Particle Simulation

HIWC Modeling Insights

30 March 2020

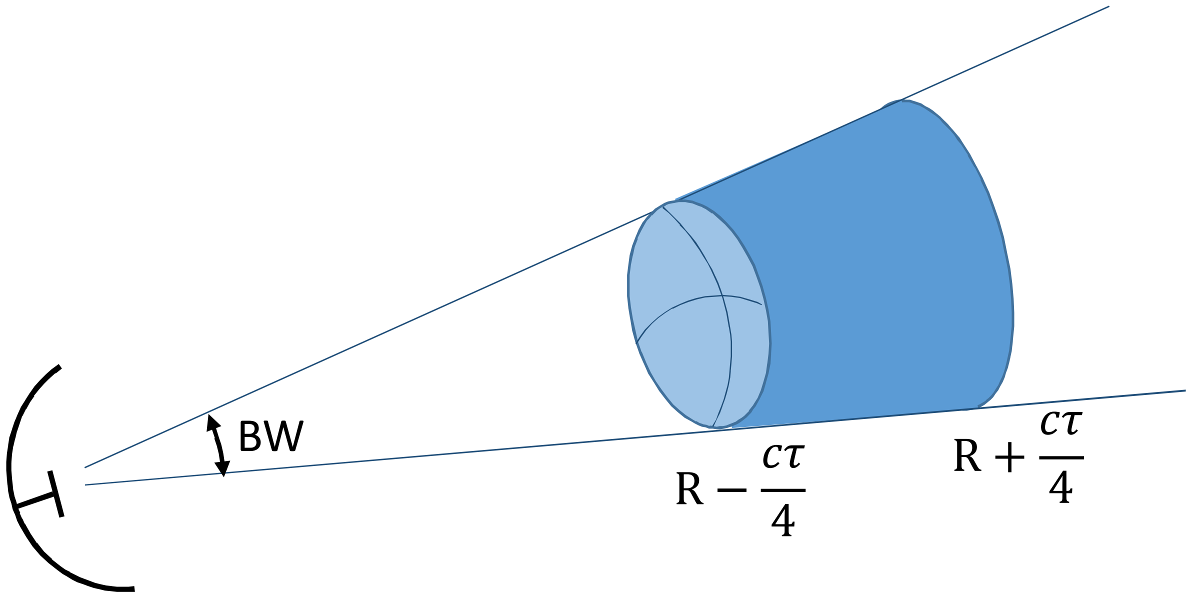
Agenda

- ADWRS modeling approach
- Detailed particle model
  - Effects of angle resolution
  - Effects of range resolution
  - Random vs stochastic processes
  - Simulating HIWC
- Findings and Results
- Conclusions and next steps

Radar Resolution Volume

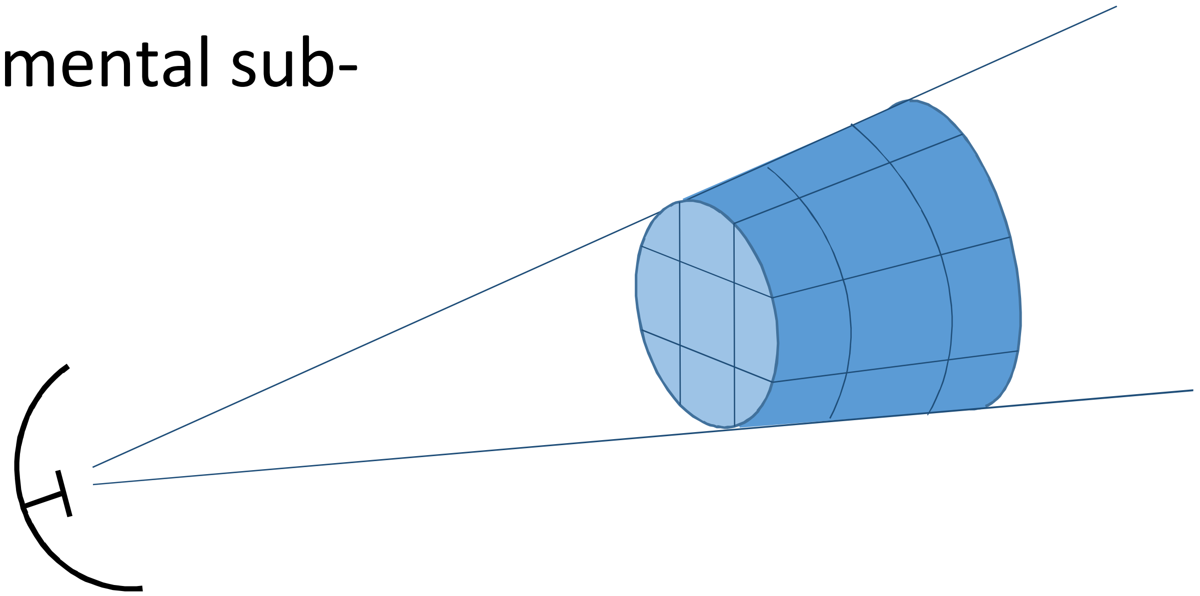
- Beam width (BW)
- Range (R)
- Pulse with (τ)

- Homogeneous cloud
  - Particle uniformly distributed throughout the volume



ADWRS Weather Modeling

- Divide resolution volume into incremental sub-volumes (cells)
  - ΔAzimuth, ΔElevation, ΔRange
- $\Delta V_i = R_i^2 \Delta Az_i \Delta El_i \Delta R_i$
- $\eta_i = \frac{\pi^5}{\lambda^4} |K_W|^2 Z_i$
- $\sigma_i = \eta_i \Delta V_i$
- $A_i = \sqrt{\frac{P_t \lambda^2 G_i^2 \sigma_i}{(4\pi)^3 R_i^4}}$
- $v(t) = \sum_i A_i e^{-j\left(\bar{\phi}_i + \frac{4\pi v_i}{\lambda} t\right)}$



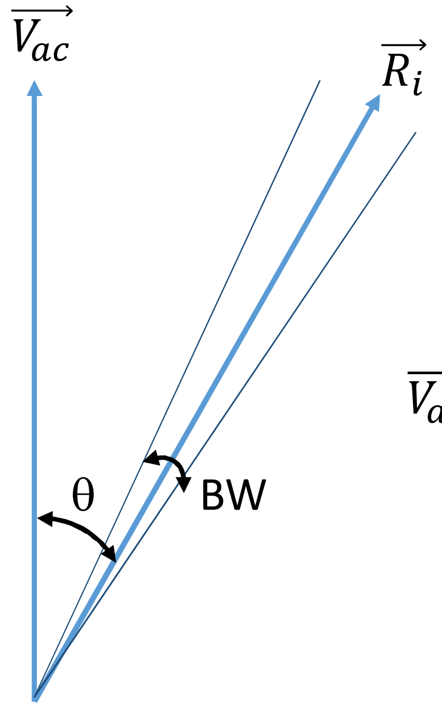
Z_i and v_i interpolated from TASS database
G_i from antenna pattern

Agenda

- ADWRS modeling approach
- Detailed particle model
  - Effects of angle resolution
  - Effects of range resolution
  - Random vs stochastic processes
  - Simulating HIWC
- Findings and Results
- Conclusions and next steps

Particle Radar Return Signal

- $\sigma_i = \frac{\pi^5}{\lambda^4} |K_W|^2 D_i^6$
- $A_i = \sqrt{\frac{P_t \lambda^2 G_i^2 \sigma_i}{(4\pi)^3 R_i^4}}$
- $\mathrm{v}(t) = \sum_i A_i e^{-j\left(\frac{4\pi R_i}{\lambda} + 2\pi f_i t\right)}$
- $f_i = \frac{2}{\lambda} \frac{dR_i}{dt}$



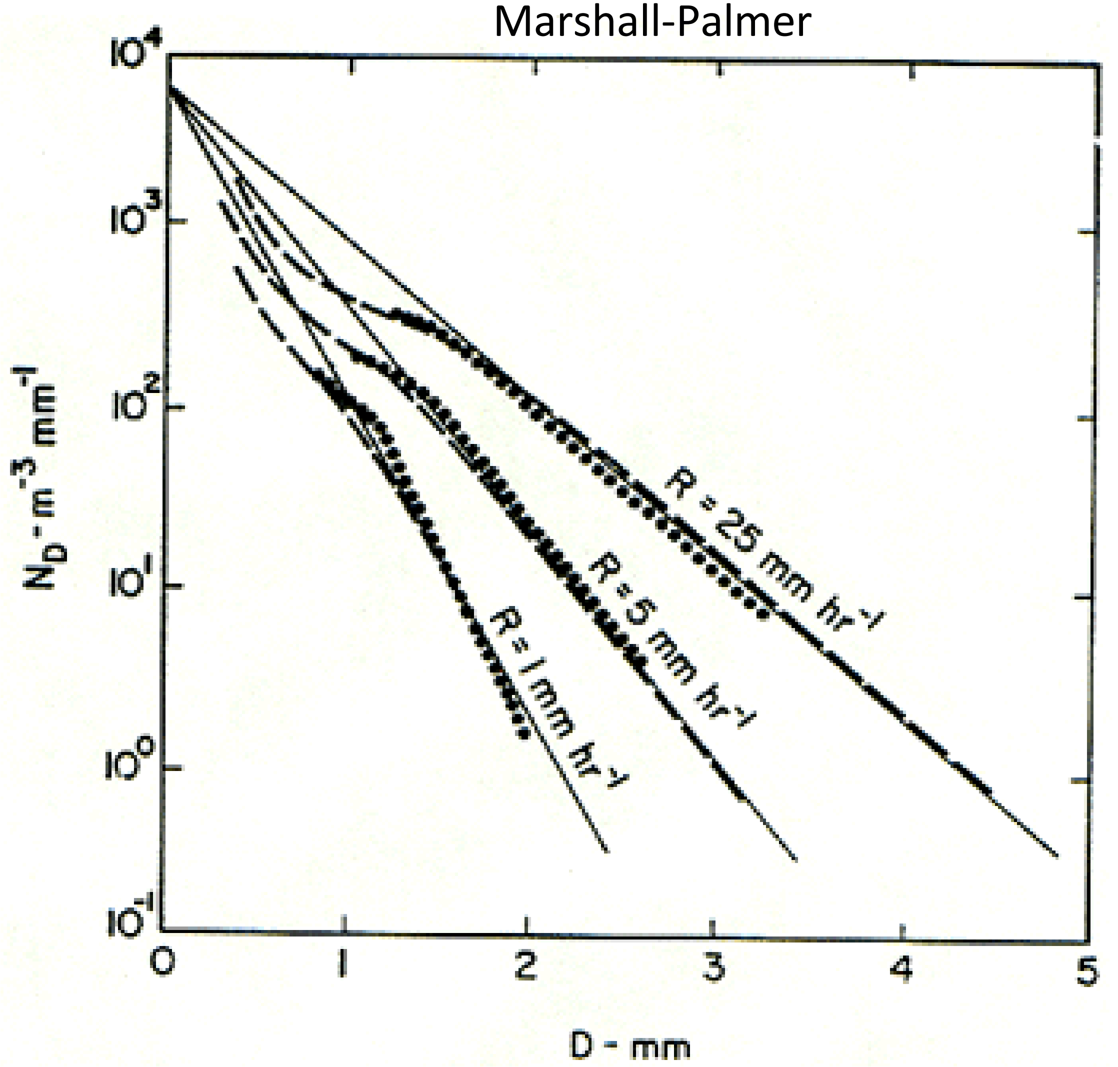
$$\overrightarrow{V_{ac}} \cdot \overrightarrow{R_i} = \left|\overrightarrow{V_{ac}}\right| \left|\overrightarrow{R_i}\right| \cos\theta_i$$

$$\frac{dR_i}{dt} = V_{ac} \cos\theta_i$$

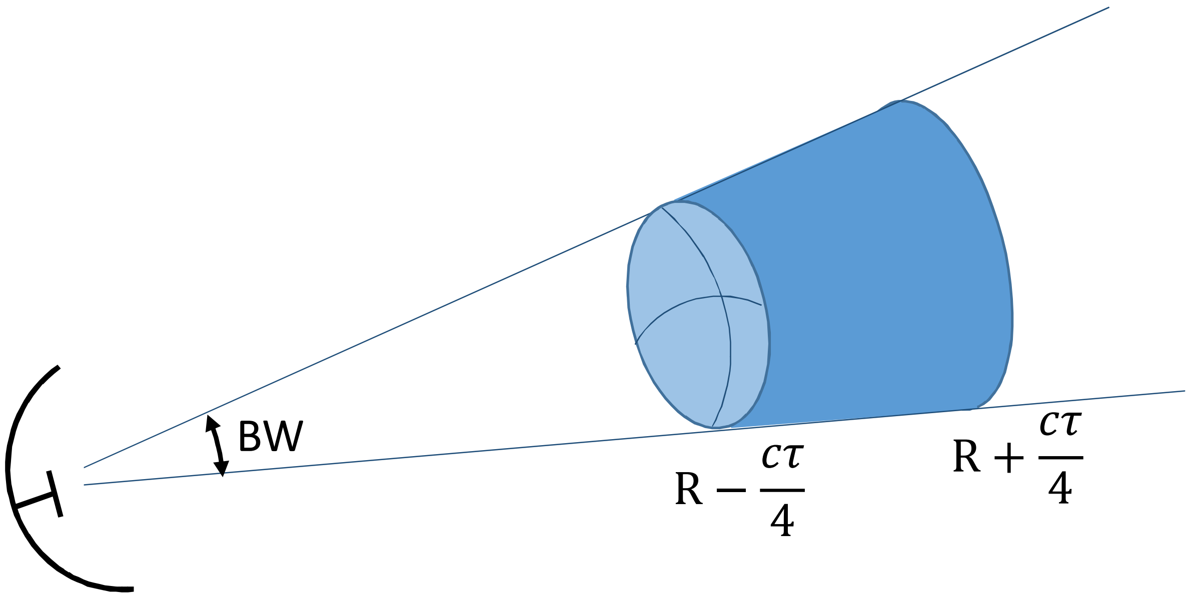
Detailed Particle Model

- Particle/Drop size distribution
  - $N(D) = N_0 e^{-\Lambda D}$
- Number of particles of a given size in a unit volume
- Theoretical/Truth information
  - Mass = $\left(\frac{\pi\rho}{6}\right)\int_0^\infty D^3 N(D)dD = \frac{\pi\rho N_0}{\Lambda^4}$
  - RRF = $\int_0^\infty D^6 N(D)dD = \frac{N_0(6!)}{\Lambda^7}$

Marshall-Palmer

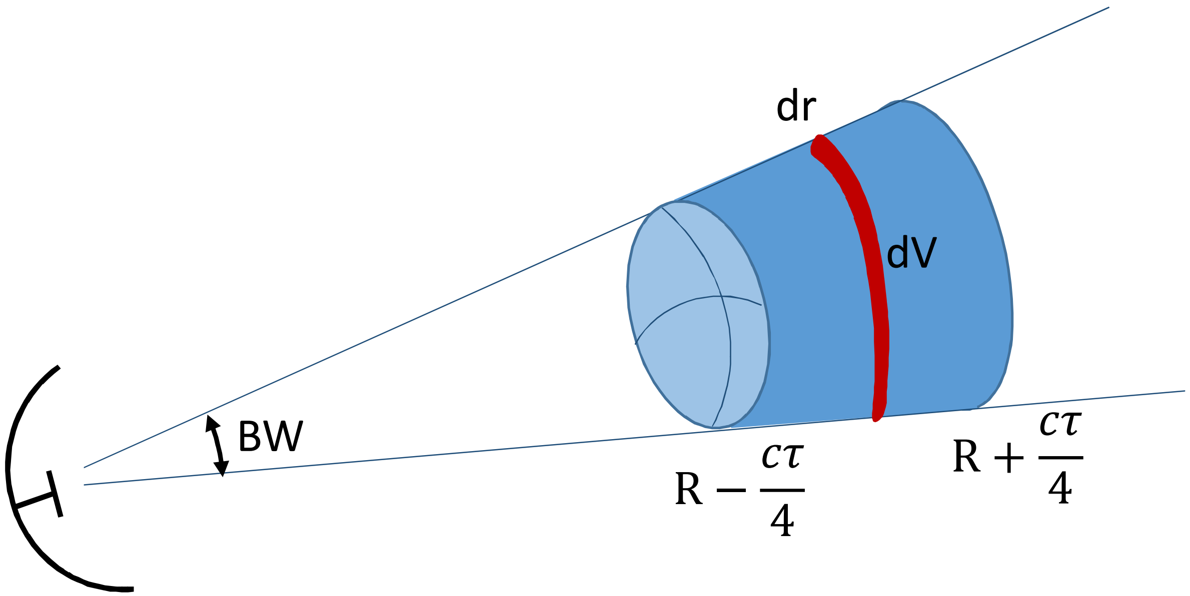


Resolution Volume



- $V = \int_{R-\frac{c\tau}{2}}^{R+\frac{c\tau}{2}} \int_0^{\pi} \int_0^{2\pi} r^2 g(\theta, \phi) \sin\theta \, d\theta d\phi dr$
- Accounting for Gaussian antenna gain pattern, $g(\theta,\phi)$
  - Doviak and Zrnic (4.3)
- $V = R^2 \frac{\pi BW^2 c\tau}{16(\ln 2)}$

Volume Distribution



- Area of a spherical cap
  - $A = 2\pi r^2(1 - \cos\theta)$
- Differential volume
  - $dV = 2\pi r^2(1 - \cos BW)dr$
- Distribution of volume in resolution cell
  - $V(r) = \int_{R-\frac{c\tau}{4}}^{r} dV = \frac{2}{3}\pi(1 - \cos BW)\left[r^3 - \left(R - \frac{c\tau}{4}\right)^3\right]$
- Inverse transform sampling
  - Generate random range resulting in uniform particle density
  - $r = \sqrt[3]{U(0,1)\left[\left(R + \frac{c\tau}{4}\right)^3 - \left(R - \frac{c\tau}{4}\right)^3\right] + \left(R - \frac{c\tau}{4}\right)^3}$

Particle Simulation

- Start at smallest D
- While N(D) > 0
  - Compute $\sigma_i = f(D^6)$
  - Get random range, R_i
  - Compute amplitude, $A_i = f(\sigma_i, R_i)$
  - Get random angle
  - Compute radial velocity/Doppler, f_i
  - For pulses in CPI
    - Compute phase = $f(PRI, R_i, f_i)$
    - v(t) = sum complex returns for all i
  - Increment D
  - Compute N(D)

Agenda

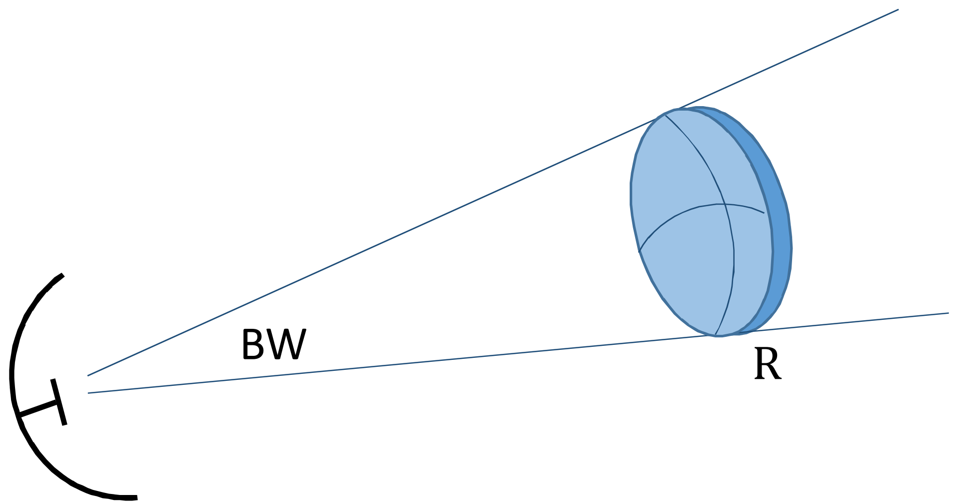
- ADWRS modeling approach
- Detailed particle model
  - Effects of angle resolution
  - Effects of range resolution
  - Random vs stochastic processes
  - Simulating HIWC
- Findings and Results
- Conclusions and next steps

Doppler Frequency Distribution

- Particles uniformly distributed in angle
  - Let X = U($\theta - \frac{BW}{2}, \theta + \frac{BW}{2}$)
  - Let Y = f = $\frac{2V_{ac}}{\lambda}\cos X$
- Transformation of Random Variable
  - Probability distribution function (PDF)
  - $p(f) = \frac{\lambda}{2BWV_{ac}\sqrt{1-\left(\frac{f\lambda}{2V_{ac}}\right)^2}}$, $f_{min} \le f \le f_{max}$
  - $f_{min} = \frac{2V_{ac}}{\lambda}\cos\left(\theta - \frac{BW}{2}\right)$ & $f_{max} = \frac{2V_{ac}}{\lambda}\cos\left(\theta + \frac{BW}{2}\right)$

Amplitude Distribution

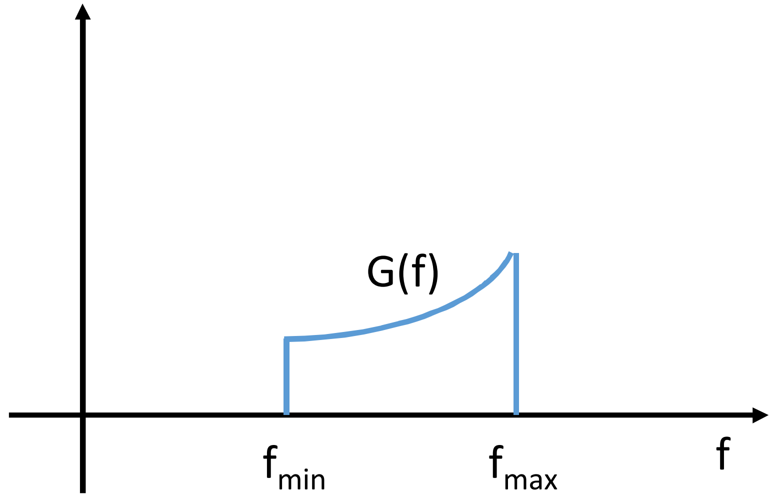
- $A = \sum_i A_i = \sqrt{P_t}\frac{\pi|K_w|G}{4\lambda}\sum_i \frac{D_i^3}{R_i^2}$



- Assume small range resolution
  - R_i = R for all i
- $A = \sqrt{P_t}\frac{\pi|K_w|G}{4\lambda R^2} V \int_0^\infty D^3 N(D) dD$
- $A = \sqrt{P_t}\frac{3\pi^2 BW^2 c\tau|K_w|G N_0}{32(\ln 2)\lambda R^2 \Lambda^4}$

Frequency Domain

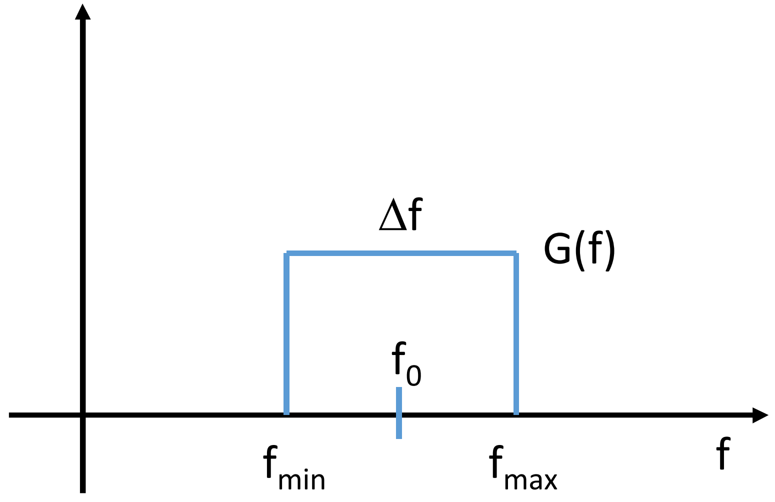
- $G(f) = Ap(f) = \frac{A\lambda}{2BWV_{ac}\sqrt{1-\left(\frac{f\lambda}{2V_{ac}}\right)^2}}$



- Inverse Fourier transform
- $g(t) = \int_{-\infty}^{\infty} G(f) e^{2\pi jft} df$
- $g(t) = \frac{A\lambda}{2BWV_{ac}} \int_{f_{min}}^{f_{max}} \frac{e^{2\pi jft}}{\sqrt{1-\left(\frac{f\lambda}{2V_{ac}}\right)^2}} df$

Time Domain

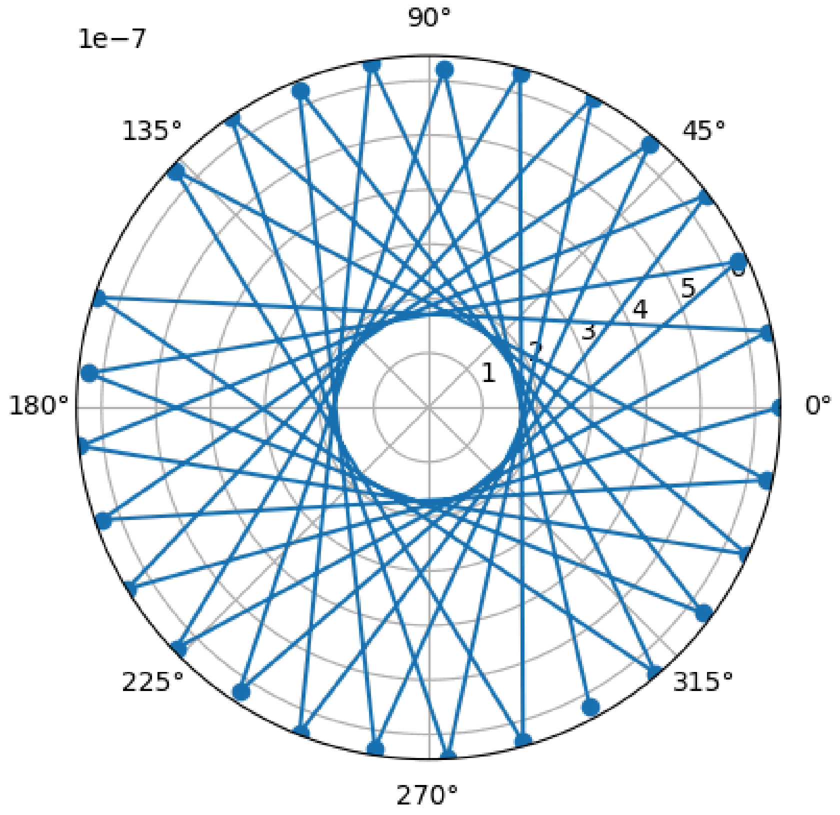
- Small Angle Approximation
  - $G(f) = Arect(F)$
  - $F = \frac{f-f_0}{\Delta f}$
- Inverse Fourier transform
- $g(t) = \int_{-\infty}^{\infty} G(f)e^{2\pi jft}df$
- $g(t) = A\int_{-\frac{1}{2}}^{\frac{1}{2}} e^{2\pi j(F\Delta f+f_0)t}dF$
- $g(t) = Asinc(\Delta ft)e^{2\pi jf_0t}$

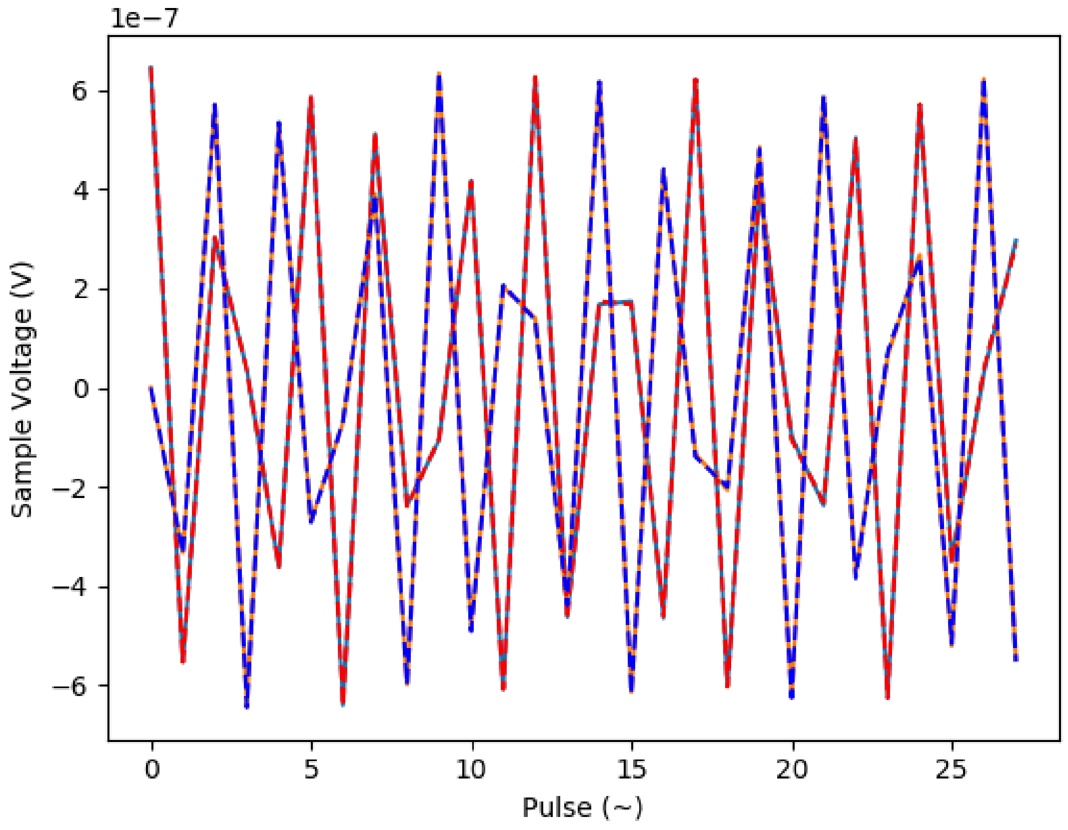


$f_0=(f_{min} + f_{max})/2$

$\Delta f= f_{max} - f_{min}$

Particle Simulation Results

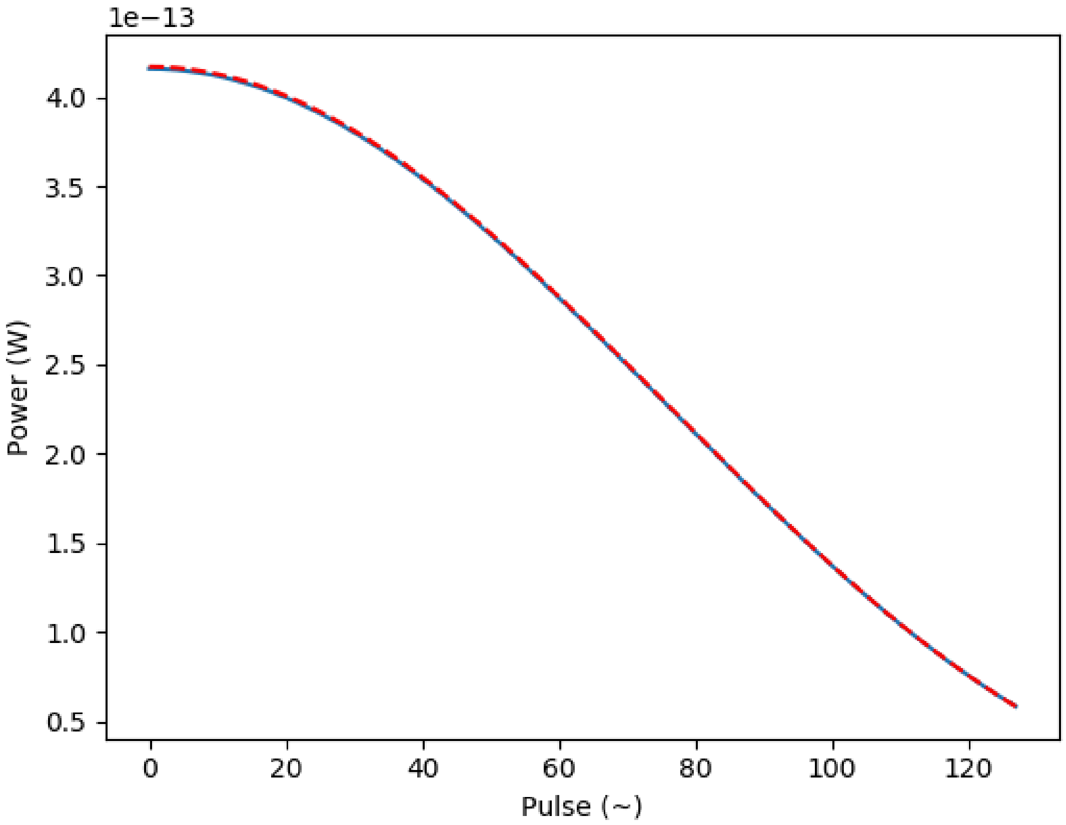


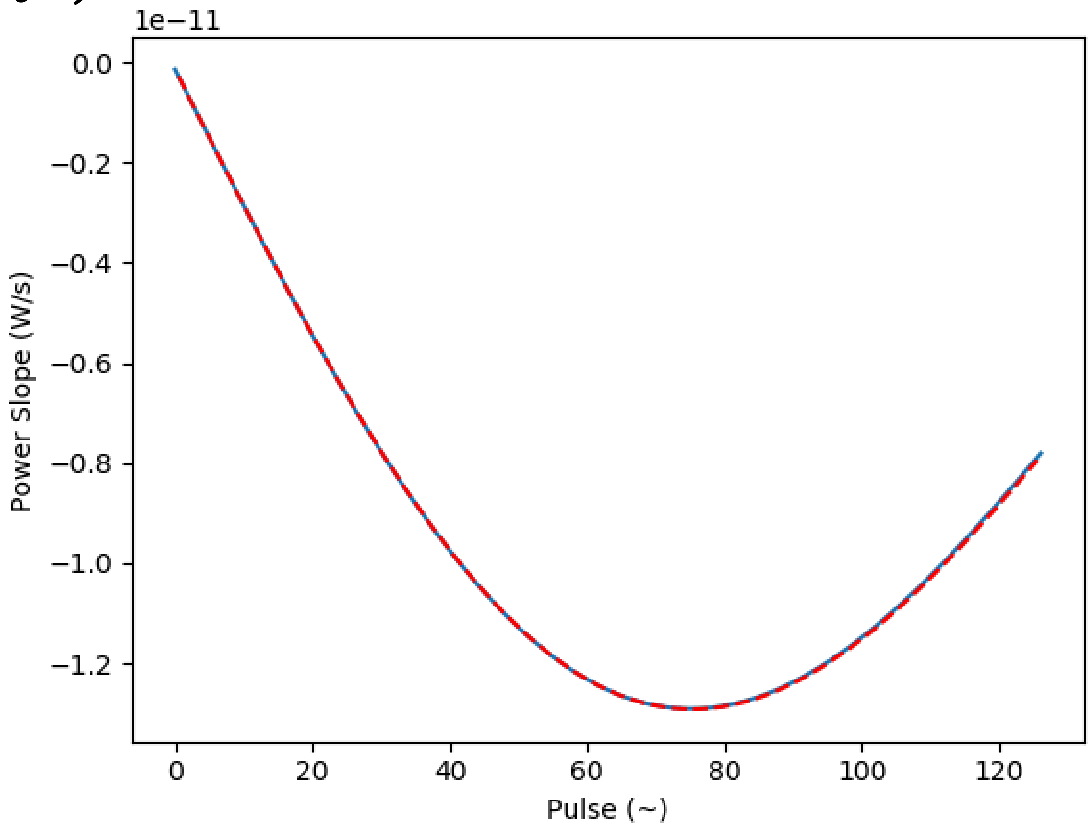


Power Fluctuations

$$P(t) = |g(t)|^2 = A^2 sinc^2(\Delta f t)$$

- $\frac{dP}{dt} = \left(\frac{2A^2}{t}\right) sinc(\Delta f t)[\cos(\pi \Delta f t) - sinc(\Delta f t)]$





Assessment of HIWC PSD

- Mass =$\frac{\pi \rho N_0}{\Lambda^4}$
- RRF = $\frac{N_0(6!)}{\Lambda^7}$

- Consider HIWC and non-HIWC conditions with equivalent RRF

- $\frac{Mass_{HIWC}}{Mass_{non-HIWC}} = \left(\frac{N_{0_{HIWC}}}{N_{0_{non-HIWC}}}\right)^{\frac{3}{7}}$

Rate of Change in Received Power

- $\frac{P_{HIWC}}{P_{non-HIWC}} = \left(\frac{A_{HIWC}}{A_{non-HIWC}}\right)^2 = \left(\frac{N_{0_{HIWC}}}{N_{0_{non-HIWC}}}\right)^{\frac{3}{7}}$
- $\frac{\left(\frac{dP}{dt}\right)_{HIWC}}{\left(\frac{dP}{dt}\right)_{non-HIWC}} = \left(\frac{A_{HIWC}}{A_{non-HIWC}}\right)^2 = \left(\frac{N_{0_{HIWC}}}{N_{0_{non-HIWC}}}\right)^{\frac{6}{7}}$

Particle Simulation

- Non-HIWC
  - N_0 = 8000 m^{-3} mm^{-1} (Marshall-Palmer)
  - RRF = 26 dBZ
  - Mass = 0.1 g m^{-3}
- HIWC
  - $1000N_0$
  - Mass = 2.1 g m^{-3}
- $\frac{Mass_{HIWC}}{Mass_{non-HIWC}} = (1000)^{\frac{3}{7}} = 19$
- $\frac{\left(\frac{dP}{dt}\right)_{HIWC}}{\left(\frac{dP}{dt}\right)_{non-HIWC}} = (1000)^{\frac{6}{7}} = 372$

Agenda

- ADWRS modeling approach
- Detailed particle model
  - Effects of angle resolution
  - Effects of range resolution
  - Random vs stochastic processes
  - Simulating HIWC
- Findings and Results
- Conclusions and next steps

Random vs Stochastic Process

- Random Processes
  - Range bin to Range bin are independent samples
  - Independent resolution volumes
- Stochastic Processes
  - Random variation in time
  - Pulse to pulse within a CPI?
- PRI = O(0.1 ms)
  - At 400 kts, aircraft changes position by > λ between pulses
  - However, relative particle motion small

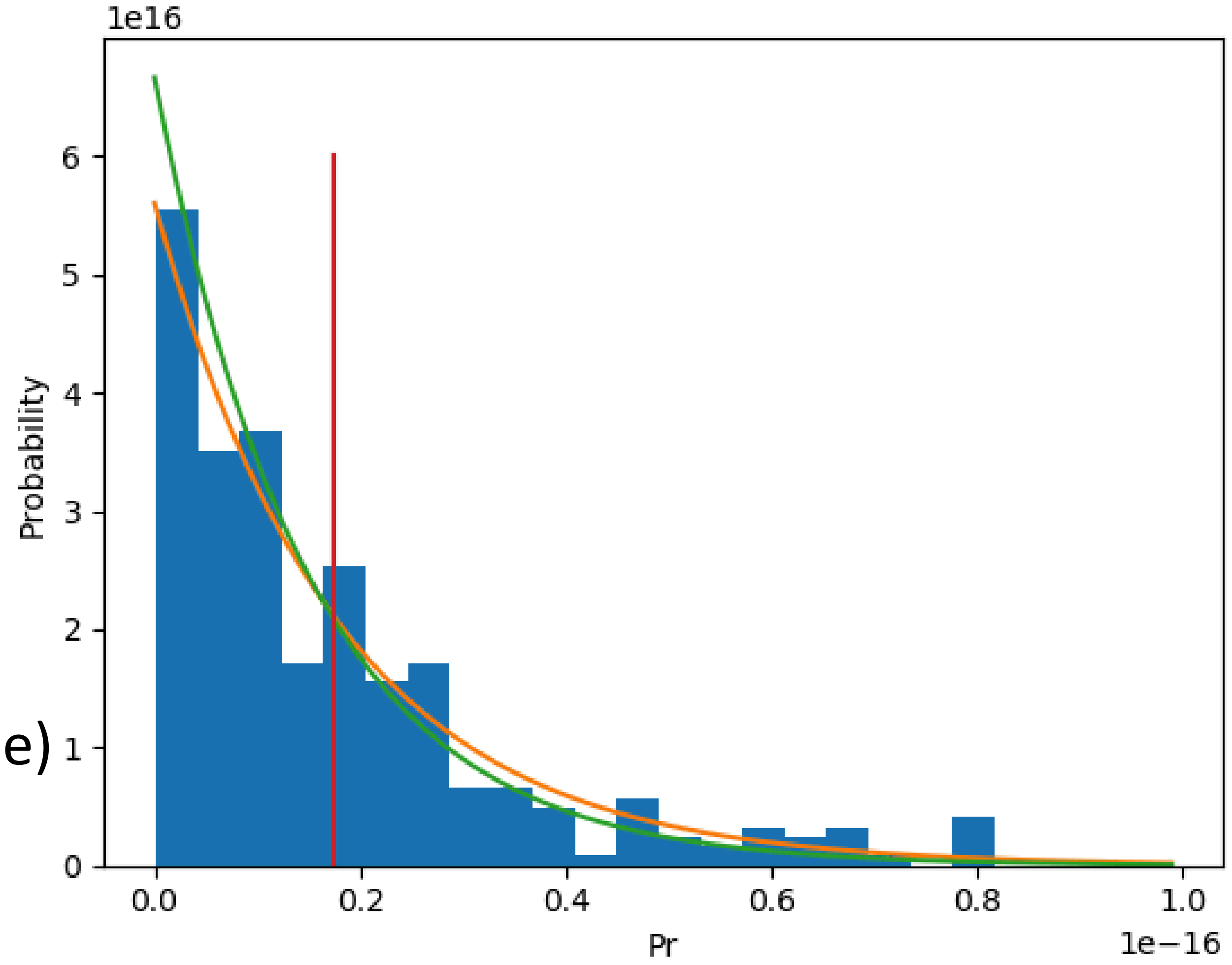
Fluctuation Echo from Randomly Distributed Scatterers

- Independent Samples
- Rayleigh Distribution of Signal Magnitude
  - Marshall and Hitschfield (1953)
  - Doviak and Zrnic (4.2)
- Exponential Power Distribution
  - $p(P) = \lambda e^{-\lambda P}$
  - Where rate parameter $\lambda = \frac{1}{\langle P_r \rangle}$
- Mean = $\frac{1}{\lambda}$
- Variance = $\frac{1}{\lambda^2}$

- $I_D = \frac{Var}{Mean} = \frac{1}{\lambda}$ = mean
- RIWC ≈ $\log_{10}(I_D) = \log_{10}(RRF)$

Particle Simulation

- 300 independent samples
- RIWC
  - I_D equivalent to mean RRF
  - Insensitive to IWC
- Flight data
  - I_D varies widely from mean RRF
  - Sensitive to IWC
  - Samples within a CPI (pulse-to-pulse) are not independent



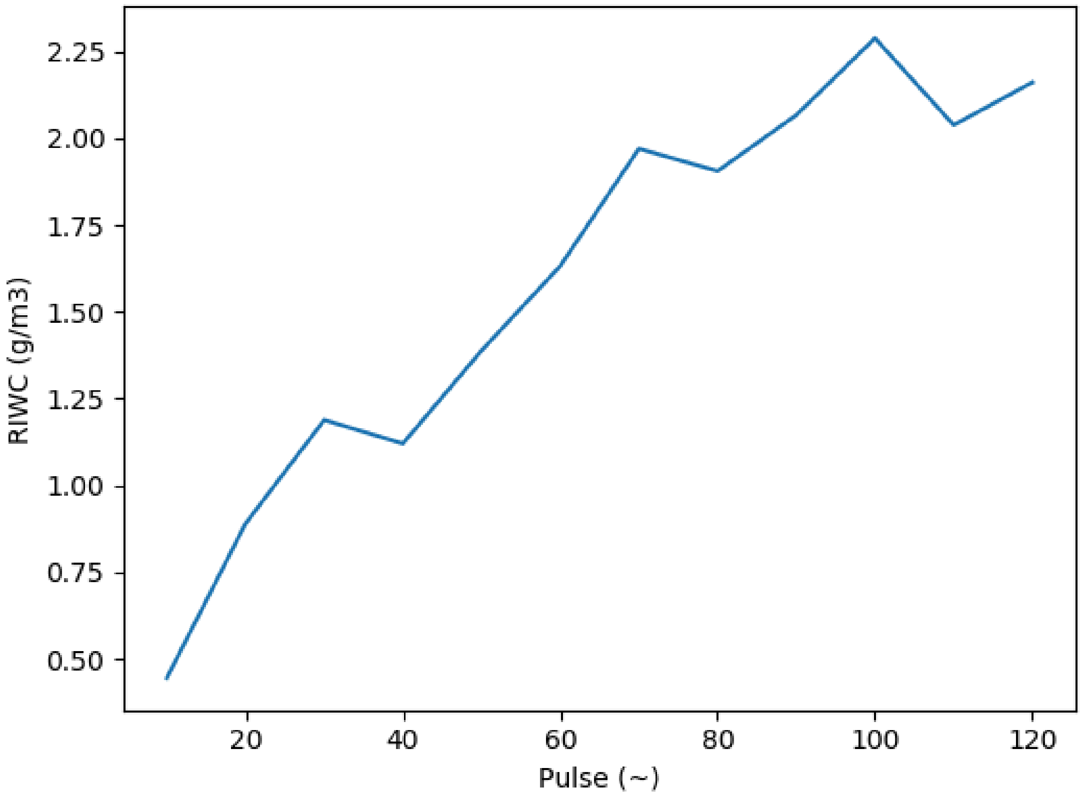
Agenda

- ADWRS modeling approach
- Detailed particle model
    - Effects of angle resolution
    - Effects of range resolution
    - Random vs stochastic processes
    - Simulating HIWC
- Findings and Results
- Conclusions and next steps

Particle Simulation

- Full sized resolution volume
  - Particles randomly distributed in angle and range
  - Uniform density
- Monte-Carlo runs
  - Multiple CPI’s in a measurement
    - Beamwidth averaging, spatial filtering
  - Mean values from n pulses in a CPI
  - Mean values from 10 CPI’s

Effect of CPI Duration



Simulating HIWC Conditions

- Mean values from 8 pulses in a CPI
- Non-HIWC condition
  - RRF 25-29 dBZ, RIWC ≈0 g m^{-3}
- HIWC conditions
  - RRF 25-27 dBZ, RIWC ≈0 g m^{-3}

Findings and Results

- Detailed particle model verifies ADWRS approximation
  - Independent samples in random process follow exponential distribution
- Theoretical solution assuming small range extent
  - Suggests variation in stochastic process should increase with N_0
- Detailed particle model and ADWRS output
  - Coherency de-correlates with time (long duration CPI)
- Detailed particle model HIWC simulation
  - variation in stochastic process insensitive to N_0

Conclusions and Next Steps

- Doppler distribution due to beamwidth results in “slow” power fluctuations
  - Return signals decorrelate over long durations
- Uniformly distributed particles do not result in “rapid” power fluctuations with increasing mass
  - Return signals remain equally correlated regardless of PSD
- Observed power variation must come from another source
  - Non-uniformly distributed particles?
- Investigate effect of clustering