Formulation of Plasticity Models through Symbolic Regression

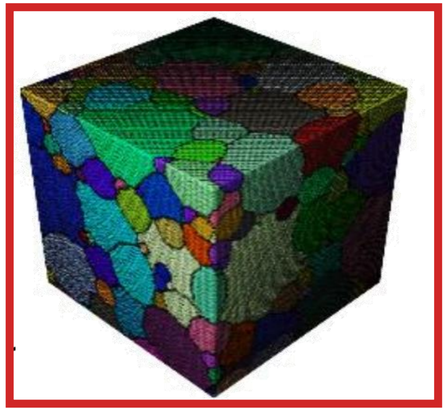
Geoffrey Bomarito
NASA Langley Research Center

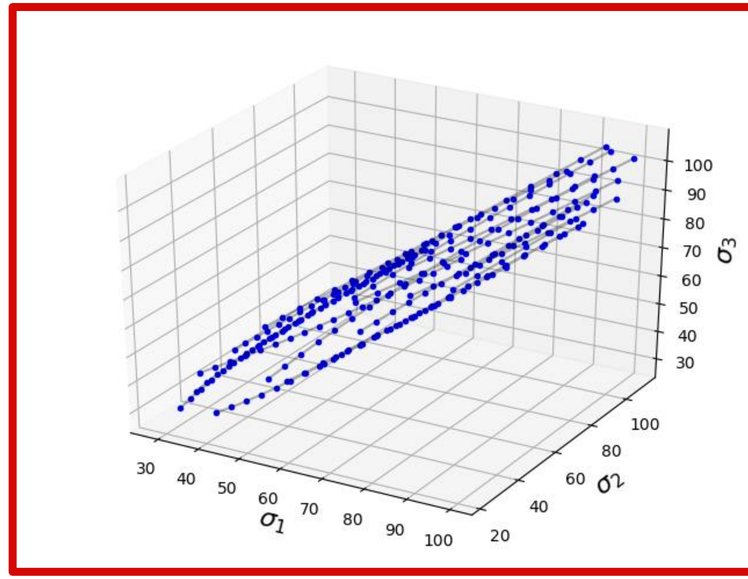
Tyler Townsend
University of Central Florida

Kathryn Esham
The Ohio State University

Ethan Adams
Brigham Young University Idaho

Motivation for homogenization



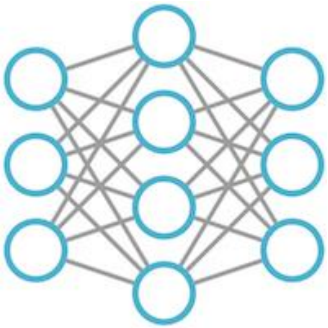


homogenization

$$\Phi = f(\sigma)$$

Homogenization methods

$$\text{Response} = f(loading)$$



Human developed/derived models

- Pros:
  - Can be physically based
  - Transferability
  - Compact and quick to evaluate
- Cons:
  - Can take decades in development

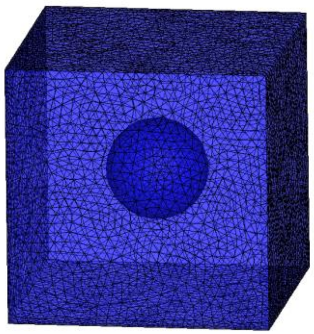
Typical machine learning models

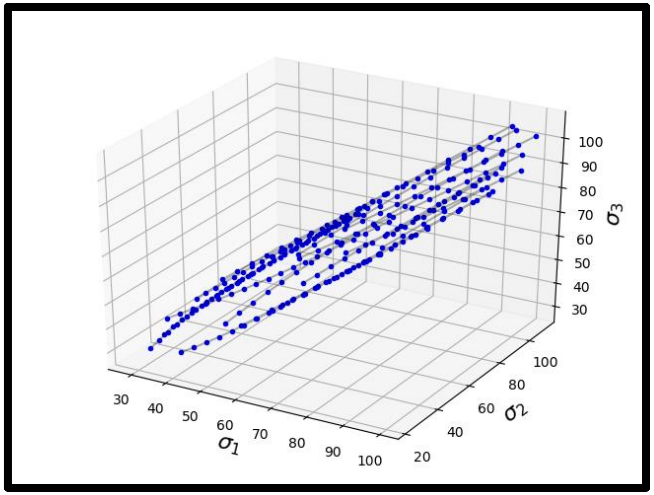
- Pros:
  - Rapid development (training)
  - More input → more accurate
- Cons:
  - Not transferable
  - Not insightful (black box)
  - Evaluation is relatively expensive

Can we have the best of both worlds?

Human developed homogenization models

Example: GTN porous plasticity model





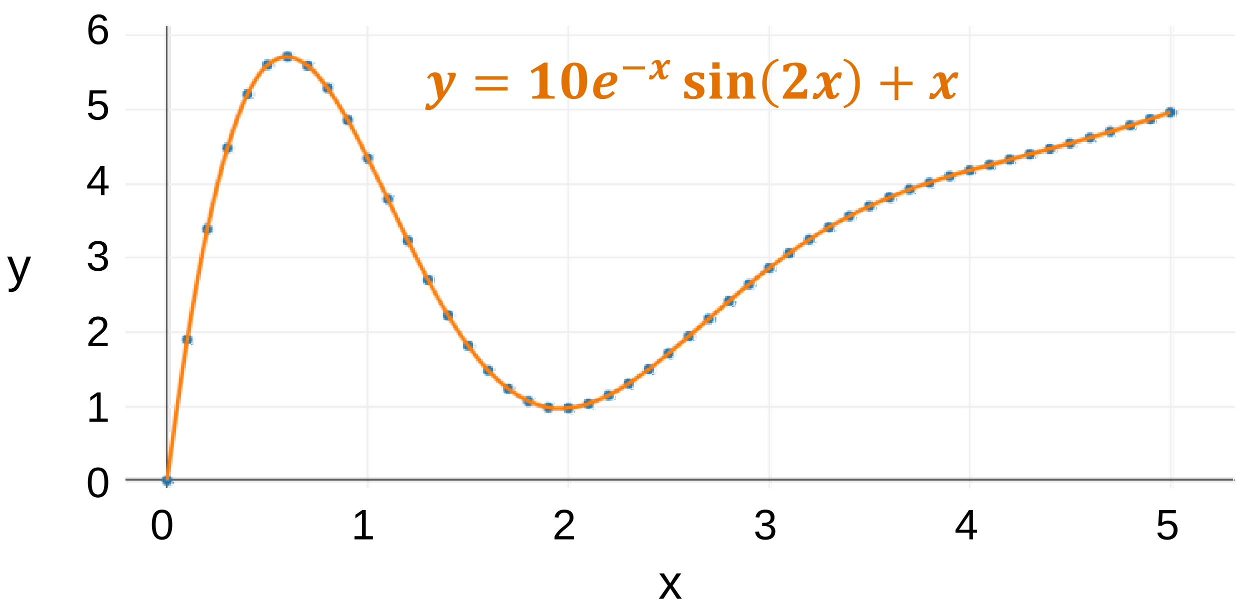
- Choose functional form
- Fit parameters (in red)
- Model misfit identified
- ~~Abuse parameters~~
- Add more physics to functional form

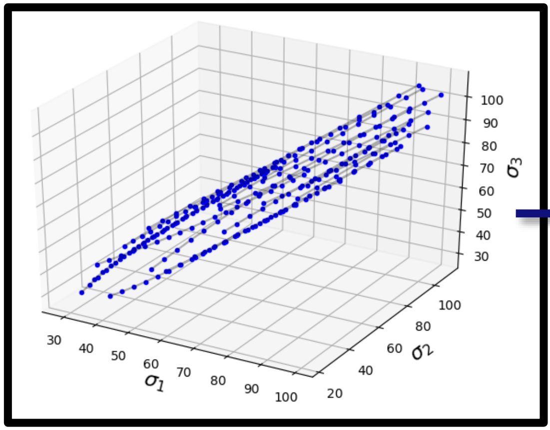
Time consuming step!

$$\Phi = \left(\frac{\bar{\sigma}}{\sigma_y}\right)^2 + 2q_1 f^* cosh\left(\frac{3}{2} q_2 \frac{\sigma_h}{\sigma_y}\right) - (1 + (q_1 f^*)^2)$$

$$\Phi = \left(\frac{\bar{\sigma}}{\sigma_y}\right)^2 + 2q_1(\sigma) f^* cosh\left(\frac{3}{2} q_2(\sigma) \frac{\sigma_h}{\sigma_y}\right) - (1 + (q_1(\sigma) f^*)^2)$$

Symbolic regression and homogenization





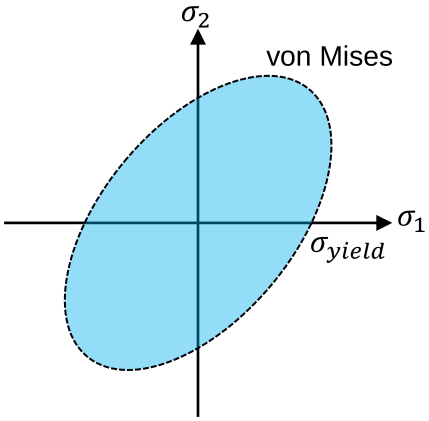
$$\Phi = f(\sigma)$$

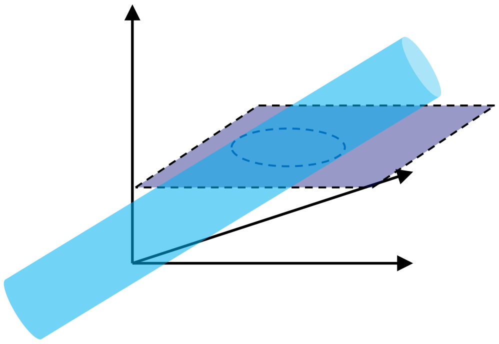
- Find **best fit functional forms** and parameters simultaneously!
  1. Decide what data to use
  2. Define fitness to data
  3. (Decide how much data to use)

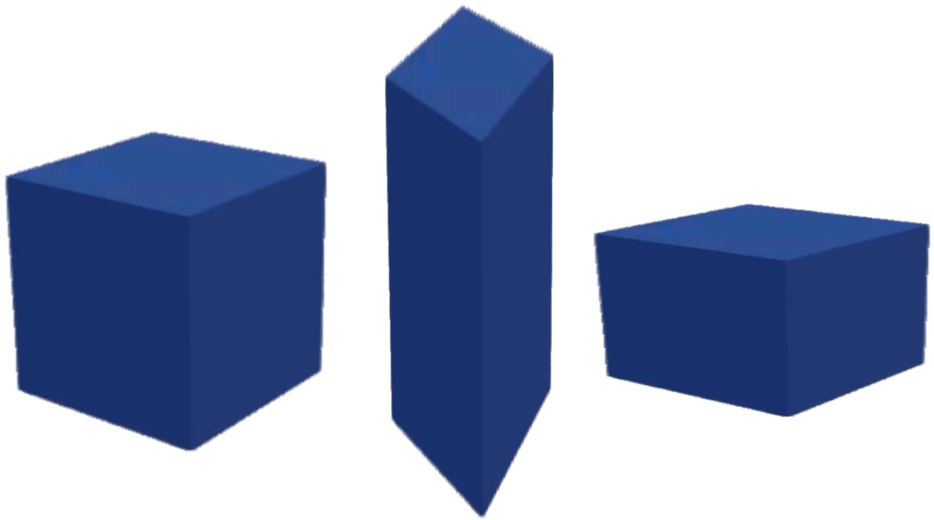
- Attribute physics to portions of equations

Verification problem: von-Mises plasticity

$$\Phi = (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 - 2\sigma_{yield}^2 = 0$$







RVE:

- Single element model
- von-Mises plasticity

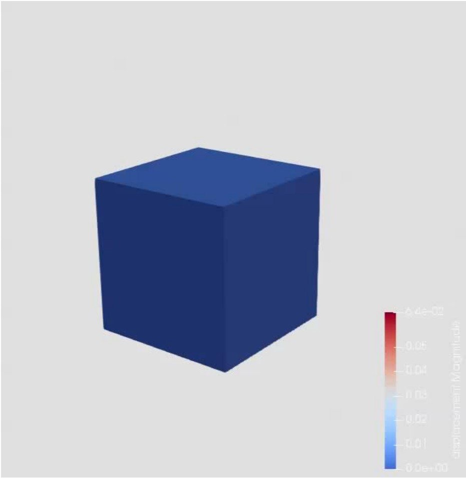
Verification:

- Can we recover ϕ from looking at response data?

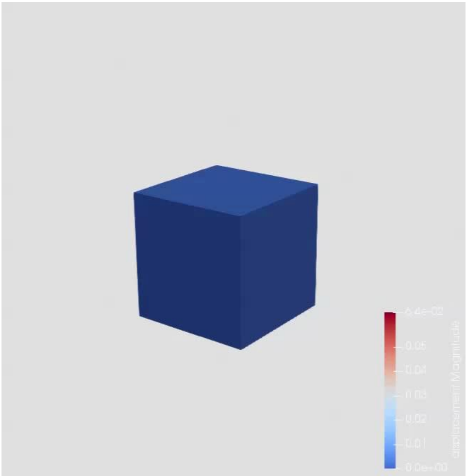
Symbolic regression problem definition

1. **Decide what data to use**
2. Define fitness to data

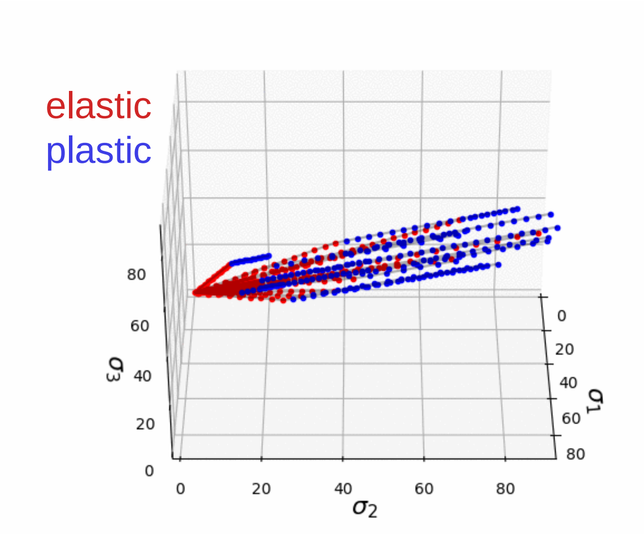
$Loading\ ratio: [0.675 \quad 1.0 \quad 0.8250]$

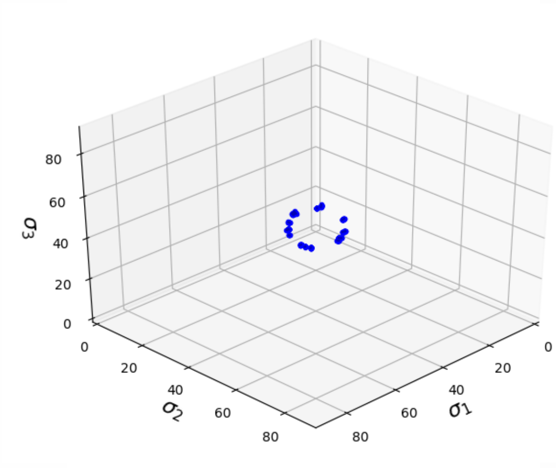


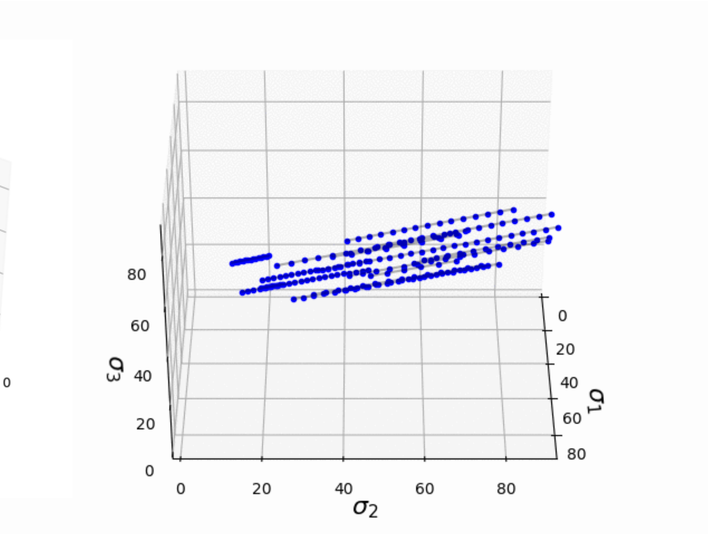
$Loading\ ratio: [0.278 \quad 0.238 \quad 1.0]$



- proportional loading
- Data for each loading case:
  - Principle stresses: σ_i
  - Principle strains: ϵ_i
  - Equivalent plastic strain: $\bar{\epsilon}^p$

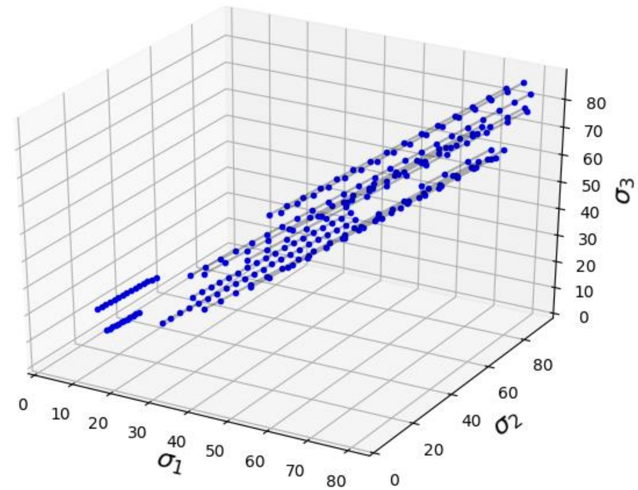




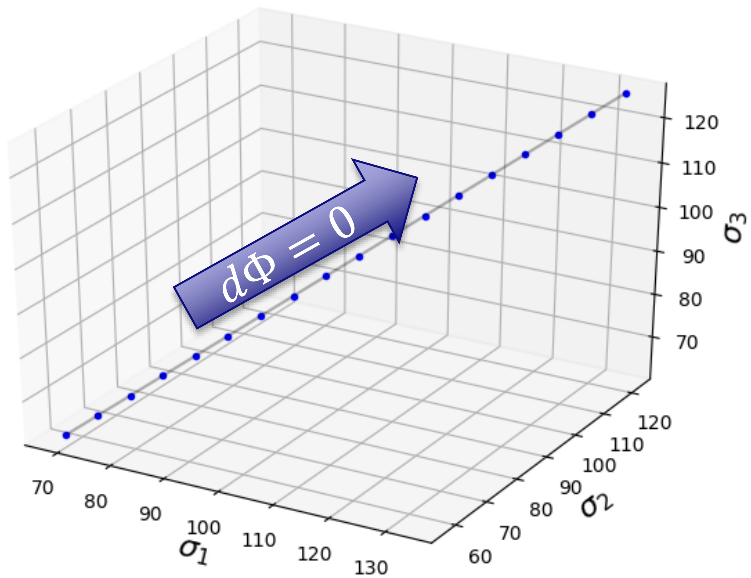


Symbolic regression problem definition

1. Decide what data to use
2. **Define fitness to data**



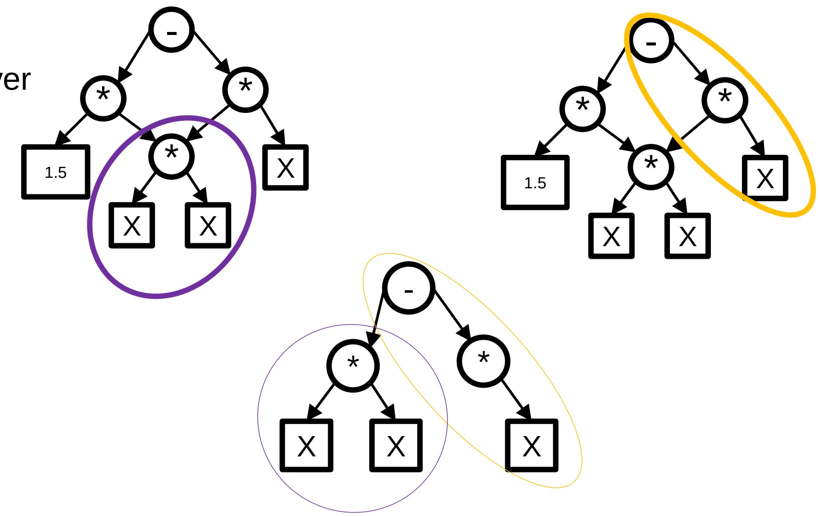
- $\Phi = f(\sigma) = 0$ (on yield surface)
- Implicit regression
- $E = \sum \frac{\frac{df(\sigma)}{d\sigma}:\frac{d\sigma}{dt}}{\left\|\frac{df(\sigma)}{d\sigma}:\frac{d\sigma}{dt}\right\|} \rightarrow 0$
- $\boldsymbol{\Phi(\sigma)} =$ **constant for each loading case**

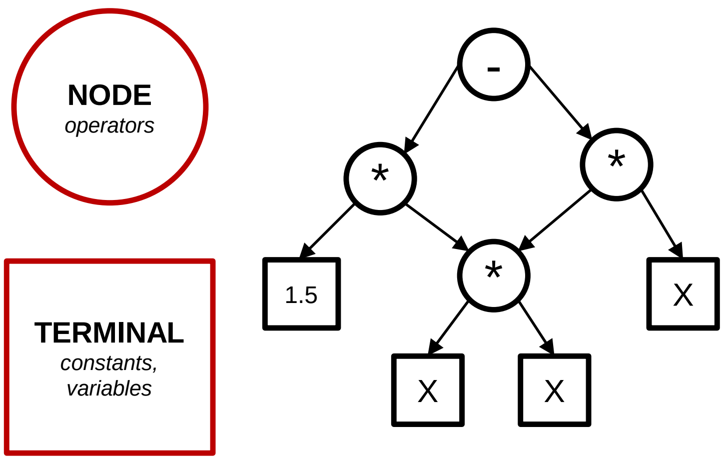


Solving the symbolic regression problem

- Using genetic programming
  - (Genetic algorithms of computer programs)
- Equations evolve untill they fit the data
- In-house code: bingo

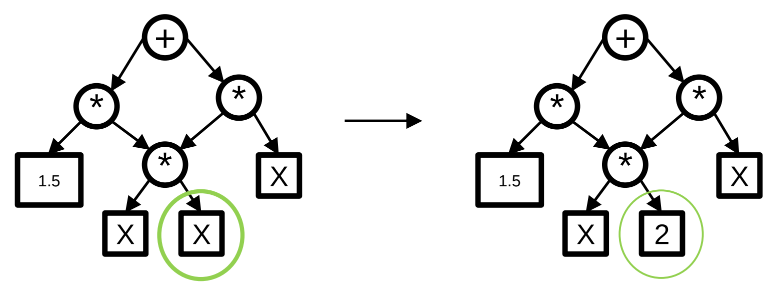
Crossover





$$y = 1.5x^2 - x^3$$

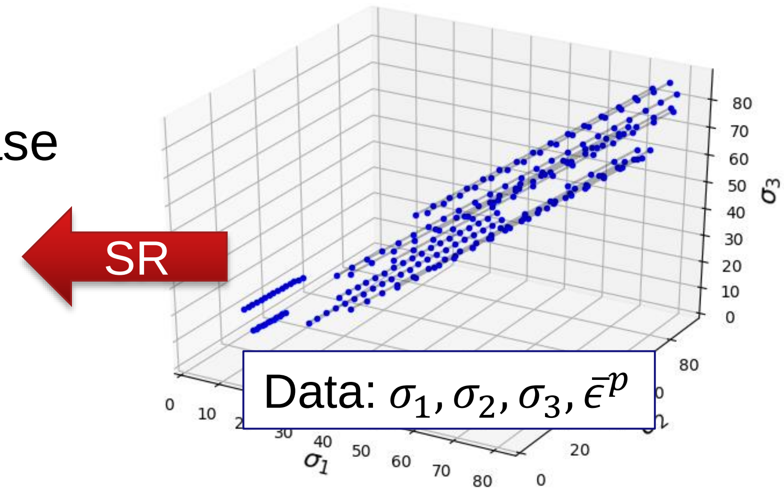
Mutation



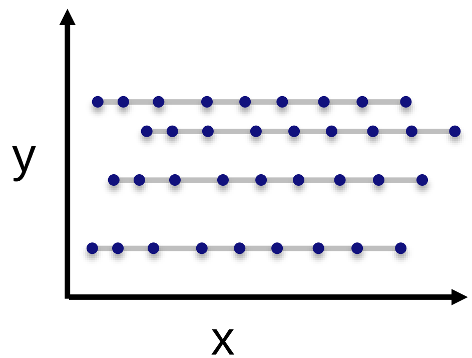
Early results

- Looking for yield surface: $\Phi(\sigma) = 0$
- $\Phi(\sigma)$ = constant for each loading case

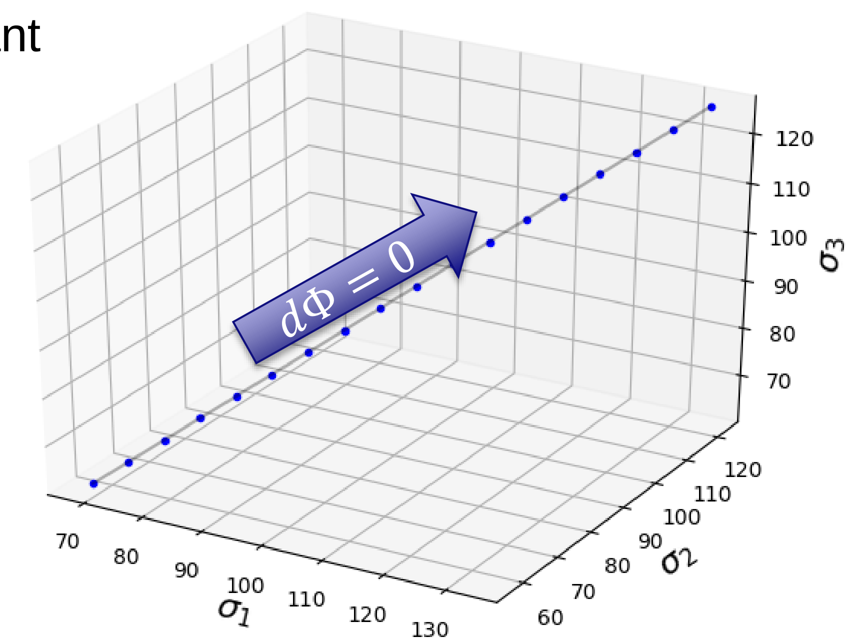
$$\sigma_1 - (\sigma_3 + \sigma_2) + \sigma_1 = constsant$$



- **Issue:** all loading cases are parallel!



$\Phi(x, y) = y$ = constant

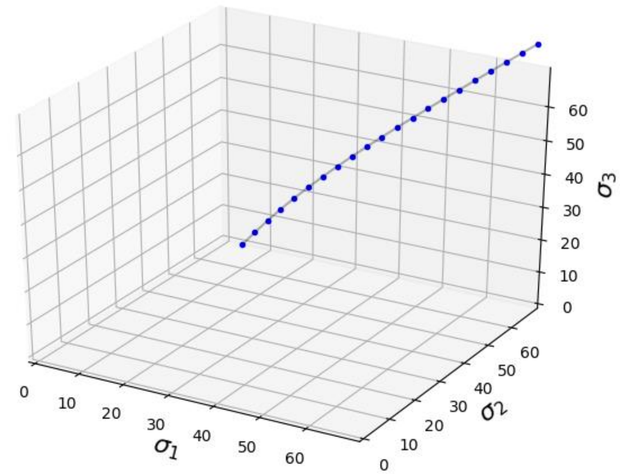


- Solution: more complex loading cases

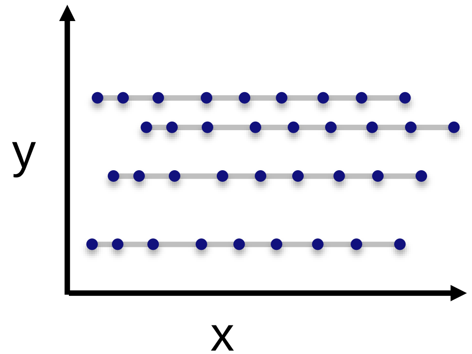
Early results

- Looking for yield surface: $\Phi(\sigma) = 0$
- $\Phi(\sigma) = \text{constant}$ for each loading case

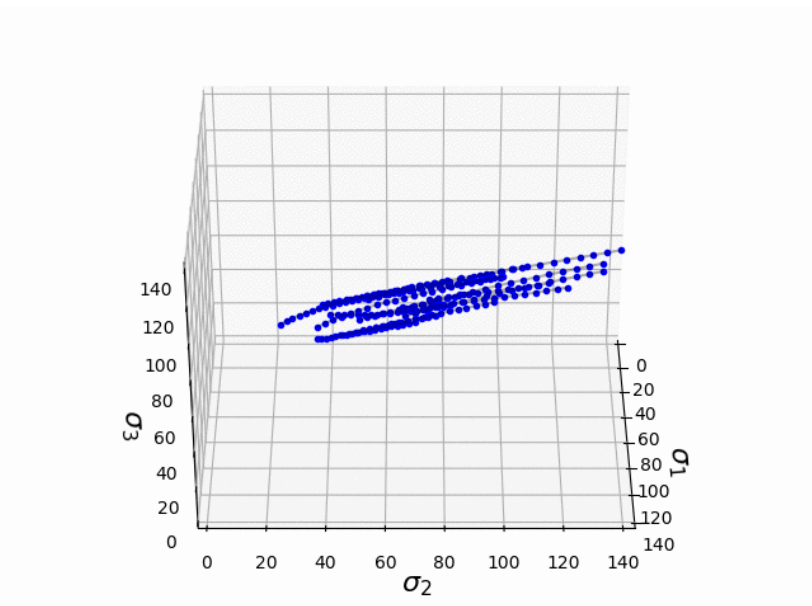
$$\sigma_1 - (\sigma_3 + \sigma_2) + \sigma_1 = constsant$$



- **Issue:** all loading cases are parallel!



$\Phi(x, y) = y = \text{constant}$



- Solution: more complex loading cases

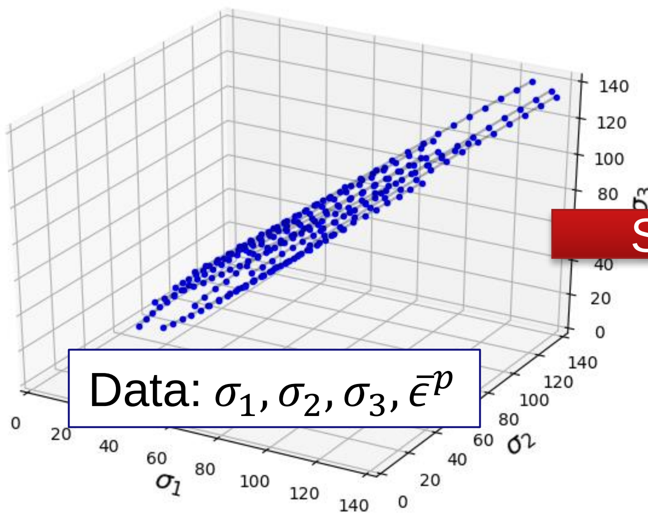
Stage 1

$Loading\ ratio$: [0.675   1.0   0.825]

Stage 2

$Loading\ ratio$: [1.0   0.825   0.675]

Yield surface from 2 stage loading data



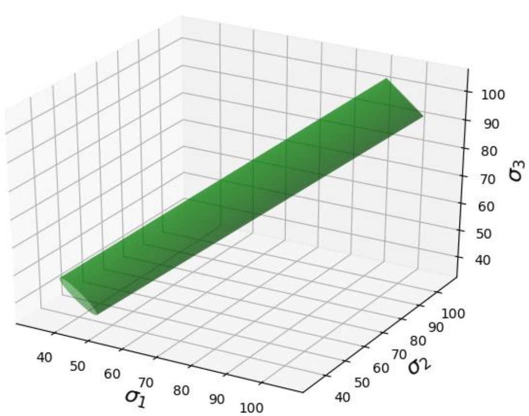
Data: $\sigma_1, \sigma_2, \sigma_3, \bar{\epsilon}^p$

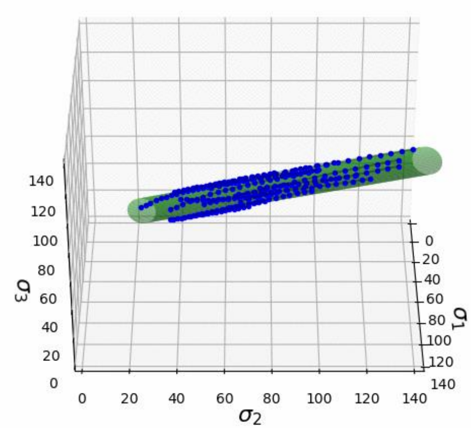
Computation time: 32s on 40 processors

SR

$$(\sigma_3 - \sigma_2)(\sigma_3 - \sigma_2) - (\sigma_2 - \sigma_1)(\sigma_3 - \sigma_1) = constant$$

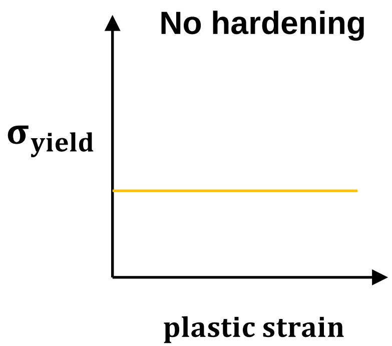
$$\Phi = (\sigma_3 - \sigma_1)^2 + (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 = constant$$

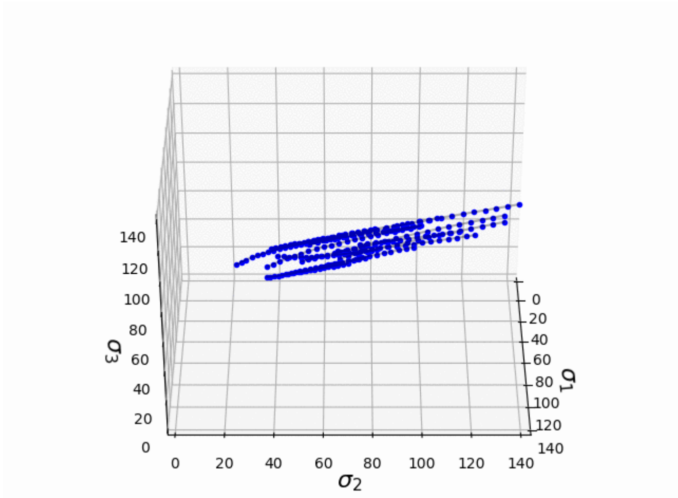


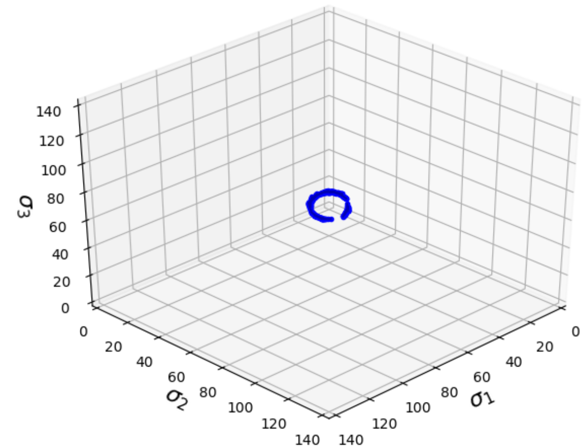


Adding hardening

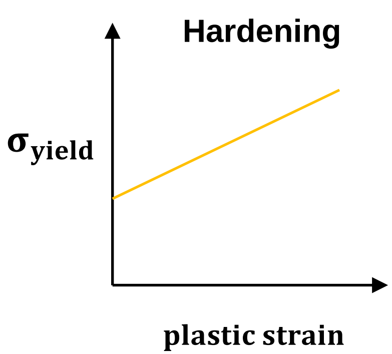
$$\Phi = (\sigma_3 - \sigma_1)^2 + (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 = constant$$

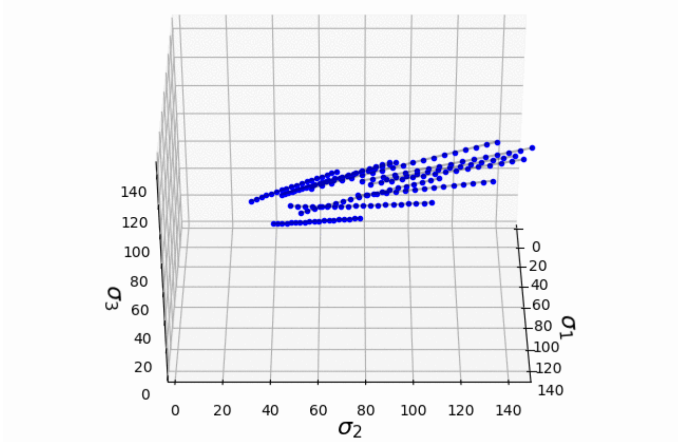


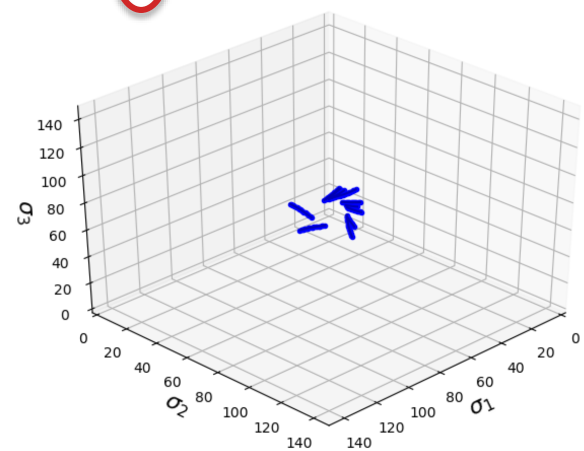




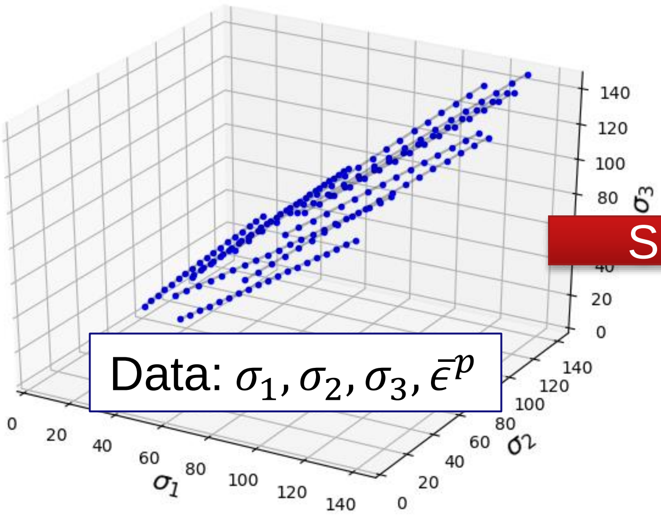
$$\Phi = (\sigma_3 - \sigma_1)^2 + (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 - c_1\bar{\epsilon}^p - c_2(\bar{\epsilon}^p)^2 = constant$$







Hardening yield surface from 2 stage loading data



SR

Computation time: 1.5h on 160 processors

$$((\sigma_1 - \sigma_3 + \sigma_3 - \sigma_2)(\sigma_3 - \sigma_2 + \sigma_1) + 47602 + \sigma_1 + (\sigma_3 - \sigma_1 - \sigma_3)(\sigma_3 + \sigma_3 -$$

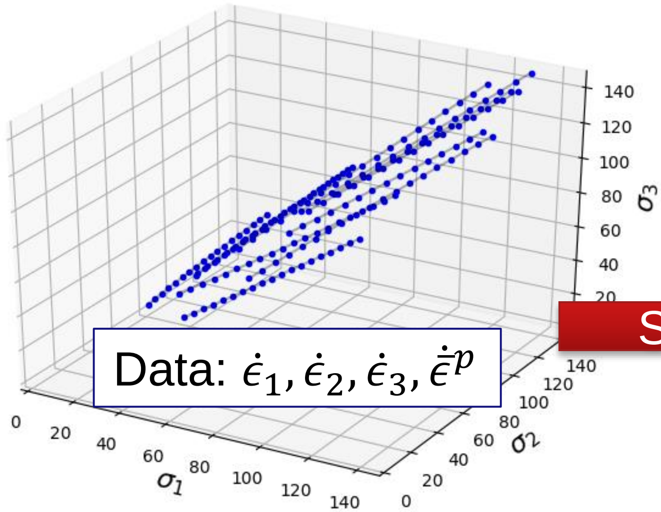
$$\Phi = (\sigma_3 - \sigma_1)^2 + (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 - 19797\bar{\epsilon}^p - 980185(\bar{\epsilon}^p)^2 = constant$$

$$\Phi = (\sigma_3 - \sigma_1)^2 + (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 - c_1\bar{\epsilon}^p - c_2(\bar{\epsilon}^p)^2 = constant$$

Yield surface now depends on a state variable!
Now it needs a state evolution equation

Hardening yield surface from 2 stage loading data

Assuming incremental elastic strains are small



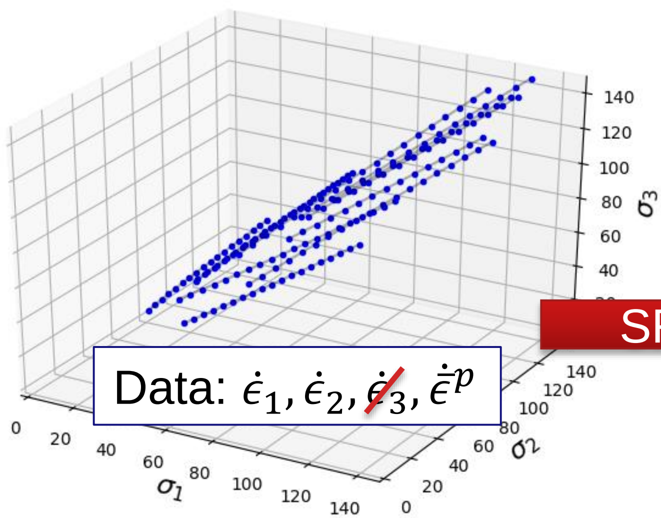
Data: $\dot{\epsilon}_1, \dot{\epsilon}_2, \dot{\epsilon}_3, \dot{\bar{\epsilon}}^p$

Goal: $\dot{\bar{\epsilon}}^p = \sqrt{\frac{2}{9}[(\dot{\epsilon}_1 - \dot{\epsilon}_2)^2 + (\dot{\epsilon}_2 - \dot{\epsilon}_3)^2 + (\dot{\epsilon}_3 - \dot{\epsilon}_1)^2]}$

SR →

$$\dot{\epsilon}_1 + \dot{\epsilon}_2 + \dot{\epsilon}_3 = \text{constant}$$

$$\dot{\epsilon}_1 + \dot{\epsilon}_2 + \dot{\epsilon}_3 = 0$$ deviatoric plastic strains



Data: $\dot{\epsilon}_1, \dot{\epsilon}_2,$ ~~$\dot{\epsilon}_3$~~$, \dot{\bar{\epsilon}}^p$

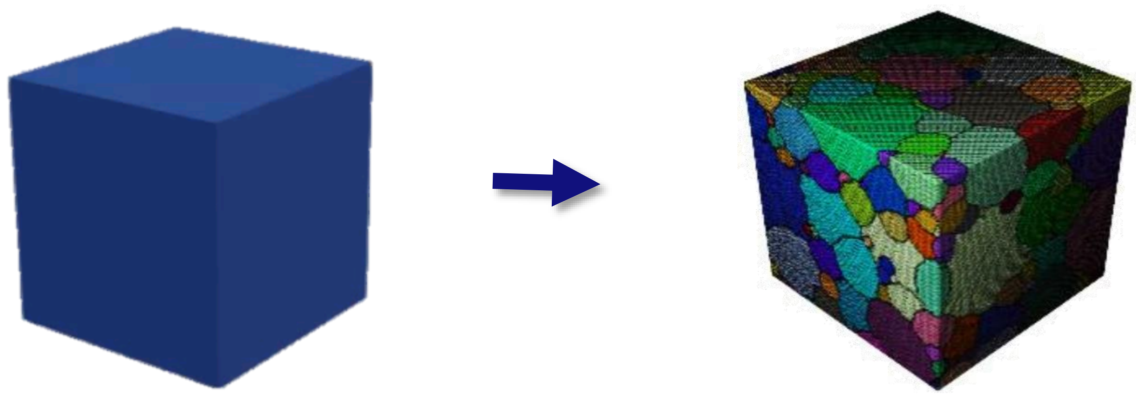
Goal: $\dot{\bar{\epsilon}}^p = \sqrt{\frac{4}{3}[\dot{\epsilon}_1^2 + \dot{\epsilon}_1\dot{\epsilon}_2 + \dot{\epsilon}_2^2]}$

SR →

$$\dot{\epsilon}_1^2 + \dot{\epsilon}_1\dot{\epsilon}_2 + \dot{\epsilon}_2^2 - (\dot{\bar{\epsilon}}^p)^2 = \text{constant}$$

Quick Recap

- Verification of von-Mises plasticity
  - 👍 Non-hardening yield surface
  - 👍 Hardening yield surface
  - 👍 State evolution
- Moving Forward:

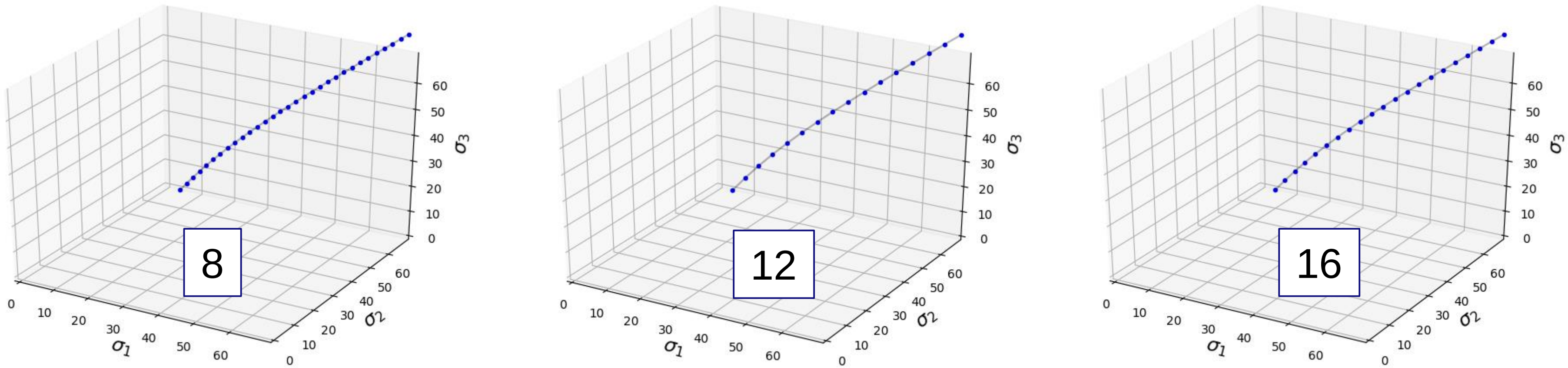


Seconds (on single processor) hours (on multiple processors)

- How much data is required?

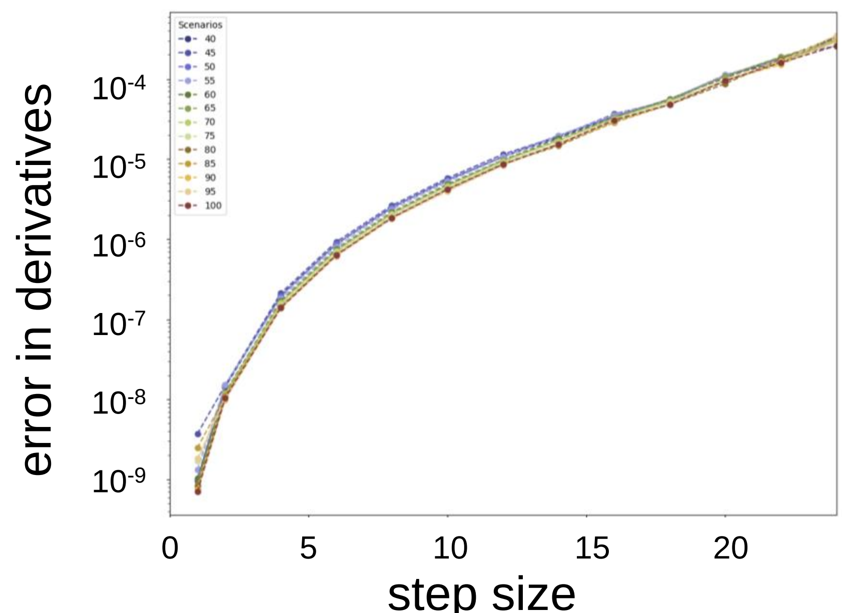
How much data is needed?

- **Step size**



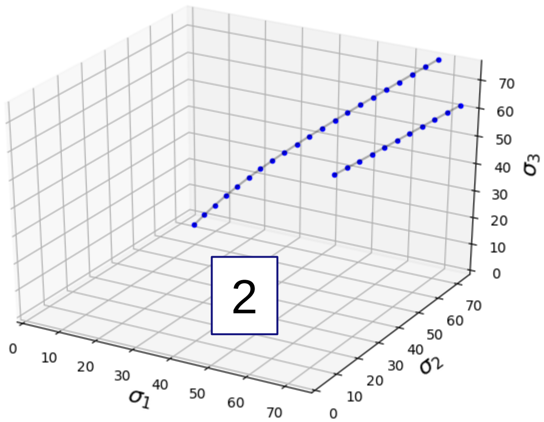
- More dense data =
  - more computation time
  - more accurate derivative calculations

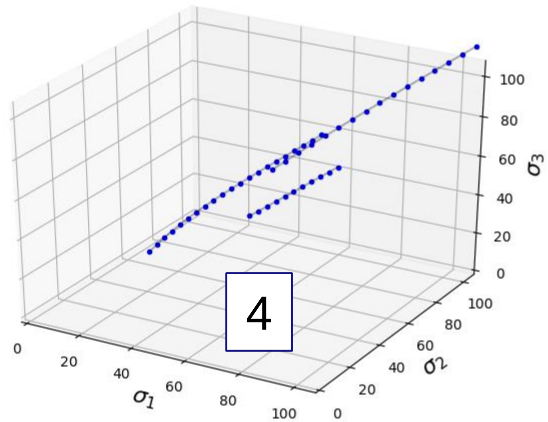
- Density needed will depend on
  - complexity of loading scenarios
  - Complexity of yield surface

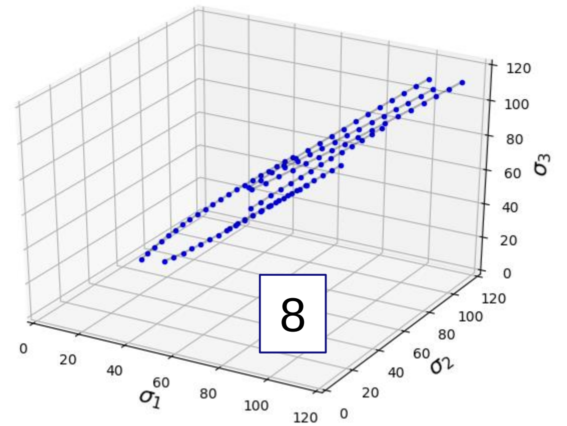


How much data is needed?

- **Number of loading scenarios**

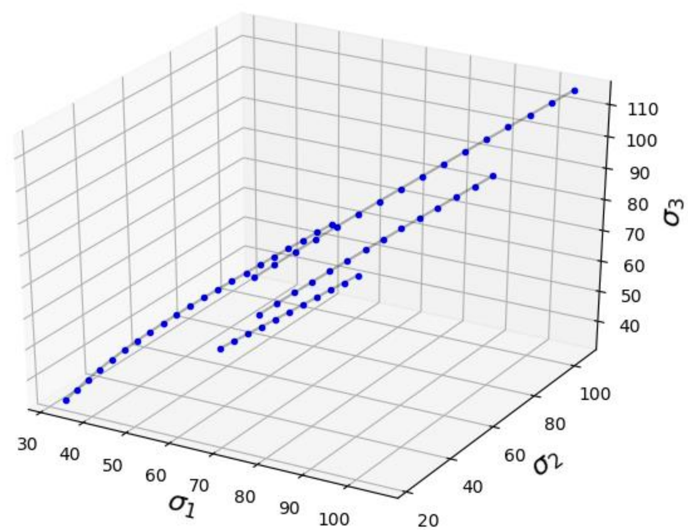






- No real trend
  (except very low values)

- Minimum case found:

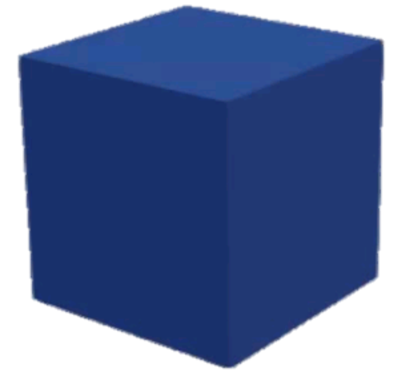


Summary

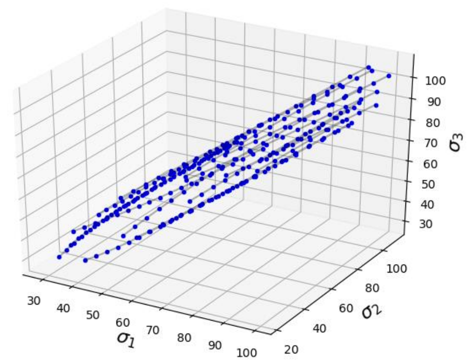
- Set up framework for SR formulation of plasticity models
- Implicit symbolic regression of yield surface
- Use non-proportional loading
- Von-Mises verification problem

- Surprisingly little data needed
  - will depend on complexity of yield surface

RVE



Data



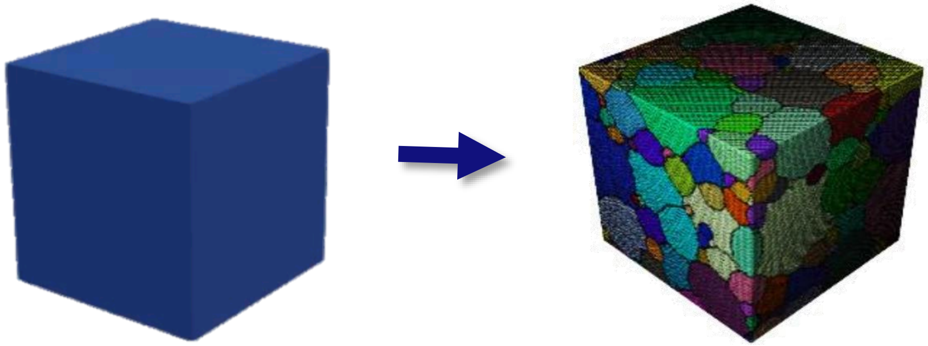
Symbolic Regression

homogenization

$$\Phi = f(\sigma)$$

Future work

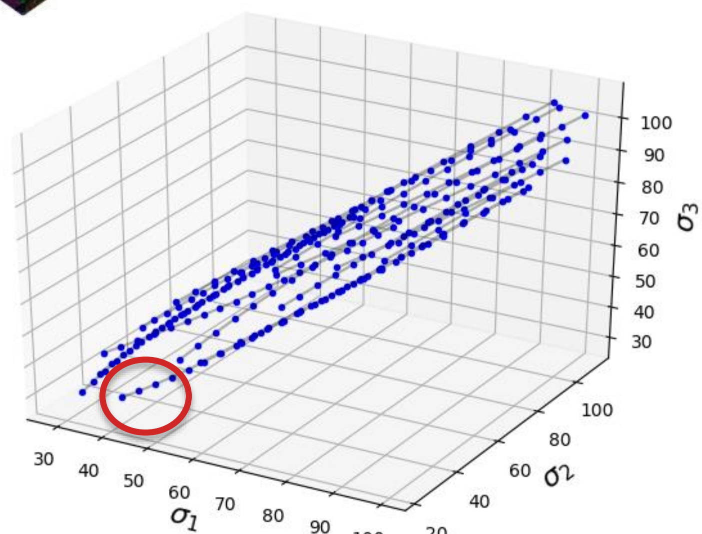
- application to real materials



- adaptive data generation
- bingo (soon to be open source)
  - python & c++
  - Features:
    - Coevolution of fitness predictors
    - Island parallelization scheme
    - Acyclic graph representation
    - Constant optimization
    - Age-fitness Pareto selection

performance (Coevolution of fitness predictors, Island parallelization scheme, Acyclic graph representation)

robustness (Constant optimization, Age-fitness Pareto selection)



thank you!

geoffrey.f.bomarito@nasa.gov