

Wind Tunnel Force Balance Characterization for Hypersonic Research Applications

Case Study in Split-Plot Experiments

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Overview



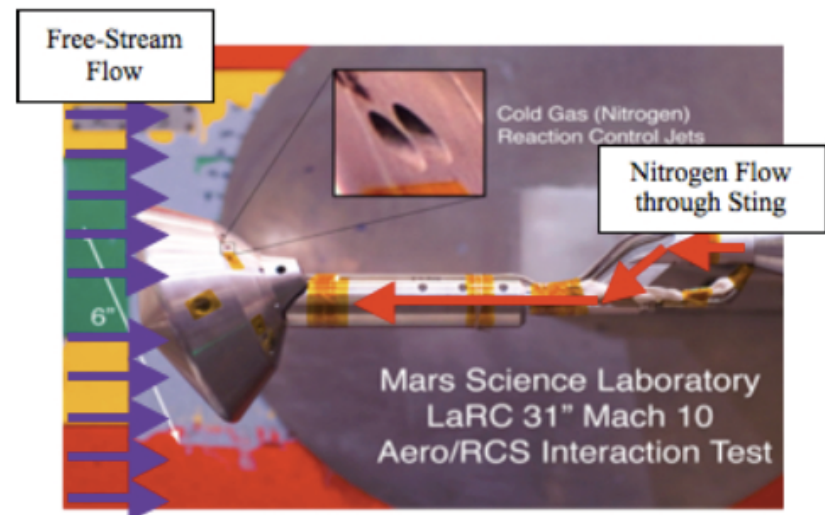
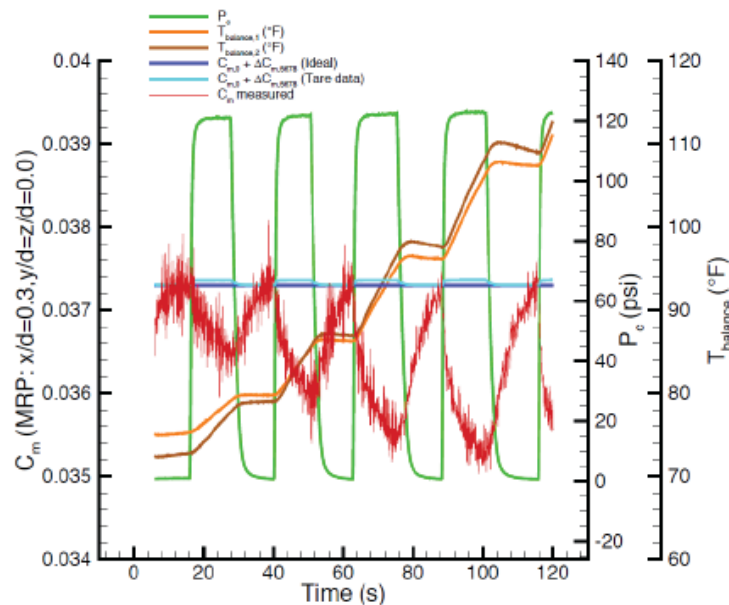
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 - What is a Force Balance?
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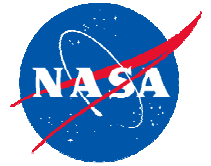
Motivation



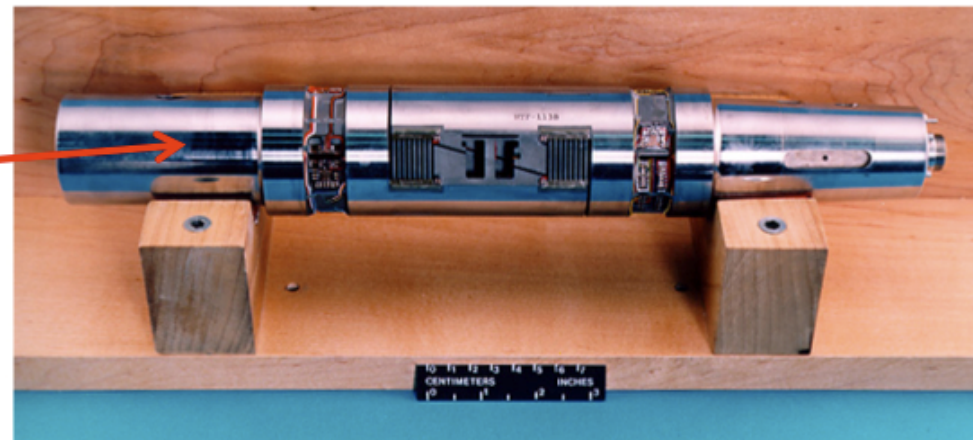
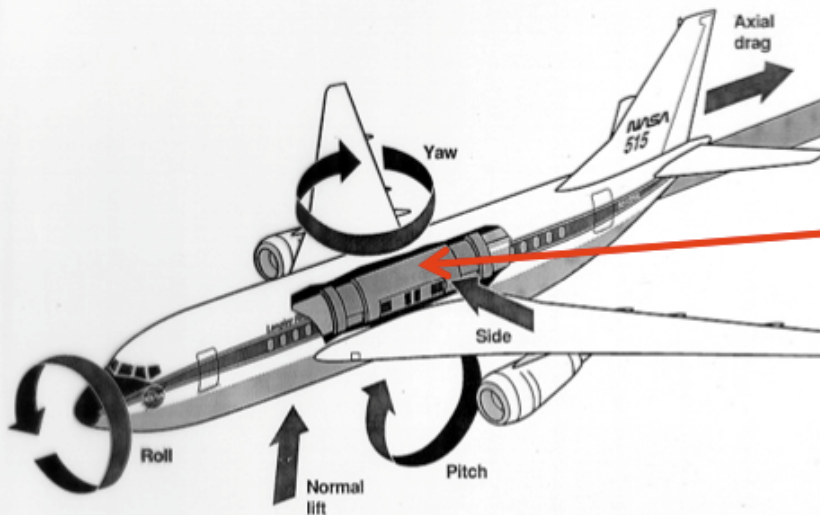
- Previous testing at the NASA LaRC 31-Inch Mach 10 facility with the Mars Science Laboratory (MSL) entry vehicle revealed several thermal related issues
 - Primary issue involved temperature drift of the force balance over the duration of each wind tunnel run
 - Worst-case temperature drift observed during RCS pressure cycling runs (~ 40 deg. F over a 120 second run)
- MSL research team proposed the following problem statement: Investigate a method for characterizing the outputs of the strain-gauges within the force balance subject to force, moments, pressures, and temperatures



What is a Force Balance?



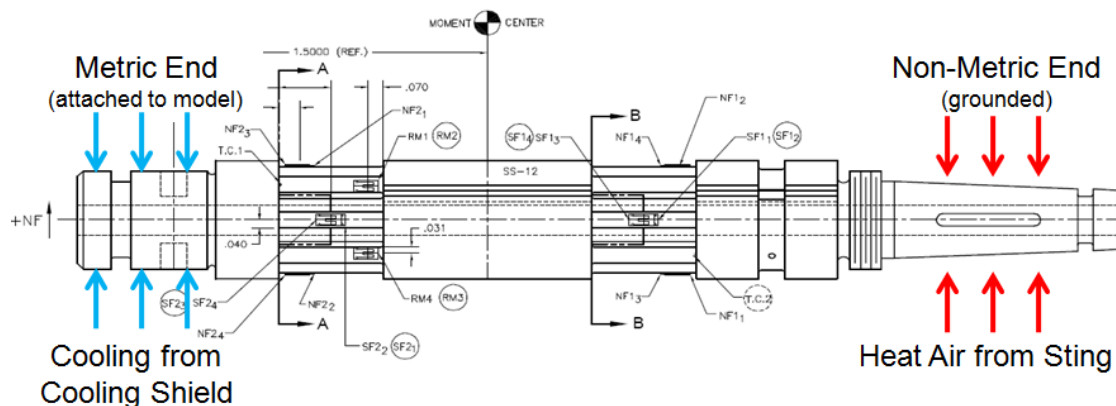
- Force balances are transducers used to measure the 6-DOF aerodynamic loads encountered by a wind tunnel model during a wind tunnel test
- Balances are complex, structural spring elements composed of flexures – only structural component between model (metric end) and sting (non-metric end)
- Flexures are instrumented with foil-resistive strain-gauges that output an electrical signal which is proportional to the strain level induced onto the flexure elements
- Balance structure and instrumentation designed to be sensitive to only single component applied loads and moments but imperfections (in machining and instrumentation) requires us to characterize interactions



SS-12 Force Balance



- SS-12 is a single-piece, 5-component, water-cooled, flow-through force balance (no axial force)
- Balance has a concentric hole down its center, allowing flow of gas through balance and out to a model attached at the metric end
- Normal force and side force components re-gauged from direct read to force type balance configuration (N1, N2, S1, S2, RM)
 - N1/N2, S1/S2 bridges at a single axial station along the balance
 - Single station bridging of strain-gauges aid in reducing sensitivity of measurement bridges to thermal effects
- Balance design features an active cooling shield that covers balance during use and actively cycles water around balance (typical for balances used in supersonic and hypersonic applications)



Component	Design Loads
NF	100 lbs.
AF	n/a
PM	150 in-lbs.
RM	32 in-lbs.
YM	40 in-lbs.
SF	30 lbs.

Experimental Design



- Fundamentals of Statistical Design of Experiments
 - Randomization: defends against systematic errors (i.e. hysteresis) in an experiment
 - Replication: provides information on the pure experimental error in the responses, which sets the lower bound for uncertainty
 - Blocking: limits the effects of any nuisance (controlled or uncontrolled) factors
- Force balances support higher-order mathematical models to adequately characterize the responses over the design space
 - Postulated mathematical model based on a second-order Taylor series expansion
- Two experimental designs were developed and executed
 - Crossed Design & IV-Optimal Design

Design Factors:

Design Factors, units	Range
Normal Force, lbs.	–100 to 100
Pitching Moment, in-lbs.	–150 to 150
Rolling Moment, in-lbs.	–32 to 32
Yawing Moment, in-lbs.	–40 to 40
Side Force, lbs.	–30 to 30
Avg. Balance Temperature, deg. F	70 to 120
Pressure, psia.	14.7 to 400

Measured Responses:

Response	Response Type, units
1	Normal Force Bridge Output, $\mu\text{V/V}$
2	Pitching Moment Bridge Output, $\mu\text{V/V}$
3	Rolling Moment Bridge Output, $\mu\text{V/V}$
4	Yawing Moment Bridge Output, $\mu\text{V/V}$
5	Side Force Bridge Output, $\mu\text{V/V}$

Mathematical Model



Consider a second-order Taylor-series expansion in 7 factors:

$$\begin{aligned} y = & \beta_0 + \beta_1 T + \beta_2 P + \beta_3 NF + \beta_4 PM + \beta_5 RM + \beta_6 YM + \beta_7 SF \\ & + \beta_{22} P^2 + \beta_{33} NF^2 + \beta_{44} PM^2 + \beta_{55} RM^2 + \beta_{66} YM^2 + \beta_{77} SF^2 \\ & + \beta_{12} T \times P + \beta_{13} T \times NF + \beta_{14} T \times PM + \beta_{15} T \times RM + \beta_{16} T \times YM + \beta_{17} T \times SF \\ & + \beta_{23} P \times NF + \beta_{24} P \times PM + \beta_{25} P \times RM + \beta_{26} P \times YM + \beta_{27} P \times SF \\ & + \beta_{34} NF \times PM + \beta_{35} NF \times RM + \beta_{36} NF \times YM + \beta_{37} NF \times SF \\ & + \beta_{45} PM \times RM + \beta_{46} PM \times YM + \beta_{47} PM \times SF \\ & + \beta_{56} RM \times YM + \beta_{57} RM \times SF \\ & + \beta_{67} YM \times SF + \epsilon + \delta \end{aligned}$$

β 's are the calibration coefficients estimated from the calibration data and

ϵ & δ are the experimental errors

Mathematical Model Example



Next, consider the mathematical model for the normal force response

$$\begin{aligned} y_{NF} = & \beta_0 + \beta_1 T + \beta_2 P + \beta_3 NF + \beta_4 PM + \beta_5 RM + \beta_6 YM + \beta_7 SF \\ & + \beta_{22} P^2 + \beta_{33} NF^2 + \beta_{44} PM^2 + \beta_{55} RM^2 + \beta_{66} YM^2 + \beta_{77} SF^2 \\ & + \beta_{12} T \times P + \beta_{13} T \times NF + \beta_{14} T \times PM + \beta_{15} T \times RM + \beta_{16} T \times YM + \beta_{17} T \times SF \\ & + \beta_{23} P \times NF + \beta_{24} P \times PM + \beta_{25} P \times RM + \beta_{26} P \times YM + \beta_{27} P \times SF \\ & + \beta_{34} NF \times PM + \beta_{35} NF \times RM + \beta_{36} NF \times YM + \beta_{37} NF \times SF \\ & + \beta_{45} PM \times RM + \beta_{46} PM \times YM + \beta_{47} PM \times SF \\ & + \beta_{56} RM \times YM + \beta_{57} RM \times SF \\ & + \beta_{67} YM \times SF + \epsilon + \delta \end{aligned}$$

Primary Sensitivity

Mathematical Model Example



Next, consider the mathematical model for the normal force response

$$\begin{aligned} y_{NF} = & \beta_0 + \beta_1 T + \beta_2 P + \beta_3 NF + \beta_4 PM + \beta_5 RM + \beta_6 YM + \beta_7 SF \\ & + \beta_{22} P^2 + \beta_{33} NF^2 + \beta_{44} PM^2 + \beta_{55} RM^2 + \beta_{66} YM^2 + \beta_{77} SF^2 \\ & + \beta_{12} T \times P + \beta_{13} T \times NF + \beta_{14} T \times PM + \beta_{15} T \times RM + \beta_{16} T \times YM + \beta_{17} T \times SF \\ & + \beta_{23} P \times NF + \beta_{24} P \times PM + \beta_{25} P \times RM + \beta_{26} P \times YM + \beta_{27} P \times SF \\ & + \beta_{34} NF \times PM + \beta_{35} NF \times RM + \beta_{36} NF \times YM + \beta_{37} NF \times SF \\ & + \beta_{45} PM \times RM + \beta_{46} PM \times YM + \beta_{47} PM \times SF \\ & + \beta_{56} RM \times YM + \beta_{57} RM \times SF \\ & + \beta_{67} YM \times SF + \epsilon + \delta \end{aligned}$$

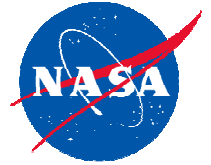
Adjustments to the Zero-Intercept

Calibration Execution



- Presence of hard-to-change (HTC) factors in an experiment can make a completely randomized experimental design impractical to implement
 - Temperature is often a time-consuming factor to change
- Split-plot designs are a technique to efficiently design an experiment with HTC factors
 - Restricted randomization
 - Concept developed from agricultural experiments
- Temperature is set and held constant while pressure, forces, and moments combinations are varied randomly
- Calibration took 10 days to complete
 - 2 days for standard calibration (ambient temperature and pressure)
 - 6 days for crossed design calibration
 - 2 days for IV optimal design calibration

Implications of Restricted Randomization I



For a Completely Randomized Design:

The general model form is $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$ where $E[\mathbf{y}] = \mathbf{X}\boldsymbol{\beta}$ and $\text{cov}[\mathbf{y}] = \sigma^2 \mathbf{I}$

The best linear unbiased estimate (BLUE) is $\hat{\boldsymbol{\beta}}_{\text{OLS}} = (\mathbf{X}'\mathbf{X})^{-1} \mathbf{X}'\mathbf{y}$

and the variance-covariance matrix of the estimates is $\text{cov}[\hat{\boldsymbol{\beta}}_{\text{OLS}}] = \sigma^2 (\mathbf{X}'\mathbf{X})^{-1}$

For a Split-Plot Design:

The general model form is $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\delta} + \boldsymbol{\epsilon}$ where $E[\mathbf{y}] = \mathbf{X}\boldsymbol{\beta}$

and $\text{cov}[\mathbf{y}] = \boldsymbol{\Sigma} = \sigma_{\epsilon}^2 \mathbf{I} + \sigma_{\delta}^2 \mathbf{J}$

The best linear unbiased estimate (BLUE) is $\hat{\boldsymbol{\beta}}_{\text{GLS}} = (\mathbf{X}'\boldsymbol{\Sigma}^{-1}\mathbf{X})^{-1} \mathbf{X}'\boldsymbol{\Sigma}^{-1}\mathbf{y}$

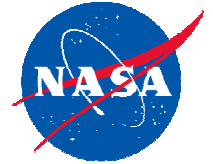
and the variance-covariance matrix of the estimates is $\text{cov}[\hat{\boldsymbol{\beta}}_{\text{GLS}}] = (\mathbf{X}'\boldsymbol{\Sigma}\mathbf{X})^{-1}$

Split-Plot Experiments can be constructed such that

$$\hat{\boldsymbol{\beta}}_{\text{OLS}} = \hat{\boldsymbol{\beta}}_{\text{GLS}}$$

However, this can only be used for estimation, not inference.

Implications of Restricted Randomization II



Correlation (of an observation):

$$\rho = \frac{\sigma_{\delta}^2}{\sigma_{\delta}^2 + \sigma_{\epsilon}^2}$$

Correlation Matrix of the Observations:

$$\frac{1}{\sigma_{\epsilon}^2 + \sigma_{\delta}^2} \Sigma = \mathbf{R} = \begin{bmatrix} 1 & \rho & \cdots & \rho & & & & \\ \rho & 1 & \cdots & \rho & & & & \\ \vdots & \vdots & \ddots & \vdots & 0 & \cdots & & 0 \\ \rho & \rho & \cdots & 1 & & & & \\ & & & & 1 & \rho & \cdots & \rho \\ & 0 & & & \rho & 1 & \cdots & \rho \\ & & & & \vdots & \vdots & \ddots & \vdots \\ & & & \rho & \rho & \cdots & 1 & \\ \vdots & & & & \vdots & & & \ddots & \vdots \\ & 0 & & & \cdots & 0 & & & 1 & \rho & \cdots & \rho \\ & & & & & & & & \rho & 1 & \cdots & \rho \\ & & & & & & & & \vdots & \vdots & \ddots & \vdots \\ & & & & & & & & \rho & \rho & \cdots & 1 \end{bmatrix}$$

Observations in different whole plots are independent while observations within a whole plot are correlated.

Development of the Experimental Design I (Crossed Design)



5-Component SVS Design:

Factorial Design Points

NF	PM	RM	YM	SF
-48	-72	± 15	-19	-13
-48	-72	± 15	19	13
-48	72	± 15	-19	13
-48	72	± 15	19	-13
48	-72	± 15	19	-13
48	-72	± 15	-19	13
48	72	± 15	-19	-13
48	72	± 15	19	13
0	0	0	0	0

Axial Design Points

NF	PM	RM	YM	SF
± 100	0	0	0	0
± 100	± 150	0	0	0
± 100	0	0	0	± 30
± 100	0	± 32	0	0
0	0	0	± 40	± 30
0	0	0	0	± 30
0	0	0	0	0

Physics-based constraint of the calibration apparatus determines possible experimental points

Factorial Points:	16
Axial Points:	20
Center Points:	6
Total:	42

Development of the Experimental Design I (Crossed Design)



5-Component SVS Design:

Factorial Design Points

Axial Design Points

NF	PM	RM	YM	SF
-1	-1	±1	-1	-1
-1	-1	±1	1	1
-1	1	±1	-1	1
-1	1	±1	1	-1
1	-1	±1	1	-1
1	-1	±1	-1	1
1	1	±1	-1	-1
1	1	±1	1	1
0	0	0	0	0

NF	PM	RM	YM	SF
±2.1	0	0	0	0
±2.1	±2.1	0	0	0
±2.1	0	±2.1	0	0
0	0	0	±2.1	±2.3
0	0	0	0	±2.3
0	0	0	0	0

Physics-based constraint of the calibration apparatus determines possible experimental points

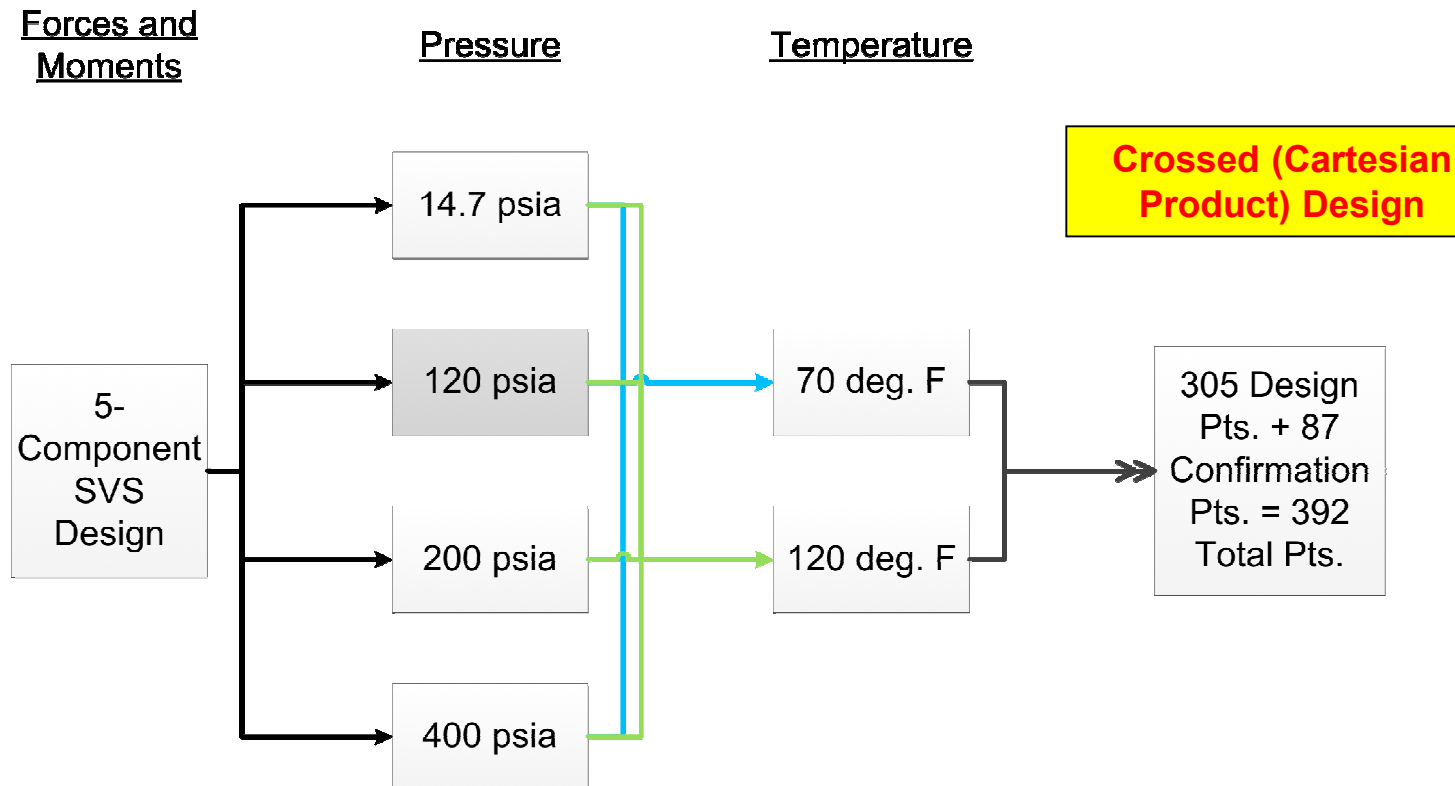
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Development of the Experimental Design I (Crossed Design)

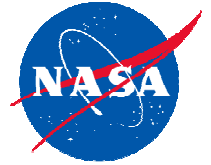


Mathematical Model Assumptions:

- First-order effect of temperature on the responses (req. 2 unique levels)
 - Assumption based on subject-matter expertise, historical data
- Second-order effect of pressure, forces, and moments on the responses (req. 3 unique levels)



Development of the Experimental Design II (IV-Optimal Design)

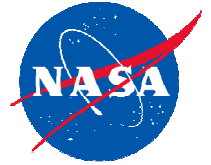


Mathematical Model Assumptions:

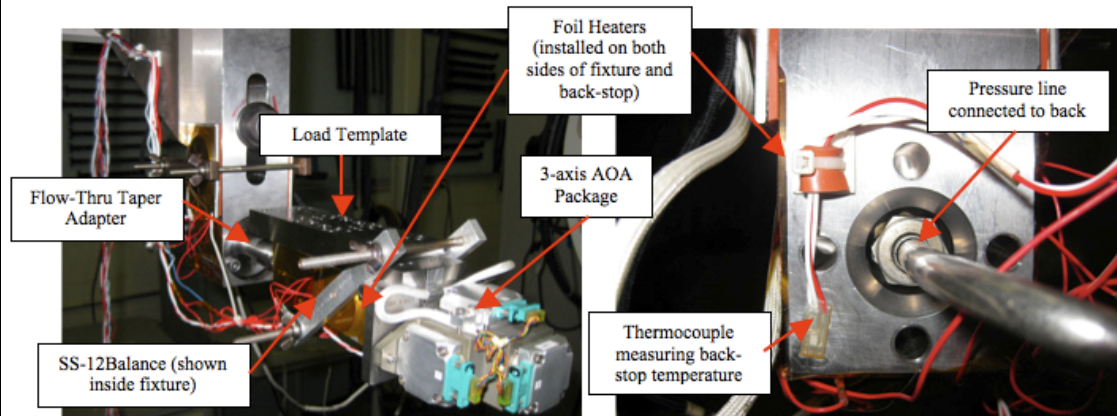
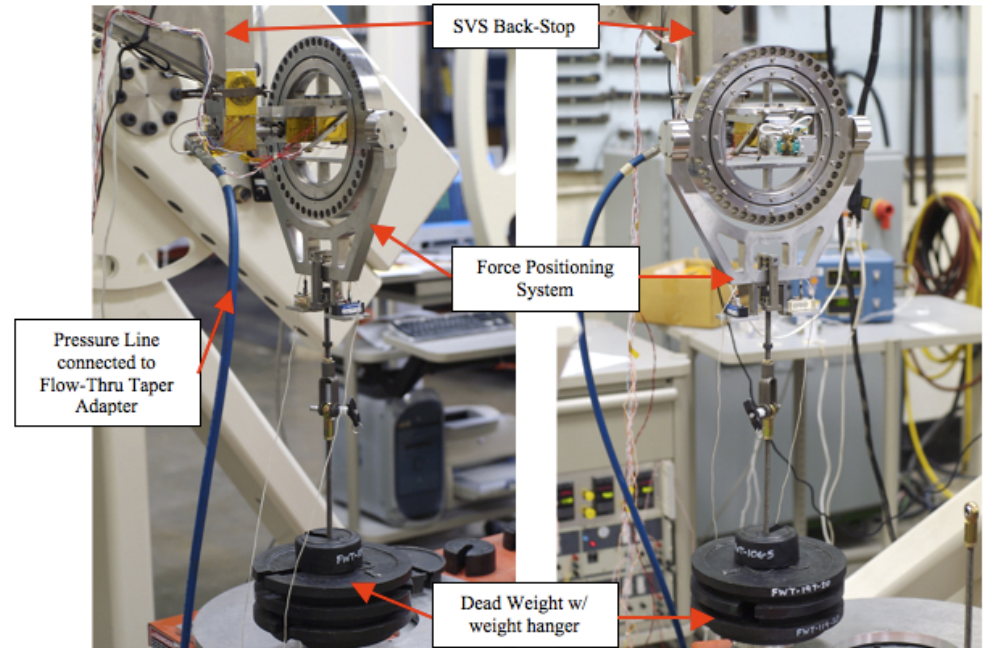
- Same as the Crossed Design (first-order in temperature, second-order in pressure, forces, and moments)
- Design points from Crossed Design used as a candidate list to generate the optimal design
 - Crossed Design contained all possible combination of design points possible using the SVS
 - IV (integrated variance) optimality minimizes the *average scaled prediction variance* over the experimental region
 - 44 design points were selected based on optimality criteria and number of terms in the model and supplemented with standard room temperature and pressure calibration
- Design points were generated in random order but re-sorted to restrict the randomization on temperature
 - Resulting design contained two (2) whole-plots but could be divided into any number of whole-plots
 - Unbalanced design

$$\text{For a CRD:} \quad \text{Min} \left[\frac{N}{K} \int_R \mathbf{x}_0' (\mathbf{X}'\mathbf{X})^{-1} \mathbf{x}_0 d\mathbf{x} \right] = \text{Min } Q$$

SS-12 Calibration Setup



- Single Vector System (SVS) used during calibration to orient balance
- Heater system configured with foil heaters on balance calibration block and SVS backstop to elevate steady balance temperature to desired settings
 - Temperature actively controlled to within 2 deg. F of desired set point
- Static pressure applied to internal balance cavity via pressure fitting in rear of stump adapter
 - Cap plate on forward end of balance calibration block sealed off system, allowing application of static pressure to balance
 - Nitrogen used to pressurize balance



Analysis of a Split-Plot Experiment



- Ordinary least squares (OLS) is no longer valid since there is correlation between the y 's
- Two (2) methods available to estimate the two (2) variance components:
 - Pure error (PE) estimation combined with method of moments
 - Mixed modeling using restricted maximum likelihood (REML) estimation

Pure Error (PE) Estimation with Method of Moments	Restricted Maximum Likelihood (REML) Estimation
Uses PE degrees of freedom to estimate the variance components; closed-form expression	Uses residual degrees of freedom to estimate the variance components; numerical approximation
Variance estimates are model <i>independent</i>	Variance estimates are model <i>dependent</i>
Smaller number of degrees of freedom available, specifically for estimation of whole-plot variance	Combines PE degrees of freedom with lack-of-fit (LOF) degrees of freedom (usually more than PE alone)
Simpler approach - can use spreadsheet or develop simple computer code to estimate	Requires advanced statistical packages to perform the analysis

Because of the small number of PE degrees of freedom, PE estimation was not used in the analysis.

Mixed Modeling using Restricted Maximum Likelihood (REML) Estimation



Full Model:

▼ Response NF 2

▼ Summary of Fit

RSquare	1
RSquare Adj	1
Root Mean Square Error	1.174839
Mean of Response	-48.7988
Observations (or Sum Wgts)	305

▼ Parameter Estimates

Term	Estimate	Std Error	DFDen	t Ratio	Prob> t
Intercept	0.3039569	0.223454	7.651	1.36	0.2125
NF(-102.71,102.704)	2720.9531	0.1059	268.6	25694	<.0001*
PM(-153.27,153.198)	43.825561	0.176765	267.9	247.93	<.0001*
RM(-31.395,31.3817)	6.2179422	0.402344	268.9	15.45	<.0001*
YM(-45.552,45.5541)	-22.12604	0.176695	268.2	-125.2	<.0001*
SF(-34.554,34.5536)	-1.73543	0.158552	268.8	-10.95	<.0001*
T(75.9619,121.5)	-0.317332	0.160367	1.638	-1.98	0.2138
P(14.371,401.092)	0.1330707	0.113199	121.8	1.18	0.2421
NF*N	0.1446552	0.25992	268.1	0.56	0.5783
NF*PM	-0.643229	0.177727	267.8	-3.62	0.0004*
PM*PM	-0.077491	0.305954	268.1	-0.25	0.8002
NF*RM	-0.202724	0.246832	268.6	-0.82	0.4122
PM*RM	-1.350646	1.396116	268.2	-0.97	0.3342
RM*RM	-0.072912	0.282545	267.9	-0.26	0.7966
NF*YM	49.29421	220.9521	269.1	0.22	0.8236
PM*YM	1.0724982	1.065249	267.9	1.01	0.3149
RM*YM	-3.266322	1.500276	268	-2.18	0.0303*
YM*YM	-0.780003	0.336138	268.4	-2.32	0.0211*
NF*SF	-0.50314	0.553773	268.6	-0.91	0.3644
PM*SF	-56.44518	250.1036	269.2	-0.23	0.8216
RM*SF	-3.163069	0.918225	267.9	-3.44	0.0007*
YM*SF	-0.090354	0.249468	268.3	-0.36	0.7175
SF*SF	1.0001693	0.269498	269.1	3.71	0.0003*
NF*T	-1.652273	0.112248	268.7	-14.72	<.0001*
PM*T	-0.450078	0.185781	268.2	-2.42	0.0161*
RM*T	0.196022	0.184126	268.6	1.06	0.2880
YM*T	-0.071005	0.188347	268.1	-0.38	0.7065
SF*T	-0.143872	0.135105	267.9	-1.06	0.2879
NF*P	-0.597004	0.142866	269.1	-4.18	<.0001*
PM*P	-0.310467	0.241366	267.8	-1.29	0.1995
RM*P	0.2469309	0.226351	269.9	1.09	0.2763
YM*P	-0.267016	0.234405	268.2	-1.14	0.2557
SF*P	-0.162518	0.171196	268.5	-0.95	0.3455
T*P	-0.206366	0.121788	115.6	-1.69	0.0929
P*P	-0.242712	0.153946	256.9	-1.58	0.1161

▼ REML Variance Component Estimates

Random Effect	Var Ratio	Component	Std Error	95% Lower	95% Upper	Pct of Total
Rep[Unique]	0.0426806	0.0589097	0.0911723	-0.119735	0.237604	4.093
Residual		1.380246	0.1192788	1.1733433	1.6474583	95.907
Total		1.4391557				100.000

-2 LogLikelihood = 960.73060638

► Covariance Matrix of Variance Component Estimates

► Iterations

Reduced Model:

▼ REML Variance Component Estimates

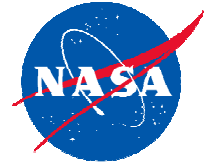
Random Effect	Var Ratio	Component	Std Error	95% Lower	95% Upper	Pct of Total
Rep[Unique]	0.1113402	0.1741066	0.1588649	-0.137263	0.4854761	10.059
Residual		1.556744	0.1288234	1.3321718	1.8436325	89.941
Total		1.7308506				100.000

-2 LogLikelihood = 1015.1155084

► Covariance Matrix of Variance Component Estimates

► Iterations

Comparison of Experimental Designs



- In force balance calibration, the back-computed residuals are used to quote the accuracy of a balance component
 - All data collected (standard, crossed, optimal) were used to estimate the accuracy
 - Accuracies are based on full calibration models (no model reduction)
- Both experimental designs show improvement in the accuracy when pressure and temperature terms are included in the model
- Small differences in accuracy between the crossed and optimal designs

Design Points	2-Sigma Accuracy (% Full-Scale)				
	NF	PM	RM	YM	SF
Standard Calibration (No Pressure or Temperature)	0.122	0.108	0.478	0.165	0.210
Crossed Design	0.085	0.070	0.464	0.119	0.163
IV-Optimal Design	0.110	0.092	0.572	0.158	0.205

Conclusions



- An engineering problem was presented to the team. A methodical approach was developed to solve this problem, which combined efforts from both engineering and statistical fields of expertise.
- Demonstrated a method to characterize the force balance in the wind tunnel environment, including temperature and pressure effects and thereby improving aerodynamic research data quality
 - Data from calibration and MSL test data reveals significant improvement (multiple sources contribute to increased data improvement)
- Both calibration designs provide similar results
 - Magnitude of the calibration coefficients
 - Accuracy of the force balance
- Appropriate metrics were used to evaluate the robustness of the experimental designs
- Resulting mathematical models were transferred to the wind tunnel and aerodynamic data collected was compensated for pressure and temperature real-time (first time implementation)

Contacts and References



- **Contacts for More Information:**

Keith (Chris) Lynn (keith.c.lynn@nasa.gov)

Sean Commo (sean.a.commo@nasa.gov)

Peter Parker (peter.a.parker@nasa.gov)

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