on a flat plate in a transitional boundary layer. In the above-mentioned effort, various aspects of the detailed flow physics are being studied using a combination of high-speed schlieren imaging, surface-flow visualization, particle image velocimetry and pressure measurements via pressure-sensitive paint (PSP). Processing of the schlieren images and measurement via PSP, have highlighted certain features of the flow that have been poorly understood and have also identified spectral content at certain planes in the flow. However, it was observed that some of the measurement techniques (such as PSP and oil-flow visualization) interfere with the development of the boundary layer, degrading the accuracy of the measurements [13].

Lindorfer et al. [15] recently performed computations for the conditions studied at UTSI. However, they assumed the flow to be either completely laminar or completely turbulent and performed computations using the steady-state Reynolds-averaged Navier-Stokes (RANS) and hybrid RANS-Large eddy simulation approach. As a result, the primary focus of the present work is to investigate the transitional configurations studied at UTSI with direct numerical simulations (DNS) in order to identify if there are any dominant instability mechanisms and the distinct frequencies that characterize them, and thereby complementing the experiments. The computations are performed with completely unstructured tetrahedral meshes as opposed to structured meshes, which is often the norm in such studies. Several additional runs corresponding to laminar interactions have also been carried out as part of this study to compare and contrast against transitional interactions. Detailed analysis of the unsteady data using different techniques and comparison of the results against data available from experiments are presented in the subsequent sections.

II. Numerical Algorithm and Code

Traditionally, most numerical SWBLI studies are carried out using either compact-differencing schemes with high-order filters [16], WENO methods [17], or a hybrid treatment [18] that uses the upwind-based WENO operator to stabilize flow discontinuities, switching to low-dissipation schemes away from the shock to resolve the unsteady waves. However, an earlier study [19] has shown that any use of an upwind-based approach to capture shock effects, when not kept to a minimum, can significantly affect the post-shock turbulence. Furthermore, all of the above-mentioned numerical schemes can only be implemented on structured meshes.

In the present research, the space-time conservation element, solution element (CESE) method [20, 21] is used for DNS with tetrahedral meshes. The CESE method is formulated with the strong space-time unity integral form of the conservation laws, making it suitable for handling waves and flow discontinuities with high accuracy [22–24]. The concept of flux conservation in time that is introduced in the CESE method removes any distinction between space and time, thereby providing uniform accuracy in space and time and that feature is important for eddy-resolving turbulent-flow computations, where numerical dissipation can be detrimental to the accuracy of the simulation [25]. Additionally, the time-accurate local time-stepping (TALTS) scheme [24, 26, 27] formulated in the CESE method is appealing for eddy-resolving turbulent simulations, due to the wide spectrum of length and time scales involved in such flows. The TALTS algorithm

allows for larger time steps to be used for large flow structures away from the wall and in the quiet freestream regions, but at the same time guaranteeing conservation of fluxes in space and time. The CESE method is a genuinely multidimensional formulation, free of approximate Riemann solvers and dimensional splitting. These features offer the capability to use triangular/tetrahedral meshes for complex geometries, even in the region of discontinuities or inside boundary layers.

The NASA Langley in-house CESE Navier-Stokes Solver, *ez4d*, was used for performing the numerical simulations in this study. For computational efficiency and ease of continual development, the *ez4d* software framework has been developed using a combination of object-oriented and generic programming paradigms in the C++ programming language. Lightweight object-oriented hierarchy is used in conjunction with heavy use of template classes and functions to allow compile-time polymorphism. Different conservation laws can be built-in via the templates feature of C++. Currently, the software supports either triangular/tetrahedral or quadrilateral/hexahedral unstructured meshes. Both multithread (based on low-level POSIX threads) and message passing interface (MPI) are used to facilitate large-scale parallel computations. Thus, each MPI process within a computational node can be executed in multithread mode to further enhance the parallel performance, especially for a memory-bound multidomain layout.

Both second- and fourth-order CESE numerical schemes have been implemented into *ez4d* for general conservation laws including Euler and Navier-Stokes equations. For Reynolds-averaged Navier-Stokes (RANS) simulations, implementations of the Spalart-Allmaras and Mentor's SST-V models [28] exist within the *ez4d* framework. Capability for large-eddy simulations with a dynamic subgrid-scale model have been recently added for the simulation of turbulent shear layers and boundary layers.

III. Cylinder-Induced Transitional Shock-Wave/Boundary Layer Interaction

A. Flow Set-up and Mesh Details

The flow configuration studied corresponds to the experimental setup [11] at UTSI. The experiment was carried out in a nominally Mach 2, low-enthalpy blowdown wind tunnel that had a freestream unit Reynolds number (Re_{unit}) of approximately $2.8 \times 10^7 \text{ m}^{-1}$ and a total temperature of 286.18K. A 3.175 mm diameter (d), 12.7 mm ($4d$) tall brass cylinder was mounted to a flat plate and the plate was inclined at a -2.9° angle of attack (to prevent leading-edge separation), reducing the incident post-shock (leading-edge shock) Mach number to 1.91. The center of the cylinder was located at a position (measured from the leading edge of the flat plate) that varied from $x/d = 6.5$ to 25.5, so that the interaction ranged from being a transitional SWBLI to a fully turbulent SWBLI. For the transitional case chosen, the center of the cylinder was located at a distance of $x/d = 8.5$ from the flat plate leading edge and the approaching boundary layer was laminar and nearly steady. It needs to be mentioned that the more recent experiments [12–14] by the UTSI research group, that included pressure measurements via pressure-sensitive paint (PSP), have been carried out for a slightly different angle of attack of -6.3° to avoid any flow separation or stalling in the tunnel. This change in angle of attack, reduced the post-shock (leading-edge shock) Mach number to 1.8. Given the small difference in Mach number, we do not expect a significant change in the flow dynamics studied and thus should be able to compare the results from the computations against these experiments.

The computational box used in the study extended $12d$ in the spanwise direction, $8d$ in the wall-normal direction, $8.5d$ upstream and $2.5d$ downstream of the desired cylinder (center) location. Tetrahedral elements were utilized to discretize the domain using the T-Rex feature of Pointwise®. Three different meshes were built, with 57 million elements, 95 million elements and 135 million elements, respectively. Based on preliminary results from the coarser mesh, the next two meshes were built with additional resolution in the region around the cylinder where the interaction between the shock and boundary layer was expected to dominate. This can be seen in the surface meshes shown in Fig. 1(a) and (b) for the 95 million element mesh. The two finer meshes had a near wall spacing of approximately 0.0064 mm ($\sim 0.002d$) that grew geometrically at the rate of 1.04 in the direction normal to the wall. There was a minimum of 65 anisotropic layers in the boundary layer part of the mesh. The 95 million and 135 million meshes only differed in the streamwise/spanwise mesh spacing near the wall and in the isotropic region outside the boundary layer. The maximum spacing in those regions was restricted to be around 0.08mm ($\sim 0.025d$). Supersonic inflow boundary condition with velocity vectors corresponding to the chosen angle of attack was imposed at the most upstream boundary; the wall was specified to be adiabatic by imposing an isothermal temperature of 286.74K; and the remaining boundaries (spanwise boundaries, the top boundary and the one downstream of the cylinder) were set with non-reflecting/outflow boundary conditions.

Only results from the 95 million mesh will be discussed here as we were able to obtain a time-averaged solution over a substantially longer time period, to capture some of the low frequency aspects of the flow, when compared to the solution obtained from the 135 million mesh. Although not shown here, no significant differences were observed in the general flow features captured by these two finer grids. A more detailed discussion on mesh convergence will be included in a future work.

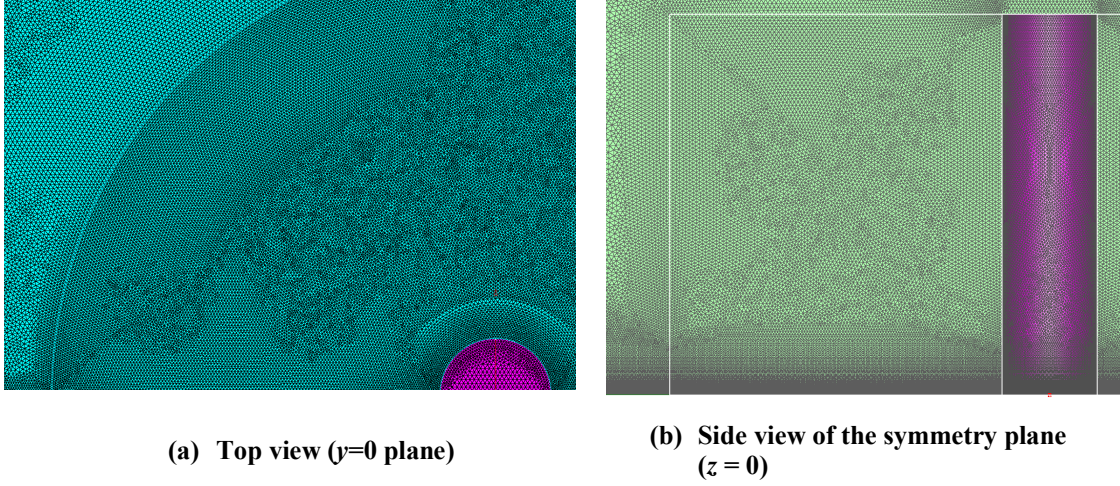


Fig. 1 Overview of the surface mesh for the mesh with 95 million elements.

B. Basic Flow Features for the Transitional and Laminar interaction

Results from the simulation of the transitional case are used to depict the general flow features involving transitional boundary-layer / normal-shock interaction in Fig. 2. The leading-edge shock and the inviscid bow shock are clearly seen from the isosurfaces of dilatation contours shown in Fig. 2. The interaction of the inviscid bow shock with the transitional boundary layer creates the series of additional oblique shocks seen in Fig. 2, as the lambda-shock system and the upstream influence (UI) shock that appear where the separation begins. The Q-criterion isosurfaces around the cylinder in Fig. 2 also show a system of horseshoe vortices emanating from the separation region, indicating possible in situ flow transition. Those vortices get swept outwards around the cylinder in the spanwise direction and convect downstream.

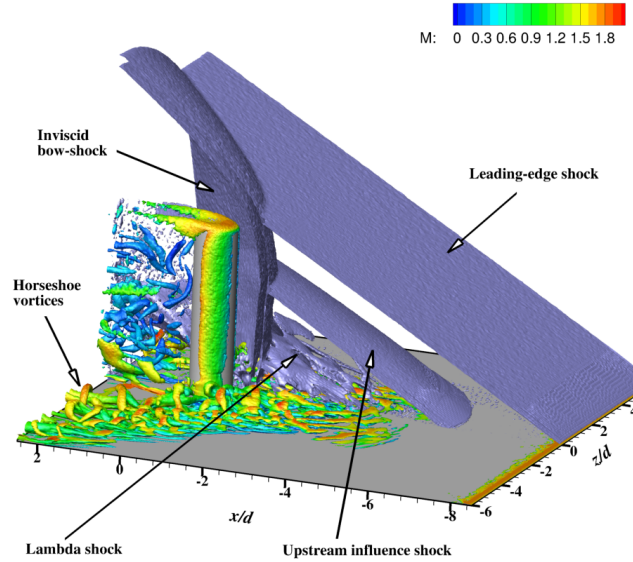


Fig. 2 Key features of the flow shown through isosurfaces of Q-criterion (colored by Mach number) and dilatation contour (shock surface). Freestream flow from right to left.

As can be seen from Fig. 3, where numerical schlieren plots at the center plane are shown for several time instances, the inclination of the UI shock is approximately parallel to the leading-edge shock (at a Mach angle corresponding to Mach 1.91), indicating that the disturbance causing the UI shock is a weak one. In contrast, the series of compression shocks that form the forward foot of the lambda shock are inclined at a higher angle, indicative of higher strength discontinuities. In the experiments [11–14], the UI shock itself was found to be intermittent in nature, both in its presence within the flow and in its movement. The UI shock appeared to emanate from the forward foot of the lambda-shock, before moving upstream toward the leading edge and disappearing completely upstream of the separation region (much before it reaches the leading-edge shock) and the entire cycle repeats itself. The computations were able to reproduce this intermittent behavior, very similar to what was observed in the experiments.

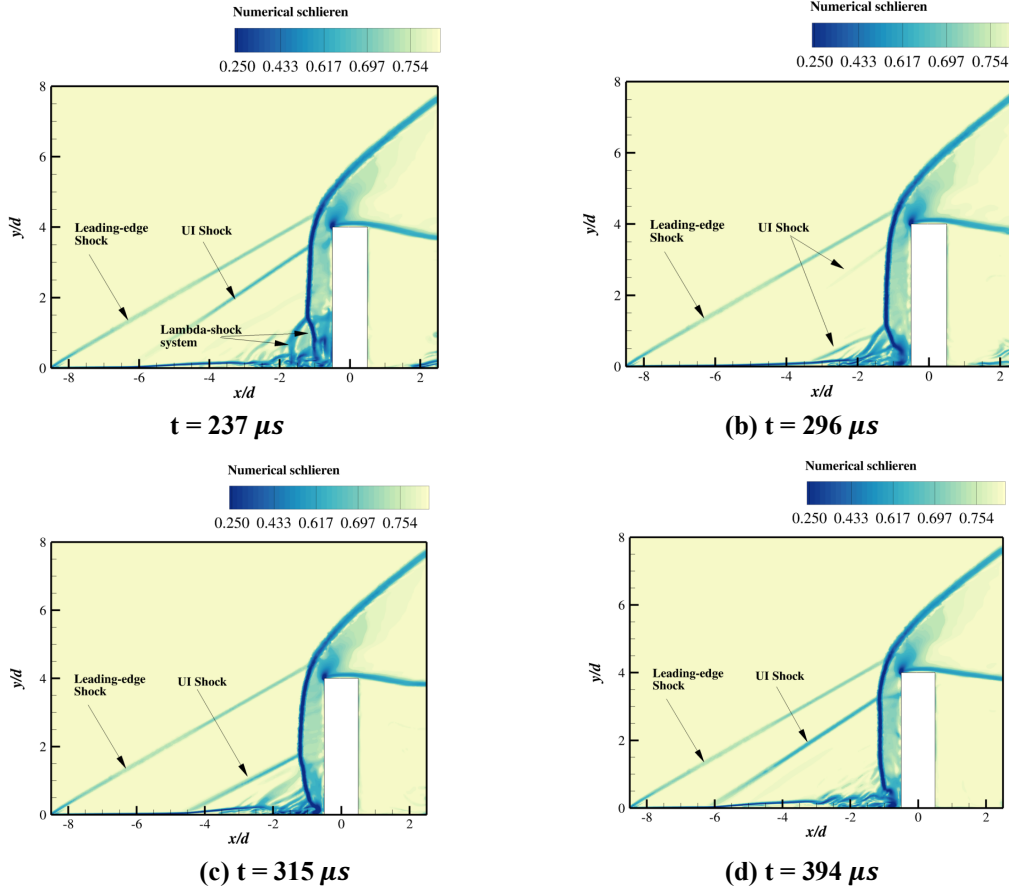


Fig. 3 Time snapshots of density gradient contours, showing the dynamics of the UI shock that appears to emanate from the forward foot of the lambda-shock. Flow from left to right.

In experimental studies featuring completely laminar or turbulent boundary layers [4–8, 29] the UI shock was not seen as a distinct entity, separate from the forward foot of the lambda-shock system. To understand this shift in behavior of the UI shock for a laminar interaction against that of a transitional interaction, additional computations were performed as part of this study. Instead of moving the cylinder upstream as was done in the experiments to create a laminar interaction, the freestream unit Reynolds number was reduced to $1/4^{\text{th}}$, half, and $3/4^{\text{th}}$ the value of that in the transitional case. This also allowed the reuse of the same computational mesh created for studying the transitional interaction case. Results from the computation of the laminar case with $1/4^{\text{th}}$ the freestream unit Reynolds number is shown in Fig. 4. The flow is only weakly unsteady in this laminar case, with the separation shock located at a fixed location and hence only one instant of the flow is shown in Fig. 4. As can be seen from Fig. 4, apart from the leading-edge shock, there is only one distinct separation shock. Thus, the intermittent behavior of the UI shock seen in the transitional case

may indicate that the UI shock may be a part of, or closely related to the lambda shock near the cylinder, as proposed by Lash et al. [11,13].

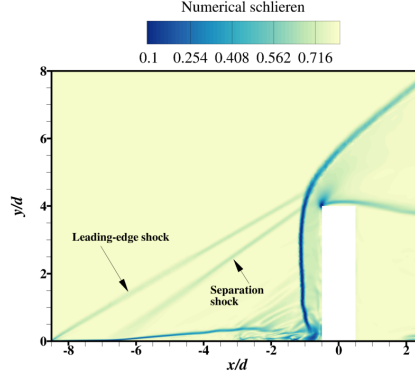


Fig. 4 Instantaneous numerical schlieren contour for the case of Laminar SWBLI ($1/4 Re_{unit}$).

Fig. 4 also shows that the size of the separation region in the laminar case is much larger than the transitional SWBLI case in terms of its upstream extent, consistent with conclusions from an earlier study by Korkegi [4]. To further demonstrate this, time-averaged centerline ($z=0$) wall pressure distribution is shown in Fig. 5 for all four Reynolds numbers computed. In general, the pressure profile along the symmetry plane has an appearance similar to that of SWBLI in two-dimensional configurations (for e.g., SWBLI in a compression corner). For all of the flow conditions, the pressure increases slowly from its baseline value around the location of separation, due to compression of the flow near the separation as it deflects over the reverse flow region. The increase in pressure is followed by a plateau, the value of which agrees with correlations determined by Hill [30] under the laminar scenario. The plateau persists for a substantial distance, beyond which laminar and transitional cases begin to depart from each other. While the pressure begins to drop for the laminar case, in the transitional case, there is a second pressure rise following the plateau region. During the second pressure rise, a series of compression waves eventually form the forward foot of the lambda-shock. This secondary rise was also clearly seen in the data obtained from experiments by Murphree [8]. In the cases of laminar interaction, the lack of these compression waves results in a pure pressure drop in the post-plateau region, possibly due to the expansion happening around the reattachment region. Further downstream, very close to the cylinder, the pressure, for all of the conditions, once again rise to a significant level near the root of the cylinder ($\sim x/d = -1.0$) due to the rearward foot of the lambda shock system.

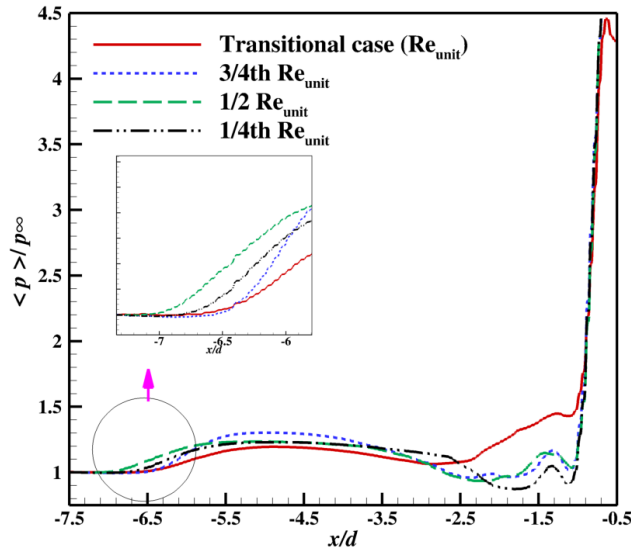


Fig. 5 Time-averaged centerline wall pressures normalized by the undisturbed value upstream of the interaction.

As the freestream unit Reynolds number scaling factor increases from $\frac{1}{4}$ to $\frac{1}{2}$, the separation length also increases (see the zoomed-in view in Fig. 5), in line with the conclusion from an earlier study [4] that the separation length weakly increases with Reynolds number for laminar interaction and is larger than the values found for transitional/turbulent interactions. As also depicted in Fig. 5, the separation length for the two lower Reynolds number cases is larger than for the transitional case. In addition, previous experiments [5] have revealed that for the case of a laminar SWBLI, the upstream separation location is around $x/d = -6$ to -9 (depending on the Reynolds number) and the separation location of $x/d = -6.5$ to -7.0 in the current computation falls within that range. In general, as the unit Reynolds number scaling factor increases from $\frac{1}{2}$ to 1, the separation length decreases indicative of transitional behavior. At first glance, results from the case with scaling factor $\frac{3}{4}$ appear to be inconsistent. Here, the separation length appears to decrease from that of the $\frac{1}{2}$ case, indicative of a transitional behavior. However, the pressure distribution lacks the distinct second pressure rise in the post-plateau region that is often found in a transitional/turbulent interaction. Upon further investigation of the computed flowfield, a weakly unsteady/intermittent appearance of a UI shock was seen (although not shown here), which partially confirms a transitional behavior. In the experiments by Lash et al. [11], transitional behavior was also seen when the cylinder was located at $x/d = 6.5$, which corresponds closely to the current case with scaling factor $\frac{3}{4}$. Based on these observations, the conditions with the scaling factor $\frac{3}{4}$ could belong to the borderline transitional case.

C. Mean Flow Behavior and Unsteady Analysis

To gain more insights into the flow dynamics and the behavior of the UI shock in the transitional SWBLI, time-series data were collected and averaged over a period of 40 domain flow-through times. As indicated by the time-averaged mean pressure plots in Fig. 6 (a), the forward foot of the lambda shock system appears to be anchored around $x/d = -2.5$ and the UI shock is anchored slightly downstream of $x/d = -5.0$ along the spanwise center plane. From the corresponding case in the original experiments [11], the location of the forward shock foot was found to vary from $x/d = -4.5$ to -2.0 and that of the UI shock was between $x/d = -5.0$ and -3.75 . In the newer experiments carried out at slightly different angles of attack [13]–[14], the location of the forward shock foot was found to vary from $x/d = -3.5$ to -2.0 , with the mean position being around $x/d = -2.5$; and the position of the UI shock was determined to vary from $x/d = -5.5$ to -3.5 . The latter experimental data apparently agree better with the current predictions.

Fig. 6 (b) shows that the pressure fluctuations are highest within the lambda shock and more interestingly, fluctuations appear to spread further upstream from the mean position of forward foot of the lambda shock. Comparing the contour plots of the pressure fluctuations for the transitional interaction case against that of the laminar interaction (see Fig. 6 (b) and Fig. 7), one observes that in the case of a transitional interaction there is a double “boomerang” region in the x - z plane. In other words, there appears to be an additional region of intense fluctuation located between $x/d = -2.0$ to -1.0 and slightly away from the wall. Fig. 8 plots the same flow structure on the y - z plane located at $x/d = -1.50$. Two intensive ring-shaped regions roughly symmetric with respect to the center plane are evident. These intensive pressure fluctuations coincide with the locations of the horse-shoe vortices already shown in Fig. 1. A closer inspection of the computed flowfield indicate that these strong pressure fluctuation regions may be the origin of strong instability waves associated with the inflectional velocity profiles present in the flow separation regions.

To highlight the significant pressure variation in the streamwise and spanwise directions, mean wall pressures at various locations are shown in Fig. 9 (a) and (b). Similar to the conclusions drawn in the study by Korkegi [4], as one moves away from the axis of symmetry, the location of initial pressure rise appears to move downstream. Also, the magnitude of pressure drop (past the initial rise) in the post-plateau region decreases as one moves away from the plane of symmetry, indicating that the vortex system ahead of the cylinder becomes weaker as one moves away in the spanwise direction. One can also observe that the spanwise variation grows in asymmetry as one marches from the location of the UI shock toward the root of the cylinder. In the case of laminar interaction, the mean pressure profiles are more symmetric in the spanwise direction everywhere.

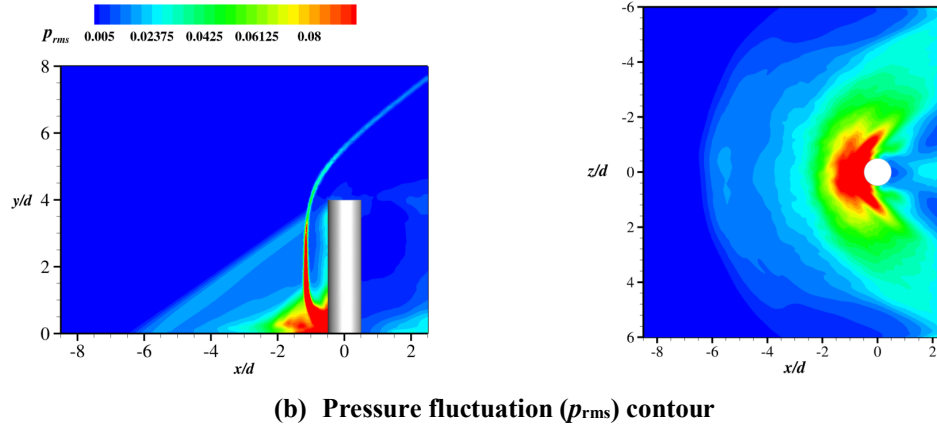
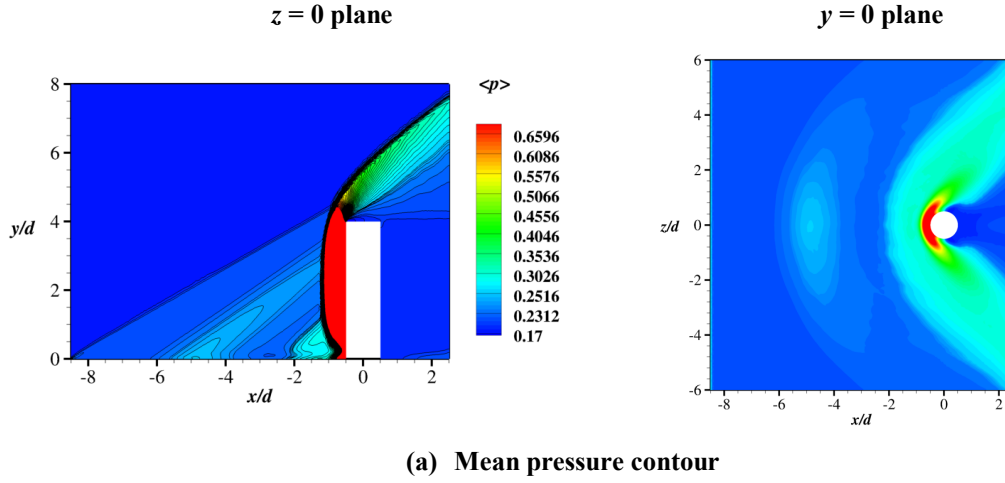


Fig. 6 Time-averaged mean pressure and its fluctuation contours (normalized by freestream values) at two different planes for the transitional SWBLI.

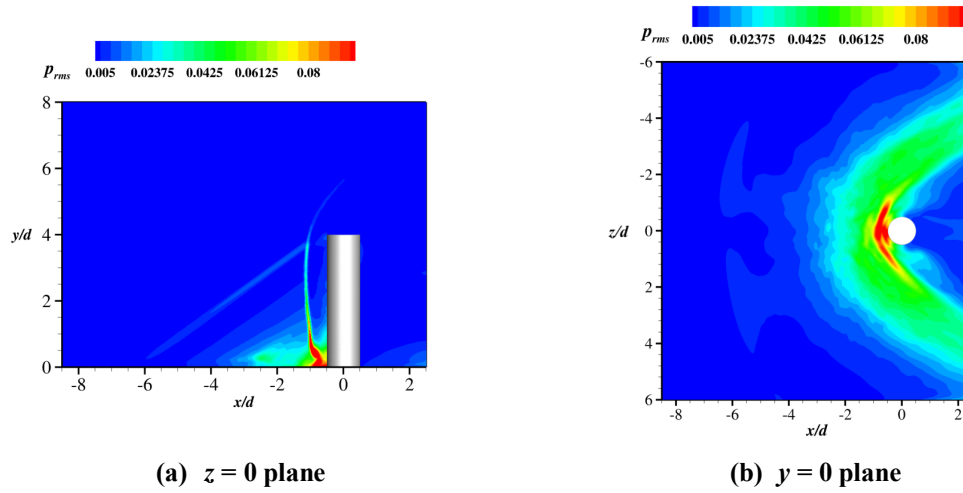


Fig. 7 Pressure fluctuation contours (normalized by freestream values) at two different planes for the laminar SWBLI interaction ($1/4^{\text{th}} Re_{\text{unit}}$).

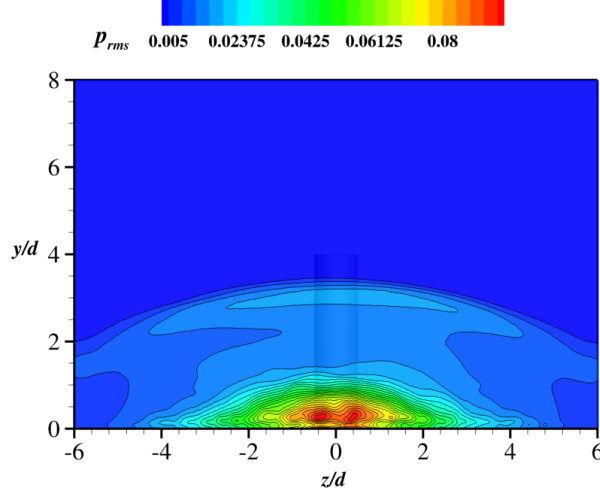


Fig. 8 Pressure fluctuation contours (normalized by freestream values) for the transitional SWBLI interaction shown for a spanwise plane at $x/d = -1.50$.

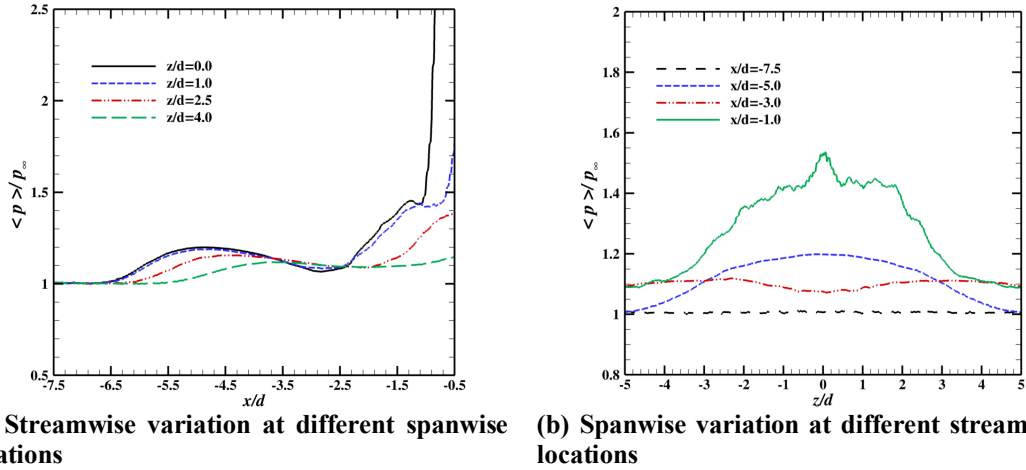


Fig. 9 Mean wall pressure (normalized by the undisturbed value upstream of the interaction) variation for the transitional SWBLI.

To further investigate the unsteady features of the transitional boundary layers, several time probes located in a streamwise direction along the symmetry plane and in spanwise locations on either side of the symmetry plane, that anchored high pressure fluctuations seen in Fig. 6 (b) and Fig. 8, respectively, were used to record time histories. These probe points were located between $y/d = 0.1$ and 0.2 as the intense perturbation appeared to be located away from the wall. Data were collected over 40 domain flow through times at a frequency of 160kHz. In Fig. 10, the corresponding pressure spectra are shown for three groups of probes: (a) near the foot of the UI shock, (b) in between the UI shock and the forward foot of the lambda-shock, and (c) around the forward foot of the lambda-shock. In more recent experiments [13], carried out at a slightly different angle of attack, a dominant frequency of around 4.7kHz was observed for the UI shock motion, based on the analyses conducted using the schlieren images. In contrast, analysis based on PSP data appeared to indicate a dominant frequency of around 5.2kHz. Unfortunately, spectral content information was not available for the old experimental conditions analyzed in the present work. As can be seen from the spectral content of the region around the foot of the UI shock in Fig. 10 (a), the dominant frequency in that region appears to be around 4.6kHz. Although not easily visible, there appear to be additional modes of lower amplitude around 3.9kHz and 5.2kHz as well. As one moves away from the UI shock toward the forward foot of the lambda-shock (along the centerline), the dominant peak is around 4.6kHz (see Fig. 10 (b)). Further

downstream, near the region around the foot of the lambda-shock (see Fig. 10 (c)), the dominant peak continues to be around 4.6kHz. However, additional peaks between 3–20kHz also appear in spanwise locations away from the centerline. Given the dominance of the 4.6kHz mode in all of the probe locations, there appear to be a potential relation between the lambda shocks and the UI shock.

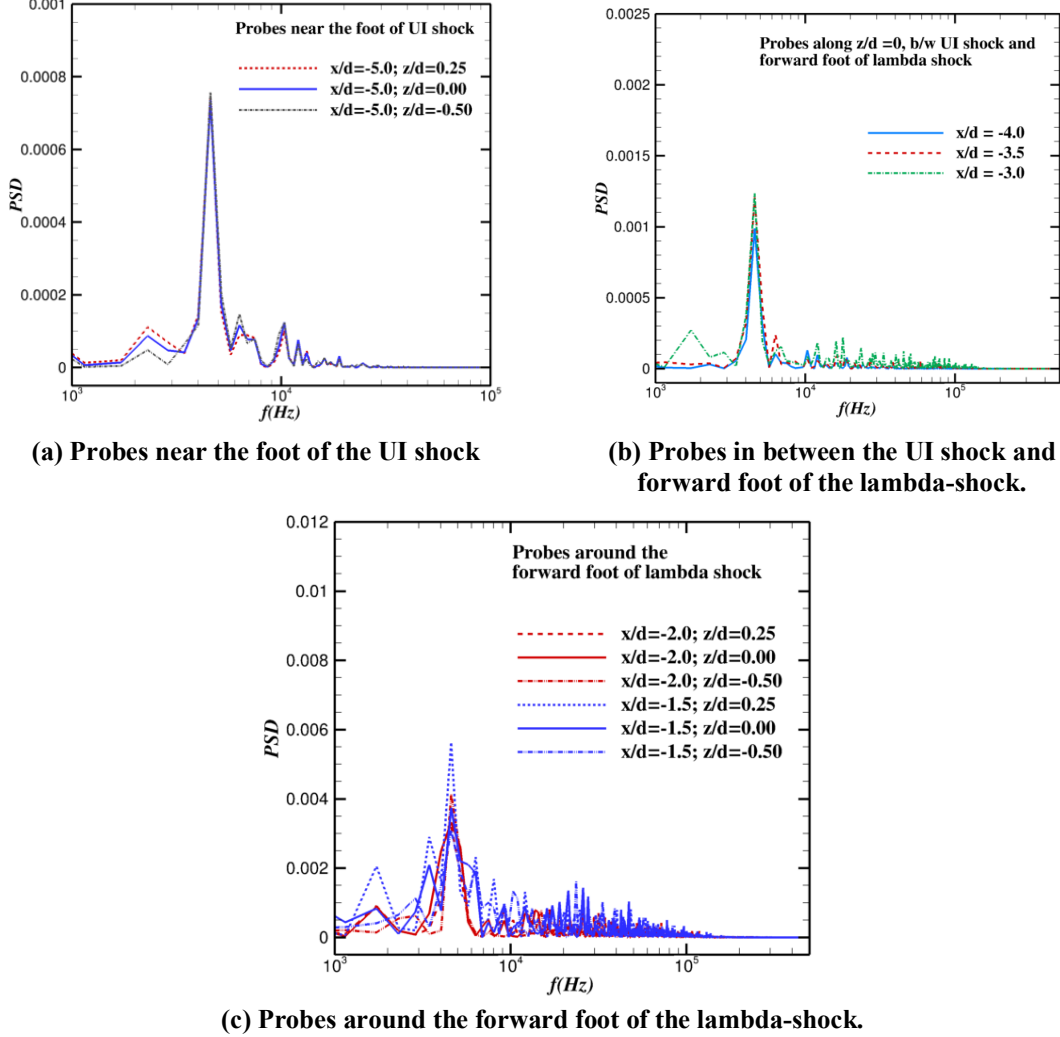


Fig. 10 Power spectral density at various x/d locations in the flow.

To get additional insights, cross-correlation was performed using the time signal from a probe point located near the UI shock ($x/d = -6.0$) as a reference and the time signals from other locations downstream of the UI shock. The resulting distributions are shown in Fig. 11, for several pairs of points as indicated in the plot legend. As can be seen from Fig. 11, as one moves from $x/d = -6.0$ to $x/d = -4.0$, the cross-correlation factor still remains high above 0.7, indicating a strong correlation between the flow in this region. Also, the lag increases as one moves from $x/d = -5.0$ to $x/d = -4.0$, indicating that the signal at $x/d = -4.0$ precedes that at $x/d = -6.0$. The cross-correlation shifts in phase (negative correlation coefficient of approximately 0.4) when one finally reaches $x/d = -3.0$ and beyond. Even at $x/d = -1.5$, the cross-correlation coefficient remains significant (~ 0.4). All of the above analyses provide a weak indication for a disturbance originating somewhere around $x/d = -3.0$ and propagating upstream, thereby strengthening the suggestion by Lash et al. [13] and Combs et al. [14] that the UI shock may simply be a manifestation of the forward foot of the lambda shock.

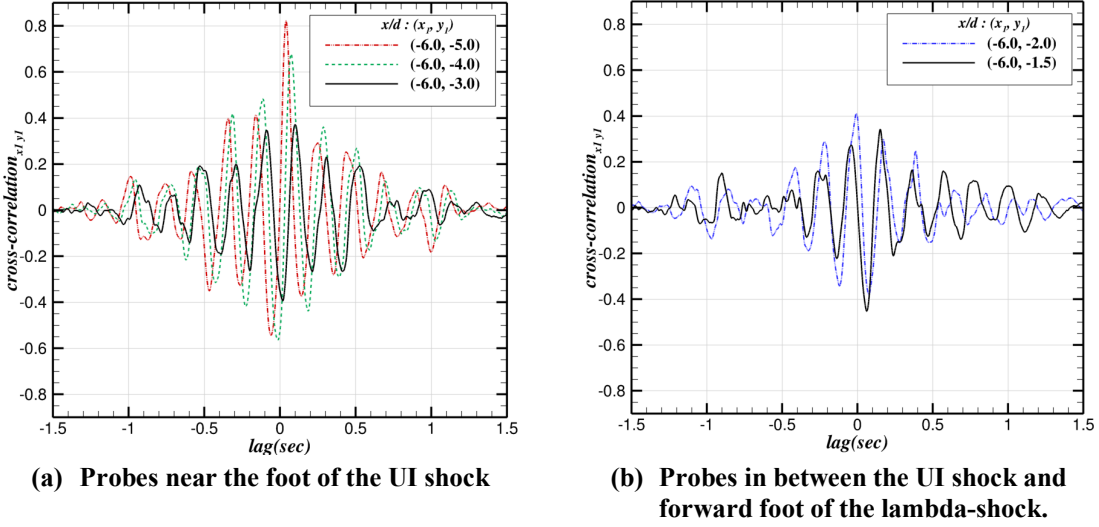


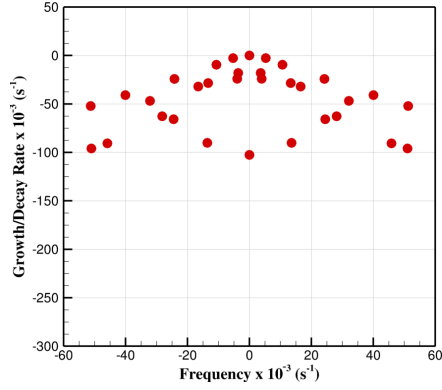
Fig. 11 Cross-correlation for probes located along the centerline for the transition SWBLI. Probe located at $x/d = -6.0$ was used as reference.

D. Dynamic Mode Decomposition

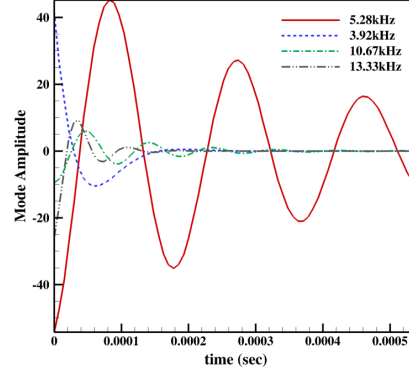
Ever since its introduction to the fluids community by Schmid [31], dynamic mode decomposition (DMD) has become a popular data analysis technique to extract coherent features from otherwise complicated experimental and numerical flowfield data. With its relation to the Koopman operator [32], it has become an important tool for analyzing nonlinear flow phenomena. DMD provides a means to decompose time-resolved data into spatial modes that oscillate with a single frequency, thereby allowing one to extract spatio-temporal modes that may potentially describe the dynamics of the flow. More details on the DMD technique can be found in Ref. [33]. The DMD technique was employed here in an attempt to enhance our understanding of the dynamics of the transitional SWBLI problem studied as part of this work and to possibly extract any dominant dynamics in the flow, if present.

For performing DMD, unsteady flow data were extracted for a subset of the computational domain described earlier. The subdomain extended upstream from the center of the cylinder to $x/d = -6.0$. In the spanwise direction, it went from $z/d = -5.0$ to 5.0 , and in the wall normal direction, it extended up to the top of the cylinder ($y/d = 4.0$). A total of 75 snapshots of the flow was acquired over 15 domain flow-through time. The snapshots were obtained at equal time intervals allowing for a maximum resolvable frequency of 67kHz. DMD was performed using the pressure variable as well as total energy. Only the analysis performed using the pressure variable is discussed below as a similar trend was obtained using total energy.

Although not shown here, all of the eigenvalues of the best-fit operator that approximates the dynamics of the flow was found to lie within the unit circle. Frequency and growth/decay rates of most of the modes (obtained from the eigenvalues) are shown in Fig. 12 (a). Four of the most important modes (based on their amplitude) were identified to correspond to frequencies (positive and negative) 5.28kHz, 10.67kHz, 3.96kHz and 13.33 kHz, respectively. Dynamics of these modes are plotted in Fig. 12 (b) to show which of the modes are likely to play a dominant role in the dynamics of the flow. Unlike what was observed from the fast Fourier analysis of the time signals from the probe data, the 4.6kHz mode that was found to be dominant does not appear here. Up on further investigation, it was identified that the time period over which the data was collected for the Fourier analysis allowed for a frequency resolution of around 0.6kHz, while the period over which the data were collected for the DMD analysis allows only for a frequency resolution of 1.8kHz. As a result, the DMD analysis should be treated as only a preliminary one. Data collection over a longer time period is currently underway for performing a more accurate DMD analysis and will be reported as part of a future work. Also, it is important to note that although Fourier analysis of the probe signals indicated a dominant frequency of 4.6 kHz, modes corresponding to roughly 3.9kHz and 5.2 kHz were also observed (with lower amplitudes) in the probe data located near the UI shock. Thus, the modes corresponding to 3.96 kHz and 5.28kHz from the DMD analysis can be considered to be an approximate representation of coherent structures of frequency range 3.5–5.5 kHz in the flow.



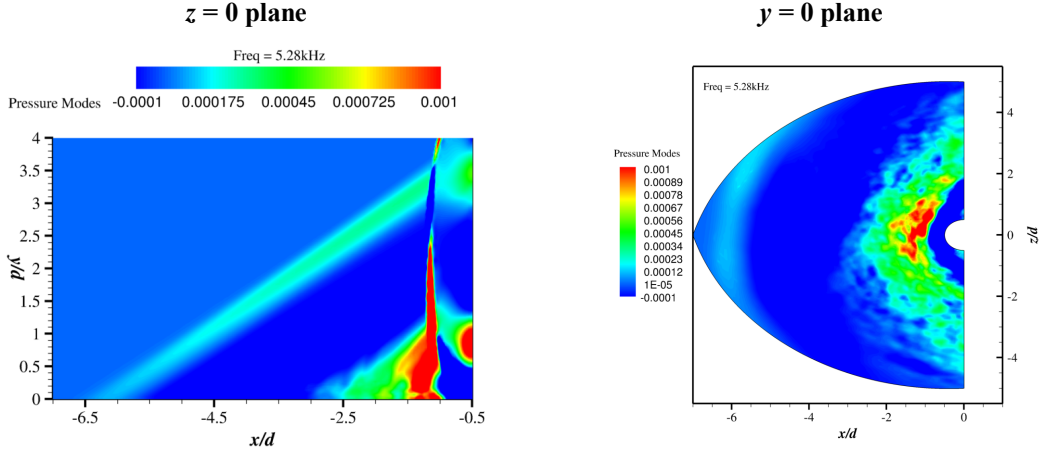
(a) Frequency and growth/decay rates of some of the DMD modes



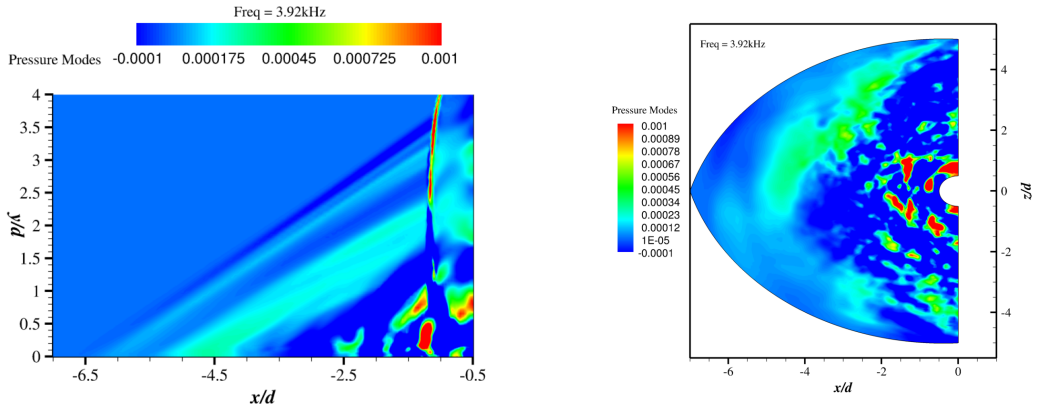
(b) Dynamics of the important DMD modes.

Fig. 12 Preliminary results for frequency, growth rate and dynamics of some of the detected critical modes.

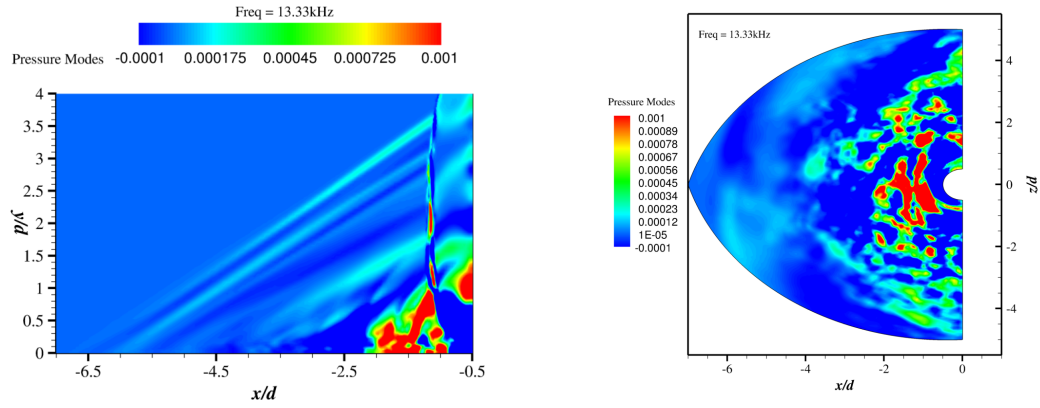
As can be seen from Fig. 12 (b), the 5.28kHz mode is expected to play a dominant role in the flow and to persist for a considerable duration of time. Some of the other modes, even though their initial amplitudes are high, don't exhibit cyclic behavior while some others decay faster. To gain a better understanding of how some of these modes manifest themselves, a planar view of the pressure modes corresponding to 5.28kHz, 3.96kHz and 13.33 kHz, respectively, are shown in Fig. 13 (the real component alone is shown for brevity). As can be seen from Fig. 13 (a), the 5.28kHz mode appears to represent the bulk of the dynamics of the flow, including the bow-shock, the UI-shock and a substantial portion of the region that showed significant pressure fluctuation in Fig. 6 (b). The regions with high amplitude are just upstream of where the rearward foot of the lambda shock system is located. As all of the core features of the flow appear to be coherent at this frequency, and based on earlier probe time signal analysis, there appears to be a strong indication of the presence of a global instability-like mode. The 3.96 kHz mode (see Fig. 13 (b)) appears to represent some aspects of the transient UI shock due to the fact that it is the only mode that exhibits higher amplitude in the UI shock region (albeit asymmetric). The 13.3 kHz mode (see Fig. 13 (c)) is mostly concentrated near the rearward foot of the lambda shock system. Fig. 10 (c) also revealed the presence of high frequency modes in this region. As a side note, the reader is reminded that, in the experiments by Lash et al. [13], data from PSP also indicated a dominant mode of around 5.2kHz. Furthermore, the PSP data indicated that this mode was dominant in the region near where the second pressure rise occurs in the pressure distribution, similar to what is seen in Fig. 13 (a). However, for reasons mentioned earlier, since the DMD data analysis lacks adequate frequency resolution, one can only assume that the dominant frequency that describes this flow is somewhere between 3.5 kHz and 5.5 kHz.



(a) Freq = 5.28kHz



(b) Freq = 3.92kHz



(c) Freq = 13.33kHz

Fig. 13 Planar view of some of the important pressure modes found from the preliminary DMD analysis of the transitional SWBLI.

IV. Conclusions

Numerical computations of a transitional SWBLI have been carried out using the space-time CESE method and an unstructured mesh with tetrahedral elements. The computations were performed to complement earlier measurements at UTSI that used high-speed schlieren imaging and PSP techniques. The primary focus of this work is to better understand the dynamic features of the transitional SWBLI as it has never been studied computationally before, and to identify if there were any dominant instability mechanisms or distinct frequencies that characterized such interactions in the UTSI experiment. Similar to what was observed in the experiments, it was found that the UI shock appears to start from the forward foot of the lambda-shock system, travels upstream toward the leading edge before disappearing and the cycle repeated itself. It was also found that the presence of an intermittent UI shock only takes place when the flow is transitional. Our computations confirm that at sufficiently lower than the experimental transitional Reynolds numbers, the flow remains laminar throughout and no moving UI shock was observed. Unsteady analysis based on Fourier analysis of probe data in the regions between the UI shock and the lambda foot suggests a dominant frequency of 4.6 kHz (for flow conditions equivalent to a cylinder center location of $8.5d$ from the leading edge in the UTSI experiment) similar to experimental measurements based on schlieren imaging (4.7kHz). On the other hand, experimental analysis based on PSP data and DMD analysis of current computational data reveal the dominant frequency to be approximately 5.2 kHz. This discrepancy needs to be resolved, both computationally as well as experimentally, through a detailed investigation in the future.

Despite the uncertainty about the dominant frequency characterizing the flow, all of the analyses indicate that the UI shock seen in the case of a transitional interaction is simply part of the system of compression waves that make up the forward foot of the lambda shock system, strengthening the arguments made by Lash et al. [13] and Comb et al. [14]. Furthermore, the analysis points to the system having an instability that is probably global and three-dimensional in nature, and is located in the region where the pressure distribution undergoes a second rise. This region was also identified in the computations to be the region from which a system of horse-shoe vortices appears to emanate, thereby closely matching the hypothesis provided by Kaufman et al. [5] for the source of unsteadiness in such a flow configuration. With the data from transitional SWBLI experiments carried out in the NASA Langley 20-inch Mach 6 tunnel for a similar configuration set to come online, more corresponding computations may be performed in the future to ascertain if some of the conclusions gained about transitional interaction from the current study carries over to the hypersonic regime as well.

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