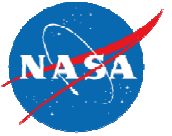




sBOOM Technical Details

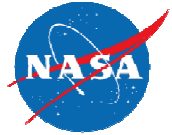
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Outline

- Overview
- Technical details on sBOOM propagation mechanisms
- Loudness prediction



sBOOM¹

- Propagation based on lossy Burgers equation
- Accepts dp/P , F-function or A_e
- Features
 - Under-track, off-track signatures
 - Horizontally stratified winds
 - Acceleration, trajectory, maneuvers
 - Three-dimensional Earth effects

Uses

- High-fidelity analysis and design optimization capability
- Used at NASA, and some industry and academic partners
- Demonstrated over multiple shape optimization exercises
- Adjoint sensitivities with respect to atmospheric conditions for robust design optimization

Unique Features

- Discrete-adjoint² based design capability
 - Ground loudness optimization
 - Ground target signature matching
 - Equivalent area matching
 - Target equivalent area generation
 - Atmospheric sensitivities

Other Features

- *Interfaces with Lossy Nonlinear Tricomi Equation (LNTE) solver from NASA SCAMP effort to predict focus/post-focus booms*
- *Lateral cut-off angle prediction of primary boom carpet*

¹Rallabhandi, S. K., "Advanced Sonic-Boom Prediction Using the Augmented Burgers Equation", *Journal of Aircraft*, Vol. 48, pp: 1245-1253, 2011

²Rallabhandi, S. K., Nielsen, E. J., Diskin, B., "Sonic-Boom Mitigation Through Aircraft Design and Adjoint Methodology", *Journal of Aircraft*, Vol. 51, pp: 502-510, 2014

Model equation

- Current implementation of sBOOM based on numerical solution of the augmented Burgers Equation
- Original non-dimensional version of augmented Burgers equation

$$\frac{\partial P}{\partial \sigma} = P \frac{\partial P}{\partial \tau} + \frac{1}{\Gamma} \frac{\partial^2 P}{\partial \tau^2} + \sum_{\nu} C_{\nu} \frac{\frac{\partial^2}{\partial \tau^2}}{1 + \theta_{\nu} \frac{\partial}{\partial \tau}} P - \frac{\frac{\partial A}{\partial \sigma}}{2A} P + \frac{\frac{\partial(\rho_0 c_0)}{\partial \sigma}}{2\rho_0 c_0} P$$

- Solution process involves operator splitting scheme

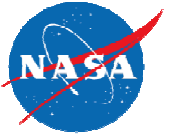
$$\frac{\partial P}{\partial \sigma} = p \frac{\partial P}{\partial \tau} [\text{Non-Linearity}]$$

$$\frac{\partial P}{\partial \sigma} = \frac{1}{\Gamma} \frac{\partial^2 P}{\partial \tau^2} [\text{Absorption}]$$

$$\frac{\partial P}{\partial \sigma} = \sum_{\nu} \frac{C_{\nu} \frac{\partial^2}{\partial \tau^2}}{1 + \theta_{\nu} \frac{\partial}{\partial \tau}} P [\text{Molecular Relaxation}]$$

$$\frac{\partial P}{\partial \sigma} = -\frac{1}{2G} \frac{\partial G}{\partial \sigma} P [\text{Ray Tube Spreading}]$$

$$\frac{\partial P}{\partial \sigma} = \frac{1}{2\rho_0 c_0} \frac{\partial(\rho_0 c_0)}{\partial \sigma} P [\text{Atmospheric Stratification}]$$



Model equation update with mean-flow terms

- Updated Burgers' equation accounting for mean-flow terms

$$\frac{\partial p}{\partial \sigma} = \frac{\frac{\partial B}{\partial \sigma}}{B} p + \frac{\beta \Omega}{2 \rho_0 c_0^3} \left(\frac{c_0}{v_g} \right) \frac{\partial p^2}{\partial \tau} + \frac{\delta \Omega^2}{2 \rho_0 c_0^3} \left(\frac{c_0}{v_g} \right) \left(\frac{1}{D^2} \right) \frac{\partial^2 p}{\partial \tau^2} + \sum_{\nu} \frac{m_{\nu}}{2 c_0} \frac{t_{\nu} \Omega^2}{1 + \frac{t_{\nu}}{D} \Omega \frac{\partial}{\partial \tau}} \left(\frac{c_0}{v_g} \right) \left(\frac{1}{D^2} \right) \frac{\partial^2 p}{\partial \tau^2}$$

$$B = \sqrt{\frac{\rho c S_0 D_0^2}{\rho_0 c_0 S D^2} \left(\frac{\frac{c v_{g0}}{D_0}}{\frac{c_0 v_g}{D}} \right)}, C_{\nu} = \frac{m_{\nu} t_{\nu} \Omega^2}{2 c_0} \left(\frac{c_0}{v_g} \right) \left(\frac{1}{D^2} \right), \theta_{\nu} = \frac{t_{\nu}}{D} \Omega, \frac{1}{\Gamma} = \frac{\delta \Omega}{2 \beta p_0} \left(\frac{c_0}{v_g} \right) \left(\frac{1}{D^2} \right), v_g = |c \cdot \bar{n} + \bar{w}|, D = 1 + \frac{\bar{w} \cdot \bar{n}}{c}$$

- Solution process still involves operator splitting strategy
- D = Doppler mean-flow wind effect

Nonlinear term

- Non-linear term

$$\frac{\partial p}{\partial \sigma} = \frac{\beta \Omega}{2\rho_0 c_0^3} \left(\frac{c_0}{v_g D} \right) \frac{\partial p^2}{\partial \tau}$$

- After non-dimensionalization

$$\frac{\partial P}{\partial \sigma} = P \left(\frac{c_0}{v_g D} \right) \frac{\partial P}{\partial \tau}$$

- Follow Cleveland's approach to solve using the Poisson solution

$$P(\sigma + \Delta\sigma, \tau) = P(\sigma, \tau + P \left(\frac{c_0}{v_g D} \right) \Delta\sigma)$$

- The waveform is mapped to a distorted grid and re-interpolated
- Multi-valuedness is avoided using:

$$\Delta\sigma < \frac{1}{\max(\frac{\partial P}{\partial \tau})}$$

Absorption term

- Updated augmented Burgers equation to account for mean-flow terms

$$\frac{\partial p}{\partial \sigma} = \frac{\delta \Omega^2}{2\rho_0 c_0^3} \left(\frac{c_0}{v_g} \right) \left(\frac{1}{D^2} \right) \frac{\partial^2 p}{\partial \tau^2}$$

- Non-dimensionalization results in the following equation

$$\frac{\partial P}{\partial \sigma} = \frac{1}{\Gamma} \left(\frac{c_0}{v_g} \right) \left(\frac{1}{D^2} \right) \frac{\partial^2 P}{\partial \tau^2}$$

- Crank-Nicolson central difference scheme used for RHS

$$\frac{\partial^2 P}{\partial \tau^2} = \frac{P(\tau + \Delta\tau) - 2P(\tau) + P(\tau - \Delta\tau)}{(\Delta\tau)^2} + O[(\Delta\tau)^2]$$

- First order forward difference scheme used for LHS

$$\frac{\partial P}{\partial \sigma} = \frac{P(\sigma + \Delta\sigma) - P(\sigma)}{\Delta\sigma} + O[(\Delta\sigma)^2]$$

- Results in a tridiagonal system, efficiently solved using Thomas Algorithm

Relaxation terms

- Updated augmented Burgers equation to account for mean-flow terms

$$\frac{\partial p}{\partial \sigma} = \sum_v \frac{m_v}{2c_0} \frac{t_v \Omega^2}{1 + \frac{t_v}{D} \Omega \frac{\partial}{\partial \tau}} \left(\frac{c_0}{v_g} \right) \left(\frac{1}{D^2} \right) \frac{\partial^2 p}{\partial \tau^2}$$

- Non-dimensionalization results in the following equation

$$\left(1 + \frac{\theta_v}{D} \frac{\partial}{\partial \tau} \right) \frac{\partial P}{\partial \sigma} = C_v \left(\frac{c_0}{v_g} \right) \left(\frac{1}{D^2} \right) \frac{\partial^2 P}{\partial \tau^2}$$

- Forward-space, centered-time finite difference

$$\frac{\partial^2 P}{\partial \tau \partial \sigma} = \frac{P_{i+1}^{j+1} - P_{i-1}^{j+1} - (P_{i+1}^j - P_{i-1}^j)}{2\Delta\tau\Delta\sigma} + O[\Delta\sigma^2, \Delta\tau^2]$$

- RHS uses the second order with regularization parameter α
- Turns into a Thomas algorithm (just as in Cleveland's Thesis), except with slightly modified terms

$$C_v = C_{v,orig} \left(\frac{c_0}{v_g} \right) \left(\frac{1}{D^2} \right) \quad \theta_v = \theta_{v,orig} \left(\frac{1}{D} \right)$$

- Currently accounting for Nitrogen and Oxygen relaxation

Ray tube spreading, stratification

- Updated augmented Burgers equation to account for mean-flow terms

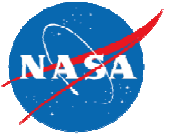
$$\frac{\partial p}{\partial \sigma} = \frac{\frac{\partial B}{\partial \sigma}}{B} p$$

- Non-dimensionalization to account for raytube area spreading, atmospheric stratification, and mean-flow effects

$$\frac{\partial P}{\partial \sigma} = -\frac{1}{2S} \frac{\partial S}{\partial \sigma} P + \frac{1}{2\rho c} \frac{\partial(\rho c)}{\partial \sigma} P + \frac{Dv_g}{2c} \frac{\partial \left(\frac{c}{Dv_g} \right)}{\partial \sigma} P$$

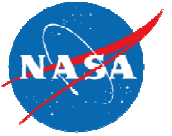
- This is solved exactly using under the assumption that the step size ($\Delta\sigma$) is small

$$P(\sigma + \Delta\sigma, \tau) = \sqrt{\frac{S(\sigma)\rho(\sigma + \Delta\sigma)c(\sigma + \Delta\sigma)}{S(\sigma + \Delta\sigma)\rho c}} \sqrt{\frac{v_g(\sigma)D(\sigma)c(\sigma + \Delta\sigma)}{v_g(\sigma + \nabla\sigma)D(\sigma + \Delta\sigma)c}} P(\sigma, \tau)$$



Trajectory and focus boom prediction

- sBOOM handles flight trajectories similar to PCBOOM i.e. through specification of $M, \dot{M}, \ddot{M}, \eta, \dot{\eta}, \ddot{\eta}, \gamma, \dot{\gamma}, \ddot{\gamma}$
- When focus is encountered, computational domain is set-up for LNTE solver developed through the SCAMP effort. Involves:
 - Calculating locus of focus locations
 - Calculation of the needed ray curvatures
 - Calculation of the incoming waveform at the edge of the computational domain for the LNTE solver
 - Generation of input file for running LNTE solver
- Focus, post-focus and evanescent boom prediction itself relies on LNTE solver

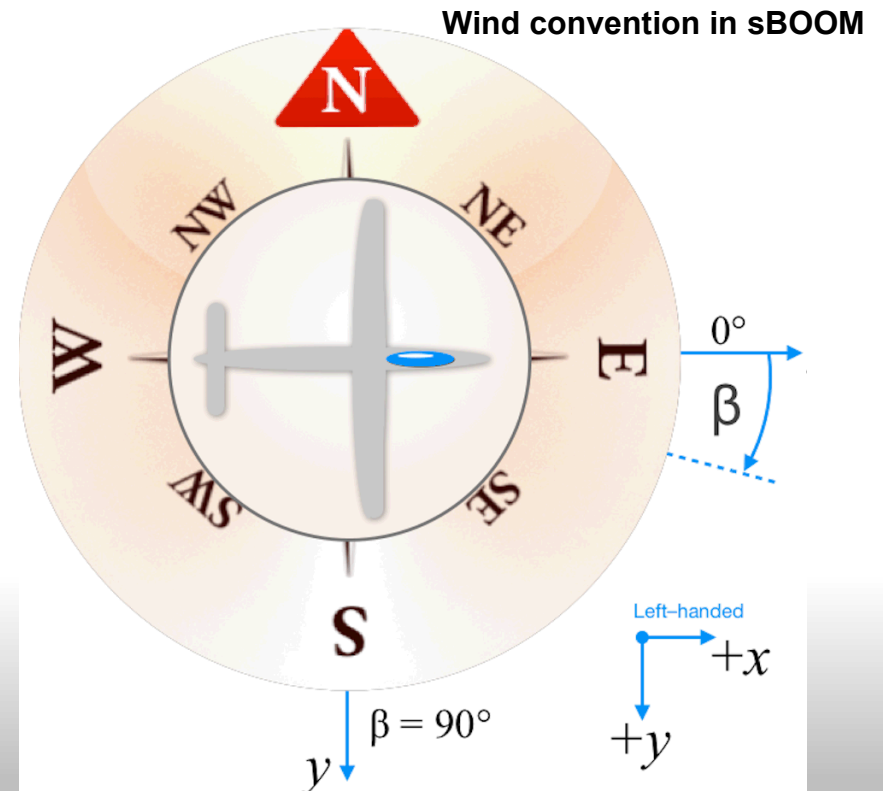
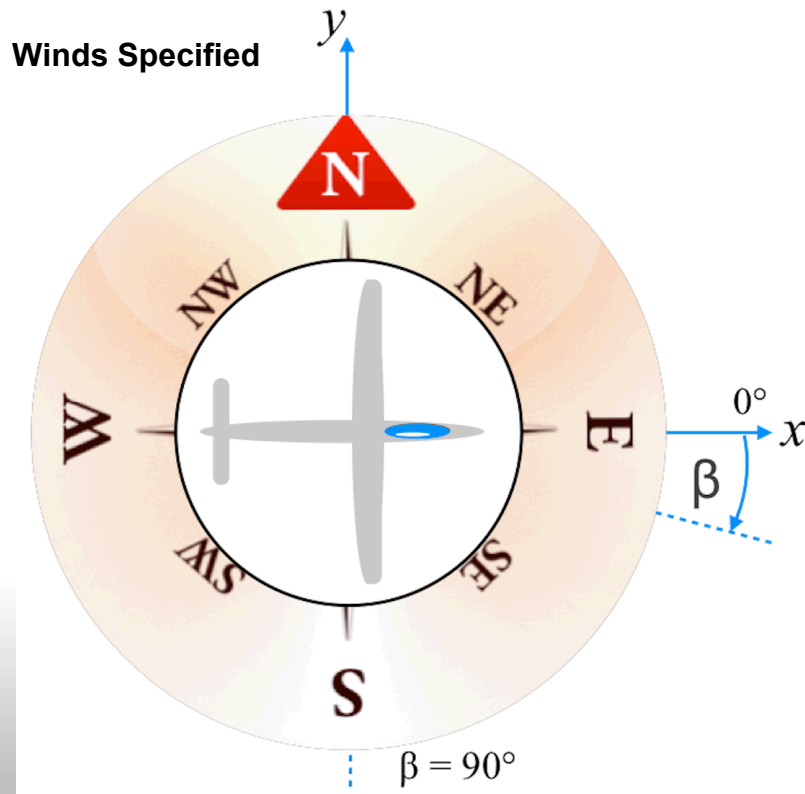


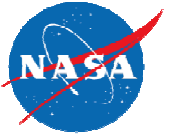
Other details

- Waveform sampling
 - Linear interpolation (or) monotonic cubic spline interpolation
- Ability to read in F-function or Equivalent area for propagation
- Ability to scale wind-tunnel waveforms
- Ability to read CFD cylinder information for propagation
- Ability to predict carpet widths and loudness metrics
- Atmospheric pressures are calculated internally based on hydrostatic equation
 - However, user can override that by specifying atmospheric pressure data
- Ability to use specific humidity instead of relative humidity
- Ability to include three dimensional Earth effects during propagation

Wind convention

- Winds convention within sBOOM is different from standard
- Standard definition
 - Meridional wind: positive when blowing from south toward north
 - Zonal wind: positive when blowing from west to east
- sBOOM: Zonal wind is same as standard, Meridional wind is reversed





Loudness prediction

Loudness calculation

- NASA Langley's LCASB³ code
- Used at NASA, and some industry and academic partners

³*"A loudness calculation procedure applied to shaped sonic booms" K.P. Shepherd and B.M. Sullivan, NASA-TP-3134, 1991*