

Autonomous Real-Time Global Aerodynamic Modeling in the Frequency Domain

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A method for autonomous real-time global aerodynamic modeling is developed and explained. Local real-time frequency-domain estimates of stability and control derivatives are assembled over the flight envelope as a function of nominal flight condition, accounting for uncertainty in the individual local model parameter estimates. Combining this with real-time estimates of low-frequency aerodynamics in the time domain produces a real-time global aerodynamic model with good prediction capability, simple evaluation of local stability and control derivatives, insight into the aerodynamic dependencies, and the ability to capture local variations in the aerodynamic dependencies throughout the flight envelope. The technique is demonstrated using data from an F-16 nonlinear simulation and flight test data from a subscale aircraft.

Nomenclature

a_x, a_y, a_z	=	body-axis translational accelerometer measurements, g
b	=	wing span, ft
$\bar{c}$	=	wing mean aerodynamic chord, ft
C_X, C_Y, C_Z	=	body-axis nondimensional aerodynamic force coefficients
C_l, C_m, C_n	=	body-axis nondimensional aerodynamic moment coefficients
I_x, I_y, I_z, I_{xz}	=	inertia tensor elements, slug-ft ²
m	=	mass, slug
N	=	number of data points
p, q, r	=	body-axis roll, pitch, and yaw rates, rad/s
$\bar{q}$	=	dynamic pressure, lbf/ft ²
rms	=	root mean square
S	=	wing reference area, ft ²
V	=	true airspeed, ft/s
α	=	angle of attack, rad or deg
β	=	sideslip angle, rad or deg
$\delta_a, \delta_e, \delta_r, \delta_f$	=	aileron, elevator, rudder, and flap deflections, rad or deg

Subscripts

cg	=	center of gravity
l	=	left
o	=	nominal or reference
r	=	right

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Superscripts

T	= transpose
$\dagger$	= complex conjugate transpose
$\hat{\cdot}$	= estimate
$\dot{\cdot}$	= time derivative
$\sim$	= finite Fourier transform
-1	= matrix inverse
$-$	= mean

I. Introduction

ACCURATE global aerodynamic models for aircraft are critical for flight simulation, dynamic analysis, and flight control system development. Identifying global aerodynamic models using autonomous methods in real time onboard an aircraft, instead of post-flight processing by an analyst, introduces significant complications to an already complex and difficult task. However, many important practical applications are enabled by this capability, including rapid aircraft prototype testing, robust adaptive flight control, efficient flight testing, fault detection, reliable flight operations for unmanned air vehicles, and rapid development of flight simulators, among others.

Autonomous onboard real-time global aerodynamic modeling was first developed and demonstrated in flight during the NASA Learn-to-Fly project [1-3]. The goal was to identify a global aerodynamic model autonomously in real time to enable full-envelope real-time adaptive flight control and guidance, without ground testing or efforts from a human analyst. In that work, autonomous onboard real-time global nonlinear aerodynamic modeling was achieved using orthogonal optimized multisine inputs [4-7] to efficiently collect informative multi-axis flight data, and a recursive orthogonalization approach in the time domain for real-time global nonlinear aerodynamic model structure determination and model parameter estimation [2-3]. The Learn-to-Fly modeling algorithm built upon previous NASA research on global nonlinear aerodynamic modeling and flight test experiment design [6-12].

Prior to the NASA Learn-to-Fly project, the state-of-the-art for global aerodynamic modeling was iterative ground-based testing, Computational Fluid Dynamics (CFD) calculations, and/or numerous flight test maneuvers, along with significant efforts in post-flight analysis [13-17]. This is expensive and time-consuming. Furthermore, wind-tunnel testing has inherent fidelity limitations arising from factors such as model scale and geometry differences relative to the full-scale aircraft, wall and sting interference, test-section flow angularity, and Reynolds number differences. CFD can be inaccurate due to flow modeling deficiencies, inadequate turbulence models, computational limitations, and grid geometry approximations for both the aircraft and the flow field. Using flight test methods to generate a global aerodynamic model directly avoids all of these problems, although the typical problems associated with any flight test remain, e.g., sensor data quality, achieving adequate data information from the flight test maneuvers, flight test risk and expense, and practical constraints. Autonomous onboard real-time global aerodynamic modeling bypasses the approximations and analyst efforts inherent in ground-based methods by autonomously identifying global aerodynamic models in real time based on flight data from the real aircraft. This approach also automatically provides the requisite flight test validation for high-fidelity simulation as part of the modeling process.

In this work, a novel autonomous real-time global aerodynamic modeling method is developed, based on real-time local modeling results obtained in the frequency domain during global modeling maneuvers. The approach is different from the time-domain approach used in the Learn-to-Fly project, and can be considered as statistically combining and assembling local frequency-domain modeling results obtained in real time throughout the flight envelope. Localized variations in the global nonlinear aerodynamic dependencies can be captured, and the global modeling is a smoothed and integrated memory for the local real-time modeling results. The result is a global model for the stability and control derivatives, obtained in a straightforward and efficient way, and assembled in a convenient form for important applications such as robust adaptive flight control, flight envelope protection, flight envelope expansion, rapid prototype evaluation, flight simulation, and dynamic analysis, among others.

The next section describes the F-16 nonlinear simulation and the E1 flight test aircraft. Section III contains a description of the flight test data, including an explanation of the excitation input design, global maneuvering, and some discussion and evaluation of the flight data. Section IV describes the real-time global aerodynamic modeling technique applied in flight, based on local real-time modeling in the frequency domain. Practical aspects and modeling results are presented and discussed in Section V, followed by conclusions in Section VI.

The real-time modeling software developed and applied in this work was written in MATLAB[®]. Some of the real-time modeling software came from System IDentification Programs for AirCRAFT, or SIDPAC [7,18].

II. Aircraft

A. F-16 Nonlinear Aircraft Simulation

The F-16 is a single-seat, multi-role fighter with a blended wing / body and a cropped delta wing planform with leading edge sweep of 40 deg. Thrust is provided by one General Electric F110-GE-100 or Pratt & Whitney F100-PW-220 afterburning turbofan engine mounted in the rear fuselage. Figure 1 is a photograph of the F-16 in flight. Aircraft geometry and nominal mass properties are given in Table 1.



Figure 1. F-16 Aircraft

Credit: NASA Langley Research Center

The F-16 was modeled with controls for throttle δ_{th} , stabilator δ_s , aileron δ_a , and rudder δ_r . Speed brake and flaps were assumed fixed at zero deflection.

Nondimensional aerodynamic force and moment coefficient data were derived from a low-speed static wind tunnel test and a dynamic forced oscillation wind tunnel test [19], both conducted using a 16% scale model of the F-16. The aerodynamic database applies to the F-16 flown out of ground effect, with landing gear retracted, and no external stores. Static aerodynamic data were in tabular form as a function of angle of attack and sideslip angle over the ranges $-10^\circ \leq \alpha \leq 45^\circ$ and $-30^\circ \leq \beta \leq 30^\circ$, respectively. Dynamic data were provided in tabular form at zero sideslip angle over the angle of attack range $-10^\circ \leq \alpha \leq 45^\circ$. Dependence of the nondimensional coefficients on $\dot{\alpha}$ and $\dot{\beta}$ were included with the q and r dependencies, respectively, because of the manner in which the data were collected in the wind tunnel.

The engine model was based on ground test data in tabular form, with thrust given as a function of altitude, Mach number, and engine power level. Engine power level was computed from throttle position and throttle gearing, including first-order lag dynamics for the thrust response.

The F-16 nonlinear simulation was programmed completely in MATLAB[®]. Full nonlinear equations of motion, including turbine engine gyroscopic effects, were used. Complete details on the F-16 nonlinear simulation can be found in Appendix D of Ref. [7].

B. E1 Aircraft

A subscale aircraft designated E1 was used for flight testing. The E1 aircraft is a commercially-available 40% scale Extra 330 SC remotely-piloted fixed-wing airplane, shown in Fig. 2. E1 is powered by an electric motor driving a fixed-pitch tractor propeller. Cruising speed is approximately 50 kts. Control surfaces are conventional ailerons and trailing-edge flaps on the wings, along with a conventional rudder and a split elevator. Trailing-edge down is positive deflection for all control surfaces except the rudder, for which trailing-edge left is positive

Table 1. Aircraft Geometry and Mass Properties

	F-16	E1
$\bar{c}$, ft	11.32	1.97
b , ft	30	10.17
S , ft ²	300	19.26
x_o , ft	$0.35 \bar{c}$	3.012
y_o , ft	0	0.000
z_o , ft	0	0.000
x_{cg} , ft	$0.25 \bar{c}$	3.002
y_{cg} , ft	0	0.042
z_{cg} , ft	0	-0.248
m , slugs	637.16	1.906
I_x , slugs-ft ²	9,496	2.959
I_y , slugs-ft ²	55,814	8.682
I_z , slugs-ft ²	63,100	11.617
I_{xz} , slugs-ft ²	982	0.750

deflection. Aircraft geometry and mass properties are given in Table 1. More information on the E1 aircraft can be found in Refs. [1] and [20].

E1 is equipped with an INS, which provides 3-axis translational accelerometer measurements, angular rate measurements, Euler attitude angles, and GPS position and velocity. Air flow angle vanes attached to a boom mounted on the right wingtip (visible in Fig. 1) measured angle of attack and sideslip angle. Accelerometer and air flow angle data were corrected to the aircraft center of gravity in real time. A pitot tube mounted on the left wing was used to measure static and dynamic pressure. Onboard static pressure and ambient temperature measurements were used to compute air density and altitude. Propeller speed was measured using a Hall-effect sensor. Mass properties were determined from ground-based mass and inertia measurements. Control surface deflections were measured with potentiometers mounted on each control surface servo shaft. Pilot radio-control stick, rudder, and throttle commands were also measured and recorded. Flight data were recorded onboard the aircraft at a sampling rate of 50 Hz.



Figure 2. E1 aircraft in flight

Credit : NASA Langley Research Center

A research pilot executed take-offs and landings using conventional radio control. Flight test maneuvers were initiated by the pilot flying the aircraft to a desired initial flight condition, then activating components of the onboard autonomous real-time modeling, guidance, and control algorithms, using switches on the radio control box. The maneuvers were terminated by the pilot switching back to conventional radio control and flying the airplane manually.

The flight computer on the E1 aircraft had the capability to inject automated control surface perturbations, called Programmed Test Inputs (PTI), to excite the aircraft dynamic response and thereby generate informative flight data for real-time modeling. Unique PTI were applied to each control surface simultaneously, by summing the PTI with control commands from the pilot and/or feedback control, just before the limiting on actuator command rates and positions. The flight data used in this work were collected with the autonomous onboard real-time modeling algorithm operating, PTI active, and the radio-control pilot flying the aircraft through global modeling maneuvers. Details of the PTI design and global modeling maneuvers are provided in the next section. Other flight tests were conducted with onboard automatic guidance and control algorithms operating simultaneously with the onboard real-time modeling [21-22], in which the radio-control pilot inputs were not used.

III. Flight Test Data

To enable accurate identification of the aircraft global aerodynamics, flight data must cover a wide range of the explanatory variables, with sufficient information content and signal-to-noise ratios, as well as low correlations among the explanatory variables. Explanatory variables are quantities such as angle of attack, sideslip angle, body-axis angular rates, and control surface deflections. The explanatory variables are used in the modeling to characterize the aircraft aerodynamics, commonly expressed as nondimensional aerodynamic force and moment coefficients. Informative flight data for all rigid-body degrees of freedom must be collected rapidly because real-time models are needed quickly for purposes such as autonomous control and guidance. Prior flight research [2-8] has demonstrated that applying automated orthogonal optimized multisine perturbation inputs to the control surfaces during transits through a range of flight conditions is an excellent and efficient method for collecting global aerodynamic modeling data with good information content. Automated orthogonal optimized multisine perturbation inputs were applied to all control surfaces simultaneously for this work. These inputs were the PTI for both the F-16 nonlinear simulation and the E1 flight tests. Detailed description of the design procedure for orthogonal optimized multisine perturbation inputs can be found in Refs. [2-7,23]. A brief summary is given next. SIDPAC program mkmswp.m [7,18] was used to design the PTI used in this work.

A. Orthogonal Optimized Multisine Inputs

The general idea is to excite the aircraft using perturbation inputs with wideband frequency content encompassing the expected modal frequencies of the aircraft dynamic response. The excitations are implemented as perturbations to the control surface deflections by summing designed perturbation inputs with the actuator commands from the pilot and/or feedback control, just before the actuator limiting on command rate and position.

This implementation is important for achieving the required excitation and decorrelation in the explanatory variables.

Each designed perturbation input is a sum of sinusoids with unique harmonic frequencies, optimized phase shifts, and specified power distribution. The harmonic frequencies are selected to cover a frequency band of interest. The wideband frequency content of the inputs is important because there is naturally some uncertainty as to what the modal frequencies are for the aircraft in flight, and the wideband inputs provide robustness to that uncertainty. Phase shifts for the sinusoidal components of each input are optimized to achieve low peak-to-peak amplitude for the sum of sinusoids used for each input. Amplitudes of the individual sinusoidal components for each input can be selected to achieve a specific power distribution with frequency.

Multiple inputs are designed to be mutually orthogonal in both the time domain and the frequency domain simultaneously, and are designed for high data information content in all axes in a short time period, while minimizing excursions from the nominal flight condition. The mutual orthogonality of the inputs allows simultaneous application of multiple inputs, which reduces the required excitation time for a given amount of input energy, or equivalently, increases the amount of input energy injected into the dynamic system over a given time period. Even more important for this work, the inputs provide continuous, effective, multi-axis excitation as the aircraft flies through various flight conditions.

Each perturbation input $\mathbf{u}_j$, which is to be applied to the j th individual control surface, is a sum of harmonic sinusoids with individual phase shifts ϕ_k ,

$$\mathbf{u}_j = \sum_{k \in \{1, 2, \dots, M\}} A_k \sin\left(\frac{2\pi k \mathbf{t}}{T} + \phi_k\right) \quad j = 1, 2, \dots, n_i \quad (1)$$

where M is the total number of available harmonic frequencies, T is the time length of the excitation, A_k is the amplitude for the k th sinusoidal component, and $\mathbf{t}$ is the time vector. Each of the n_i inputs is the sum of selected components from the pool of M harmonic sinusoids with frequencies $\omega_k = 2\pi k/T$, $k = 1, 2, \dots, M$, and $\omega_M = 2\pi M/T$ represents the upper limit of the frequency band for the excitation inputs. The interval $[\omega_1, \omega_M]$ rad/s specifies the range of frequencies where the aircraft dynamics are expected to lie.

The mutual orthogonality of the PTI in the time domain comes from the fact that each input is composed of harmonic sinusoids with the same base period T , but unique harmonic frequencies. Orthogonality in the frequency domain comes from using unique frequencies for the component sinusoids in each multisine input.

If the phase angles ϕ_k in Eq. (1) were chosen at random on the interval $(-\pi, \pi]$ rad, then in general, the various harmonic components would add together at some points to produce a multisine input $\mathbf{u}_j$ with relatively large amplitude excursions. This is undesirable because such inputs can move the dynamic system too far from the reference condition for the test. To prevent this, the phase angles ϕ_k for each of the selected harmonic components are optimized to minimize the relative peak factor RPF , defined by

$$RPF(\mathbf{u}_j) = \frac{[\max(\mathbf{u}_j) - \min(\mathbf{u}_j)]/2}{\sqrt{2(\mathbf{u}_j^T \mathbf{u}_j)/N}} = \frac{[\max(\mathbf{u}_j) - \min(\mathbf{u}_j)]}{2\sqrt{2} \text{rms}(\mathbf{u}_j)} \quad j = 1, 2, \dots, n_i \quad (2)$$

Relative peak factor is a measure of the efficiency of an input for dynamic modeling purposes, in terms of the amplitude of the input divided by a measure of the input energy. Low values of relative peak factors are desirable for highly efficient and effective aerodynamic modeling because the objective is to excite the dynamic system with good input energy over a variety of frequencies while minimizing the input amplitudes in the time domain, to avoid driving the dynamic system too far away from the reference condition. For each multisine input $\mathbf{u}_j$ in Eq. (1), minimum RPF is achieved by adjusting the phase angles ϕ_k for each individual sinusoidal component of the input. The resulting optimization problem is non-convex; however, a simplex algorithm can be applied to find a solution [4-7].

The integers k specifying the harmonic frequencies for the j th input $\mathbf{u}_j$ are selected to be unique to that input, but are not necessarily consecutive. A good approach for multiple inputs is to assign integers k to each input alternately. This is illustrated in Fig. 3 for PTI design on the E1 aircraft. There are 6 inputs in this case: elevator, rudder, and ailerons and flaps on the left and right wings. The harmonic frequencies were interleaved among these six inputs to achieve wideband frequency content in each input, to provide robustness in the excitation and to enable accurate individual control surface effectiveness estimates. Because each input has wideband frequency content, the same input design can be applied at various flight conditions, which simplifies the excitation strategy and reduces flight computer memory requirements. Figure 4 shows time series for the PTI designed using the frequency content depicted in Fig. 3.

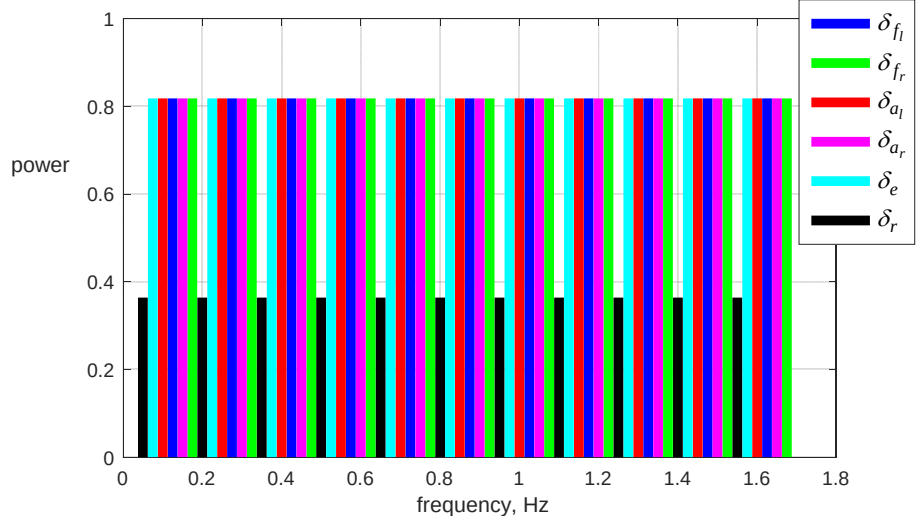


Figure 3. E1 orthogonal optimized multisine input power spectra

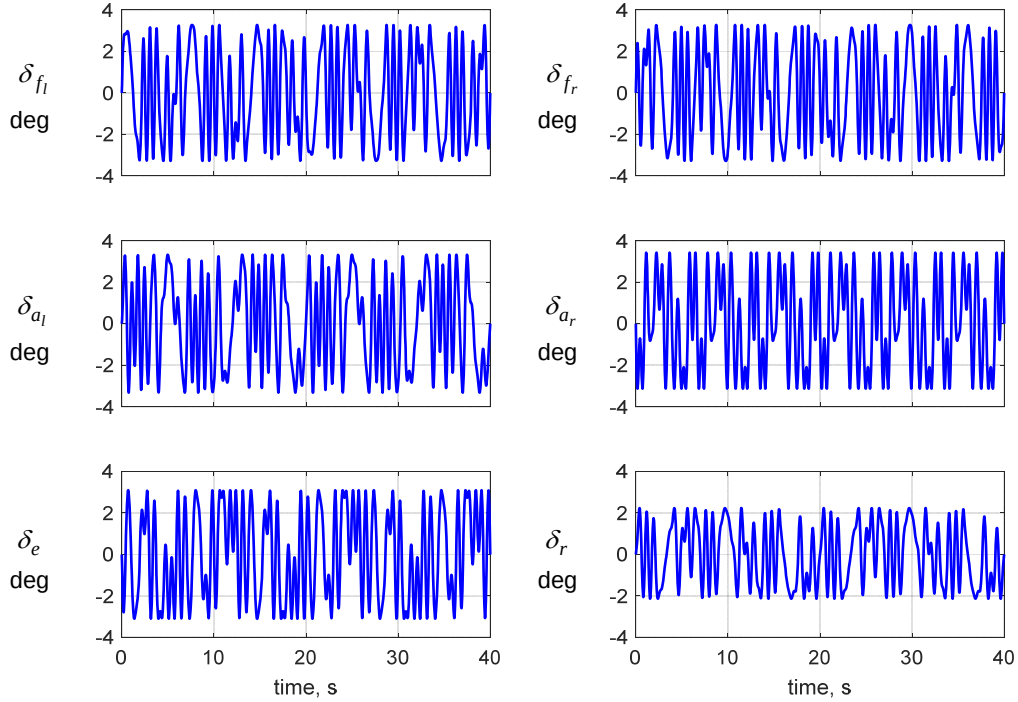


Figure 4. E1 orthogonal optimized multisine inputs

To achieve a uniform power distribution, the A_k are selected as

$$A_k = \frac{A}{\sqrt{n}} \quad (3)$$

where n is the number of sinusoidal components included in the summation of Eq. (1) for $\mathbf{u}_j$, and A is the amplitude of the multisine input $\mathbf{u}_j$. With uniform power distribution, selection of the A_k reduces to selecting a single value for the input amplitude A . Each input $\mathbf{u}_j$ can have arbitrary amplitude A , subject to practical flight test and modeling constraints. The different heights of the bars in Fig. 3 result from the different amplitudes used for the PTI applied to each individual control surface, which reflect different control effectiveness.

The PTI design shown in Fig. 4 was used for the E1 flight testing, with only input amplitude adjustments made after the first flight, to achieve acceptable signal-to-noise ratios for the measured aircraft responses. This adjustment was necessary because there was no prior information for either the measurement noise levels or the control surface effectiveness. Because the PTI are sums of harmonic sinusoids with a common base period T , they are periodic for the excitation period T , so that the PTI can be applied repeatedly without any discontinuities in magnitude or slope. The PTI design has minimum *RPF* with various frequencies and phase angles, which keeps the aircraft response close to the nominal flight condition and produces responses similar to flight in light to moderate turbulence. The aircraft response also stays near the nominal flight condition because each perturbation input is a sum of harmonic sinusoids, which are all balanced about zero (equal area above and below zero). The result is rich dynamic multi-axis response about the nominal flight condition. In practice, pilot inputs and/or automated guidance and control act to spoil the perfect orthogonality (zero pairwise correlations) of the PTI. However, good modeling results require only low correlations, not zero correlations, so that the slightly correlated inputs that result from applying orthogonal PTI with pilot inputs and/or automated guidance and feedback control active work very well in practice.

B. Global Modeling Maneuvers

The PTI described in the preceding subsection were used to provide multi-axis flight data with high information content and low correlations among the explanatory variables over a large range of flight conditions, as required for accurate and efficient global aerodynamic modeling. The multiple-axis orthogonal input design is important because the aircraft dynamics can be excited rapidly and effectively in all axes simultaneously, to collect informative data quickly and thereby enable accurate real-modeling as soon as possible. The same approach was used in the NASA Learn-to-Fly project [2,3], for the same reasons.

Figure 5 shows cross plots of selected explanatory variable data from E1 flight 17. These plots demonstrate that a wide range of the explanatory variables was swept through during the E1 flight tests with PTI active. Note that the cross plots generally do not resemble straight lines, which means that the explanatory variable data had low pairwise correlations. Low correlations mean that the dependencies of the aircraft aerodynamics on the explanatory variables can be identified accurately and without ambiguity. Cross plots for other pairs of explanatory variables were similar.

The E1 flight data shown in Fig. 5 were generated using a sequence of global maneuvers performed by the pilot, which were intended to cover a wide range of nominal flight conditions within the aircraft flight envelope. These maneuvers were flown with the PTI active, so that the task for the pilot was to fly the aircraft slowly through nominal flight conditions, allowing the PTI to efficiently fill in the explanatory variable space with informative dynamic data. Global modeling is most accurate when the explanatory variable space is covered as completely as possible, to avoid effects such as low information content and functional aliasing [3],[7]. The objective of the global modeling maneuvers from the pilot's perspective was to fly slowly through nominal flight conditions with PTI active, covering as much of the flight envelope as possible, so that the aircraft dynamic response from the PTI would fill in the explanatory space with data points as completely as possible. The cross-plots in Fig. 5 show that this approach was very effective.

The global maneuvers flown by the pilot with the PTI active were figure-eight patterns and slow approaches to stall and recovery. The pilot was flying the aircraft from the ground using conventional radio control, so it was not possible to fly precision maneuvers. However, precision maneuvering was not required; only maneuvers that traversed nominal flight conditions were required, and the nominal flight conditions could be flown approximately and spaced loosely because of the wide swath of the explanatory variable data coverage provided by the PTI. In effect, the active PTI reduced the pilot workload to only flying the aircraft slowly through nominal flight conditions. This approach has also been used very effectively in past flight test programs [2-3],[6-8],[24].

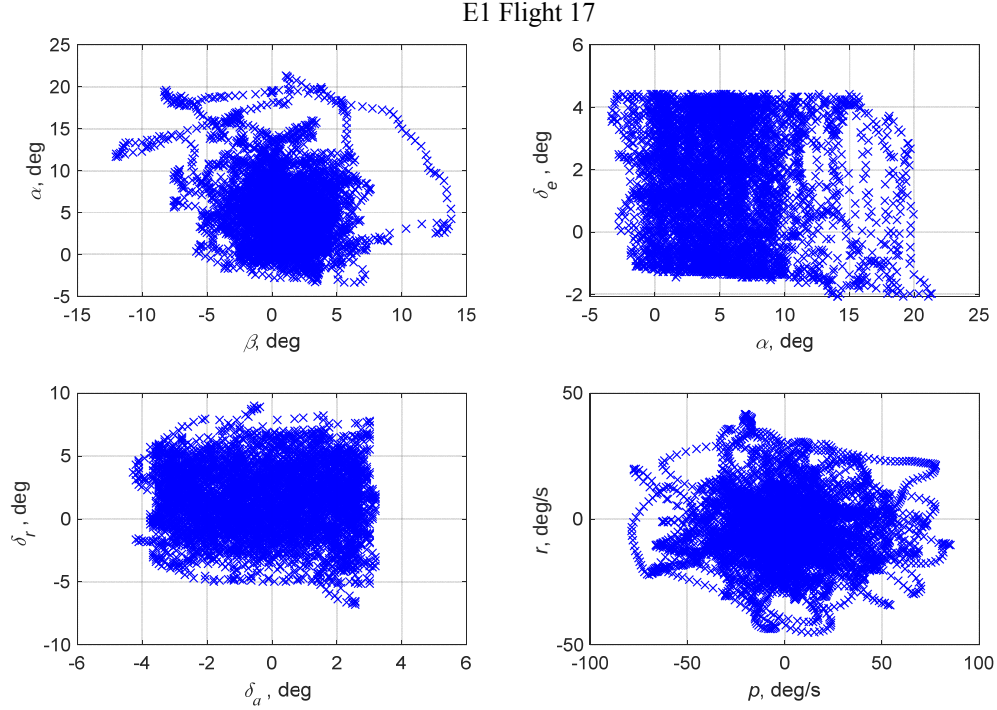


Figure 5. E1 flight data cross plots

Figure 6 shows E1 flight data from a level figure-eight maneuver, turning first to the right, then to the left. During the level turns, the nominal angle of attack elevates slightly, and the nominal sideslip drifts toward the inside of the turn because the turn cannot be perfectly coordinated by the pilot on the ground with PTI active. These slow changes in nominal angle of attack and nominal sideslip angle with PTI active help to fill in the explanatory variable space with data, which is the main objective of the global maneuvers.

Figure 7 shows another type of global maneuver, called a slow approach to stall and recovery. In this case, the objective is to cover a wide range of nominal angle of attack while the PTI move the explanatory variables in an uncorrelated manner.

For the flight test data used in this work, two level figure-eight maneuvers were flown first, followed immediately by a slow approach to stall and recovery. This sequence was often repeated. Flight data for one such sequence of global maneuvers was used in Fig. 5. This global maneuver sequence is very effective and efficient for collecting global modeling data because the simultaneous multi-axis excitations from the automated PTI provide rich data information for global modeling in all six rigid-body degrees of freedom for the nominal flight conditions traversed during the global maneuvers.

For E1, the propeller advance ratio was the only explanatory variable that was not changed efficiently and in an uncorrelated manner by the automated PTI during the global maneuvers. In principle, this could have been done, but in practice it was simpler and safer to have the pilot manually implement random step changes to the throttle during the global maneuvers. This approach decorrelated changes in the propeller advance ratio from the changes made by the automated PTI, which were acting continuously and mainly in a higher frequency range. In addition, the pilot had more control and awareness of the aircraft energy state as a result of manually implementing the random throttle changes. Only occasional random step changes to the throttle were required every 5-10 seconds at the pilot's discretion, because the engine/propeller dynamic response was generally slower and less complex than the aircraft dynamic response. For some of the level figure-eight maneuvers, the pilot was instructed to implement a random step change in throttle at the midpoint, for simplicity and safety in the maneuver execution, and so that the power level would be roughly constant during each loop of the figure-eight maneuver. During the slow approach to stall and recovery, the magnitude and timing of the random step changes to the throttle were at the pilot's discretion.

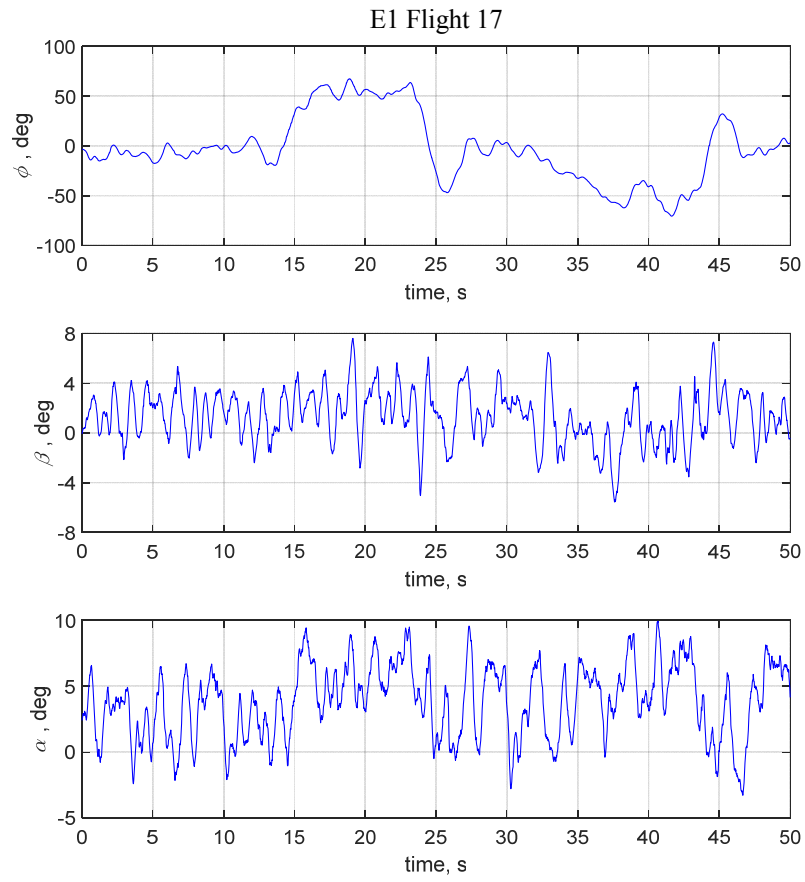


Figure 6. E1 level figure-eight maneuver

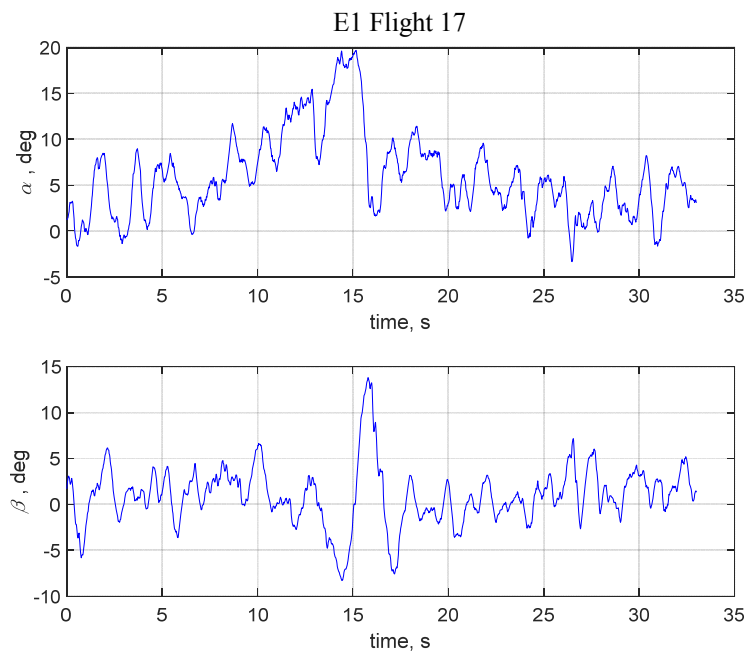


Figure 7. E1 slow approach to stall and recovery maneuver

IV. Real-Time Global Aerodynamic Modeling in the Frequency Domain

The overall modeling objective is to autonomously identify a global nonlinear model in real time for each nondimensional aerodynamic force and moment coefficient as a function of the explanatory variables. There are two main difficulties in doing this: 1) designing an experiment to collect informative dynamic data rapidly, which was addressed in Section III, and 2) identifying an accurate global nonlinear model in real time, which is the subject of this section.

Global aerodynamic modeling involves identifying (possibly complex) connections between the explanatory variables (such as angle of attack and control surface deflections) and the quantities to be predicted (such as nondimensional pitching moment coefficient), often called the dependent variables. An effective approach to identifying such connections from flight data is to evaluate candidate linear and nonlinear functions of the explanatory variables for effectiveness in accurately characterizing the dependent variable, using real-time data transformations and statistical criteria in the time domain. This real-time global modeling approach was developed and flown successfully in the NASA Learn-to-Fly project [2-3], with all computations done in real time by the onboard flight computer, to rapidly identify global nonlinear aerodynamic models over large ranges of the explanatory variables.

In the present work, an alternate global modeling method was developed, based on real-time local modeling in the frequency domain. The general idea is demonstrated in Fig. 8, which shows global nonlinear dependence of aerodynamic lift coefficient C_L on angle of attack α for the F-16 aircraft, based on wind tunnel data used in the F-16 nonlinear simulation described earlier. The solid line shows the data, and the short dashed lines show local linear approximations of the global nonlinear dependence. Used collectively, the local linear approximations provide an accurate characterization of the global nonlinear dependence of C_L on α .

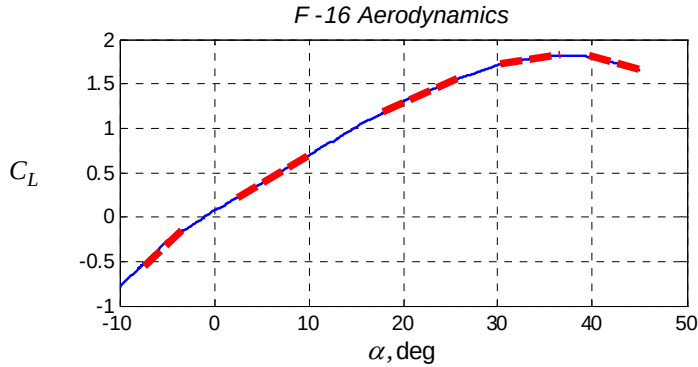


Figure 8. Global nonlinear modeling approach using local linear models

In practice, the dependencies in the data are multivariate and generally nonlinear, so that the local linear approximations are hyperplanes in multiple explanatory variables instead of straight lines. However, the basic concept is the same.

The next subsections describe a frequency-domain method for identifying global aerodynamic models for the nondimensional aerodynamic force and moment coefficients. The method assembles real-time local modeling results in the frequency domain, combined with a low-frequency model identified in the time domain, to produce an accurate global aerodynamic model.

A. Real-Time Modeling

For global aerodynamic modeling from flight data, nondimensional aerodynamic force and moment coefficients were used as the dependent variables. A separate modeling problem was solved for each nondimensional force or moment coefficient, corresponding to minimizing the squared equation error in each individual equation of motion for the six rigid-body degrees of freedom of the aircraft [7]. Values for the nondimensional aerodynamic force and moment coefficients cannot be measured directly in flight, but instead must be computed from measured and known quantities using the following equations [7]:

$$C_X = m a_x / (\bar{q} S) \quad (4a)$$

$$C_Y = m a_y / (\bar{q} S) \quad (4b)$$

$$C_Z = m a_z / (\bar{q} S) \quad (4c)$$

$$C_l = \frac{I_{xx}}{\bar{q} S b} \left[\dot{p} - \frac{I_{xz}}{I_{xx}} (pq + \dot{r}) + \frac{(I_{zz} - I_{yy})}{I_{xx}} qr \right] \quad (5a)$$

$$C_m = \frac{I_{yy}}{\bar{q} S \bar{c}} \left[\dot{q} + \frac{(I_{xx} - I_{zz})}{I_{yy}} pr + \frac{I_{xz}}{I_{yy}} (p^2 - r^2) \right] \quad (5b)$$

$$C_n = \frac{I_{zz}}{\bar{q} S b} \left[\dot{r} - \frac{I_{xz}}{I_{zz}} (\dot{p} - qr) + \frac{(I_{yy} - I_{xx})}{I_{zz}} pq \right] \quad (5c)$$

These expressions retain the full rigid-body nonlinear dynamics in the aircraft equations of motion. Because the E1 aircraft had no separate method for calculating thrust from the propeller, the a_x measurement, and therefore C_X calculated from Eq. (4a), included both thrust and aerodynamic effects.

Equations (4)-(5) were used to compute values of the nondimensional force and moment coefficients in real time. Such data are often called measured force and moment coefficient data, even though the data are not measured directly, but rather computed from other measurements and known quantities. Real-time data for explanatory variables such as air flow angles, body-axis angular rates, and control surface deflections came from sensor measurements.

Because angular accelerations $(\dot{p}, \dot{q}, \dot{r})$ were not measured, a smooth local differentiation method [2,3],[7] was applied to the measured angular rate data to compute angular accelerations in real time. The method used both past and future samples of the angular rates relative to the data point being smoothed and differentiated, causing a delay of two time samples in the real-time results. For 50 Hz data, this amounted to a 0.04 s delay. The same delay was applied uniformly to all of the data used for real-time modeling, in order to maintain time synchronization of the data. This small 0.04 s delay did not impact the real-time results in any significant way because real-time model dependencies changed at a much slower rate, due to the relatively slow changes in flight condition and the gradual expansion of explanatory variable coverage as the flight test progressed.

The global aerodynamic model is a mathematical model structure with estimated model parameter values and associated uncertainties that relates the nondimensional aerodynamic force and moment coefficients to the explanatory variables. For this work, the model structure was fixed and linear, with model parameters that vary with the nominal flight condition. Assuming that the nominal flight condition can be characterized by nominal angle of attack α_o , the model structures for the nondimensional aerodynamic force and moment coefficients were:

$$C_X = C_{X_\alpha}(\alpha_o) \Delta\alpha + C_{X_q}(\alpha_o) \Delta\hat{q} + C_{X_{\delta_e}}(\alpha_o) \Delta\delta_e + C_{X_o}(\alpha_o) \quad (6a)$$

$$C_Y = C_{Y_\beta}(\alpha_o) \Delta\beta + C_{Y_p}(\alpha_o) \Delta\hat{p} + C_{Y_r}(\alpha_o) \Delta\hat{r} + C_{Y_{\delta_a}}(\alpha_o) \Delta\delta_a + C_{Y_{\delta_r}}(\alpha_o) \Delta\delta_r + C_{Y_o}(\alpha_o) \quad (6b)$$

$$C_Z = C_{Z_\alpha}(\alpha_o) \Delta\alpha + C_{Z_q}(\alpha_o) \Delta\hat{q} + C_{Z_{\delta_e}}(\alpha_o) \Delta\delta_e + C_{Z_o}(\alpha_o) \quad (6c)$$

$$C_l = C_{l_\beta}(\alpha_o) \Delta\beta + C_{l_p}(\alpha_o) \Delta\hat{p} + C_{l_r}(\alpha_o) \Delta\hat{r} + C_{l_{\delta_a}}(\alpha_o) \Delta\delta_a + C_{l_{\delta_r}}(\alpha_o) \Delta\delta_r + C_{l_o}(\alpha_o) \quad (7a)$$

$$C_m = C_{m_\alpha}(\alpha_o) \Delta\alpha + C_{m_q}(\alpha_o) \Delta\hat{q} + C_{m_{\delta_e}}(\alpha_o) \Delta\delta_e + C_{m_o}(\alpha_o) \quad (7b)$$

$$C_n = C_{n_\beta}(\alpha_o)\Delta\beta + C_{n_p}(\alpha_o)\Delta\hat{p} + C_{n_r}(\alpha_o)\Delta\hat{r} + C_{n_{\delta_a}}(\alpha_o)\Delta\delta_a + C_{n_{\delta_r}}(\alpha_o)\Delta\delta_r + C_{n_o}(\alpha_o) \quad (7c)$$

where $\hat{p}$, $\hat{q}$, and $\hat{r}$ are nondimensional angular rates defined by

$$\hat{p} \equiv pb/(2V) \quad \hat{q} \equiv q\bar{c}/(2V) \quad \hat{r} \equiv rb/(2V) \quad (8)$$

In Eqs. (6)-(7), the model parameters, such as C_{m_α} and $C_{n_{\delta_r}}$, are a function of nominal angle of attack α_o , but this dependency could be extended to include other descriptors for the nominal flight condition. The Δ notation indicates a perturbation quantity relative to a reference condition. The models in Eqs. (6)-(7) constitute a global nonlinear aerodynamic model that could be described as a local linear hyperplane model that varies with the nominal flight condition, defined in this case by nominal angle of attack. Another interpretation is a local linear multivariate Taylor series in the explanatory variables with model parameters that depend on the nominal flight condition.

B. Real-Time Frequency-Domain Data

Local real-time modeling in the frequency domain requires transforming sampled time-domain data into the frequency domain. The analytical tool used for this task is the finite Fourier transform. For an arbitrary scalar signal $x(t)$ on the time interval $[0, T]$, the finite Fourier transform is defined by

$$\mathcal{F}[x(t)] \equiv \tilde{x}(\omega) \equiv \int_0^T x(t) e^{-j\omega t} dt \quad (9a)$$

which can be approximated by

$$\tilde{x}(\omega) \approx \Delta t \sum_{i=0}^{N-1} x(i) e^{-j\omega i \Delta t} \quad (9b)$$

where $x(i) \equiv x(i\Delta t)$ and Δt is a constant sampling interval. The summation in Eq. (9b) is defined as the discrete Fourier transform,

$$X(\omega) \equiv \sum_{i=0}^{N-1} x(i) e^{-j\omega i \Delta t} \quad (10)$$

so that the finite Fourier transform approximation in Eq. (9b) can be written as

$$\tilde{x}(\omega) \approx X(\omega) \Delta t \quad (11)$$

1. Recursive Discrete Fourier Transform

For real-time modeling, the discrete Fourier transform can be computed recursively as follows. For any given frequency ω , the discrete Fourier transform at sample time $i\Delta t$ is related to the discrete Fourier transform at time $(i-1)\Delta t$ by

$$X_i(\omega) = X_{i-1}(\omega) + x(i) e^{-j\omega i \Delta t} \quad (12a)$$

where

$$e^{-j\omega i \Delta t} = e^{-j\omega \Delta t} e^{-j\omega (i-1) \Delta t} \quad (12b)$$

This is simply calculating the discrete Fourier transform in Eq. (10) as a running sum. The quantity $e^{-j\omega\Delta t}$ is constant for a given frequency ω and constant sampling interval Δt . It follows that the discrete Fourier transform can be computed for a given frequency at each time step using one addition in Eq. (12a) and two multiplications – one in Eq. (12b) using the stored constant $e^{-j\omega\Delta t}$ for frequency ω , and one in Eq. (12a). There is no need to store the time-domain data in memory when computing the discrete Fourier transform in this way because each sampled data point is processed immediately as a projection onto Fourier basis functions. Previous time series data can be incorporated in all subsequent analysis by simply continuing the recursive calculation of the Fourier transform. In this sense, the recursive Fourier transform acts as memory for the information in the data. Additional time-domain data improves the quality of the data in the frequency domain without increasing memory requirements to store it. Furthermore, the Fourier transform is available at any time $i\Delta t$. The approximation to the finite Fourier transform can be completed using Eq. (9),

$$\tilde{x}(\omega, t_i) \approx \Delta t X_i(\omega) \quad (13)$$

Analysis frequencies ω in Eqs. (12) can be selected arbitrarily. If the selected frequencies are closely spaced and are limited to a frequency band where the aircraft dynamics are expected to reside, then the recursive Fourier transform can capture fine detail in the frequency domain data while also implementing automated data smoothing, which produces accurate modeling results. The recursive discrete Fourier transform implemented in this way has been used successfully in flight tests for real-time dynamic modeling in the frequency domain [4],[6-7],[23-27].

2. Detrending

Prior to transforming time-domain data into the frequency domain, any low-frequency signal components with relatively high amplitude, such as the constant part (called the bias) and part that is linearly related to time (called the linear trend or drift), are removed from the time series. This process is called detrending. The purpose of detrending is to prevent leakage from these relatively large low-frequency components, which can pollute the frequency-domain data at the lower end of the frequency range of interest [7],[23]. For this reason, detrending is applied before any Fourier transform is applied in practice.

Detrending can be implemented in real time using a high-pass filter. Desirable characteristics for the high-pass filter are sharp frequency cutoff between the frequencies passed through and those that are not (generally associated with higher-order filters), and small amplitude and phase changes to the components passed through the filter. Good results have been obtained using a 4th-order Butterworth high-pass filter with cutoff frequency set at $1/(2T)$ Hz, where T is the period of the lowest frequency of interest. Other high-pass filter design solutions are also possible. This approach has been applied successfully in flight for real-time dynamic modeling in the frequency domain [7],[23-26]. Note that detrending also removes systematic bias errors from the explanatory variable data.

3. Time Variation and Data Forgetting

The recursive discrete Fourier transform in Eqs. (12) implements a data information memory for as long as the running sum is continued. When the situation changes, older data should be discounted or forgotten in some way. If this is not done, then the speed of response to the changes is progressively degraded, as new information has to overwhelm an increasingly longer memory. Consequently, there is a trade-off between the desired rapid response to changes versus retaining enough data information for accurate data analysis and modeling.

The simplicity of Eqs. (12) and the linearity of the Fourier transform with respect to the time series data $x(i)$ allow easy implementation of exponential data forgetting in the recursive discrete Fourier transform, by modifying Eq. (12a) to

$$X_i(\omega) = \lambda X_{i-1}(\omega) + x(i) e^{-j\omega\Delta t} \quad (14)$$

This can be interpreted as multiplying each past value of the time series by a forgetting factor λ , $0 < \lambda < 1$, at each time step. Old data are gradually devalued and eventually discarded for practical purposes. Use of a constant forgetting factor in this way corresponds to exponential data forgetting with respect to time [7]. Note that $\lambda = 1$ makes Eq. (14) the same as Eq. (12a), corresponding to no data forgetting, and $\lambda = 0$ would be the impractical case of forgetting all data except the current data point. In practice, typical values of the forgetting factor are

$$0.95 \leq \lambda < 1.00 \quad (15)$$

The forgetting factor λ can be related to the effective data window τ in seconds by [7]

$$\lambda = 1 - \Delta t / \tau \quad (16)$$

Partial forgetting can be implemented by subtracting saved values of the running sums at earlier times. Alternatively, the recursive discrete Fourier transform can be simply restarted by setting all $X(\omega)$ values to zero (complete forgetting) based on an event, such as a flight condition change, start of a new maneuver, change in the aircraft configuration, or some type of failure or damage.

C. Real-Time Local Equation-Error Modeling in the Frequency Domain

The equation-error method calculates aerodynamic parameter estimates that minimize the sum of squared differences between values of nondimensional force and moment coefficients determined from measured flight data and corresponding model values computed using the model structure in Eqs. (6)-(7). Equation-error parameter estimation can be applied in the frequency domain using a similar approach to equation-error parameter estimation in the time domain. However, the simple first step of transforming the data into the frequency domain using a limited frequency band with high resolution in the frequency domain provides important practical and computational advantages. These advantages include automatic data smoothing, faster computation because the number of frequencies is relatively small and constant regardless of the maneuver time length, and accurate parameter estimates and uncertainties from simple calculations [7],[23],[25-26].

For each of Eqs. (6)-(7), substituting force or moment coefficient values computed from Eqs. (4)-(5) on the left side, along with corresponding measured states and controls on the right, results in an over-determined set of equations for the unknown aerodynamic parameters. This problem can be solved using a standard least-squares method. Because the data are detrended prior to applying the Fourier transform, bias terms such as C_{m_0} are omitted from the modeling in the frequency domain. These terms must be estimated in the time domain, as will be discussed later.

A typical frequency band for the analysis of a 20-second data record for a full-scale aircraft might be

$$f = [0.100 \ 0.125 \ \dots \ 1.475 \ 1.500]^T \text{ Hz} \quad \omega = 2\pi f \quad (17)$$

The lower bound is chosen as $2/T$, where T is the time length of the maneuver, so that the lowest analysis frequency will have 2 complete cycles in T seconds, and the upper bound is chosen to include the dynamics of interest, typically identified as the upper bound of the frequency range for large components relative to the noise level in the frequency domain.

For example, the least-squares problem for the pitching moment coefficient C_m is formulated in the frequency domain using the model structure in Eq. (7b) as

$$\tilde{z} = \tilde{X}\theta + \tilde{\varepsilon} \quad (18)$$

where

$$\tilde{z} = [\tilde{C}_m(1) \ \tilde{C}_m(2) \ \dots \ \tilde{C}_m(M)]^T = M \times 1 \text{ vector of dependent variable data}$$

$$\theta = [C_{m_\alpha} \ C_{m_q} \ C_{m_{\delta_e}}]^T = 3 \times 1 \text{ vector of unknown parameters}$$

$$\tilde{X} = [\tilde{\alpha} \ \tilde{q} \ \tilde{\delta_e}] = M \times 3 \text{ matrix of modeling function vectors or regressors}$$

$$\tilde{\varepsilon} = [\tilde{\varepsilon}(1) \ \tilde{\varepsilon}(2) \ \dots \ \tilde{\varepsilon}(M)]^T = M \times 1 \text{ vector of equation errors}$$

and M is the number of analysis frequencies or data points in the frequency domain, e.g., $M = 57$ using the frequency vector in Eq. (17).

The matrix $\tilde{X}$ is assembled using frequency-domain data, with each column representing a modeling function, also called a regressor. Equation (18) is a set of M over-determined algebraic equations involving complex numbers.

The best estimate of θ in a least-squares sense comes from minimizing the sum of squared differences between the dependent variable data $\tilde{z}$ and the model output [7],

$$J(\theta) = \frac{1}{2} (\tilde{z} - \tilde{X}\theta)^\dagger (\tilde{z} - \tilde{X}\theta) \quad (19)$$

The least-squares solution for the unknown parameter vector is

$$\hat{\theta} = \left[\text{Re}(\tilde{X}^\dagger \tilde{X}) \right]^{-1} \text{Re}(\tilde{X}^\dagger \tilde{z}) \quad (20)$$

so that the model output is

$$\hat{y} = \tilde{X}\hat{\theta} \quad (21)$$

The estimated parameter covariance matrix is computed from [23]

$$\Sigma(\hat{\theta}) = \hat{\sigma}^2 \left[\text{Re}(\tilde{X}^\dagger \tilde{X}) \right]^{-1} \quad (22a)$$

$$\hat{\sigma}^2 = \frac{1}{2T} \text{Re} \left[(\tilde{z} - \hat{y})^\dagger (\tilde{z} - \hat{y}) \right] / [\max(f) - \min(f)] \quad (22b)$$

The residual variance expression in Eq. (22b) is needed when the analysis frequencies are chosen in a limited frequency band within $[0, f_N]$ Hz, where f_N is the Nyquist frequency, $f_N = 0.5/\Delta t$ Hz.

The standard errors of the estimated parameters are the square root of the diagonal elements of the covariance matrix,

$$s(\hat{\theta}_j) = \sqrt{\Sigma_{jj}} \quad j = 1, 2, \dots, n_p \quad (22c)$$

where n_p is the number of model parameters, e.g., $n_p = 3$ for the pitching moment example in Eq. (18).

For the dependent variable, transforming into the frequency domain using a limited frequency band improves the signal-to-noise ratio by removing most of the wideband noise, but also leads to model residuals in the frequency domain that can be approximated as constant amplitude with frequency, which is characteristic of white noise. As a consequence, if equation-error is applied in the frequency domain with the transform frequencies chosen to approximately correspond to the large magnitude components in the frequency domain, then the simple calculations in Eqs. (22) can be used to accurately quantify the uncertainty in the parameter estimates. This characteristic improves the global modeling, which is an assembly of the local modeling results accounting for the local model uncertainties, as described later.

In principle, the parameter estimation calculations in Eqs. (20)-(22) could be done at every time step. However, this is neither necessary nor practical because the additional information in the frequency domain from one added time-domain data point is negligible, and the matrix inversion in Eq. (20) would have to be computed at each time step. Local model parameters typically do not change that rapidly in flight. In this work, the calculations in Eqs. (20)-(22) were done every 2 s (0.5 Hz), and the forgetting factor was set for an effective data window of 2 s, corresponding to approximately 2 cycles of the dominant dynamic modes for the linear modeling. More generally, the forgetting factor could be linked to the rate of change of the nominal flight condition during the global modeling maneuvers. Other values can be used for the effective data window length, but for the F-16 and E1 aircraft, a 2-second data window worked well. Longer data windows can degrade local model accuracy because a larger range of nominal flight conditions are covered, whereas shorter data windows can lead to reduced accuracy because of

fewer data points and consequent lower data information content. However, a relatively wide range of values for the effective data window length gave acceptable results, so that this choice was not difficult to make.

Real-time local equation-error modeling in the frequency domain uses relatively simple calculations, and produces accurate modeling results. The modeling can be done using any of the Eqs. (6)-(7) (or analogous versions of these equations, with different model structures), analyzed individually. Because the data are detrended prior to applying the Fourier transform, the bias parameters, e.g., C_{m_0} in Eq. (7b), are excluded from the analysis. Fortunately, the bias parameters can be estimated easily and accurately in the time domain, as will be shown next.

D. Low-Frequency Component Modeling in the Time Domain

The bias term in a local model is not observable in the frequency domain because the time series for the model terms and the dependent variable are detrended prior to Fourier transformation. Stated another way, frequency-domain data include only dynamic information, and not static information. Consequently, a separate subsequent step must be included to estimate the bias parameter in the model. The estimated bias parameter then corresponds to the aerodynamic bias term in a multivariable Taylor series expansion for the aerodynamic coefficient, which is generally of interest for aerodynamics and flight dynamics. The process also restores the static information that was discarded when the data were transformed into the frequency domain.

Each bias term in Eqs. (6)-(7) is simply the mean value of the dependent variable over the effective data window used for the local modeling. This can be computed inexpensively using a sliding data window containing the most recent measured values of the dependent variable. The mean value and standard deviation are computed as

$$\bar{z} = \sum_{i=1}^{n_w} z(i) \quad \sigma_z = \sqrt{\sum_{i=1}^{n_w} [z(i) - \bar{z}]^2 / (n_w - 1)} \quad (23)$$

where n_w is the number of data points in the sliding data window.

For local modeling in the frequency domain, the time length of the sliding data window used to estimate the nominal flight condition was set at 2 seconds, to match the effective data window associated with the forgetting factor used for the recursive Fourier transform, as well as the time interval for local parameter estimation results using Eqs. (20)-(22). A sliding window was used for the low-frequency component modeling to avoid the significant time lag that would result from using a forgetting factor to compute low-frequency results. For the local real-time modeling in the frequency domain, the model parameters were assumed to change slowly with nominal flight condition, and the modeling task was more extensive and difficult, so that a forgetting factor was applied to retain small amounts of information from more than 2 seconds earlier.

E. Real-Time Global Modeling

Local modeling results are assembled for a variety of nominal flight conditions to produce the global model. Essentially, the collection of stability and control derivatives and bias terms in Eqs. (6)-(7) as a function of nominal flight condition constitute the global model, as demonstrated in Fig. 8. Originally, the idea was to characterize the local model parameters as a function of nominal flight condition using polynomial and/or spline functions of the nominal flight condition variable(s). However, it became clear that it was easier and more efficient to just tabulate the local model parameters as a function of the nominal flight condition, then simply interpolate the values in those tables for the global model. This works well, and keeps the real-time onboard computations simple. Of course, the tables of local parameter estimates assembled in real time can be modeled in more compact ways during post-flight analysis using analytic functions of the nominal flight condition variable(s).

As the nominal flight condition changed during the flight test maneuvers with PTI active, results from real-time local modeling in the frequency domain were stored in the table location (or bin) associated with the nominal flight condition nearest to the estimated nominal flight condition for the current sliding data window. This was done by simply rounding the estimated nominal flight condition to the nearest bin value. If that bin contained previous results from a different transit through the same nominal flight condition, then the local modeling results were combined using statistical weighting [28] as follows.

Using w for the weightings and s for the standard error of parameter estimate θ , let subscripts 0 and 1 denote the stored table value and the current flight estimate for the same bin, respectively, and subscript 2 denote the updated value to be stored in the table,

$$w_0 = 1/s_0^2 \quad w_1 = 1/s_1^2 \quad (24a)$$

$$\theta_2 = (w_0\theta_0 + w_1\theta_1)/(w_0 + w_1) \quad (24b)$$

$$s_2 = 1/\sqrt{(w_0 + w_1)} \quad (24c)$$

If the transit through nominal flight conditions was such that some values in the table were not populated by local modeling results in real time, then the missing values were interpolated using local modeling results in the table and the statistical expressions in Eqs. (24). Table values for nominal flight conditions outside of those populated with local modeling results in real time were filled in using linear extrapolation based on the local modeling results in the table.

At the start, model parameters and uncertainties from the first real-time frequency-domain local model were used to populate the entire global model table. After that, the uncertainty for any interpolated or extrapolated parameter values in table were increased by a factor of 2, to approximately model the added uncertainty in those values.

The final form of the global model was a set of tables for the local model parameters in Eqs. (6)-(7) as a function of nominal flight condition. This form provides easily-updated modeling memory and the capability to model localized variations, based on partitioning local modeling results according to nominal flight condition. In contrast, data partitioning approaches partition time-domain data according to defined boundaries in the explanatory variable space [14],[29].

To use the global model, values for the parameters in Eqs. (6)-(7) were interpolated from the tables as a function of nominal flight condition, and high-pass filtered explanatory variable data were substituted into Eqs. (6)-(7) to compute nondimensional aerodynamic force and moment coefficients for the global model.

V. Results

The global aerodynamic modeling approach described in the preceding sections was applied first to data from the F-16 nonlinear simulation, then to E1 flight test data.

A. F-16 Nonlinear Simulation

Figure 9 shows simulated F-16 flight test data for a slow approach to stall and recovery with multi-axis PTI active. The vertical aerodynamic force coefficient C_Z was chosen for this demonstration because this aerodynamic coefficient exhibits significant nonlinearity over an extended flight envelope. Global modeling for the other aerodynamic coefficients was similar. The frequency-domain global modeling method described earlier was applied to this data, assuming that the nominal flight condition can be characterized by nominal angle of attack only, and using the analysis frequency vector $\mathbf{f} = [0.100 \ 0.125 \ \dots \ 1.975 \ 2.000]^T$ Hz. Nominal angle of attack in the table of local modeling results was discretized at intervals of 1 degree, reflecting an assumption that local model parameters can be assumed constant over that range of variation in nominal angle of attack.

Figure 10 shows the global model fit to the modeling data from a global modeling maneuver. Note that the fit degrades somewhat near large negative values of C_Z (associated with high nominal α), mainly because of the increased nonlinearity and smaller amount of flight time spent in that flight regime.

In Fig. 11, the identified global model was applied to predict data from a different maneuver. Prediction accuracy is similar to the model fit quality shown in Fig. 10. The model is consistent and accurate overall, but has a slight deficiency at high nominal angles of attack.

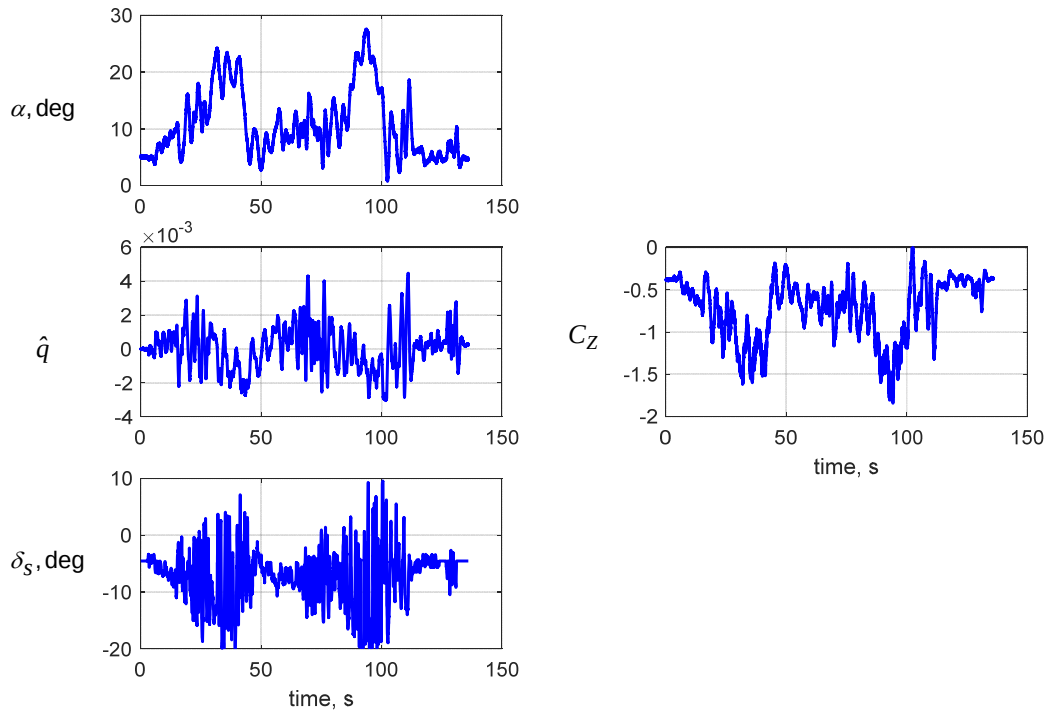


Figure 9. F-16 global modeling maneuver data

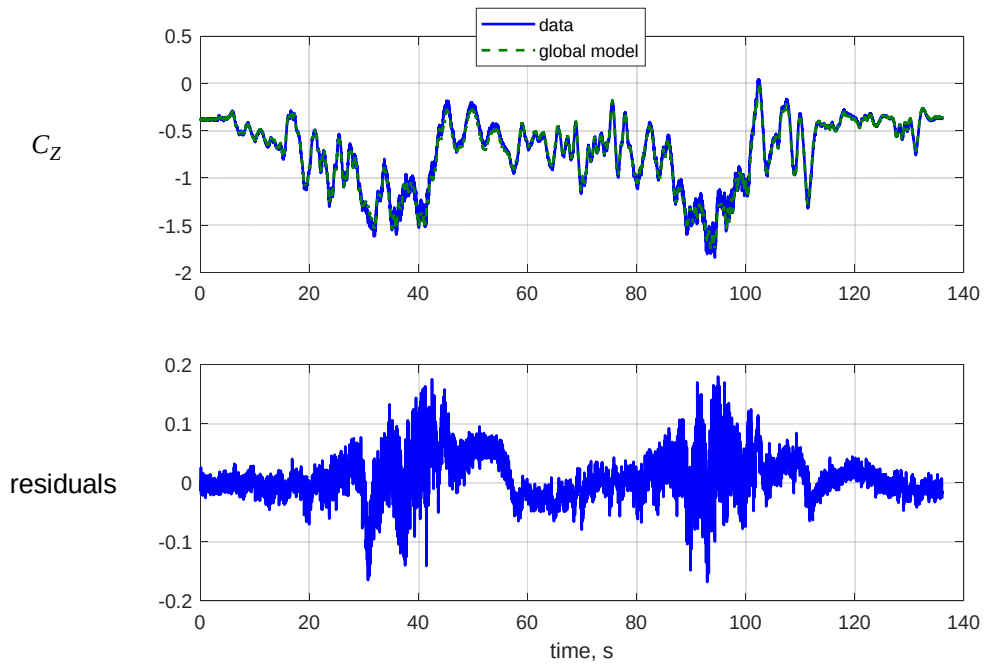


Figure 10. F-16 global modeling

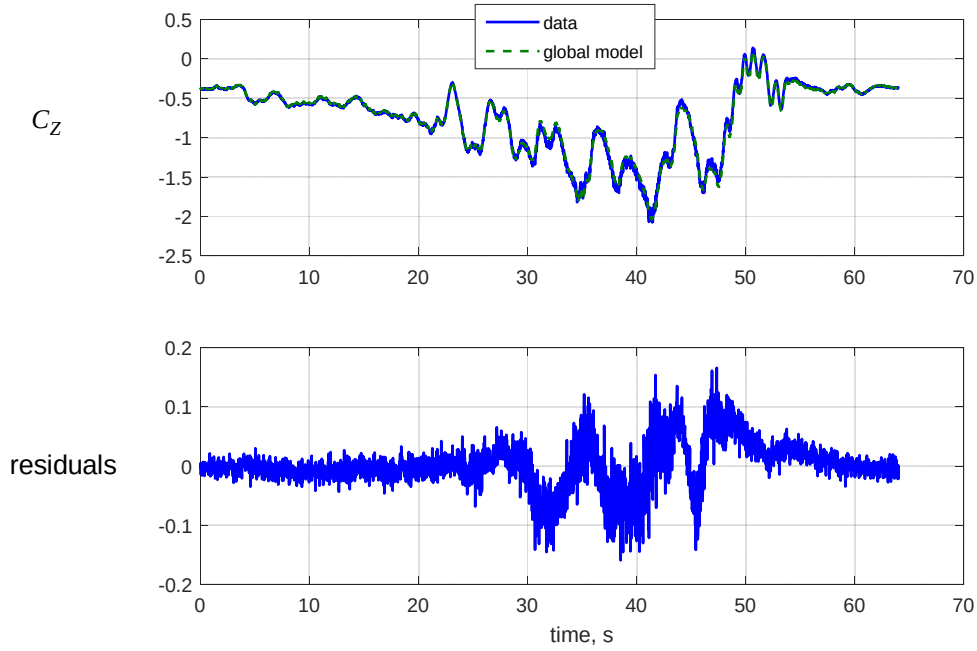


Figure 11. F-16 global model prediction

More flight time with PTI active at high nominal angles of attack should improve the local linear modeling in that flight regime, and thereby improve the global model fit and prediction capability. This could be achieved with either repeated global maneuvers that reach high nominal angles of attack, or a slower transit through the higher nominal angles of attack. In practice, modeling results at higher nominal angles of attack can typically be improved with increased PTI amplitudes, due to reduced control surface effectiveness in that flight regime.

B. E1 Flight Test

E1 flight testing was conducted at Beaver Dam Airpark near Elberon, VA, using the instrumented E1 aircraft shown in Fig. 2 and described in Section II. Figure 12 shows E1 flight test data for a sequence of figure-eight and slow approach to stall and recovery maneuvers. All of the control surfaces were moved simultaneously using PTI, which were mutually orthogonal optimized multisine excitation inputs, shown in Fig. 4. This demonstration uses only the vertical force aerodynamic coefficient, for the same reasons given in the F-16 simulation example. The vector of analysis frequencies was the same as for the F-16 simulation, and nominal flight condition was again assumed to be characterized by nominal angle of attack only, with 1 degree discretization.

Figure 13 shows the global model fit to C_z for E1 flight 27 data shown in Fig. 12. As in the F-16 nonlinear simulation case, the global model fit is degraded at high nominal angles of attack. In Fig. 14, the identified global model was used to predict flight data from a sequence of figure-eight and slow approach to stall and recovery maneuvers from E1 flight 17. As before, the prediction accuracy is consistent with the model fit quality, but has a slight deficiency at high nominal angles of attack. Similar results were obtained when predicting flight 27 data using the global model identified from flight 17 data, i.e., swapping the modeling and prediction data relative to the results shown in Figs. 13 and 14.

Figure 15 shows the tabulated values of the local linear model parameters as a function of nominal angle of attack. The solid triangle markers are frequency-domain local modeling results or updated results using Eqs. (24). The open circle markers are interpolated or extrapolated values based on the flight-estimated or updated local modeling results in the table. It is apparent that more flight data with PTI active are needed to fill in the tables with flight estimates. The global model could be improved with additional informative data from more global maneuvers, or by moving more slowly through the nominal flight conditions, or possibly by estimating the local models at a higher rate than every 2 s.

An improvement to the global modeling and prediction might result from including nominal propeller advance ratio μ_o to more completely characterize the nominal flight condition. This would require making all of the tables for the linear model parameters in Eqs. (6)-(7) into 2-dimensional tables, e.g., $C_{m_\alpha}(\alpha_o, \mu_o)$. This general idea could be extended to include additional variables to characterize the nominal flight condition and local model dependency, requiring mostly additional bookkeeping and onboard computer memory.

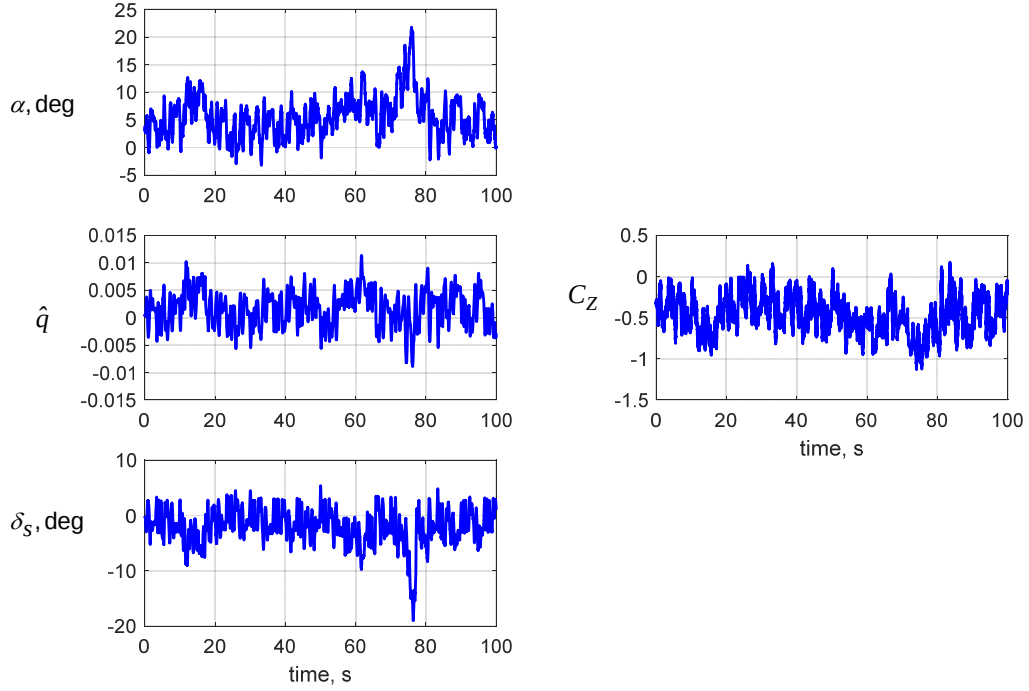


Figure 12. E1 global modeling maneuver data

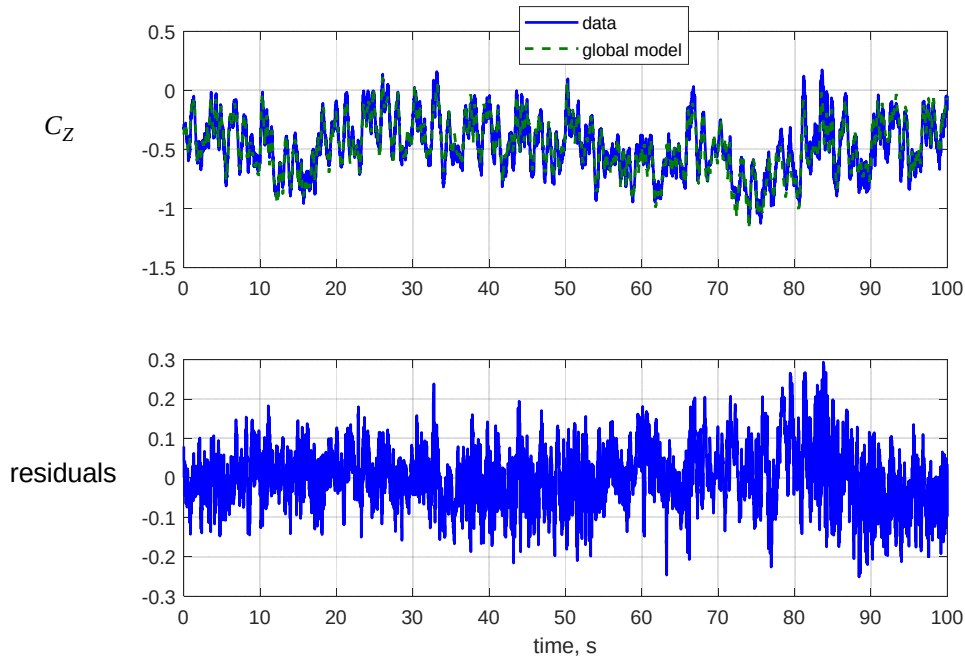


Figure 13. E1 global modeling

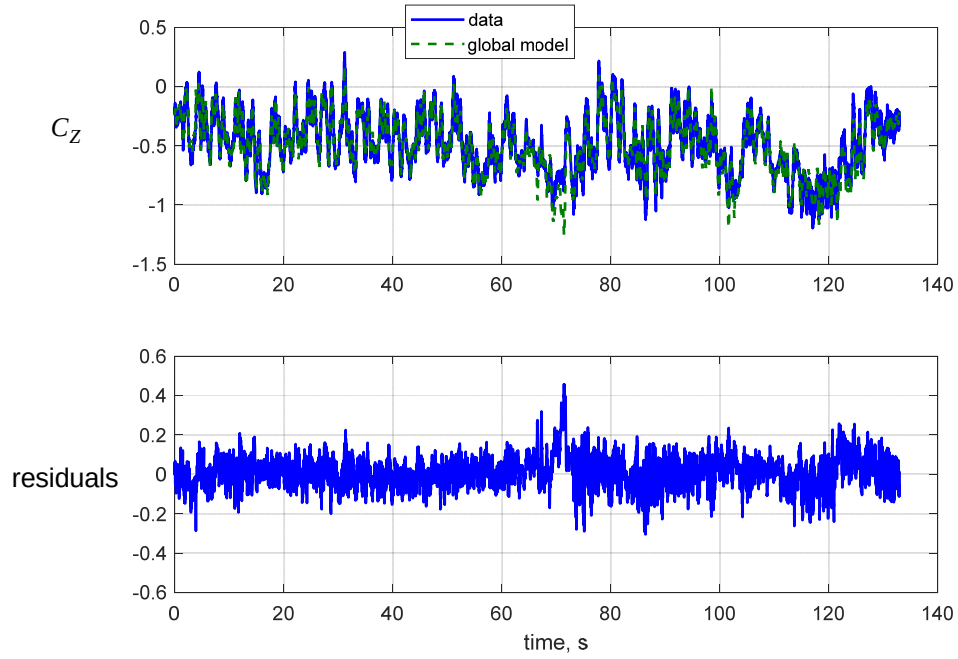


Figure 14. E1 global model prediction

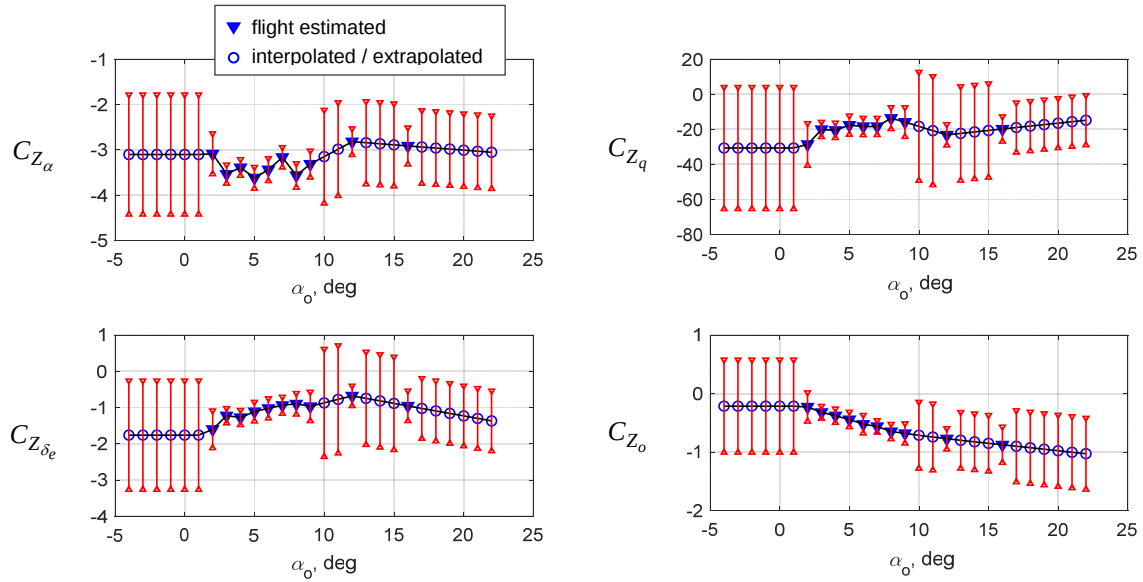


Figure 15. E1 global model parameter values

Compared to the time-domain real-time global nonlinear aerodynamic modeling method that was successfully flown during Learn-to-Fly flight testing [2-3], the frequency-domain global modeling described in this work has lower computational requirements and approximately the same requirements for onboard computer memory.

VI. Conclusions

A new autonomous real-time frequency-domain global aerodynamic modeling method was explained and demonstrated using F-16 nonlinear simulation data and flight test data from a subscale aircraft called E1. Local real-time frequency-domain estimates of stability and control derivatives were assembled over the flight envelope as a function of nominal flight condition, accounting for uncertainty in the individual local model parameter estimates, then combined with real-time estimates of low-frequency aerodynamics in the time domain to produce a global nonlinear aerodynamic model autonomously in real time.

The benefits of the frequency-domain global modeling approach are:

1. Flight-validated real-time parameter estimation in the frequency domain is used for the local modeling. This provides accurate model parameters and uncertainty estimates using efficient calculations, which is important for real-time global modeling based on local modeling results.
2. Computing local stability and control derivatives from the identified global model is easy because local stability and control derivative estimates are used directly to produce the global model.
3. Local variations over the flight envelope can be modeled in a straightforward and general way.
4. There is no need to postulate a pool of candidate model terms or to use real-time statistical model structure determination.
5. The recursive Fourier transform implemented using a limited frequency band rejects noise and data dropouts automatically, and ignores bias errors in the explanatory variables.
6. The frequency-domain global modeling approach is easy to generalize for additional controls.
7. The current form of the real-time frequency-domain global modeling algorithm requires less computation and approximately the same amount of flight computer memory as the Learn-to-Fly real-time global nonlinear modeling in the time domain, which was flown successfully on E1.

The drawbacks are:

1. The low-frequency part of the global model must be estimated separately in the time domain and combined with local frequency-domain models. These steps are straightforward and well-conditioned, but they are extra steps.
2. Local models must be associated with a nominal flight condition for the global modeling. Consequently, the nominal flight condition must be computed in real time because it changes during global modeling maneuvers. If the nominal flight condition changes rapidly, the accuracy of local real-time modeling is degraded.
3. Explanatory variables that affect the nondimensional force or moment coefficients must be specified in advance. For example, does the dynamic part of the nondimensional pitching moment coefficient C_m depend on sideslip angle β ? The quantities that define the flight condition must also be specified in advance, e.g., nominal angle of attack α_o .
4. Local modeling and nominal flight condition estimation must discount past data with a forgetting factor or use a sliding data window. This introduces some design parameters to select, but the selections are not difficult to make and good results can be obtained with a wide range of values.
5. Very complicated local nonlinear aerodynamic dependencies will be approximated. However, this is true for any practical real-time global modeling technique.

Global modeling accuracy requires global maneuvers that transit the nominal flight conditions slowly with multi-axis excitations active, to provide sufficient data information for accurate and complete local modeling information for the global model. This requirement can also be met by using multiple global maneuvers with repeated transits through the nominal flight conditions. Because local modeling results and uncertainties are computed in real time, it should be possible to autonomously determine how fast the nominal flight conditions can be transited. Informative data with dense and extensive coverage of the explanatory variable space lead to more accurate and useful global models.

The real-time frequency-domain global modeling approach is easy to implement, partly because SIDPAC software [7,18] can be used for multi-axis excitation design, real-time high-pass filtering, and local real-time modeling in the frequency domain. The simplicity, accuracy, and extensibility of the frequency-domain global modeling approach make it an attractive option for a wide variety of flight test vehicles and flight conditions.

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