Design of Experiments in Measurement System Characterization and Uncertainty

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1930

Outline

1. In-Flight Force Measurement Method
2. Non-monolithic Calibration Design
3. Variable Acceleration Calibration System
4. Center of Gravity Determination Method

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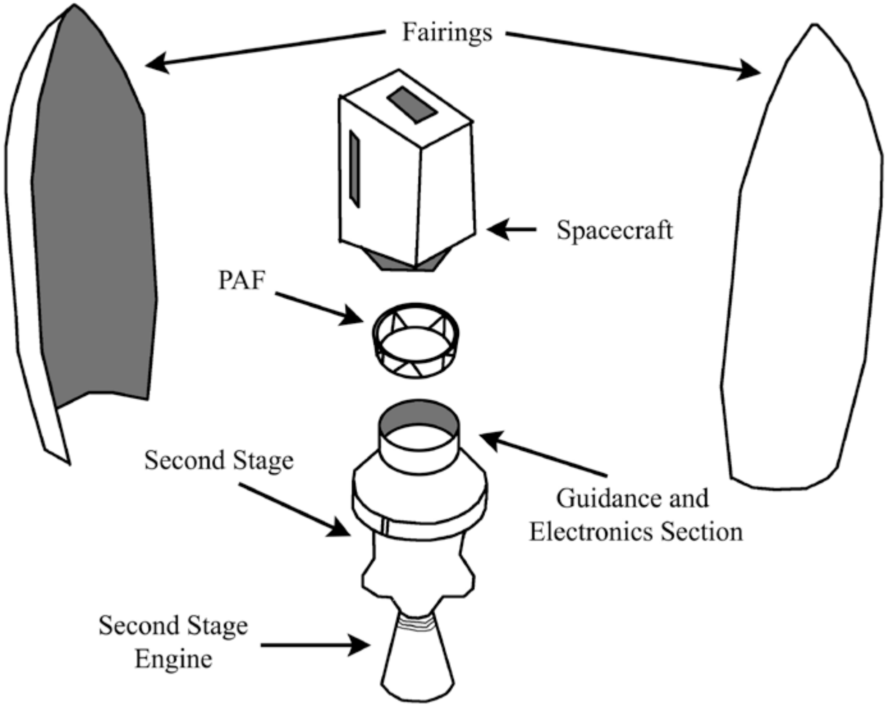
In-Flight Force Measurement Method

- What is the problem?
  - There is a lack of confidence in the measurement of the loads exerted on a spacecraft during launch.
- Why does it matter?
  - To properly understand the physics of the problem
  - To ensure the safety of the spacecraft
  - to achieve a successful launch
- Who does it matter to?
  - Engineers developing simulation models
  - Customers using the delta II rocket
  - Boeing to maintain a reliable track record
- Project Objective
  - Monitor loads exerted on spacecraft during launch (pre-determined)
  - Adapt a structural piece of a Boeing delta II rocket, called a Payload Attachment Fitting (PAF), into multi-component force transducer (also pre-determined)



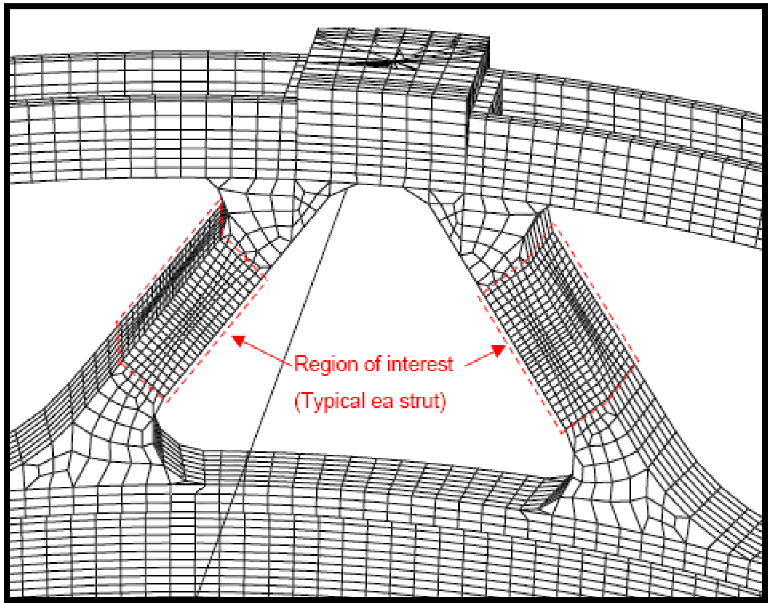
“A Multi-Component Force Transducer Design from an Existing Rocket Payload Attachment Fitting,” Johnson,T.; Landman,D; Parker, P.; AIAA-2009-1716, AIAA USAF T and E Days 2009, Albuquerque, NM, February 10-12, 2009.

In-Flight Force Measurement Method

- *Proposed Solution Framework*
  1. First, optimize strain gauge locations on the PAF using computer simulation (Finite Element Analysis)
  2. Instrument PAF according to strain gauge location optimization study
  3. Perform a ground based calibration
  4. Use in-flight data with the calibration models to obtain in-flight forces

- *Strain gauge optimization method*
  - Objective: determine strain gauge locations that maximize the sensitivity of the reading for a given force component, while minimizing interactions effects due to other forces.
  - Using design of experiment, a factorial design was run to model strain as a function of applied forces at each element in the FEA model.
  - Factors (6): predicted max loads
  - Responses (>10,000): strain at each element

$$y_n = \beta_0 + \sum_{j=1}^{k} \beta_j x_j + \sum_{i<j}\sum \beta_{ij} x_i x_j + \epsilon$$



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In-Flight Force Measurement Method

- optimization method (continued)
    - The next step of the problem is to find combinations of 4 strain locations that maximizes sensitivity while minimizing interaction effects
    - Since wheatstone bridges require 4 gauges
    - Numerical search method used to find best combination of gauges for each model
    - 4 gauges for each component resulted in 24 gauge locations total
- Optimization results
    - Approximate xxx lb resolution
- Proposed next steps
    - Instrument the PAF
    - Perform ground based calibration
- Conclusion
    - A completely uniqe method for determining gauge location methods was demonstrated
    - Design of experiments was used to make efficient use of computational

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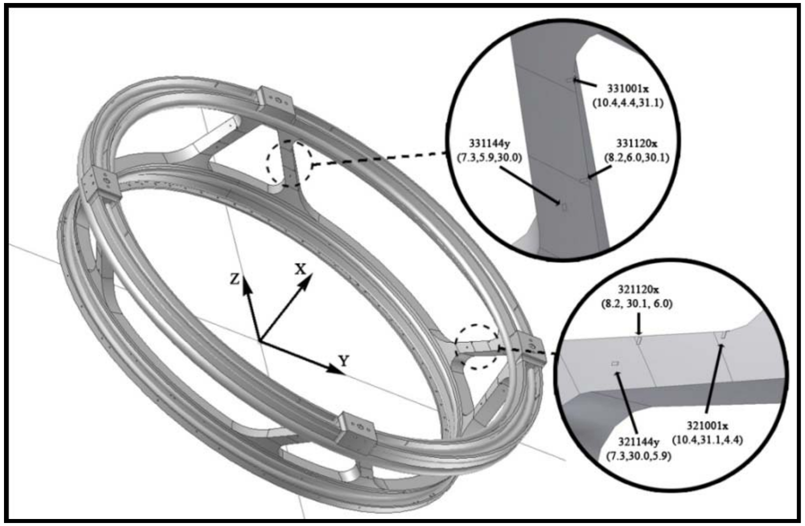

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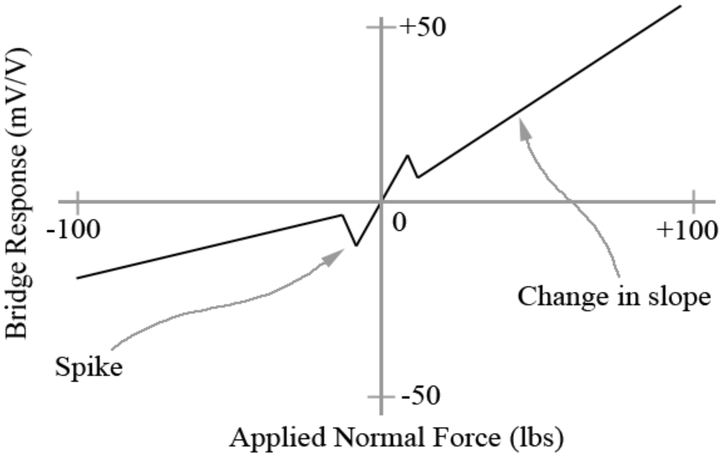
Non-Monolithic Calibration Design

- What is the problem?
  - It is nationally recognized that current methods used to model non-monolithic force balances is inadequate
- Why does it matter?
  - There are many non-monolithic balances currently being used to characterize the performance of tomorrows space vehicles
  - The performance of future missions relies on the accuracy of wind tunnel tests
- Who does it matter to?
  - Force measurement community
  - AIAA reccommended standard calibration practices document
  - Project leaders
- Project Objective
  - Demonstrate shortcomings of current recommended procedure
  - Propose alternative solutions

- What is a non-monolithic balance?

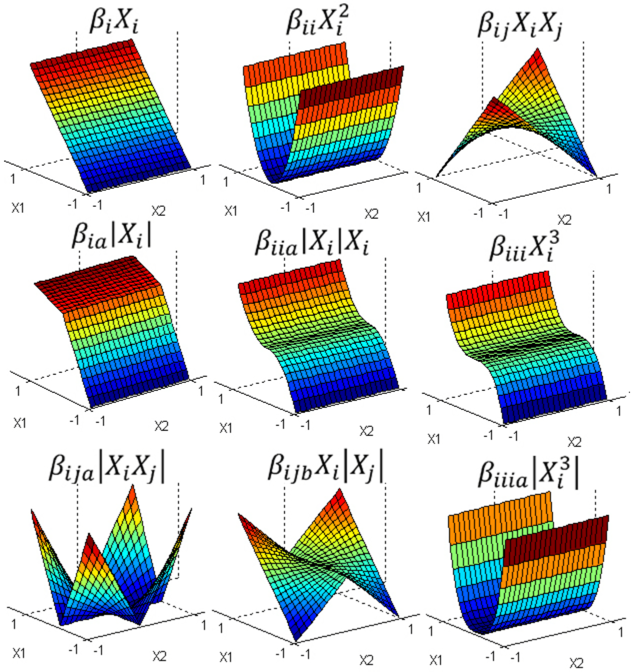


Johnson, T. H., Parker, P.A., Landman, D., "Calibration Modeling of Nonmonolithic Wind-Tunnel Force Balances," AIAA-46356-110 , AIAA Journal of Aircraft, Vol. 47, No. 6, Nov-Dec 2010.

Non-Monolithic Calibration Design

- Current standard procedure recommends using the model shown below
- Takes a heavily parameterized approach
- Includes absolute value terms to model asymmetry in the response
- The problem with the model is that it is over parameterized
- Certain parameters in the model should not co-exist no matter what experimental design is used
- The figure to the bottom right shows response surfaces of various effects from the model
- Variance Inflation Factors are used to show multi-collinearity between model parameters

$$
\begin{aligned}
R_i = a_i &+ \sum_{j=1}^{n} b1_{i,j} F_j + \sum_{j=1}^{n} b2_{i,j} |F_j| + \sum_{j=1}^{n} c1_{i,j} F_j^2 \\
&+ \sum_{j=1}^{n} c2_{i,j} F_j |F_j| + \sum_{j=1}^{n} \sum_{k=j+1}^{n} c3_{i,j,k} F_j F_k \\
&+ \sum_{j=1}^{n} \sum_{k=j+1}^{n} c4_{i,j,k} |F_j F_k| + \sum_{j=1}^{n} \sum_{k=j+1}^{n} c5_{i,j,k}\, F_j |F_k| \\
&+ \sum_{j=1}^{n} \sum_{k=j+1}^{n} c6_{i,j,k}\, |F_j| F_k + \sum_{j=1}^{n} d1_{i,j}\, F_j^3 \\
&+ \sum_{j=1}^{n} d1_{i,j}\, |F_j^3|
\end{aligned}
$$



Non-Monolithic Calibration Design

Eq #	Model	Parameters	Design	# of Runs
1	Independent	28	2 CCDs	128
2	Cubic	55	Draper	228
3	Absolute Value	34	Draper	228
4	Indicator Variable	28	2 CCDs	128

$$R_i = a_i + \sum_{j=1}^{n} b1_{i,j}F_j + \sum_{j=1}^{n} c1_{i,j}F_j^2 + \sum_{j=1}^{n}\sum_{k=j+1}^{n} c3_{i,j,k}F_jF_k \tag{1}$$

$$R_i = a_i + \sum_{j=1}^{n} b1_{i,j}F_j + \sum_{j=1}^{n} b2_{i,j}F_j^2 + \sum_{j=1}^{n}\sum_{k=j+1}^{n} b3_{i,j,k}F_jF_k + \sum_{m}^{n}\sum_{j=m+1}^{n}\sum_{k=m+j+1}^{n} b4_{i,m,j,k}F_mF_jF_k + \sum_{j=1}^{n} b5_{i,j}F_j^3 \tag{2}$$

$$R_i = a_i + \sum_{j=1}^{n} b1_{i,j}F_j + \sum_{j=1}^{n} b2_{i,j}|F_j| + \sum_{j=1}^{n} b3_{i,j}F_j|F_j| + \sum_{j=1}^{n}\sum_{k=j+1}^{n} b4_{i,j,k}F_jF_k \tag{3}$$

$$R_i = a_i + \sum_{j=1}^{n} b1_{i,j}F_j + \sum_{j=1}^{n} \Psi_{i,j}Z_{i,j}F_j + \sum_{j=1}^{n}\sum_{k=j+1}^{n} b4_{i,j,k}F_jF_k \tag{4}$$

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Variable Acceleration Calibration System

- What is the problem?
  - Calibration of large-scale internal wind tunnel force balances is expensive and inefficient.
- Why does it matter?
  - Large balances are needed for experimentation in NASA wind tunnels
  - Needed for full-scale wind tunnel test or semi-span tests
- Who does it matter to?
  - Force balance community, wind tunnel researchers, project engineers
- Project Objective
  - Design, fabricate and test two proof of concept variable acceleration calibration systems
  - Verify the applied load accuracy is within the predicted bounds
  - Propose next stage of system development
- How is this related to statistical engineering?
  - Mechanical system designed for an efficiently designed experiment
    - Helps clarify mechanical design objectives
    - Makes efficient use of resources
  - Uncertainty analysis used to identify problems and to validate the expected accuracy

$$\boldsymbol{F} = m\boldsymbol{g} + m\boldsymbol{r}\boldsymbol{\omega}^2$$

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Variable Acceleration Calibration System

1. Design Experiment
2. Physics Model
3. Mechanical Design
4. Run Experiment
5. Verification

- 3 Factors: Normal Force (NF) (lbs), Axial Force (AF) (lbs), Pitching Moment (PM) (in-lbs)
- 3 Responses: NF (volts), AF (volts), PM (volts)
- Fully replicated central composite design in two blocks

	NF (lbs)	AF (lbs)	PM (in-lbs)
Balance Design Loads	100	60	800
Calibration Loads	30	20	120

Variable Acceleration Calibration System

1. Design Experiment

2. Physics Model

3. Mechanical Design

4. Run Experiment

5. Verification

- Develop a physics-based prediction model to determine independent variable settings required to apply loads

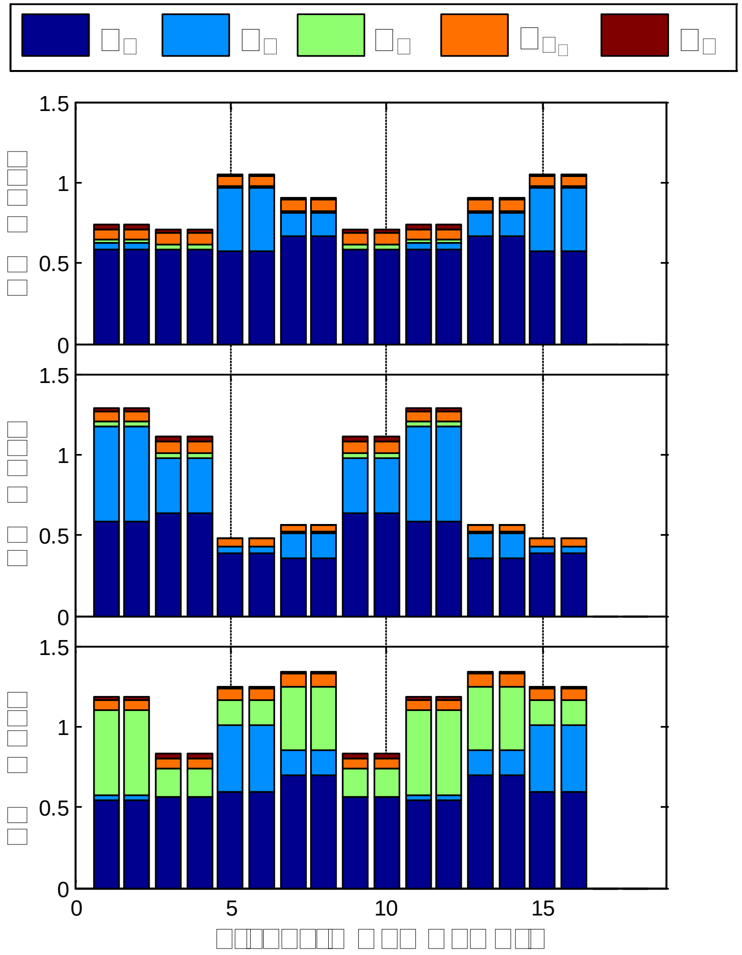
$$\boldsymbol{X} = [\omega \quad R \quad D \quad T_x \quad L \quad \theta \quad \phi \quad \alpha \quad m]$$

- Determine predicted uncertainty using propagation of uncertainty analysis

$$U_{pred,NF} = \sqrt{\sum_{j=1}^{9} \left(\frac{\partial NF}{\partial X_j} u_{X_j} \right)^2}$$

$$\boldsymbol{u_X} = [u_\omega \quad u_R \quad u_D \quad u_{T_x} \quad u_L \quad u_\theta \quad u_\phi \quad u_\alpha \quad u_m]$$

- Predicted independent variable uncertainty contributions shown to right for each run in calibration experiment



Variable Acceleration Calibration System

1. Design Experiment

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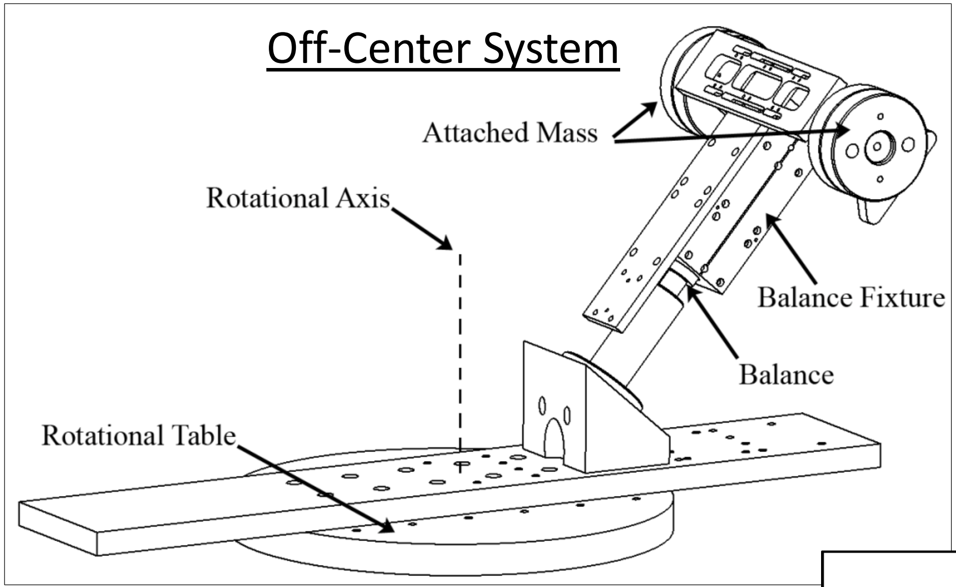
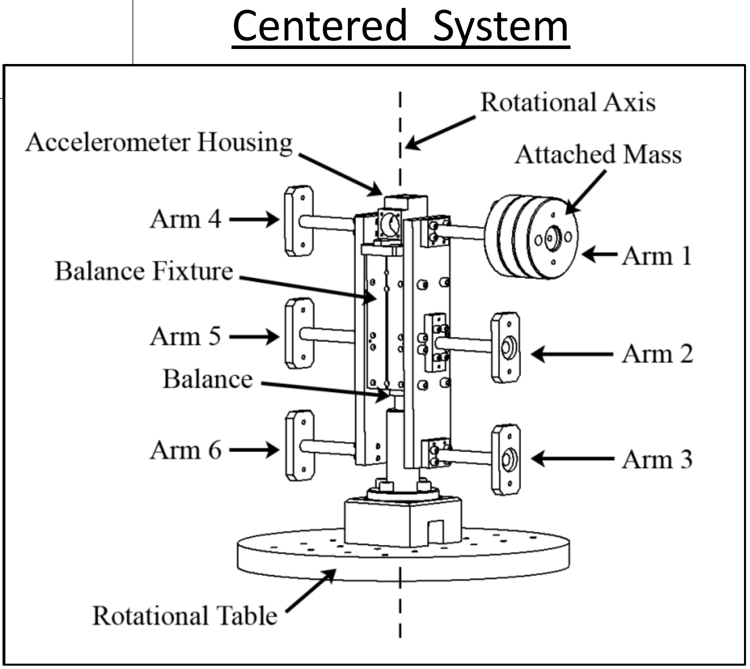
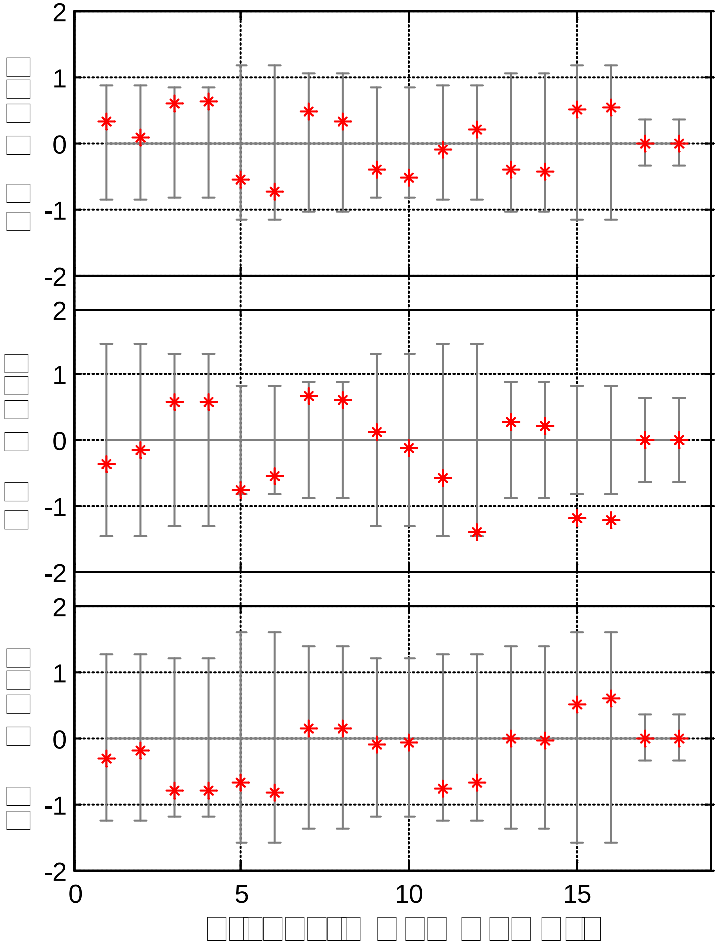




Variable Acceleration Calibration System

1. Design Experiment
2. Physics Model
3. Mechanical Design
4. Run Experiment
5. Verification

- Verify applied load error is within predicted uncertainty
- Predicted uncertainty contains
  - Uncertainty predicted using propagation analysis
  - Balance measurement uncertainty
  - Pure error (noise)
- Applied Load error is the physics model predicted loads minus balance measured loads (shown in red in figure to right)
- Residual Analysis
  - plot applied load error vs. independent variable
  - Plot pure error vs. independent variable graphs



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Center of Gravity Determination Method

- What is the problem?
  - CAD center of gravity results are not perfect. Experimental verification is often required for space vehicles.
- Why does it matter?
  - Center of gravity info is critical to guidance, navigation and control of spacecrafts.
- Objective
  - Create a cheap and efficient experimental method to determine the center of gravity of a space vehicle
  - Provide repeatable and statistically defendable results
- How does it work?
  - Geometry measurements are recorded for multiple test article hang configurations
  - A gravity vector is projected from the hang vertex in each configuration
  - The center of gravity is found by determining the "intersection point" of the multiple gravity vectors.

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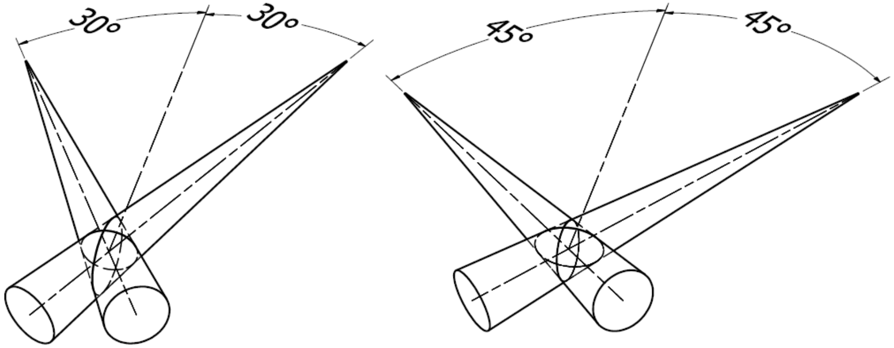
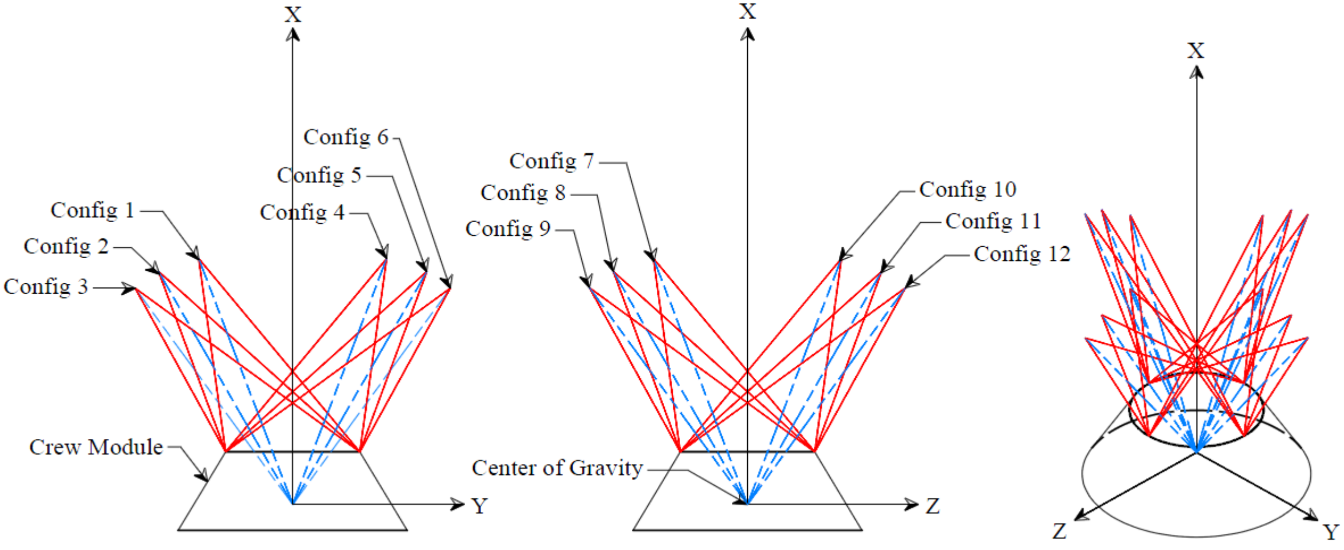
Center of Gravity Determination Method

- CG Method by Tom Jones, NASA LaRC
- My Contribution:
  - Help mature a concept and define the uncertainty
- Reduce uncertainty by reducing prediction uncertainy, quantifying experimental uncertainty
  - Proposed new hang angle to reduce intersection volume uncertainty
  - Orthogonal intersection reduces volume of uncertainty by 15% (compared to 60 deg intersection)





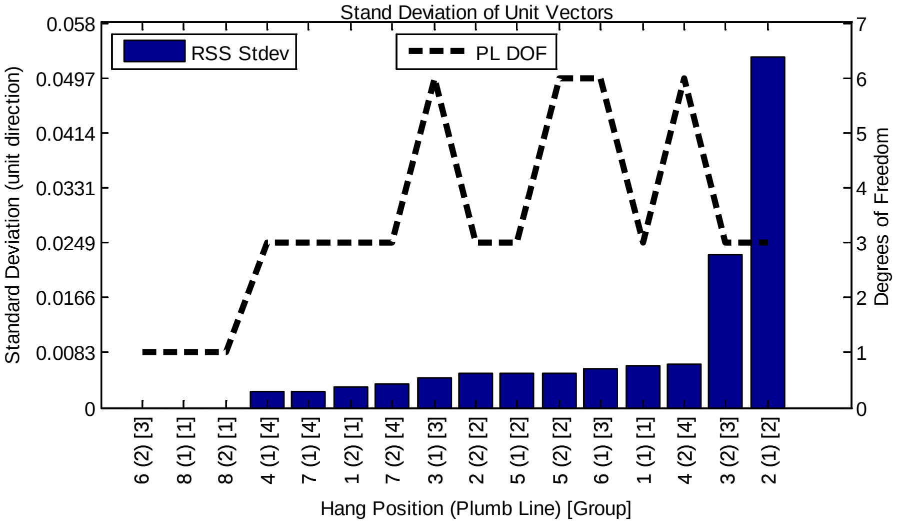
Front View Side View Isometric View

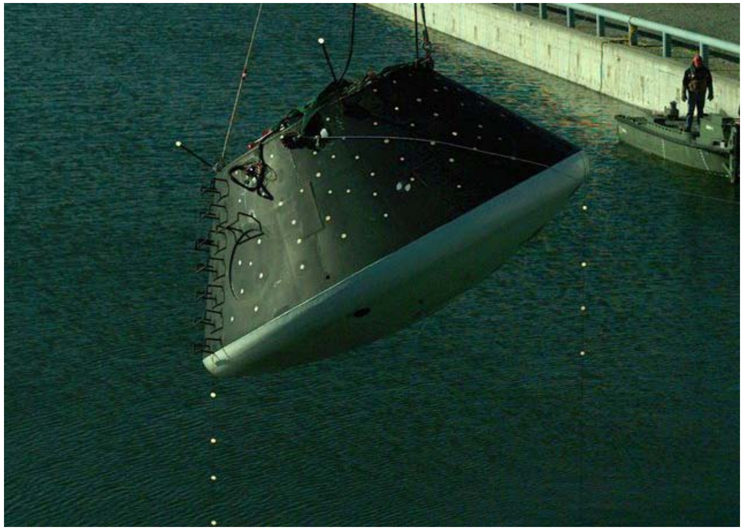
Center of Gravity Determination Method

- Gravity vector construction contains uncertainty due to wind and water effects
- Each gravity line was formed using 2-5 photogrammetry targets
- A bootstrapping method was used to determine the uncertainty in the gravity direction
  - Pairs of targets within each gravity line were use to construct gravity direction

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

  - The standard deviation was calculated for each line in each hang configuration to determine which lines had the most noise
- The lines with the least amount of noise were used for the CG calculation

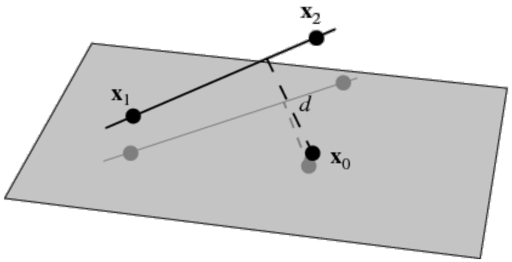




Center of Gravity Determination Method

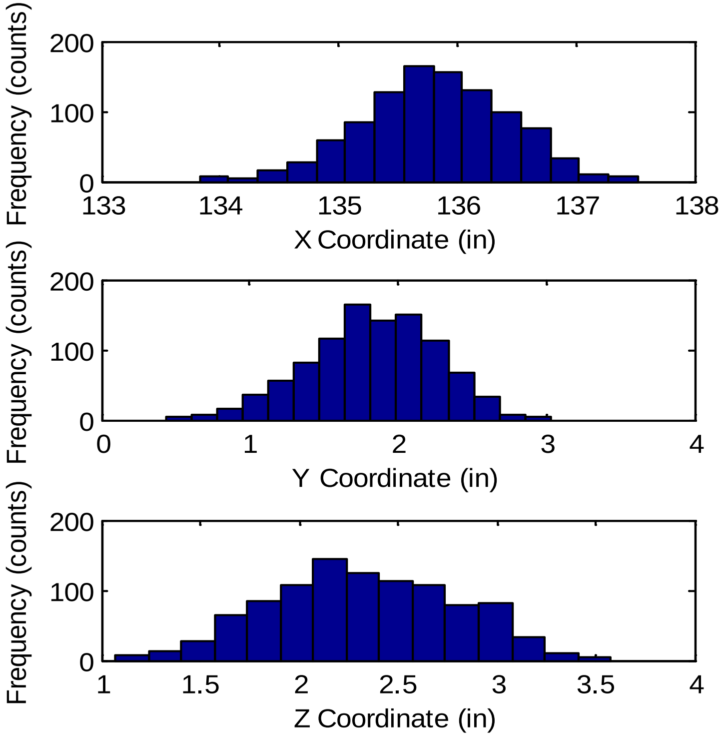
- The center of gravity calculation was solved using a numerical minimization algorithm
- Objective was to find the point that minimized the distance between the selected gravity vectors
- The minimum distance between a point and a line formed by two points is

$$d = \frac{|(\boldsymbol{x}_2 - \boldsymbol{x}_1) \times (\boldsymbol{x}_1 - \boldsymbol{x}_0)|}{|(\boldsymbol{x}_2 - \boldsymbol{x}_1)|}$$



- The minimum distance was found with respect to each gravity vector.
- The numerical algorithm minimized the sum of squares distances
- A Monte Carlo was run that perturbed the mean gravity vector directions by the standard deviations of each line

- The following results were obtained



	X (in)	Y (in)	Z (in)
Mean	135.81	1.83	2.33
2σ	1.23	0.87	0.93

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