

# Mechanisms for transition over blunt cones at hypersonic speeds

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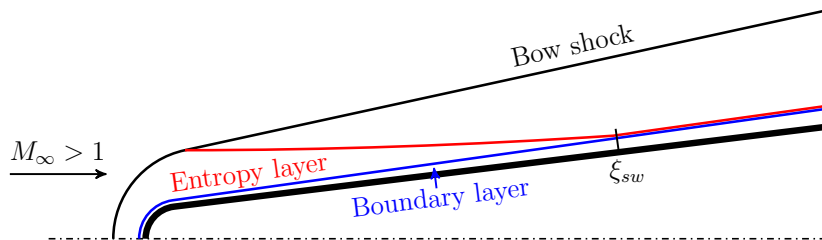
# Outline

- 1 Overview of Transition Measurements over Blunt Cones
- 2 Computational Analysis of Transition over Blunt Cones
  - Regime 2: Nonmodal growth of disturbances over the frustum
  - Regime 3: Nose-tip roughness effects on blunt cones
- 3 Summary and Concluding Remarks

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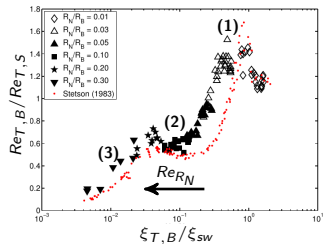
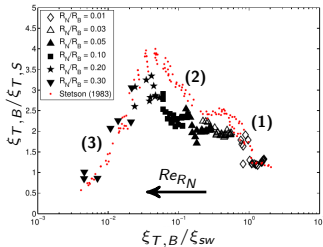
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# Flow over blunt cones at supersonic/hypersonic speeds



# Transition measurements over blunt cones<sup>1,2</sup>

- The three Stetson regimes

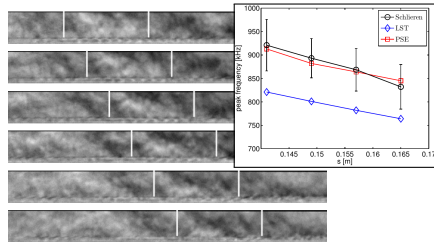
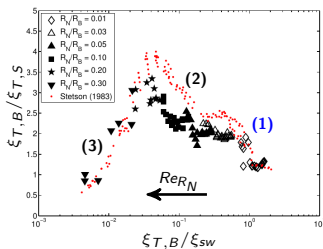


<sup>1</sup>K.F. Stetson. *Nosetip bluntness effects on cone frustum boundary layer transition in hypersonic flow.* AIAA Paper 83-1763. 1983.

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# Transition measurements over blunt cones<sup>1,2</sup>

## • The three Stetson regimes



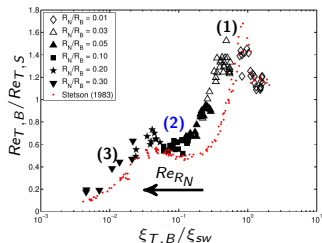
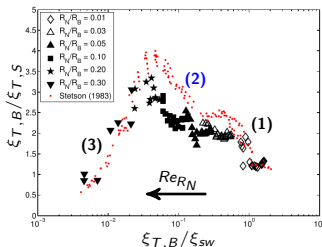
- ①  $\xi_{T,B}/\xi_{sw} \approx 1$ : MM waves dominant, but “stabilizing effect of nose-tip bluntness”

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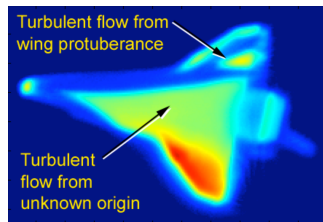
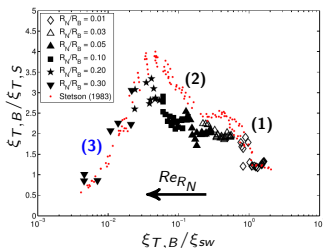
- ①  $\xi_{T,B}/\xi_{sw} \approx 1$ : MM waves dominant, but “stabilizing effect of nose-tip bluntness”
- ②  $\xi_{T,B}/\xi_{sw} \approx 0.1$ : “some unidentified effect was dominant” &  $Re_{T,B} < Re_{T,S}$ 
  - ▶ “ $\xi_T$  is easily determined, repeatable (for this wind tunnel)”

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(Horvath et al. AIAA 2010-241)

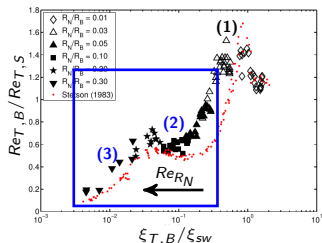
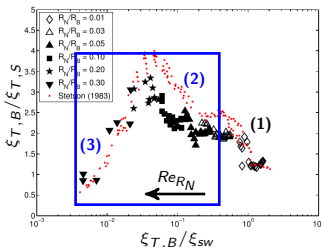
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  - ▶ “ $\xi_T$  is easily determined, repeatable (for this wind tunnel)”
- ③  $\xi_{T,B}/\xi_{sw} < 0.03$ : “dominated by nose-tip flow and surface conditions”
  - ▶ “ $\xi_T$  is unpredictable and not always well defined with long transition region”

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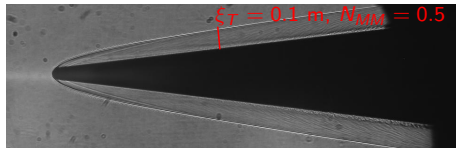
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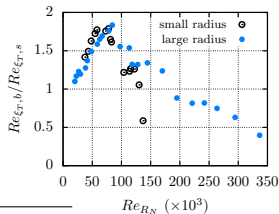
<sup>2</sup>J.S. Jewell et al. *Transition on a variable bluntness 7-degree cone at high Reynolds number*. AIAA Paper 2018-1822. 2018.

## Regime 2: Transition measurements over blunt cones

- Shadowgraph taken at AEDC Ballistic Range G in quiescent environment:<sup>3</sup>
  - Blunt - Regime 2 ( $M_\infty = 10$ ,  $Re_{R_N} = 232000$ )



- Transition locations measured during Reentry-F flight experiment:<sup>4</sup>
  - Sharp & Blunt - Regime 1-2 ( $M_\infty = 20$ ,  $Re_{R_N} = 20204 - 336550$ )



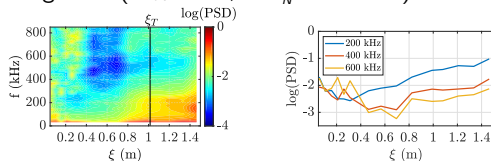
<sup>3</sup>T. Swanson and D. Daniel. *Hypersonic boundary layer transition experiments in hypervelocity ballistic range G.* . AIAA Paper 2016-2118. 2016.

<sup>4</sup>R.L. Wright and E.V. Zoby. *Flight boundary layer transition measurements on a slender cone at Mach 20.* AIAA Paper 77-719. 1977.

## Regime 2: Instability measurements over blunt cones

- PCB measurements in AEDC Tunnel 9:<sup>6</sup>

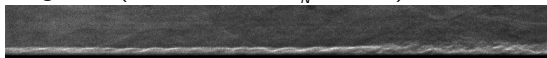
- ▶ Blunt - Regime 2 ( $M_\infty = 10$ ,  $Re_{R_N} = 215900$ )



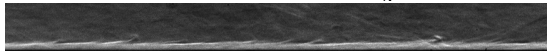
- ▶ Increased spectral amplitudes, then decrease, and finally increase

- LIF-based schlieren measurements at VKI Longshot hypersonic tunnel:<sup>7</sup>

- ▶ Sharp - Regime 1 ( $M_\infty = 11.9$ ,  $Re_{R_N} = 9275$ )



- ▶ Blunt - Regime (early) 2: ( $M_\infty = 11.9$ ,  $Re_{R_N} = 55100$ )



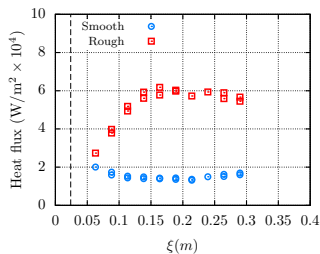
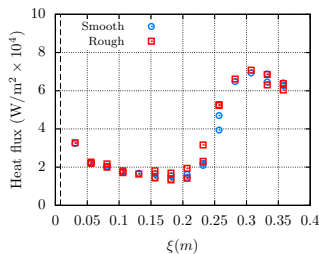
- ▶ Disturbances extending above the BL edge only observed for blunt cone

<sup>6</sup>E.C. Marineau et al. *Mach 10 boundary-layer transition experiments on sharp and blunted cones*. AIAA Paper 2014-3108. 2014.

<sup>7</sup>G. Grossir et al. *Hypersonic boundary layer transition on a 7 degree half-angle cone at Mach 10*. AIAA Paper 2014-2774. 2014.

# Regime 3: Effect of nose bluntness at fixed $Re_{kk} \approx 300$

- AFRL Mach 6 High Reynolds number facility
- $7^\circ$  half-angle cone at zero AoA;  $R_N = 5.080, 15.24$  mm
- Array of  $15\ \mu\text{m}$  cubical, laser-machined roughness elements at  $\phi = 45^\circ$ 
  - $R_N = 5.08$  mm,  $m = 210$ ,  $k/\delta_h = 0.381$
  - $R_N = 15.24$  mm,  $m = 420$ ,  $k/\delta_h = 0.225$



- **Small bluntness:** Roughness has no effect on  $\xi_T$
- **Large bluntness:** Rough tip moves  $\xi_T$  from the end of the cone to  $\approx \xi_j$
- **Transition onset associated with combined effects of bluntness and roughness**

# Outline

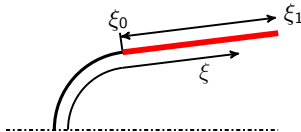
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# Computational Analysis of Transition over Blunt Cones

Regime 2: Nonmodal growth of disturbances over the frustum

# Methodology

- Linear nonmodal growth analysis<sup>8</sup>
  - ▶ linear PSE (LPSE)
  - ▶ variational formulation using direct and adjoint equations
  - ▶ initial location ( $\xi_0$ ) selected close to the juncture
  - ▶ final location ( $\xi_1$ ) selected at the measured  $\xi_T$
  - ▶ parametric analysis w.r.t. azimuthal wavenumber ( $m$ ) & frequency ( $F$ )



- Nonlinear development of nonmodal disturbances<sup>9</sup>
  - ▶ nonlinear PSE (NPSE) and 7<sup>th</sup>-order WENO DNS
  - ▶ linear nonmodal perturbations used as inflow near the nose tip

<sup>8</sup>P. Paredes et al. "Nonmodal growth of traveling waves on blunt cones at hypersonic speeds". In: *AIAA Journal* (accepted) (2019).

<sup>9</sup>P. Paredes, M.M. Choudhari, and F. Li. "Mechanism for frustum transition over blunt cones at hypersonic speeds". In: *Journal of Fluid Mechanics* (in review) (2019).

# AFRL blunt cones

- Basic state computed with the DPLR solver by Jewell & Kimmel<sup>10</sup>
- 7° half-angle cone at zero AoA with AFRL Mach 6 facility conditions
  - $M_\infty = 5.9$ ,  $T_\infty = 76.74$  K,  $T_w = 300.0$  K,  $T_w/T_{w,ad} = 0.57$

| Case | Regime | $R_N$ [mm] | $Re_\infty [\times 10^6 \text{ m}^{-1}]$ | $Re_{R_N} [\times 10^3]$ | $\xi_T$ [m] | $N_{MM}(\xi_T)$ |
|------|--------|------------|------------------------------------------|--------------------------|-------------|-----------------|
| I    | 1      | 0.508      | 91.5                                     | 46.48                    | 0.103       | 7.77            |
| II   | 1-2    | 1.524      | 30.5                                     | 46.48                    | 0.223       | 3.75            |
| III  | 2      | 1.524      | 61.0                                     | 92.96                    | 0.170       | 1.14            |
| IV   | 2      | 1.524      | 91.5                                     | 139.4                    | 0.133       | 0.04            |
| V    | 2      | 2.540      | 91.5                                     | 232.4                    | 0.147       | —               |
| VI   | 2      | 5.080      | 91.5                                     | 464.8                    | 0.161       | —               |

- Summary of linear nonmodal analysis for Cases II, III and IV
- Further linear nonmodal results for Cases II-VI can be found in Paredes et al.<sup>11</sup>

<sup>10</sup>J.S. Jewell and R.L. Kimmel. "Boundary layer stability analysis for Stetson's Mach 6 blunt cone experiments". In: *Journal of Spacecraft Rockets* 54.1 (2017), pp. 258–265.

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| <b>IV</b> | <b>2</b> | <b>1.524</b> | <b>91.5</b>                              | <b>139.4</b>             | <b>0.133</b> | <b>0.04</b>     |
| V         | 2        | 2.540        | 91.5                                     | 232.4                    | 0.147        | —               |
| VI        | 2        | 5.080        | 91.5                                     | 464.8                    | 0.161        | —               |

- Nonlinear analysis for Case IV in absence of Mack mode growth

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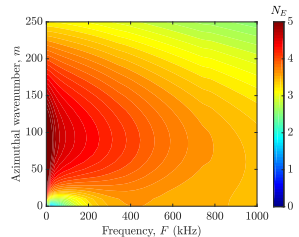
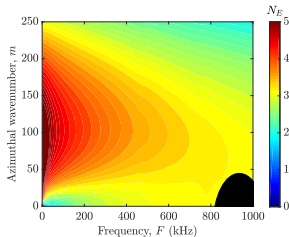
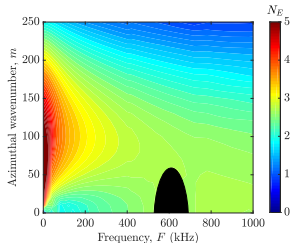
# Effect of $Re_\infty$ for fixed $R_N = 1.524$ mm

- Amplification in terms of  $N_E = 1/2 \log(G_E^{out})$

- $Re_{\infty,S} = 30.5 \times 10^6 \text{ m}^{-1}$ ,  
 $(\xi_0; \xi_1) = (0.014; 0.223) \text{ m}$

- $Re_{\infty,M} = 61.0 \times 10^6 \text{ m}^{-1}$ ,  
 $(\xi_0; \xi_1) = (0.014; 0.170) \text{ m}$

- $Re_{\infty,L} = 91.5 \times 10^6 \text{ m}^{-1}$ ,  
 $(\xi_0; \xi_1) = (0.014; 0.133) \text{ m}$



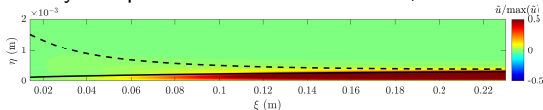
- As  $Re_\infty$  (or  $Re_{R_N}$ ) increases

- ▶ nonmodal growth persists up to increasingly higher  $F$
- ▶ peak for 2D waves appears after MM waves become stable

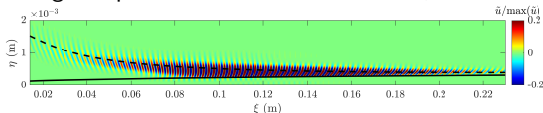
# Linear nonmodal disturbances<sup>11</sup>

$$R_N = 1.524 \text{ mm}, Re_{\infty,L} = 91.5 \times 10^6 \text{ m}^{-1}, \xi_T = 0.133 \text{ m}$$

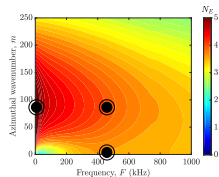
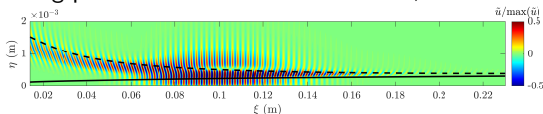
- Stationary oblique disturbance:  $F = 0 \text{ kHz}$ ,  $m = 80$



- Traveling oblique disturbance:  $F = 420 \text{ kHz}$ ,  $m = 80$



- Traveling planar disturbance:  $F = 420 \text{ kHz}$ ,  $m = 0$



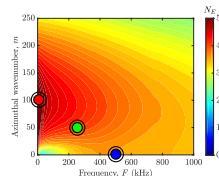
- Stationary oblique disturbances: large  $\tilde{T}$  and  $\tilde{u}$  growth & contained within BL
- Traveling oblique disturbances: large  $\tilde{T}$  and  $\tilde{u}$  growth & do not penetrate BL
- Traveling planar disturbances: large  $\tilde{T}$ , small  $\tilde{u}$  growth & penetrate BL

<sup>11</sup>P. Paredes et al. "Nonmodal growth of traveling waves on blunt cones at hypersonic speeds". In: *AIAA Journal* (accepted) (2019).

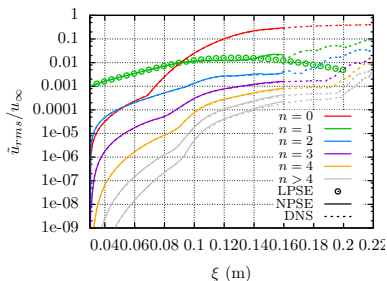
# Oblique traveling waves

$$R_N = 1.524 \text{ mm}, Re_\infty = 91.5 \times 10^6 \text{ m}^{-1}, \xi_T = 0.133 \text{ m}$$

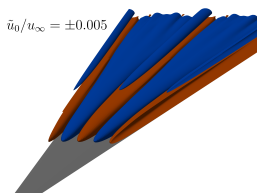
- Initial disturbance:  $F = 250 \text{ kHz}$ ,  $m = \pm 50$
- Baseline amplitude:  $\tilde{u}_{0,b} = 1.0083 \times 10^{-3} \bar{u}_\infty$



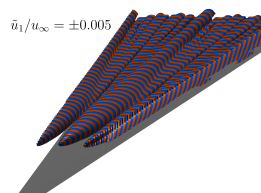
$$\tilde{u}_{rms} = \tilde{u}_{0,b}$$



$$F_0 = 0 \text{ kHz}$$



$$F_1 = 250 \text{ kHz}$$



- Excellent agreement between NPSE and DNS predictions
- Disturbances penetrate BL through generation of streaks  $(F, m) = (0, 100)$
- Possible route to transition via excitation of higher harmonics inside BL

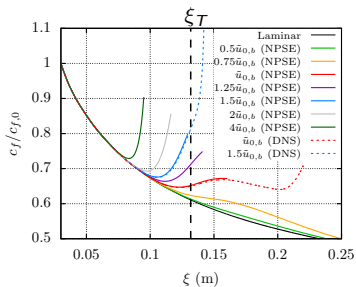
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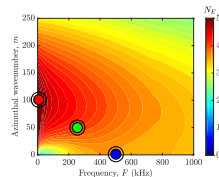
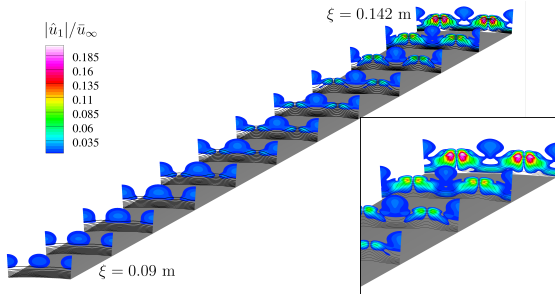
- Initial disturbance:  $F = 250 \text{ kHz}$ ,  $m = \pm 50$

$$\tilde{u}_{rms} = 1.5\tilde{u}_{0,b} = 1.5124 \times 10^{-3} \tilde{u}_\infty$$

- Skin friction



- $\tilde{u}_{rms} = 1.5\tilde{u}_{0,b}$ ,  $\bar{u} + \tilde{u}_0$  isolines &  $\tilde{u}_1$  isocontours

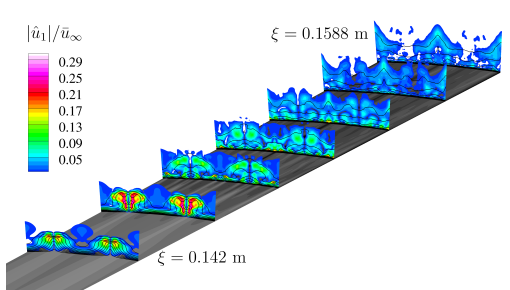
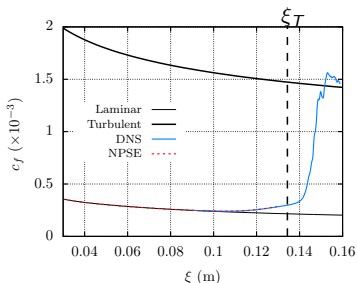
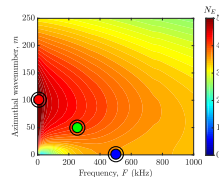


- Initial wall shear rise due to stationary streaks
- Unsteady disturbances grow close to the wall leading to final rise of wall shear

# Oblique traveling waves

$$R_N = 1.524 \text{ mm}, Re_\infty = 91.5 \times 10^6 \text{ m}^{-1}, \xi_T = 0.133 \text{ m}$$

- Initial disturbance:  $F = 250 \text{ kHz}$ ,  $m = \pm 50$
- $\tilde{u}_{rms} = 1.5\tilde{u}_{0,b} = 1.5124 \times 10^{-3}\bar{u}_\infty$
- Skin friction
- $\bar{u} + \tilde{u}_0$  isolines &  $\tilde{u}_1$  isocontours

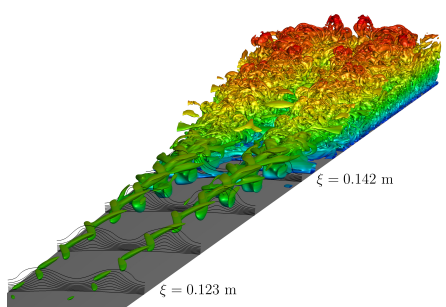
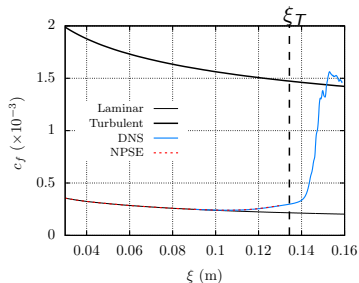
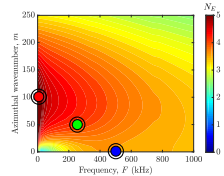


- Streak breakdown via azimuthally-subharmonic instability
- Skin-friction/heat-transfer turbulent levels are reached

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- Initial disturbance:  $F = 250 \text{ kHz}$ ,  $m = \pm 50$
- $\tilde{u}_{rms} = 1.5\tilde{u}_{0,b} = 1.5124 \times 10^{-3}\tilde{u}_\infty$
- Skin friction
- Instantaneous flow field:  $u$  isolines &  $Q$  isosurfaces

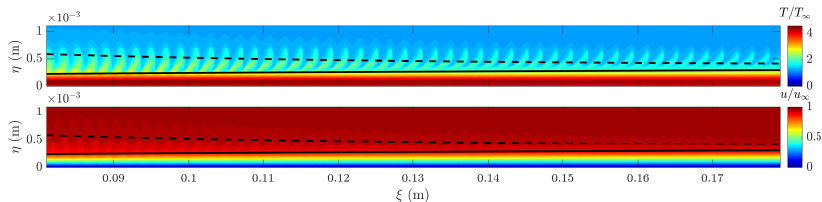
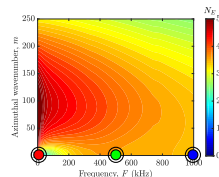


- Streak breakdown via azimuthally-subharmonic instability
- Skin-friction/heat-transfer turbulent levels are reached

# Planar traveling waves

$$R_N = 1.524 \text{ mm}, Re_\infty = 91.5 \times 10^6 \text{ m}^{-1}, \xi_T = 0.133 \text{ m}$$

- Initial disturbance:  $F = 500 \text{ kHz}$ ,  $m = 0$
- $\tilde{u}_{rms}/\bar{u}_\infty = 4.738 \times 10^{-3}$
- Contours of total flow components  $T$  and  $u$  (solid line:  $\delta_h$ , dashed line:  $\delta_S$ )

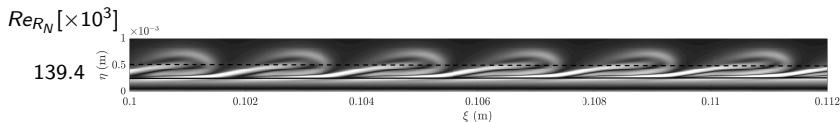
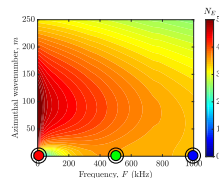


- Appearance of inclined structures above BL edge evident in  $T$  contours
- Inclined structures hardly distinguishable in  $u$  contours
- Do not appear to modify qualitative nature of oblique breakdown

# Planar traveling waves

$$R_N = 1.524 \text{ mm}, Re_\infty = 91.5 \times 10^6 \text{ m}^{-1}, \xi_T = 0.133 \text{ m}$$

- Initial disturbance:  $F = 500 \text{ kHz}$ ,  $m = 0$
- $\tilde{u}_{rms}/\bar{u}_\infty = 4.738 \times 10^{-3}$
- Contours of numerical schlieren (solid line:  $\delta_h$ , dashed line:  $\delta_S$ )



- Schlieren images taken by Kennedy et al.<sup>12</sup> (white line:  $\delta_h$ )

92.8



115.3



- Inclined structures strongly reminiscent of those observed in experiments

<sup>12</sup>R.E. Kennedy et al. *Visualizations of hypersonic boundary-layer transition on a variable bluntness cone.* AIAA Paper 2019-3079. 2019.

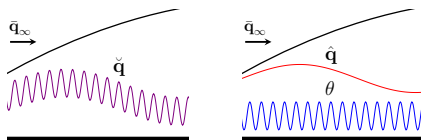
# Computational Analysis of Transition over Blunt Cones

Regime 3: Nose-tip roughness effects on blunt cones

# Methodology

- Instability analysis of modified BL flow: linear plane-marching PSE<sup>13</sup>
- Shear-layer instabilities ( $\omega \neq 0$  &  $\alpha \neq 0$ )
- Perturbation form:

$$\tilde{\mathbf{q}}(\xi, \eta, \zeta, t) = \hat{\mathbf{q}}(\xi, \eta, \zeta) \exp \left[ i \left( \int_{\xi} \alpha(\xi') d\xi' - \omega t \right) \right]$$



- Onset of transition estimated by using an  $N$ -factor criterion:

$$N_E(\xi) = \int_{\xi_0}^{\xi} \sigma_E(\xi') d\xi'; \quad \sigma_E(\xi) = -\alpha_i(\xi) + 1/2 \frac{d}{d\xi} \ln(\hat{E}(\xi)); \quad \sigma_E(\xi_0) = 0$$

<sup>13</sup>P. Paredes, "Advances in global instability computations: from incompressible to hypersonic flow". PhD thesis. Universidad Polit cnica de Madrid, 2014.

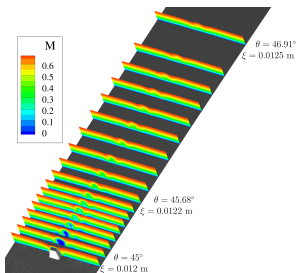
# Laminar solution with roughness elements at sonic location

$$R_N = 15.24 \text{ mm}, Re_{\infty,L} = 75.8 \times 10^6 \text{ m}^{-1}, m = 420$$

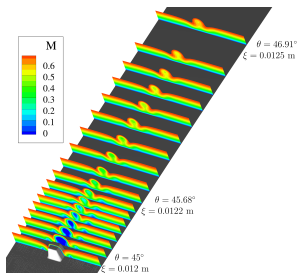
- Roughness array near sonic point at  $\phi = 45^\circ$

- 1 Cubic elements,  $k = 15 \text{ } \mu\text{m}$  ( $Re_{kk} = 336$ ,  $k/\delta_h = 0.2074$ , experiment)
- 2 Cubic elements,  $k = 20 \text{ } \mu\text{m}$  ( $Re_{kk} = 482$ ,  $k/\delta_h = 0.2765$ )
- 3 Rectangular prismatic elements,  $k = 15 \text{ } \mu\text{m}$ , 50% blockage

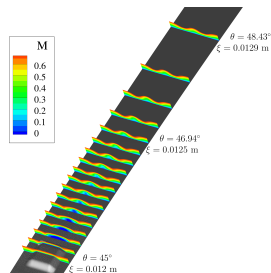
- Cubic,  $k = 15 \text{ } \mu\text{m}$



- Cubic,  $k = 20 \text{ } \mu\text{m}$



- Prismatic,  $k = 15 \text{ } \mu\text{m}$



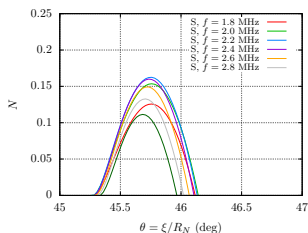
- **Cubic elements:** wake vanishes quickly & is equivalent to isolated elements
- **Prismatic elements:** longer extend of the wake as periodic streaks

# Modal instability analysis of roughness wakes

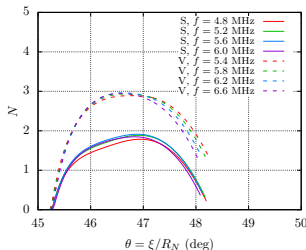
$$R_N = 15.24 \text{ mm}, Re_{\infty,L} = 75.8 \times 10^6 \text{ m}^{-1}, m = 420$$

- Roughness array near sonic point at  $\phi = 45^\circ$ 
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  - 3 Rectangular prismatic elements,  $k = 15 \text{ } \mu\text{m}$ , 50% blockage

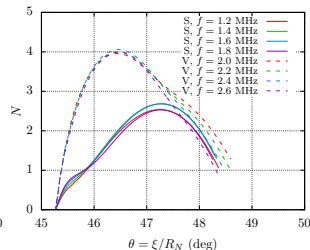
- Cubic,  $k = 15 \text{ } \mu\text{m}$



- Cubic,  $k = 20 \text{ } \mu\text{m}$



- Prismatic,  $k = 15 \text{ } \mu\text{m}$



- **Cubic elements:** marginally unstable for  $k = 15 \text{ } \mu\text{m}$  and  $\max(N)=3$  for  $20 \text{ } \mu\text{m}$
- **Prismatic elements:**  $\max(N)=4$  for  $k = 15 \text{ } \mu\text{m}$

# Outline

- 1 Overview of Transition Measurements over Blunt Cones
- 2 Computational Analysis of Transition over Blunt Cones
  - Regime 2: Nonmodal growth of disturbances over the frustum
  - Regime 3: Nose-tip roughness effects on blunt cones
- 3 Summary and Concluding Remarks

# Concluding remarks

## Regime 2: Mechanism for frustum transition

- significant nonmodal growth of 2D and 3D traveling disturbances
- traveling waves peak within entropy layer above boundary-layer edge
- oblique waves lead to generation of stationary streaks inside BL
- route to transition via streak breakdown & excitation of near-wall unsteady disturbances
- nonlinear interactions lead to formation of inclined structures visible in  $T$  &  $\rho$
- strongly reminiscent of the structures observed in the schlieren flow visualizations
- can accelerate the nonlinear growth of oblique waves

## Regime 3: Nose-tip roughness effects on transition reversal

- laminar wake distortion calculated for roughness elements at sonic location
- cubic roughness ( $15\ \mu\text{m}$ ) wake marginally unstable at experimental conditions
- prismatic roughness ( $15\ \mu\text{m}$ ) elements lead to larger  $N_{max} = 4$ .

# Thank you for your attention

## ● Acknowledgments

- ▶ This research was sponsored by the NASA Transformational Tools and Technologies (TTT) Project of the Transformative Aeronautics Concepts Program (TACP) and by the Hypersonic Technology Project (HTP) under the Aeronautics Research Mission Directorate (ARMD)
- ▶ Resources supporting this work were provided by the NASA High-End Computing (HEC) Program through the NASA Advanced Supercomputing (NAS) Division at Ames Research Center
- ▶ We thank Drs. Jewell, Kimmel, Laurence, Marineau, Chang, Corke, and Schneider, and Mr. Kennedy for various technical discussions related to this topic



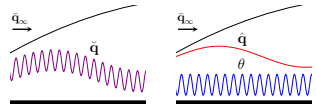
# Outline

## 4 Backup slides

# Linear stability analysis<sup>14</sup>

## Approaches

- Harmonic LNSE (HLNSE): Elliptic problem



$$\mathbf{L}_{HLNSE} \check{\mathbf{q}}(\xi, \eta) = \left( \mathbf{A} + \mathbf{B} \frac{\partial}{\partial \eta} + \mathbf{C} \frac{\partial^2}{\partial \eta^2} + \mathbf{D} \frac{\partial}{\partial \xi} + \mathbf{E} \frac{\partial^2}{\partial \xi \partial \eta} + \mathbf{F} \frac{\partial^2}{\partial \xi^2} \right) \check{\mathbf{q}}(\xi, \eta) = 0$$

- Reduced HLNSE (RHLNSE): Parabolic integration in  $\xi$

$$\mathbf{L}_{RHLNSE} \check{\mathbf{q}}(\xi, \eta) = \left( \mathbf{A} + \mathbf{B} \frac{\partial}{\partial \eta} + \mathbf{C} \frac{\partial^2}{\partial \eta^2} + \mathbf{D} \frac{\partial}{\partial \xi} + \mathbf{E} \frac{\partial^2}{\partial \xi \partial \eta} + \cancel{\mathbf{F} \frac{\partial^2}{\partial \xi^2}} \right) \check{\mathbf{q}}(\xi, \eta) = 0$$

- PSE: Parabolic integration in  $\xi$  coupled with normalization condition

$$\check{\mathbf{q}}(\xi, \eta) = \hat{\mathbf{q}}(\xi, \eta) \exp \left[ i \int_{\xi_0}^{\xi} \alpha d\xi' \right]; \mathbf{L}_{PSE}(\alpha) \hat{\mathbf{q}}(\xi, \eta) = 0 \ \& \ \int_{\eta} \hat{\mathbf{q}}^* \frac{\partial \hat{\mathbf{q}}}{\partial \xi} h_{\xi} h_{\zeta} d\eta = 0$$

- Streamwise pressure gradient:  $\frac{\partial \hat{p}}{\partial \xi} = i\alpha \hat{p} + \Omega_{PNS} \frac{\partial \hat{p}}{\partial \xi}$

- ▶ Stationary perturbation ( $\omega = 0$  &  $\alpha = 0$ ):  $\Omega_{PNS} = 0$  (RHLNS & PSE)
- ▶ Traveling perturbation ( $\omega \neq 0$ ):
  - ★ RHLNS:  $\alpha = 0$  &  $\Omega_{PNS} = \min(1, M_{\xi}^2)$
  - ★ PSE:  $\alpha \neq 0$  &  $\Omega_{PNS} = 1.0$

<sup>14</sup>P. Paredes et al. "Nosetip bluntness effects on transition at hypersonic speeds: experimental and numerical analysis". In: *Journal of Spacecraft Rockets* 56.2 (2019).

# Nonlinear Parabolized Stability Equations<sup>15</sup>

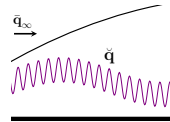
- Decomposition of flow variables

$$\mathbf{q}(\xi, \eta, \zeta, t) = \bar{\mathbf{q}}(\xi, \eta, \zeta) + \epsilon \tilde{\mathbf{q}}(\xi, \eta, \zeta, t); \quad \bar{\mathbf{q}} = \mathcal{O}(1); \quad \epsilon \ll 1$$

- Perturbation form

- Expand in terms of their truncated Fourier components in time

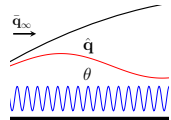
$$\tilde{\mathbf{q}}(\xi, \eta, \zeta, t) = \sum_{n=-N}^N \check{\mathbf{q}}_n(\xi, \eta, \zeta) \exp[-in\omega t]$$



- Separation of scales in streamwise direction

- Exploit slow variations in streamwise direction

$$\check{\mathbf{q}}_n(\xi, \eta, \zeta) = \hat{\mathbf{q}}_n(\xi, \eta, \zeta) \theta_n(\xi); \quad \theta_n(\xi) = \exp \left[ i \int_{\xi_0}^{\xi} \alpha_n(\xi') d\xi' \right]$$



- Nonlinear Plane-Marching PSE

$$\left( \mathbf{L}_n + \mathbf{M}_n \frac{\partial}{\partial \xi} \right) \hat{\mathbf{q}}_n(\xi, \eta, \zeta) = \mathbf{F}_n(\xi, \eta, \zeta) \exp \left( -i \int_{\xi_0}^{\xi} \alpha_n(\xi') d\xi' \right)$$

<sup>15</sup>P. Paredes et al. "The nonlinear PSE-3D concept for transition prediction in flows with a single slowly-varying spatial direction". In: *Procedia IUTAM* 14C (2015), pp. 35–44.

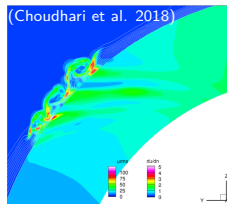
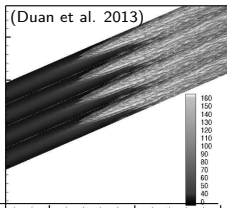
# Direct Numerical Simulations

## • Code

- ▶ Inviscid fluxes: 7<sup>th</sup>-order WENO (Jian & Shu 1996, Martin et al. 2006)
- ▶ Viscous fluxes: 4<sup>th</sup>-order central finite-difference scheme
- ▶ 3<sup>rd</sup>-order accurate low-storage R-K for time advancement (Williamson 1980)
- ▶ Previously validated for transitional and turbulent flow configurations (Duan et al. 2013, 2016, Choudhari et al. 2013)

## • Grid

- ▶  $N_\xi = 2600$ ,  $N_\eta = 301$ ,  $N_\zeta = 96 - 288$  (Total = 75 – 225 M)
- ▶ Increased streamwise resolution in strongly nonlinear region
- ▶ Grid designed on the basis of prior computation of second mode and crossflow instabilities in hypersonic boundary layers<sup>16</sup>



<sup>16</sup>M.M. Choudhari et al. *Nonlinear evolution and breakdown of azimuthally compact crossflow vortex pattern over a yawed cone.* AIAA Paper 2018-1823, 2018.