Formulation of Plasticity Models through Symbolic Regression

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Motivation for homogenization

metallic alloy

$\Phi = f(\sigma)$
Homogenization methods

Response = \( f(\text{loading}) \)

Human developed/derived models

- **Pros:**
  - Can be physically based
  - Transferability
  - Compact and quick to evaluate

- **Cons:**
  - Can take decades in development

Typical machine learning models

- **Pros:**
  - Rapid development (training)
  - More input \( \rightarrow \) more accurate

- **Cons:**
  - Not transferable
  - Not insightful (black box)
  - Evaluation is relatively expensive

Can we have the best of both worlds?
Human developed homogenization models

Example: GTN porous plasticity model

\[
\Phi = \left( \frac{\bar{\sigma}}{\sigma_y} \right)^2 + 2q_1 f^* \cosh \left( \frac{3}{2} q_2 \frac{\sigma_h}{\sigma_y} \right) - (1 + (q_1 f^*)^2)
\]

\[
\Phi = \left( \frac{\bar{\sigma}}{\sigma_y} \right)^2 + 2q_1 (\sigma) f^* \cosh \left( \frac{3}{2} q_2 (\sigma) \frac{\sigma_h}{\sigma_y} \right) - (1 + (q_1 (\sigma) f^*)^2)
\]

- Choose functional form
- Fit parameters (in red)
- Model misfit identified
- Abuse parameters
- Add more physics to functional form

Time consuming step!
Symbolic regression and homogenization

- Find best fit functional forms and parameters simultaneously!
  1. Decide what data to use
  2. Define fitness to data
  3. (Decide how much data to use)

- Attribute physics to portions of equations

\[ y = 10e^{-x} \sin(2x) + x \]

\[ \Phi = f(\sigma) \]
Verification problem: von-Mises plasticity

\[ \Phi = (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 - 2\sigma_{\text{yield}}^2 = 0 \]

**RVE:**
- Single element model
- von-Mises plasticity

**Verification:**
- Can we recover \( \phi \) from looking at response data?
1. Decide what data to use
2. Define fitness to data

- proportional loading

Data for each loading case:
- Principle stresses: $\sigma_i$
- Principle strains: $\epsilon_i$
- Equivalent plastic strain: $\epsilon^p$
Symbolic regression problem definition

1. Decide what data to use
2. Define fitness to data

- \( \Phi = f(\sigma) = 0 \) (on yield surface)
- Implicit regression
- \( E = \sum \frac{df(\sigma)d\sigma}{d\sigma dt} = 0 \) 
- \( \Phi(\sigma) = \text{constant for each loading case} \)
Solving the symbolic regression problem

- Using genetic programming
  - (Genetic algorithms of computer programs)
- Equations evolve until they fit the data
- In-house code: bingo

\[ y = 1.5x^2 - x^3 \]

**NODE operators**

**TERMINAL**
- constants, variables

Crossover

Mutation
Looking for yield surface: $\Phi(\sigma) = 0$

$\Phi(\sigma)$ = constant for each loading case

$\sigma_1 - (\sigma_3 + \sigma_2) + \sigma_1 =$ constant

**Issue:** all loading cases are parallel!

**Solution:** more complex loading cases
Early results

- Looking for yield surface: $\Phi(\sigma) = 0$
- $\Phi(\sigma)$ = constant for each loading case

$$\sigma_1 - (\sigma_3 + \sigma_2) + \sigma_1 = \text{constant}$$

- **Issue:** all loading cases are parallel!

  $$\Phi(x, y) = y = \text{constant}$$

- **Solution:** more complex loading cases

  Stage 1
  
  *Loading ratio:* [0.675 1.0 0.825]

  Stage 2
  
  *Loading ratio:* [1.0 0.825 0.675]
Yield surface from 2 stage loading data

\[ \Phi = (\sigma_3 - \sigma_1)^2 + (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 = \text{constant} \]

Computation time: 32s on 40 processors

\[ (\sigma_3 - \sigma_2)(\sigma_3 - \sigma_2) - (\sigma_2 - \sigma_1)(\sigma_3 - \sigma_1) = \text{constant} \]
Adding hardening

\[ \Phi = (\sigma_3 - \sigma_1)^2 + (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 = \text{constant} \]

- No hardening

\[ \sigma_{\text{yield}} \]

\[ \text{plastic strain} \]

- Hardening

\[ \sigma_{\text{yield}} \]

\[ \text{plastic strain} \]
Hardening yield surface from 2 stage loading data

\[
\Phi = (\sigma_3 - \sigma_1)^2 + (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 - 19797\bar{\varepsilon}^p - 980185(\bar{\varepsilon}^p)^2 = \text{constant}
\]

\[
\Phi = (\sigma_3 - \sigma_1)^2 + (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 - c_1\bar{\varepsilon}^p - c_2(\bar{\varepsilon}^p)^2 = \text{constant}
\]

Computation time: 1.5h on 160 processors

\[
((\sigma_1 - \sigma_3 + \sigma_3 - \sigma_2)(\sigma_3 - \sigma_2 + \sigma_1) + 47602 + \sigma_1 + (\sigma_3 - \sigma_1 - \sigma_3)(\sigma_3 + \sigma_3 -
\]

Data: \(\sigma_1, \sigma_2, \sigma_3, \bar{\varepsilon}^p\)

Yield surface now depends on a state variable!

Now it needs a state evolution equation
Hardening yield surface from 2 stage loading data

Assuming incremental elastic strains are small

Goal: \( \dot{\epsilon}^p = \sqrt{\frac{2}{9}[(\dot{\epsilon}_1 - \dot{\epsilon}_2)^2 + (\dot{\epsilon}_2 - \dot{\epsilon}_3)^2 + (\dot{\epsilon}_3 - \dot{\epsilon}_1)^2]} \)

\( \dot{\epsilon}_1 + \dot{\epsilon}_2 + \dot{\epsilon}_3 = \text{constant} \)

\( \dot{\epsilon}_1 + \dot{\epsilon}_2 + \dot{\epsilon}_3 = 0 \) deviatoric plastic strains

Data: \( \dot{\epsilon}_1, \dot{\epsilon}_2, \dot{\epsilon}_3, \dot{\epsilon}^p \)

Goal: \( \ddot{\epsilon}^p = \sqrt{\frac{4}{3}[\dot{\epsilon}_1^2 + \dot{\epsilon}_1 \dot{\epsilon}_2 + \dot{\epsilon}_2^2]} \)

\( \dot{\epsilon}_1^2 + \dot{\epsilon}_1 \dot{\epsilon}_2 + \dot{\epsilon}_2^2 - (\dot{\epsilon}^p)^2 = \text{constant} \)

Data: \( \dot{\epsilon}_1, \dot{\epsilon}_2, \dot{\epsilon}_3, \dot{\epsilon}^p \)
Quick Recap

- Verification of von-Mises plasticity
  - Non-hardening yield surface
  - Hardening yield surface
  - State evolution

- Moving Forward:
  - Seconds (on single processor)
  - Hours (on multiple processors)

- How much data is required?
How much data is needed?

- **Step size**
  - More dense data =
    - more computation time
    - more accurate derivative calculations

- Density needed will depend on
  - complexity of loading scenarios
  - Complexity of yield surface
How much data is needed?

- **Number of loading scenarios**
  - No real trend (except very low values)
  - Minimum case found:

![Graphs showing data points and numbers 2, 4, and 8]
Summary

- Set up framework for SR formulation of plasticity models
- Implicit symbolic regression of yield surface
- Use non-proportional loading
- Von-Mises verification problem
  - Surprisingly little data needed
    - will depend on complexity of yield surface

\[ \Phi = f(\sigma) \]

RVE

Data

Symbolic Regression

homogenization
Future work

- application to real materials
- adaptive data generation
- bingo (soon to be open source)
  - python & c++
  - Features:
    - Coevolution of fitness predictors
    - Island parallelization scheme
    - Acyclic graph representation
    - Constant optimization
    - Age-fitness Pareto selection

performance

robustness
thank you!

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