



Modeling Fatigue Crack Growth Using Symbolic Regression

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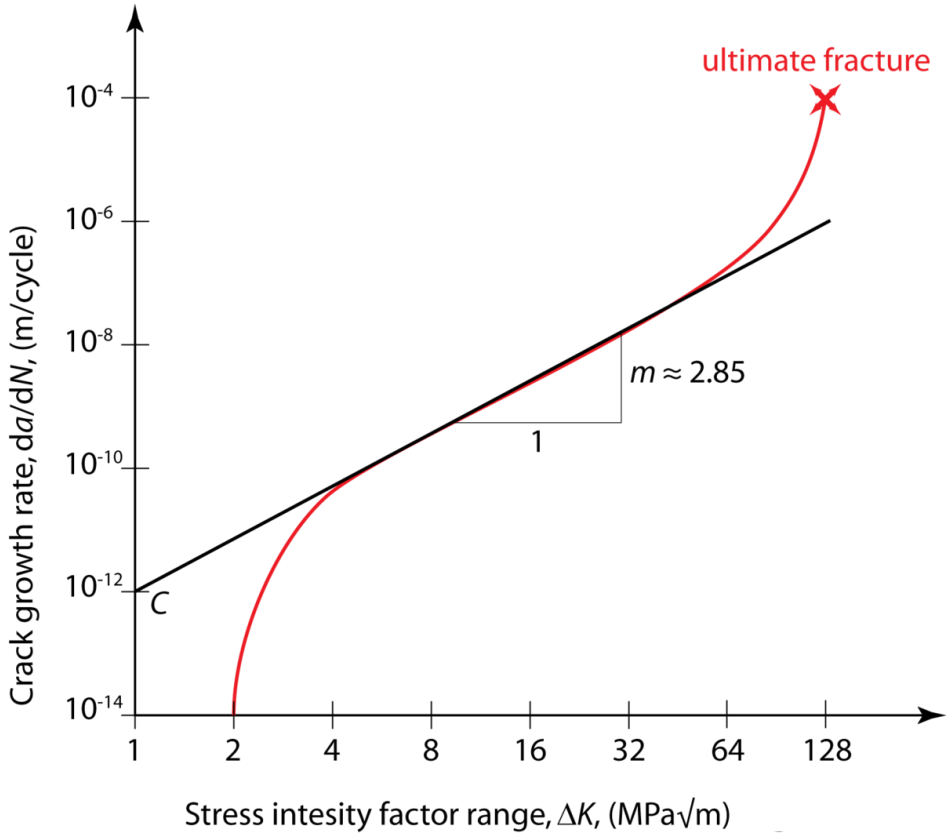
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Contents

- Background
- Objective
- Data Processing
- Results
- Conclusion
- Future Work

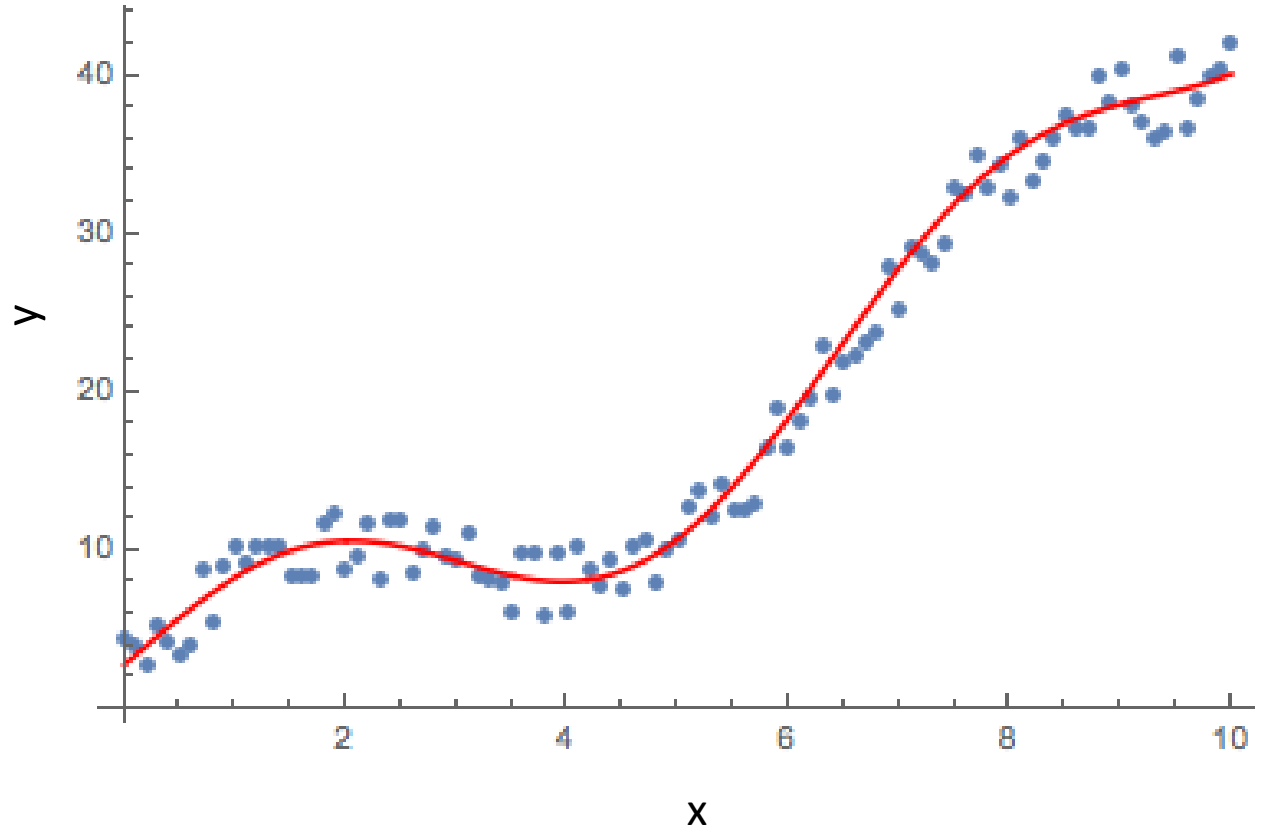
Background

- Fatigue Crack Growth is the process by which a crack extends when subjected to a cyclic load.
- Mostly rely on empirical fits to describe the crack growth rate
- Paris' law in linear regime shown
- NASGRO for full curve



Background

- Symbolic Regression is a regression method that searches the space of mathematical functions to find optimized model
- Allows closed-form solutions to be derived from inputted data.
- Can distill free-form natural laws from experimental data



Background

- Bingo is a NASA-produced-open-source symbolic regression software.
- Objective: Use Bingo to find a model that performs just as well or better than existing models to predict crack growth rate.



Data

Specimen Material	Nominal Parameters	Qualitative Parameters
2024-T3 CLAD AND BARE SHEET Direction: L-T	da/dN: crack growth rate (in/cycle) a: crack length (in) N: number of cycles ΔK: change in stress intensity factor (ksi √in) K_{max} : Maximum stress intensity factor K_{min} : Minimum stress intensity factor R: stress ratio	Type of Test: • Constant K_{max} • Constant ΔK • Constant R Environment: • LA/RT • UHV/RT

Splitting the train and test data:

- 80/20 split
- Train on all but one experiment, test on extracted experiment.

NASGRO

$$f = \frac{K_{op}}{K_{\max}} = \begin{cases} \max\left(R,\ A_0 + A_1 R + A_2 R^2 + A_3 R^3\right) & R \geq 0 \\ A_0 + A_1 R & -2 \leq R < 0 \end{cases}$$

$$A_0 = \left(0.825 - 0.34\alpha + 0.05\alpha^2\right)\left[\cos\left(\tfrac{\pi}{2} S_{\max} / \sigma_0\right)\right]^{1/\alpha}$$

$$A_1 = (0.415 - 0.071\alpha) S_{\max} / \sigma_0$$

$$A_2 = 1 - A_0 - A_1 - A_3$$

$$A_3 = 2A_0 + A_1 - 1.$$

$$\frac{da}{dN} = C\left[\left(\frac{1-f}{1-R}\right)\Delta K\right]^n \frac{\left(1 - \frac{\Delta K_{th}}{\Delta K}\right)^p}{\left(1 - \frac{K_{\max}}{K_c}\right)^q}$$

$$\Delta K_{th} = \Delta K_1^* \left[\frac{1-R}{1-f[R]}\right]^{(1+RC_{th}^p)} / (1-A_0)^{(1-R)C_{th}^p}, \quad R \geq 0$$

$$\Delta K_{th} = \Delta K_1^* \left[\frac{1-R}{1-f[R]}\right]^{(1+RC_{th}^m)} / (1-A_0)^{(C_{th}^p - RC_{th}^m)}, \quad R < 0$$

in which

$$\Delta K_1^* = \Delta K_1 \left[\frac{a}{a + a_0}\right]^{1/2}$$

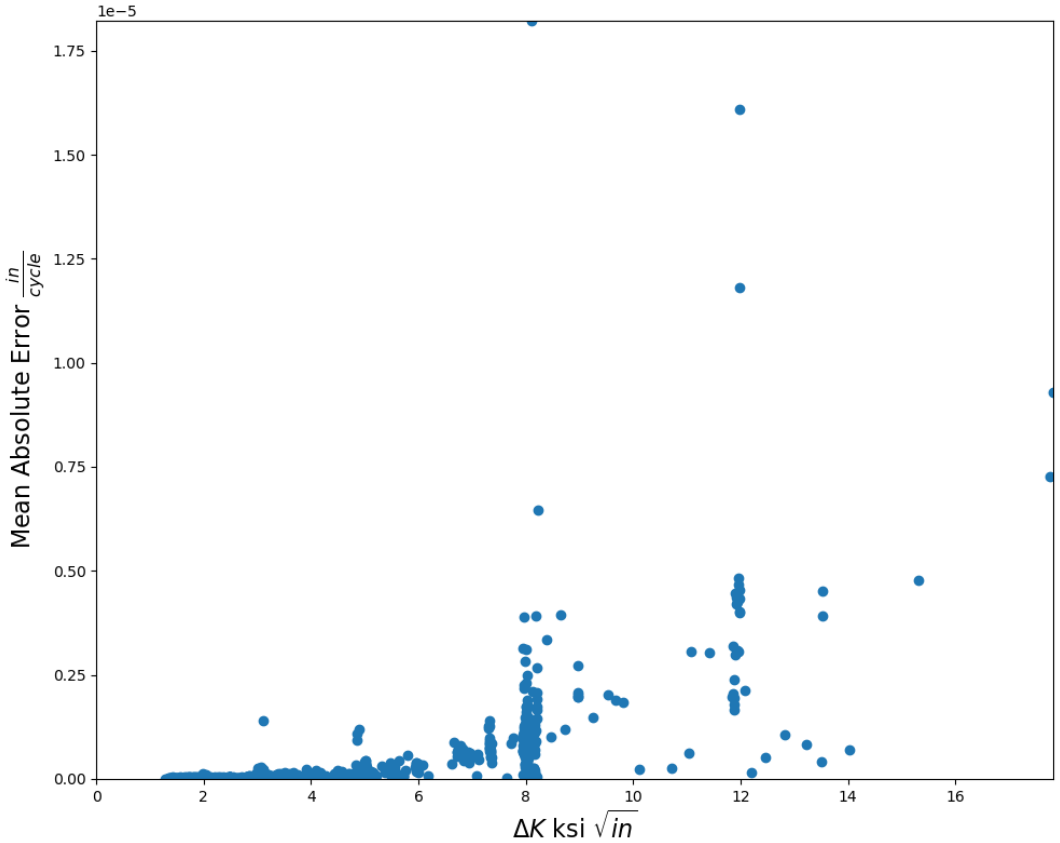
NASGRO

Model fit using 80% of data, tested on 20%

Constants found from fitting NASGRO:

- C = 7.8656e-09
- n = 3.113
- p = 0.2074
- q = 0.9524

Mean Absolute Error: 4.6969e-07



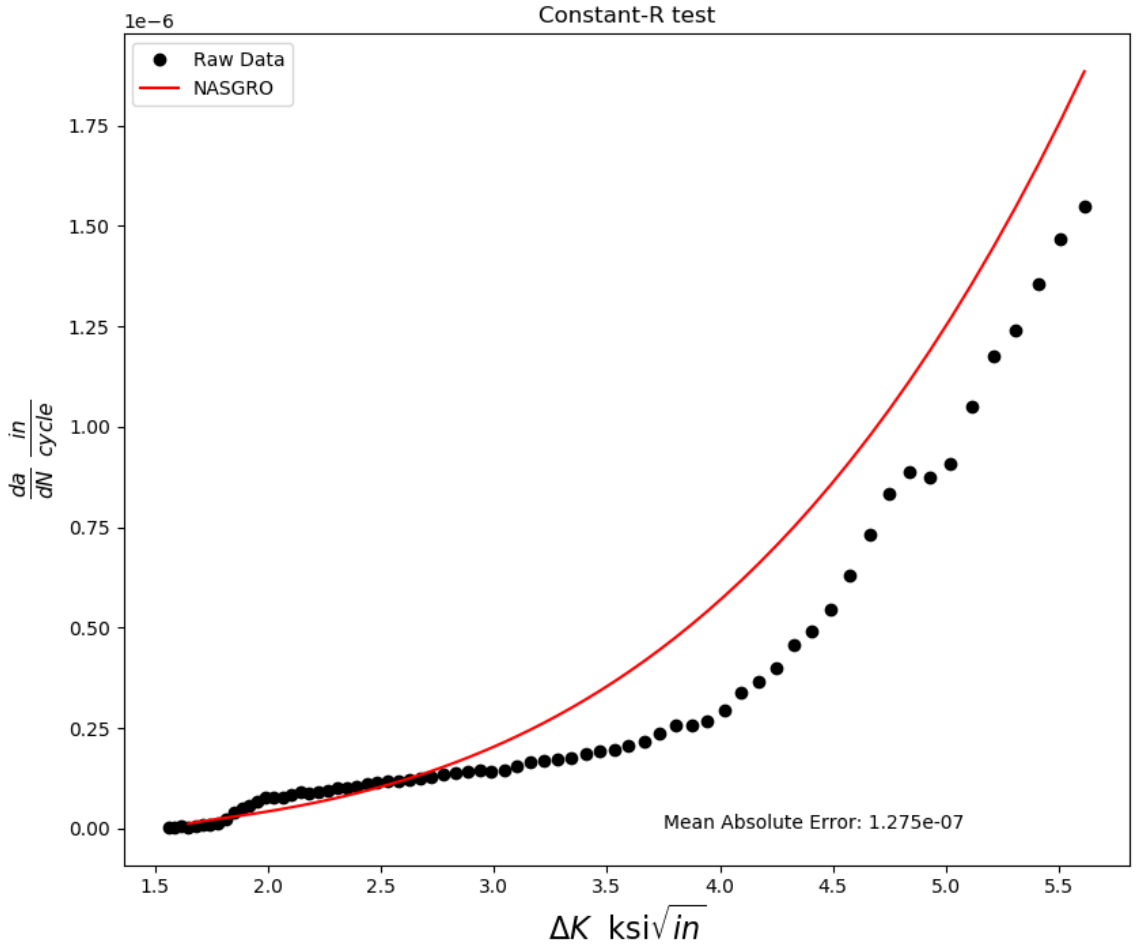
NASGRO

Model fit using all data except one test

Constants found from fitting NASGRO:

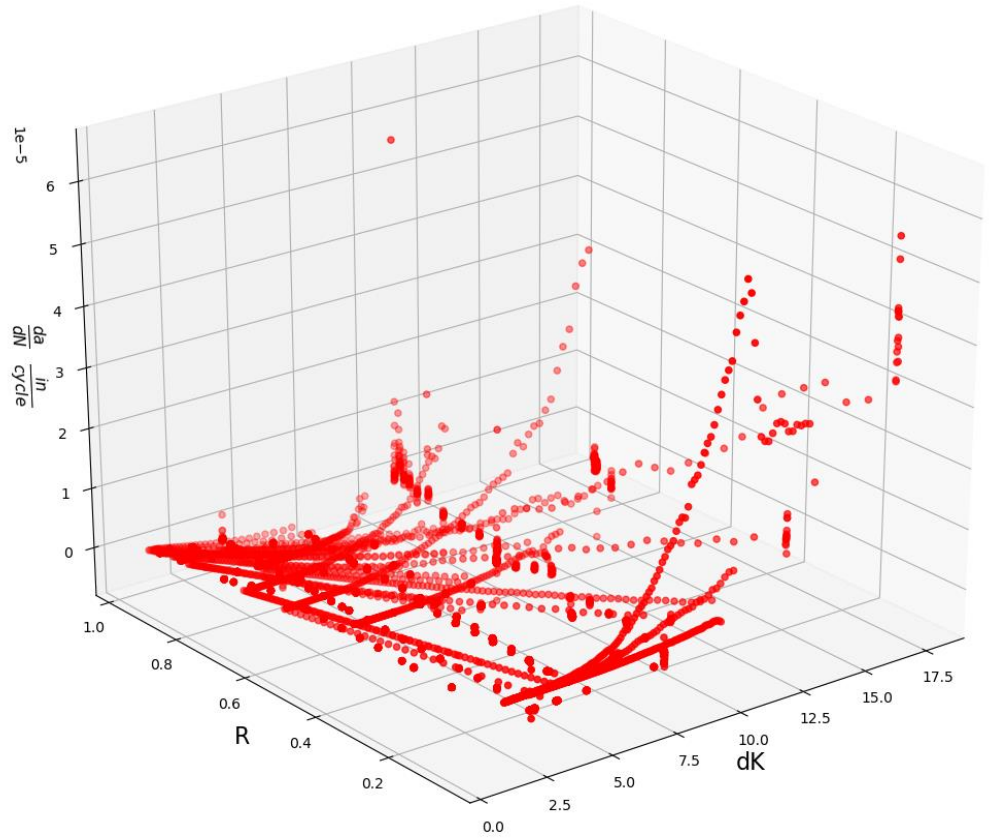
- C = 8.0e-09
- n = 3.2
- p = 0.25
- q = 1.0

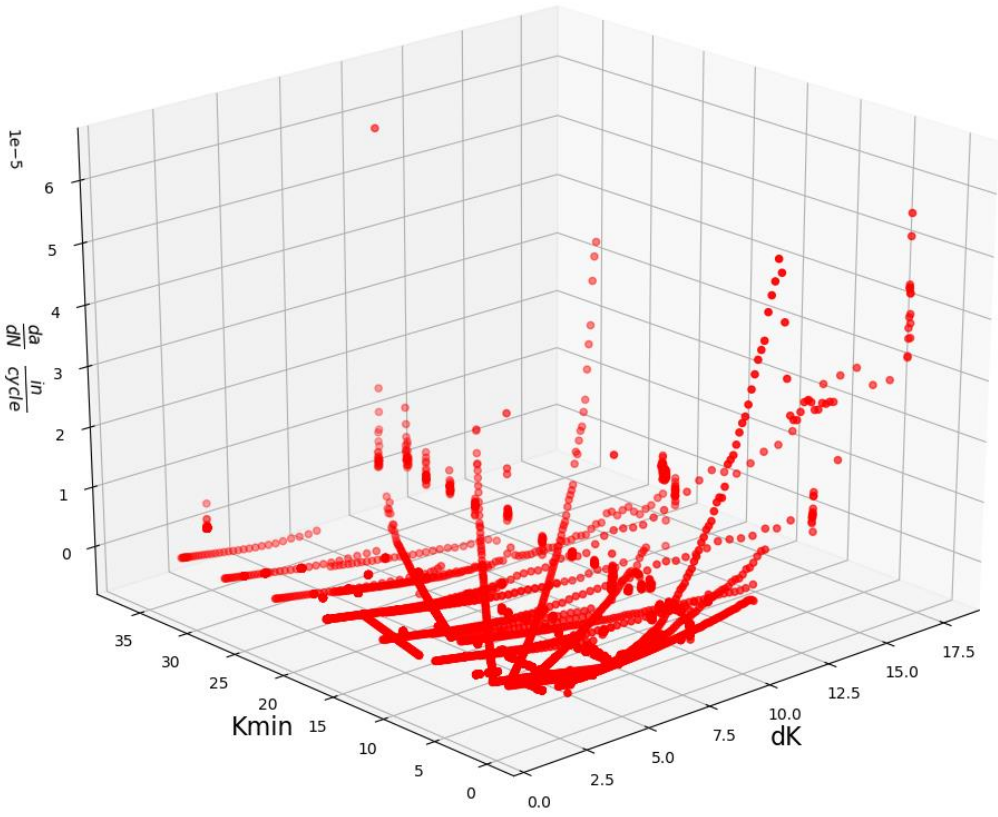
Mean Absolute Error: 1.275e-07



BINGO

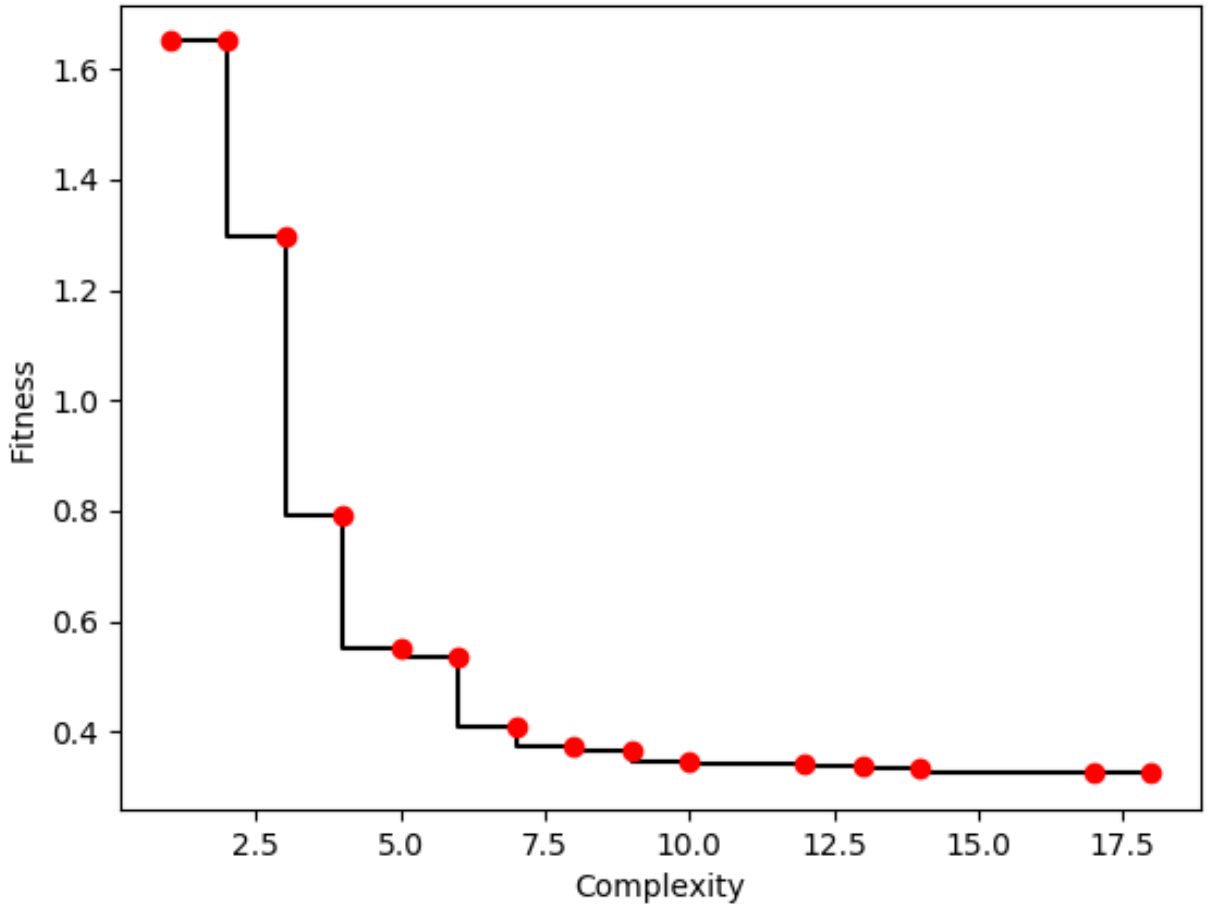
- Only two independent variables
- All combinations of ΔK, R, Kmax, and Kmin





BINGO

- Complexity/Fitness tradeoff
- Less complex equations tend to be more general
- More complex equations more prone to overfitting



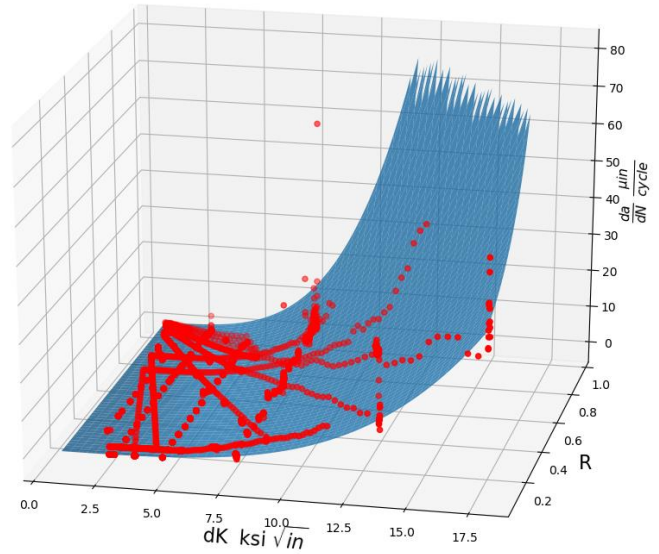
BINGO

Model fit using 80% of data, tested on 20%

$$\frac{da}{dN} = f(\Delta K, R)$$

$$\frac{-0.07313\Delta K^3}{2\Delta K R^2 - 13.6734}$$

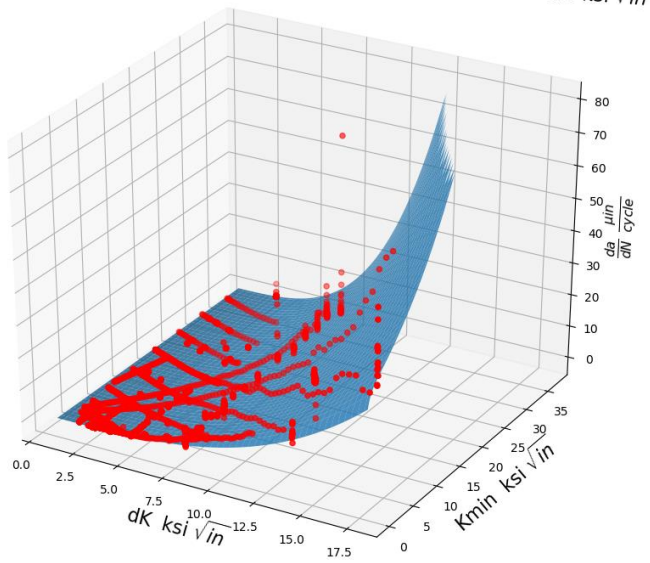
Error: 3.785e-07



$$\frac{da}{dN} = f(\Delta K, Kmin)$$

$$\Delta K^3(0.000373K_{min}(\Delta K - K_{min})^{0.01335} + 0.00535)$$

Error: 2.906e-07



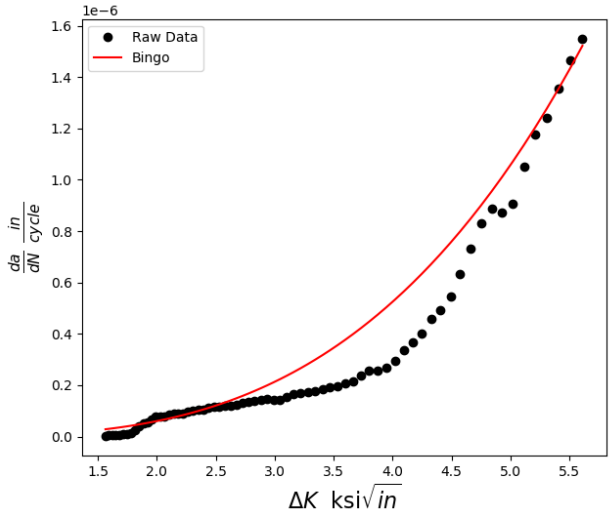
BINGO

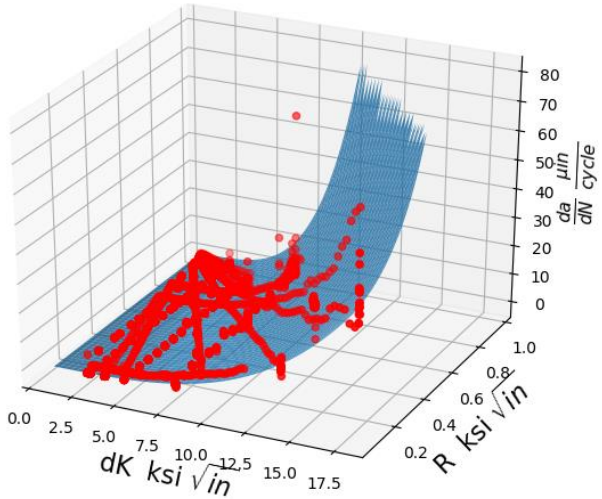
Model fit using all data except one test

$$\frac{da}{dN} = f(\Delta K, R)$$

$$0.002442\Delta K^{\frac{-13.1039}{R^2-3.916}}$$

Error: 4.692e-08

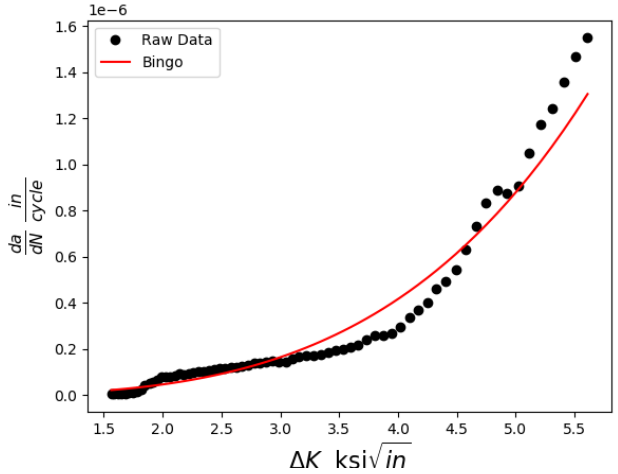


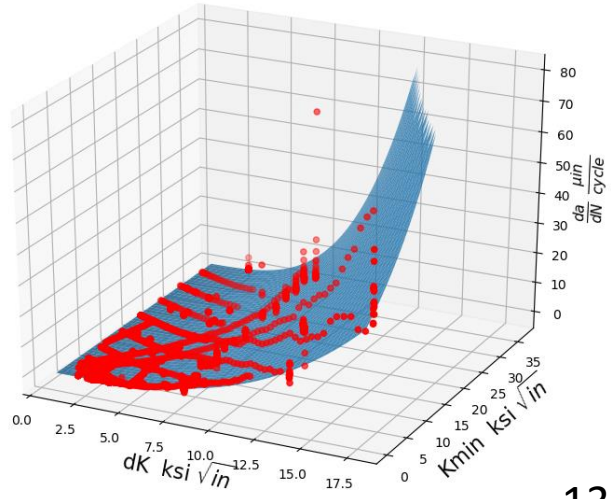


$$\frac{da}{dN} = f(\Delta K, Kmin)$$

$$0.00538\Delta K^{\frac{-561.93}{K_{min}-185.84}}$$

Error: 5.529e-08





Conclusion

- Bingo can be used to search the space of mathematical functions to find an optimal free-form solution from fatigue crack growth data.
- Models found by Bingo, when compared to a similar test dataset, have lower error than NASGRO models
- However, in the work shown, models found by Bingo still have fitting constants with unknown physical meaning.

Future Work

- Search higher dimensional space
- Include material constants in dataspace
- Use different materials, and different specimen geometry to be able to find a more general solution
- Further investigation into the many equations produced to find physical meaning