

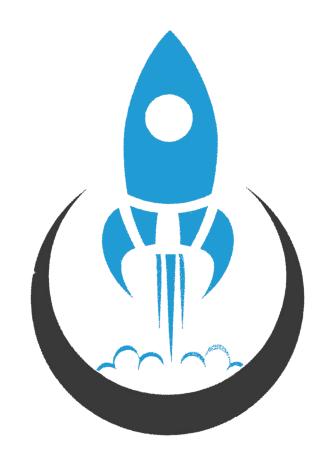
PHYSICS-INFORMED GENERATIVE ADVERSARIAL NETWORKS FOR VIRTUAL MECHANICAL TESTING

By Julian Cuevas Paniagua

Mentors – James Warner, Geoffrey Bomarito, Patrick Leser NASA Langley Research Center Summer 2019

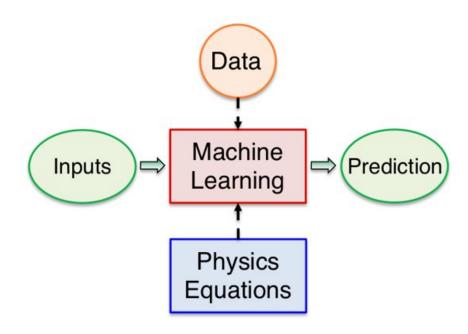
MOTIVATION

- 1. In the course of analyzing complex phenomena, the cost of data acquisition is prohibitive.
- 2. In this small data regime, data-driven methods often fail to provide convergence or quantify uncertainty associated with their predictions.
- There exists a vast amount of prior knowledge not being utilized in modern machine learning.
- 4. There's skepticism regarding the solid grounding of purely data-driven approaches.



GOAL

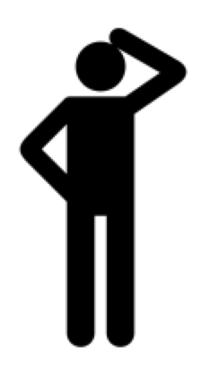
 To develop a scalable framework for uncertainty propagation in physical systems that leverages recent developments in machine learning and governing physical laws in the form of partial differential equations.



WHAT IS A PI-GAN?

A combination of:

- Generative Adversarial Networks (GANs):
 - Uses adversarial training to learn to generate new samples based on the underlying distribution of some training data.
- Physics-Informed Neural Networks (PINNs):
 - Constraints the space of admissible solutions by some stochastic partial differential equation that encodes the governing physical laws of the system being studied into the training procedure.



ADVERSARIAL TRAINING

TRAINING DATA





DISCRIMINATOR



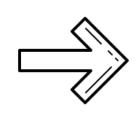






GENERATOR







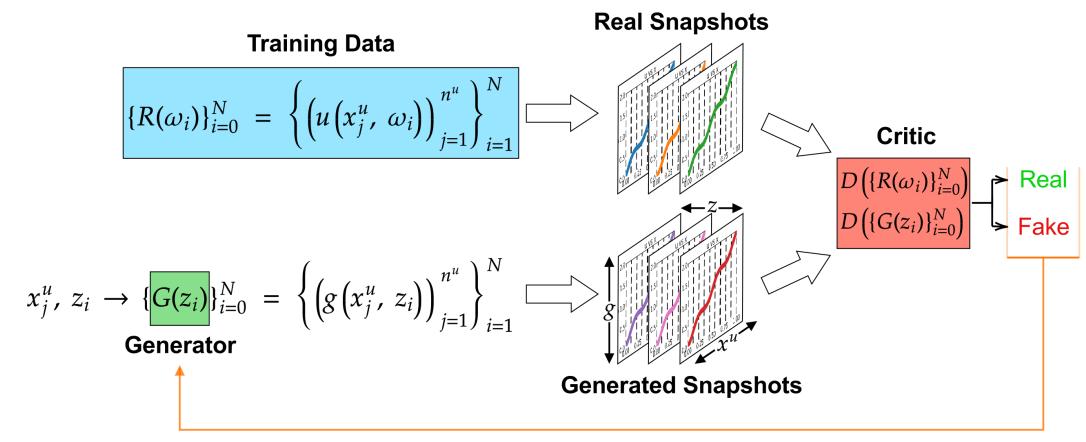




FAKE



ADVERSARIAL TRAINING



Feedback

PROBLEM SETUP

 To illustrate the main idea of PI-GANs, consider a partial differential equation of the form:

$$\mathcal{N}_x[u(x,\omega);k(x,\omega)] = f(x,\omega), \quad x \in \mathcal{D}, \quad \omega \in \Omega$$

Where:

 \mathcal{N}_x -> a general differential operator

 $\stackrel{\sim}{\mathcal{D}}$ -> d-dimensional physical domain of \mathbf{R}^d

 Ω -> probability space

 ω -> random event that denotes the random instance of the snapshot

 $u(x,\omega)$ and $k(x,\omega)$ are modeled by generators, and $f(x,\omega)$ is induced using the automatic differentiation capabilities of TensorFlow 2.

PI-GAN LOSS FORMULATION



$$\mathcal{L}_{data}(heta) = rac{1}{N_u} \sum_{i=1}^{N_u} \|\hat{u}_{ heta}(\mathbf{x}_u^i) - \mathbf{u}^i\|^2$$

PINN Loss

$$r(\mathbf{x}) = \mathcal{N}_{x}[\mathbf{u}; k] - f(\mathbf{x}) = 0, \mathbf{x} \in \Omega$$

$$\mathcal{L}_{PDE}(\theta, \phi) = \frac{1}{N_r} \sum_{i=1}^{N_r} ||\hat{r}_{\theta}(\mathbf{x}_r^i)||^2$$

$$\mathcal{L}_{PI}(\theta,\phi) = \mathcal{L}_{data}(\theta) + \mathcal{L}_{PDE}(\theta,\phi)$$

PI-GAN Loss

$$\max_{\psi} \mathcal{L}_{\mathcal{D}}(\psi)$$

$$\min_{\theta,\phi} \mathcal{L}_{\mathcal{G}}(\theta) + \beta \mathcal{L}_{PDE}(\theta,\phi)$$

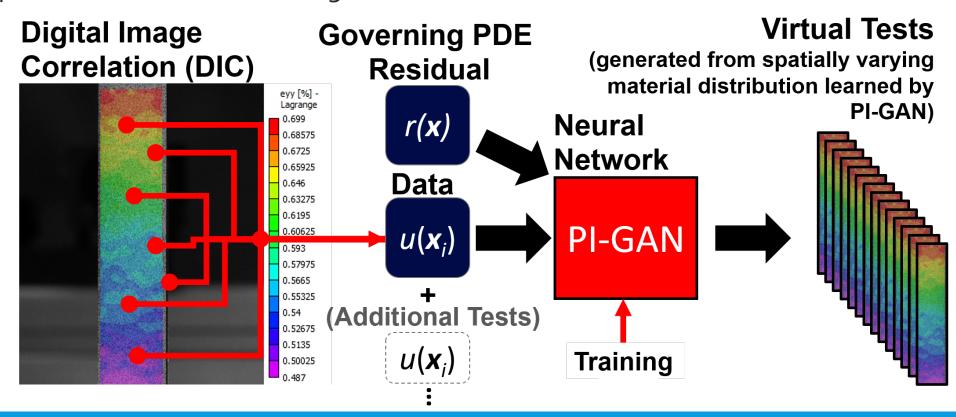
The PINN loss is a function of *u* and *k*, which are individual neural networks; i.e.,

$$\hat{k}_{\phi}(x) \approx k(x)$$

The PDE is derived through automatic differentiation of these networks.

APPLICATION

- Problem:
 - Traditional test-driven certification of novel materials and structures is expensive and time consuming.

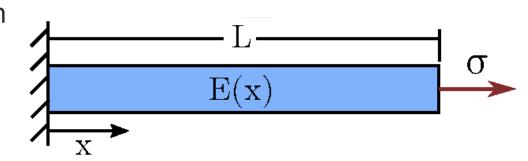


INFERING MATERIAL PROPERTIES OF A BAR IN UNIAXIAL TENSION

- A one-dimensional bar with unknown, spatially-varying elastic modulus E(x) subjected to a known stress σ .
- The target for elastic modulus was assumed to be:

$$E(x) = 1.2 + \left(\sin\left(\frac{5\pi}{L}x + \frac{\pi}{2}(\theta - \frac{1}{2})\right)\right)^{-1}$$

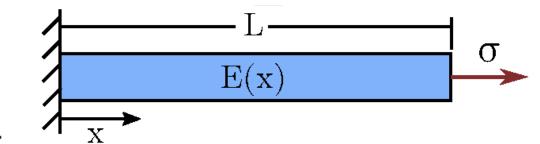
where θ is a Beta random variable, making E(x) a stochastic function whose uncertainty is to be quantified by the network.



INFERING MATERIAL PROPERTIES OF A BAR IN UNIAXIAL TENSION (Cont.)

• In this case, u(x) has a closed form expression:

$$u(x) = \sigma(1.2 * x - \frac{L}{5\pi} cos(\frac{5\pi}{L}x + \frac{\pi}{2}(\theta - \frac{1}{2})))$$

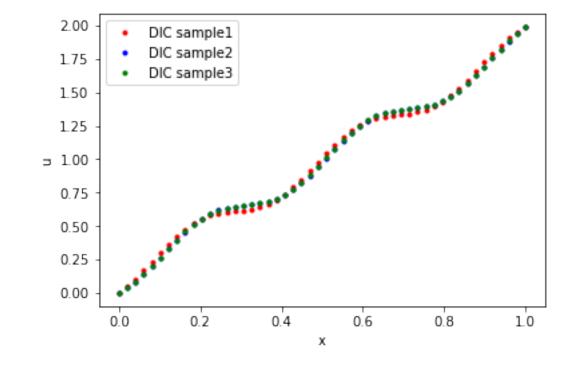


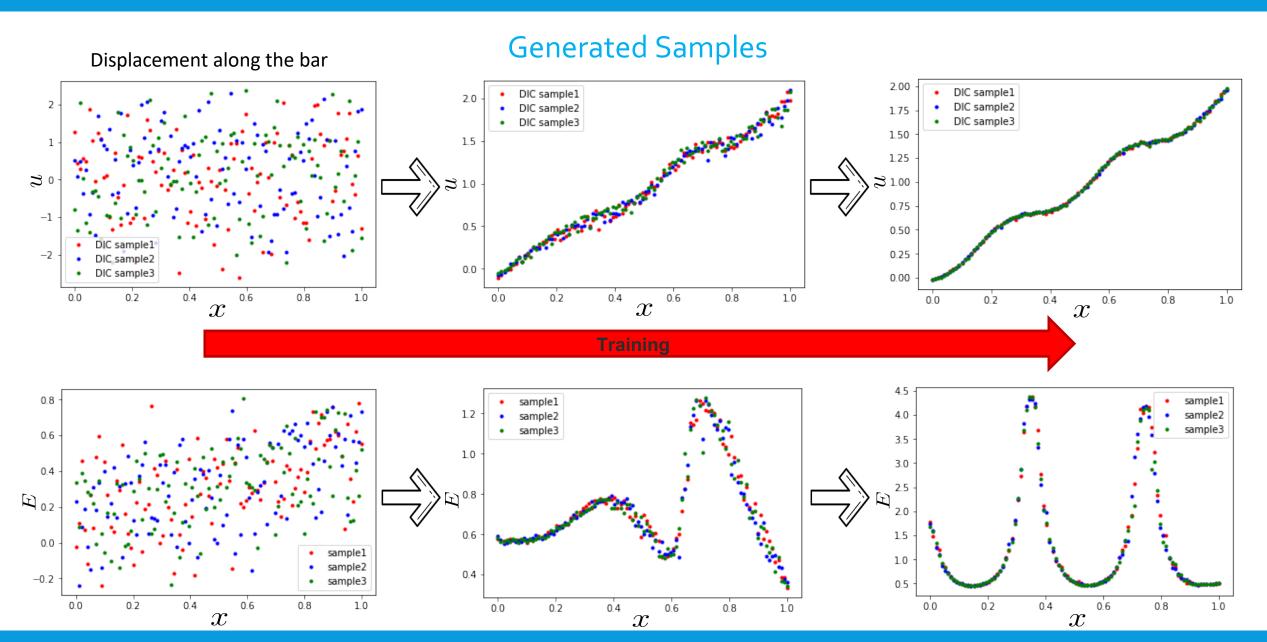
• u(x) is related to E(x)through the governing PDE:

$$r(x) = E(x)u'(x) - \sigma$$

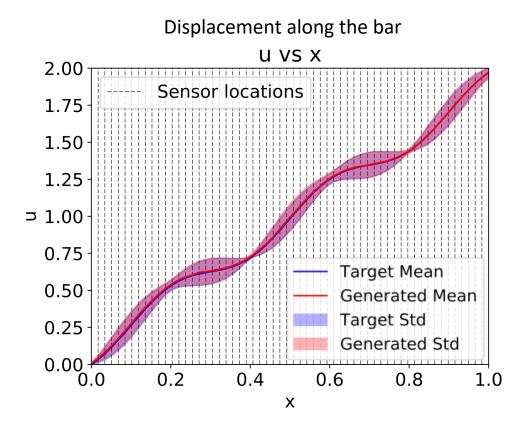
INFERING MATERIAL PROPERTIES OF A BAR IN UNIAXIAL TENSION (Cont.)

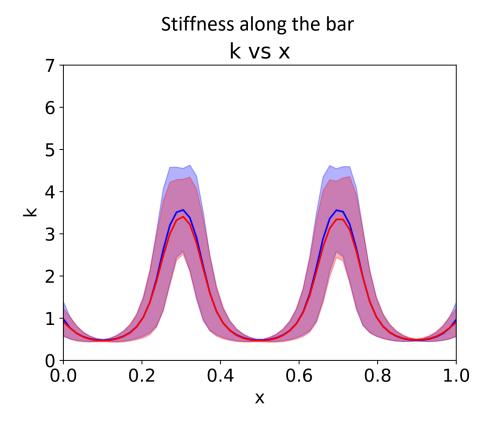
- Training Data: 1,000 snapshots of displacement measurements at 60 uniformly distributed sensors (with inherent randomness)
- PDE constraint: enforced at 10,000 uniformly distributed collocation points
- Training Steps: 50,000
- Training Time: 2h15m (Tesla V100)





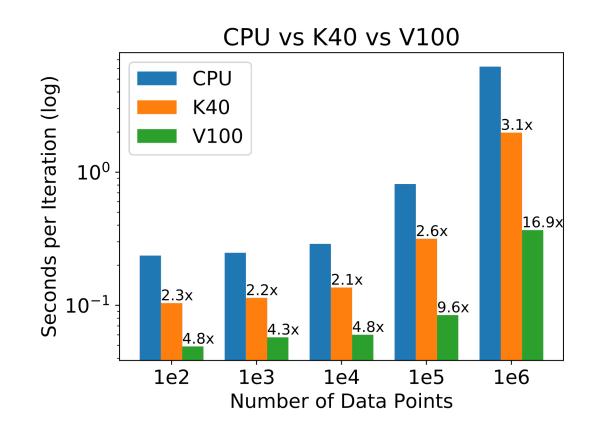
PRELIMINARY RESULTS





K-CLUSTER PERFORMANCE

- # of Data Points: # Sensors * #Snapshots
- **CPU:** Dual socket 8 core 2.6 GHz Intel E₅-2640v₃ Haswell Node
- GPUs:
 - NVIDIA Tesla K40
 - NVIDIA Tesla V100
- Future work involves higher dimensions and significantly more data. The scalability and efficiency provided by GPUs will be critical.



FUTURE WORK

- Scale architecture to 2D data.
- Expand work to Digital Image Correlation(DIC) samples of full-field displacements over the surface of a small number of test specimens.
- Explore distributed deep learning to minimize the cost of scaling the network.

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