

Analysis of the equilibrium wall model for wall-bounded high-speed turbulent flow

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Motivation

- **Wall-Modeled LES (WMLES) and hybrid RANS-LES** methods show improved predictions when compared to the state-of-the-art RANS models for **turbulent separated flows**.
- **High fidelity DNS/ WRLES too expensive/ infeasible** for realistic high Re encountered in flight.
- **Further understanding/ establishment of best practices** of WMLES for compressible canonical (and attached) flows necessary before assessing more complex flows.

**Outcome of the
NASA CFD Error
Prediction
Workshop 2018,
Suffolk, VA**

Outline of Presentation

- ❑ Wall Model Equations
- ❑ *A priori* results
- ❑ *A posteriori* WMLES results
- ❑ Summary

Equilibrium Wall Model Equations

- Compressible equilibrium BL ODEs solved in WM region (Kawai & Larsson 2012, Larsson et al. 2016).

$$\frac{d}{dy} \left[(\mu + \mu_{t,wm}) \frac{du_{||}}{dy} \right] = 0$$
$$\frac{d}{dy} \left[(\mu + \mu_{t,wm}) u_{||} \frac{du_{||}}{dy} + (\lambda + \lambda_{t,wm}) \frac{dT}{dy} \right] = 0$$

- Nonequilibrium effects (pressure gradient, acceleration, unsteady terms) neglected.

Mixing Length Models

- Eddy viscosity obtained from mixing-length model and turbulent thermal conductivity obtained assuming constant $Pr_t=0.9$.

$$\mu_{t,JK} = \rho \kappa y \sqrt{\frac{\tau_w}{\rho}} D$$

Equilibrium version of Johnson-King Model (1985)
Cabot (1995)

$$\mu_{t,Pr} = \rho (\kappa y)^2 \left| \frac{du}{dy} \right| D$$

Prandtl Model (1925)

$$\lambda_t = \frac{C_p \mu_t}{Pr_t}$$

Typically constant Pr_t of 0.9 assumed

$$D = \left[1 - \exp\left(-\frac{y^+}{A^+}\right) \right]^2$$

Van-Driest Damping function (empirical)

- Von Karman (κ) assumed constant ~ 0.41 (but varies between 0.38 – 0.41 depending on the type of flow)
- Both mixing length models reduce to the compressible law of the wall (Bradshaw 1994, Trettel & Larsson 2016).

Damping Function for Compressible Flows

- The standard Van-Driest (1956) damping function used for the Prandtl mixing length model is:

$$D = \left[1 - \exp\left(-\frac{y^+}{A^+}\right) \right]^2$$

- With $A^+ = 26$. Note that the damping function is purely empirical with the requirement that $D \sim 1$ for $y^+ > 30$ (log-law region), and $D \sim 0$ as $y^+ \rightarrow 0$. For the JK model, $A^+ = 17$ gives best predictions.
- For incompressible flows, no ambiguity in defining y^+ . For compressible flows, multiple definitions possible. Note that all these satisfy the basic requirements of the damping function!

$$y_{wall}^+ = \frac{\rho_w u_\tau y}{\mu_w}$$

Traditionally used for compressible flows
Kawai & Larsson (2012)

$$y_{SL}^+ = \frac{\sqrt{\rho_w \rho} u_\tau y}{\mu}$$

Most recently recommended scaling by :
Aupoix & Viala (1995)

Bocquet, Sagaut & Jouhaud (2012)
Lv & Yang (2018),
wmles.umd.edu

} Only tested cold wall flows!

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$$y_{local}^+ = \frac{\rho u_\tau y}{\mu}$$

$$y_{mixed}^+ = \frac{1}{2} [y_{wall}^+ + y_{SL}^+]$$

$$y_{mixed \min}^+ = \min(y_{mixed}^+, y_{SL}^+)$$

Newly proposed scalings

$$D_{VD} = \left[1 - \exp\left(-\frac{y^+}{A^+}\right) \right]^2$$

$$D_{SA} = \frac{\hat{\mu}_t^3}{\hat{\mu}_t^3 + C_{v1}^3 \mu^3}$$

Damping function used in the SA model
 Hat => undamped turbulent viscosity
 ($C_{v1} = 7.1$)

A priori analysis of DNS data

Wall Model Parameters

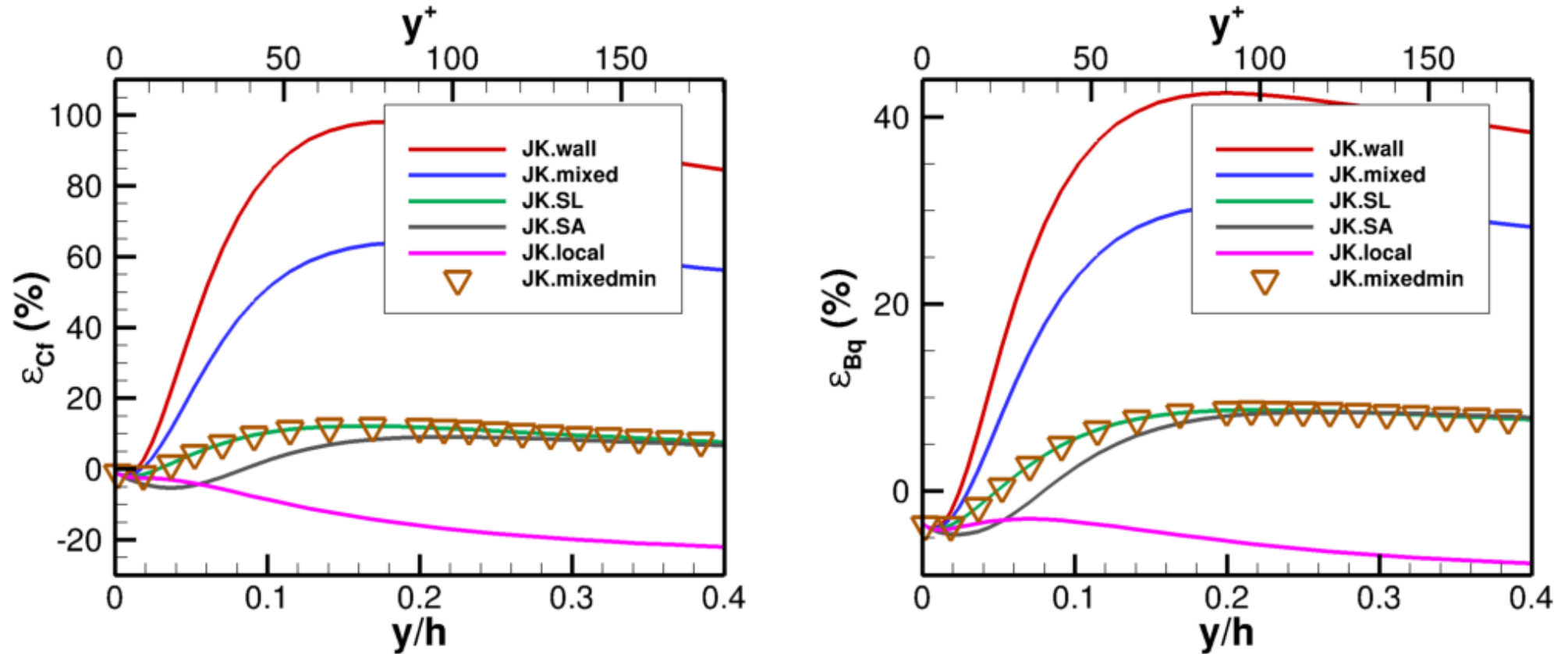
- Input to WM: u , T from DNS
- Output of WM: C_f and B_q / T_w
- WM solved using 2nd order staggered finite difference (conservative fluxes)
- Typically 40 points used within WM layer with $\Delta y_w^+ < 0.1$ for all cases. Results quite insensitive to number of grid points used in solving the WM equations.
- Typical time to solution : $< 1 \mu s$.

$$C_f = \frac{\mu_w \left. \frac{du}{dy} \right|_w}{\frac{1}{2} \rho_{ref} u_{ref}^2}$$

$$B_q = \frac{-\lambda_w \left. \frac{dT}{dy} \right|_w}{\rho_w u_\tau C_p T_w}$$

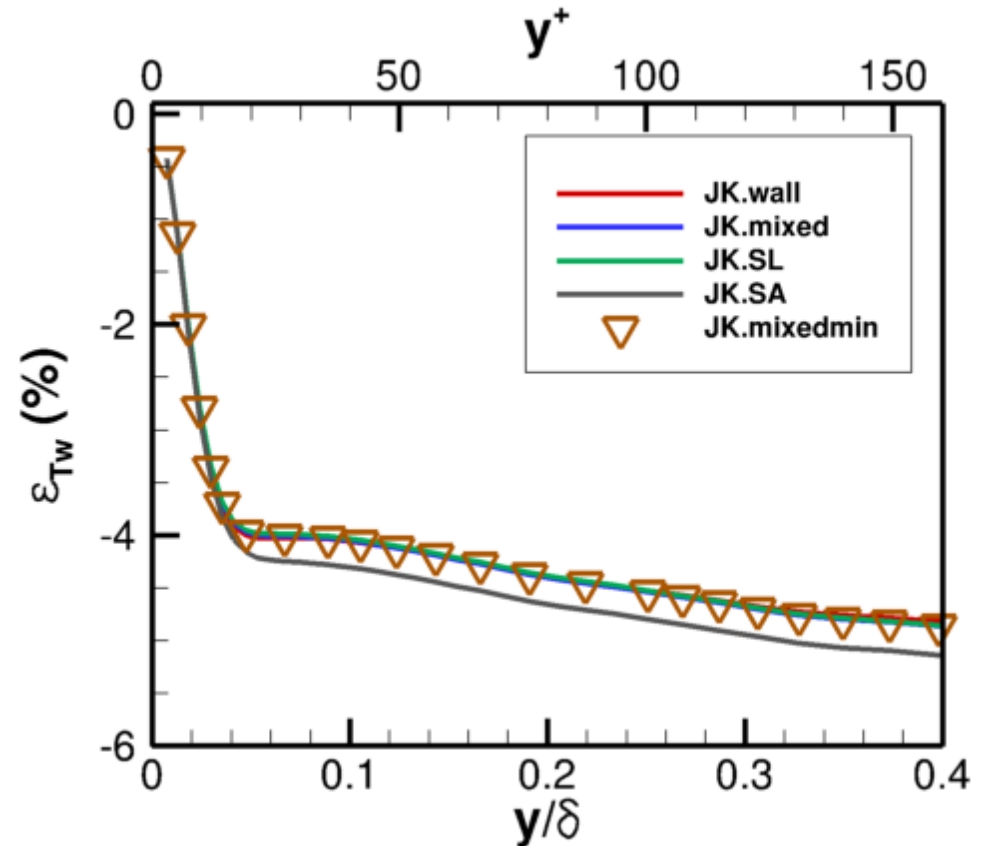
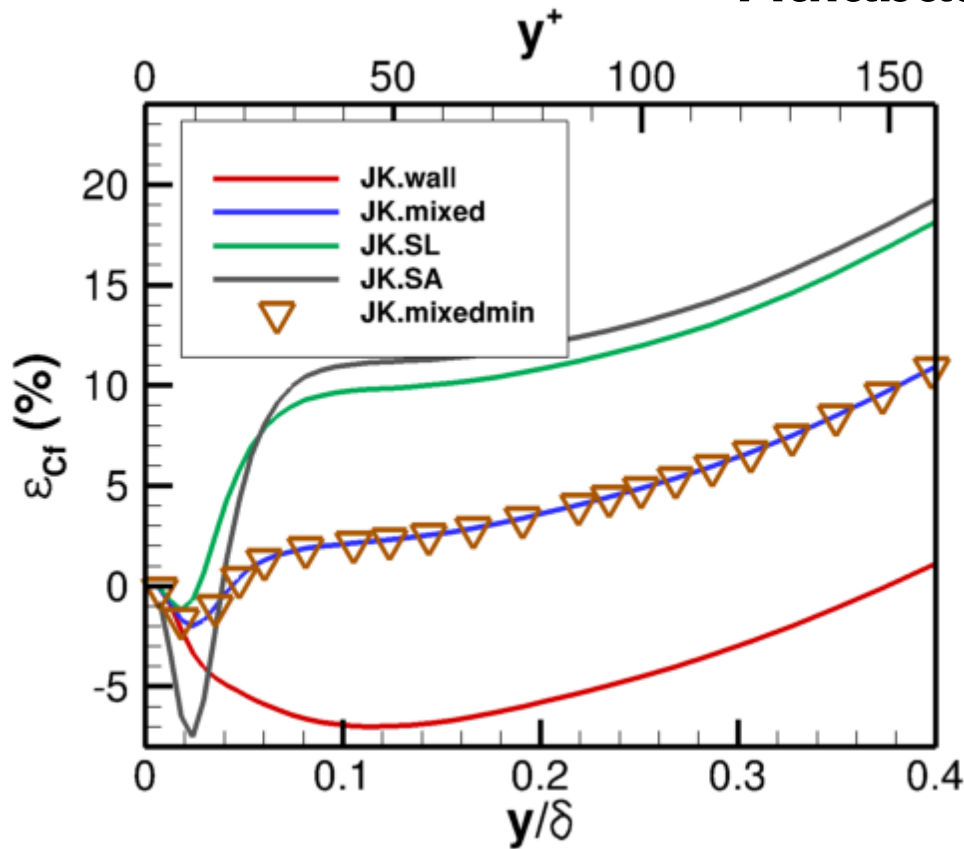
$$\varepsilon_{Cf} (\%) = \left(\frac{C_{f,wm} - C_{f,DNS}}{C_{f,DNS}} \right) \times 100 \quad \varepsilon_{Bq} (\%) = \left(\frac{B_{q,wm} - B_{q,DNS}}{B_{q,DNS}} \right) \times 100$$

Mach 3 Cold Wall Channel



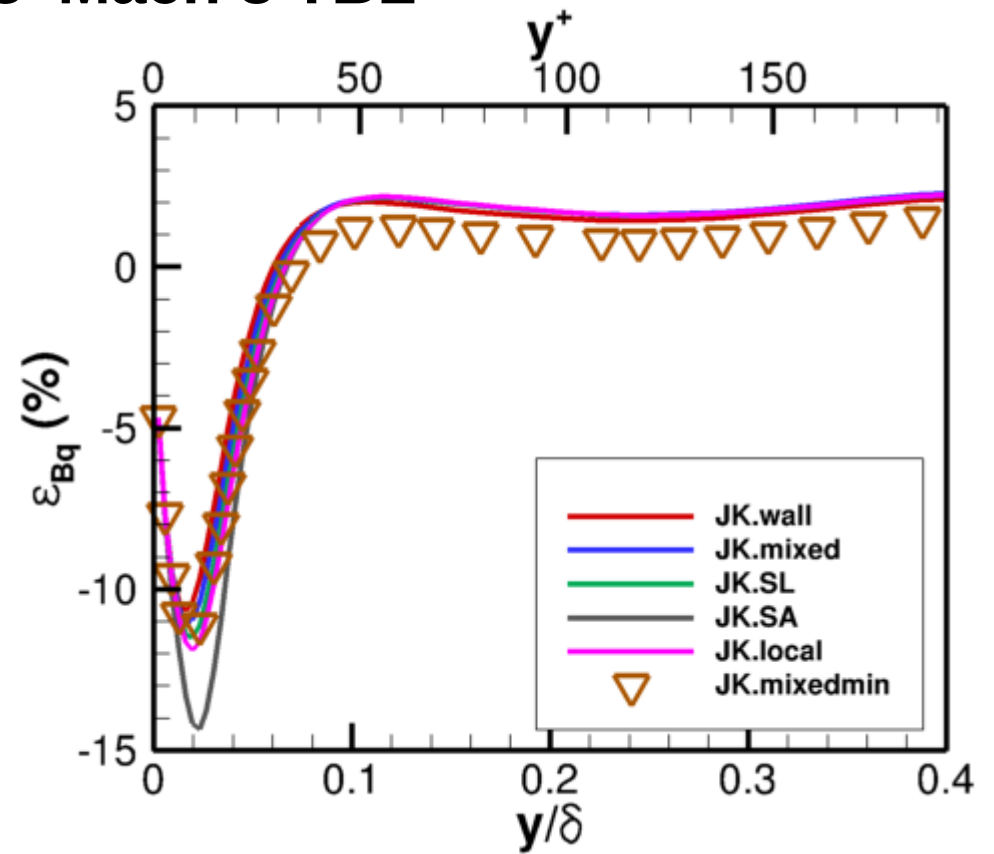
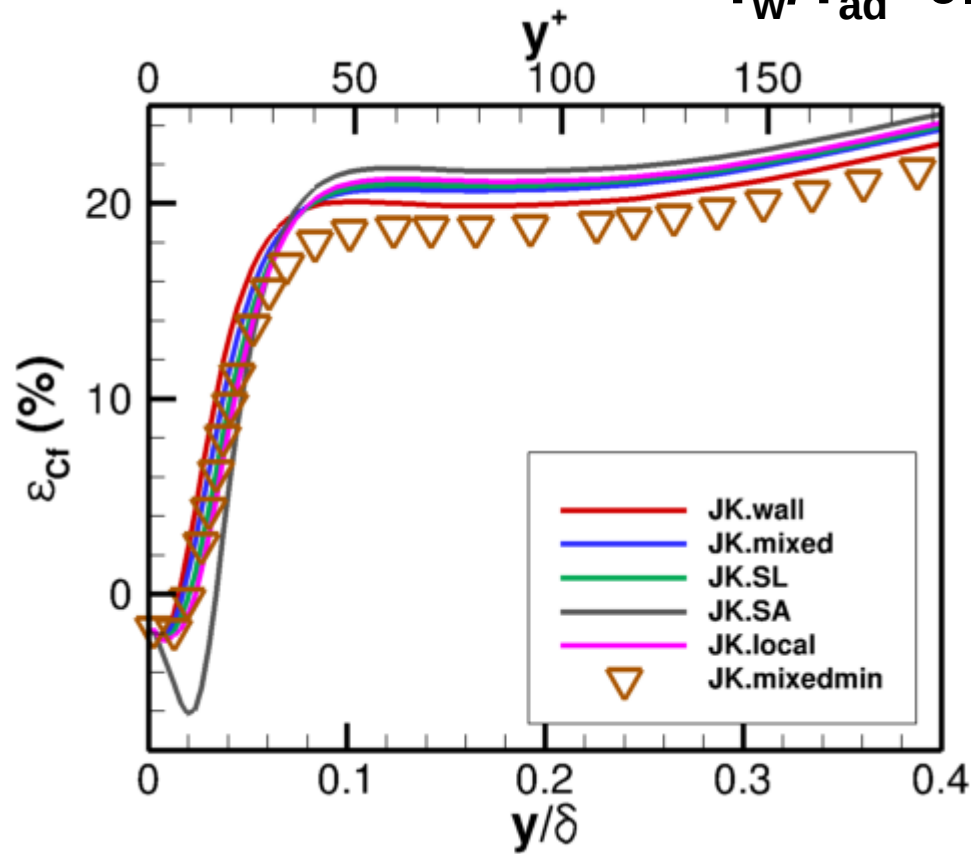
- $M = 3$, $Re_b = 2650$, $Re_\tau = 451$, $T_w/T_b \sim 0.4186$ corresponding to DNS of Coleman, Kim & Moser (1995).
- SL works best for VD damping function. Consistent with recommendations of previous studies (Bocquet et al. 2012, Lv & Yang 2018). SA damping function also works as well.
- Note that the *mixedmin* scaling gives identical results to the SL scaling for cold wall cases. Overall, error in $C_f \sim 5\text{-}10\%$ depending on the exchange location.

Adiabatic Mach 4 TBL



- $M_\infty = 4$, $Re_\tau = 400$, adiabatic wall corresponding to DNS of Pirozzoli & Bernardini (2011).
- Mixed scaling works best for VD damping function. SL & SA damping functions have 10% error.
- Note that the *mixedmin* scaling gives identical results to the mixed scaling for hot wall cases. Overall, error in $C_f < 3\%$ for mixed scaling.
- Wall temperature well predicted by all scalings!

$T_w/T_{ad}=0.48$ Mach 8 TBL

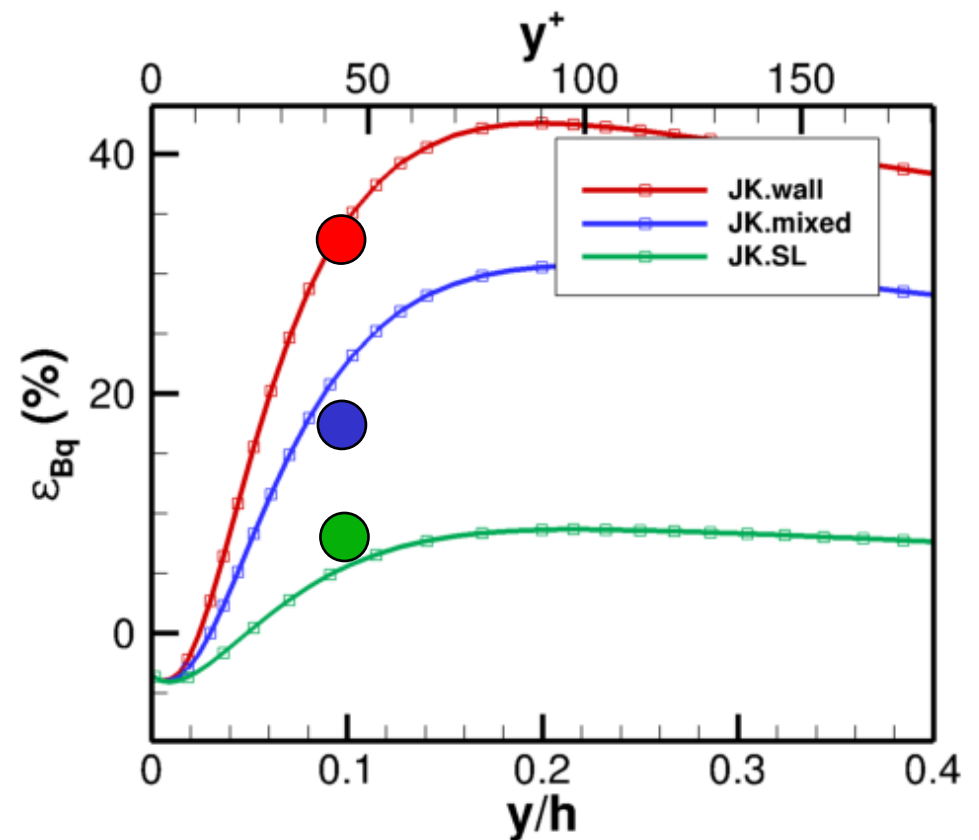
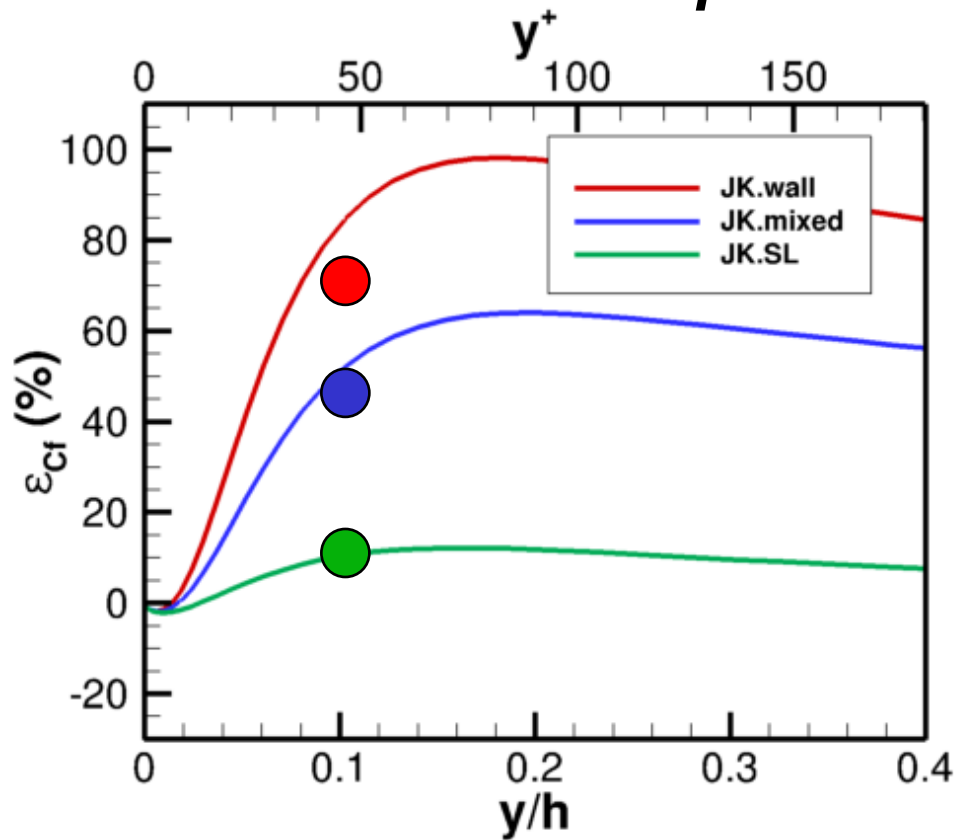


- $M_\infty = 7.87$, $Re_\tau = 480$, $T_w/T_r = 0.48$ TBL corresponding to DNS of Zhang, Duan & Choudhari (2018).
- All scalings give nearly identical results.
- Overall, error in $C_f \sim 5\text{-}20\%$ depending on the exchange location.

***A posteriori* WMLES results**

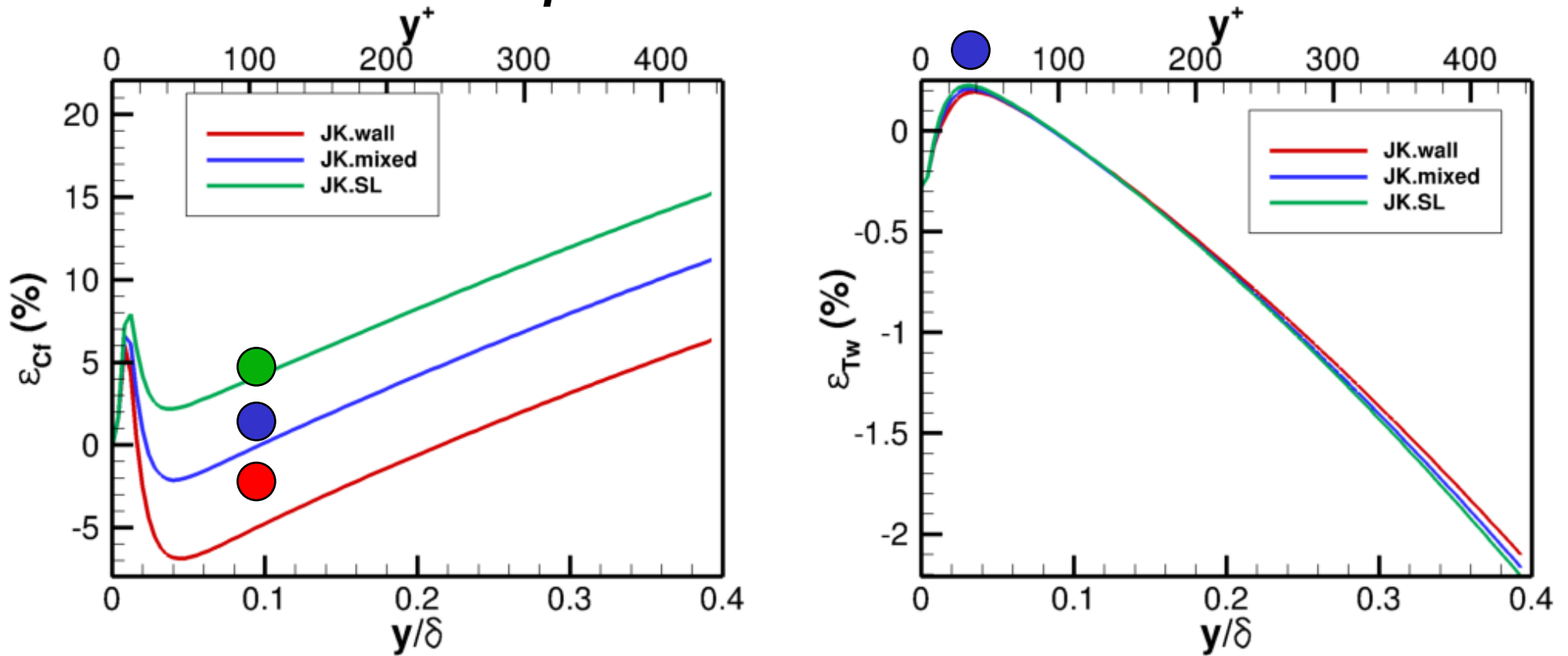
Charles* solver (2nd order unstructured FVM)
16-20 points per δ in each direction
Results not too sensitive to grid resolution

A posteriori WMLES results



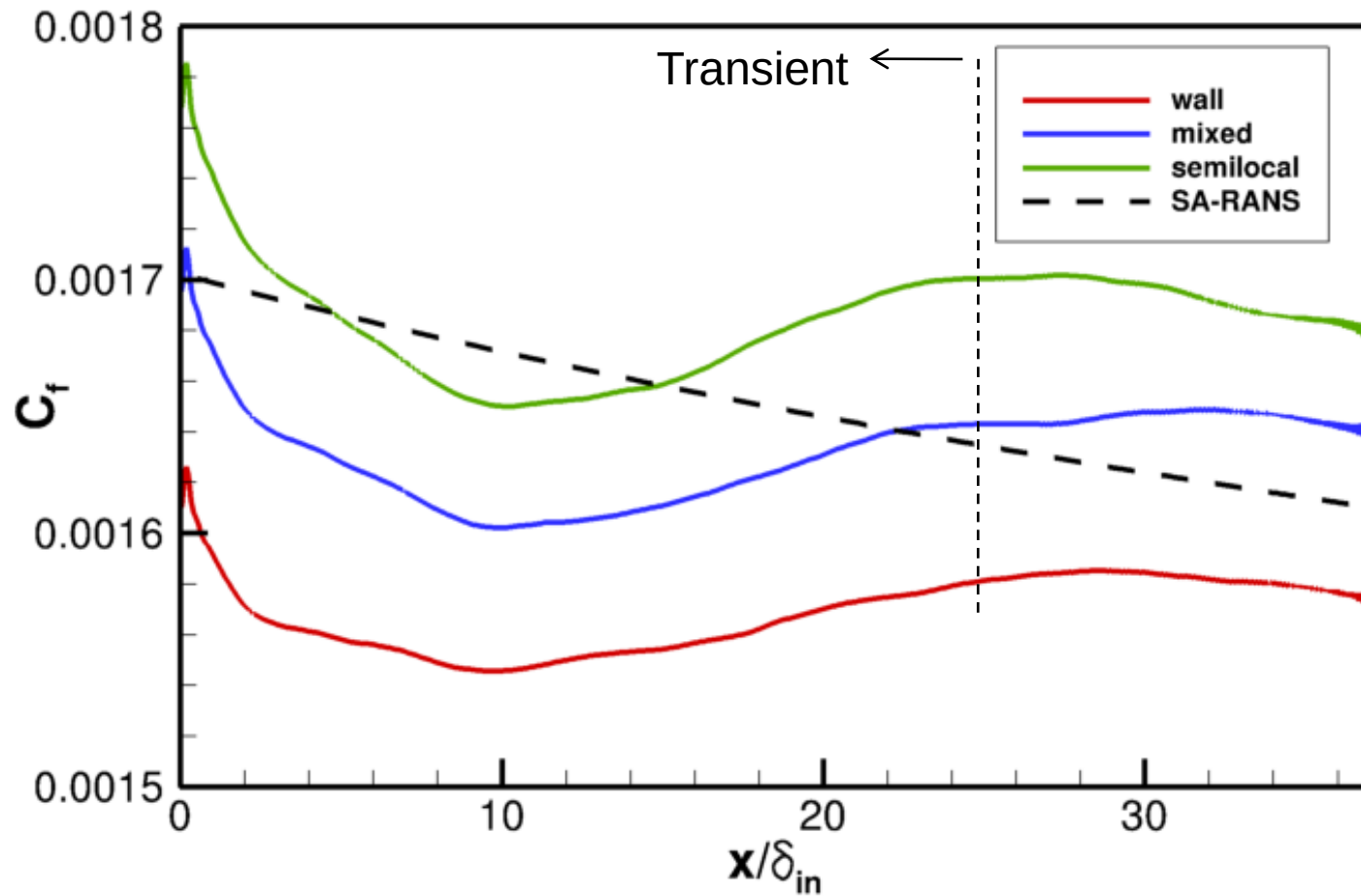
- $M = 3$, $Re_b = 2650$, $Re_\tau = 451$, $T_w/T_b \sim 0.4186$ corresponding to DNS of Coleman, Kim & Moser (1995).
- *A posteriori* WMLES results (solid circles) generally consistent with *a priori* analysis.
- Overall, SL scaling best with an error in $C_f \sim 10\%$. Decreasing exchange location to 0.05 and/ or refining the grid did not reduce the error further.

A posteriori WMLES results



- $M = 2.85$, $Re_\delta = 1.98 \times 10^5$, $Re_\theta = 11,880$, adiabatic wall corresponding to experiments of Dunagan et al.(1986)
- A posteriori WMLES results at $\sim 25\delta_{in}$ downstream (solid circles) generally consistent with a priori analysis.
- Overall, mixed scaling best with an error in $C_f < 2\%$. Inflow turbulence method also introduces some error in the results.

Mach 2.85 Adiabatic TBL



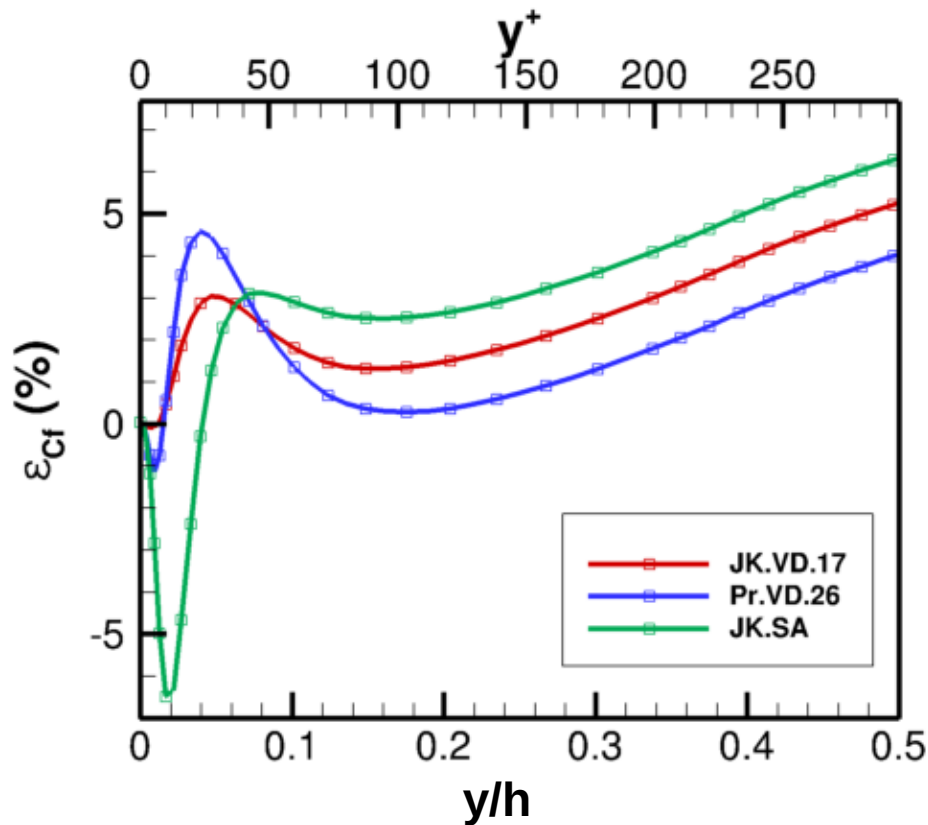
- Improved predictions by mixed scaling. Consistent with a priori analysis results.

Summary

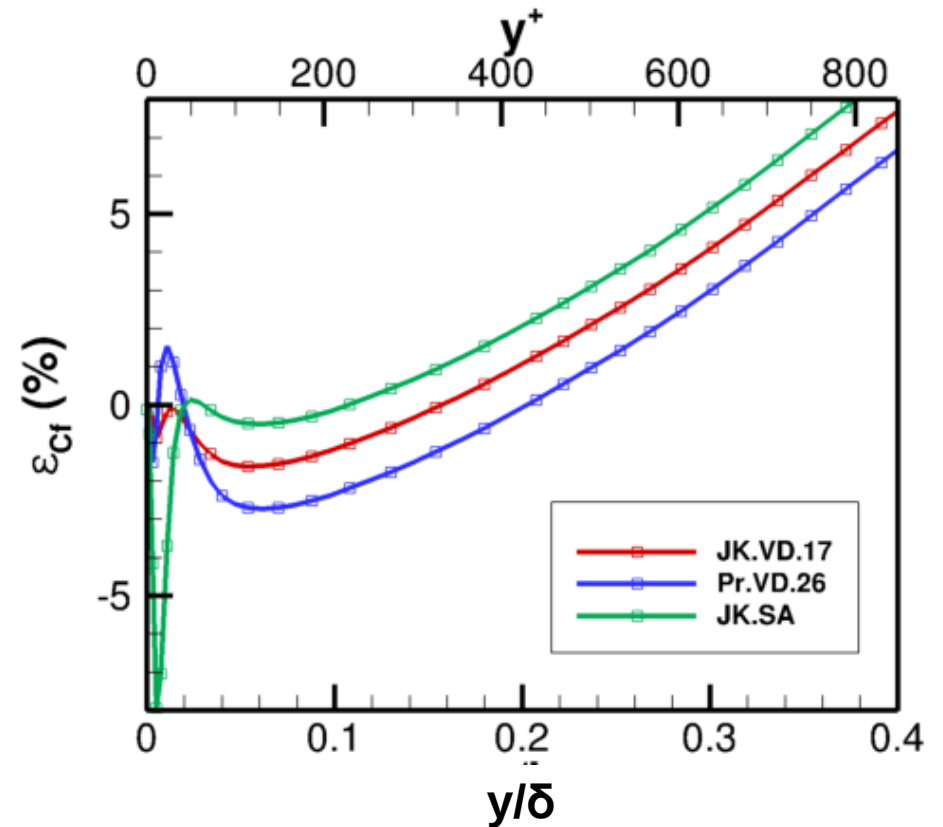
- Analyzed the predictive ability of the equilibrium wall model for compressible flows over a range of thermal BCs.
- *A priori* analysis of DNS databases showed that for $EL \sim 0.05-0.1\delta$:
 - old {
 - **wall scaling** erroneous for both cold and adiabatic wall conditions
 - **semilocal scaling** works best for cold walls with $\sim 10\%$ error in C_f
 - new {
 - **mixed scaling** works best for adiabatic walls with $< 3\%$ error
 - **mixedmin** scaling automatically switches between **mixed** and **semilocal** scaling depending on flow conditions.
- All the different scalings collapse when $T_w/T_{ad} \sim 0.5$ but with an error of 10-20%.
- *A posteriori* WMLES for cold Mach 3 channel and adiabatic Mach 2.85 TBL consistent with the *a priori* results.

Backup Slides

Effect of ML model for Incompressible Flows



$Re_T = 590$ Channel



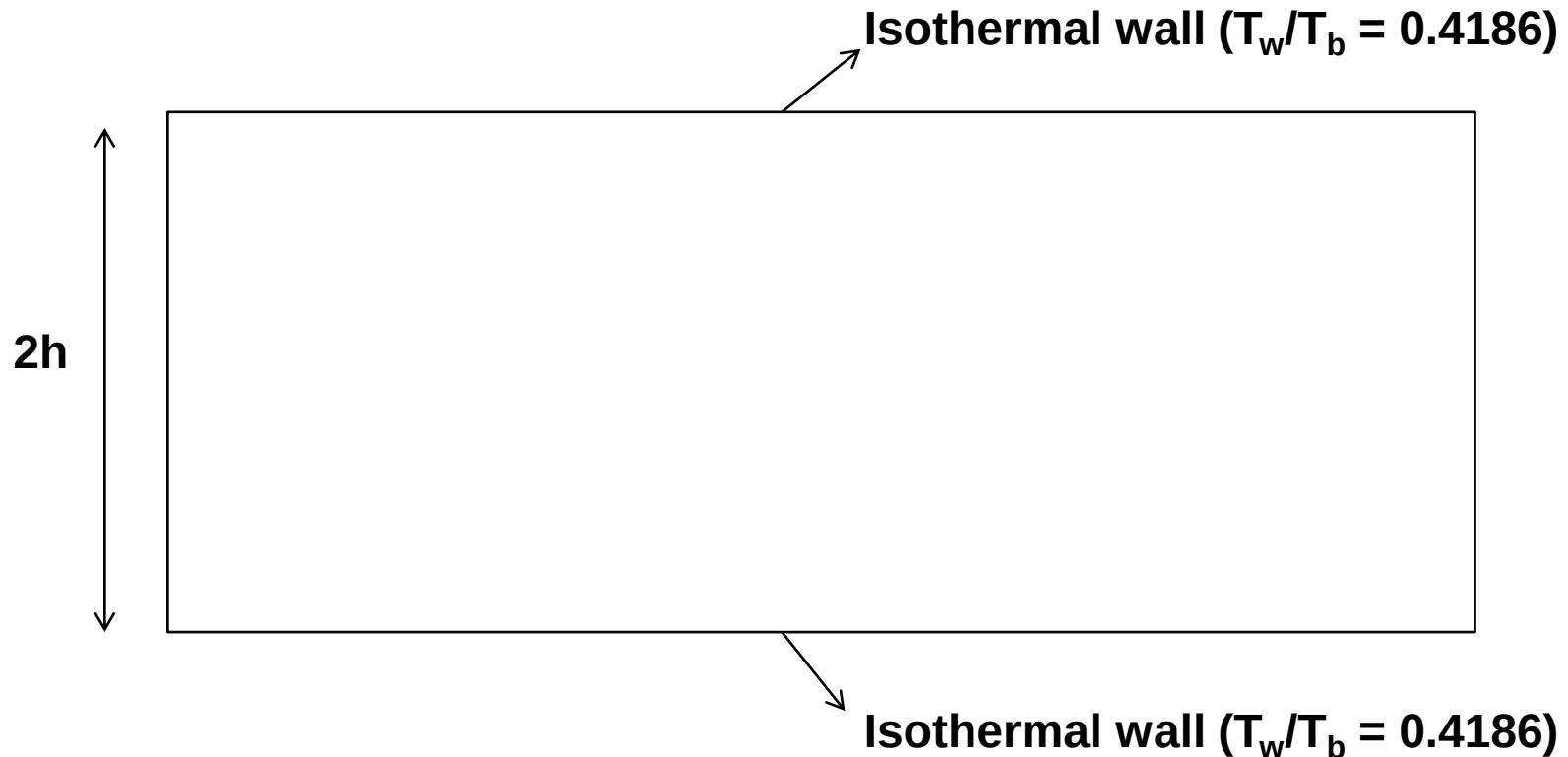
$Re_T = 2118$ TBL (Schlatter & Orlu 2010)

- Overall, EQWM works well for incompressible flow with $< 3\%$ error in C_f .
- Typical EL ~ 0.05 to $0.1 h$ (or δ). SGS & numerical errors will likely show up if EL is closer to the wall (Kawai & Larsson 2012).

Numerical Details

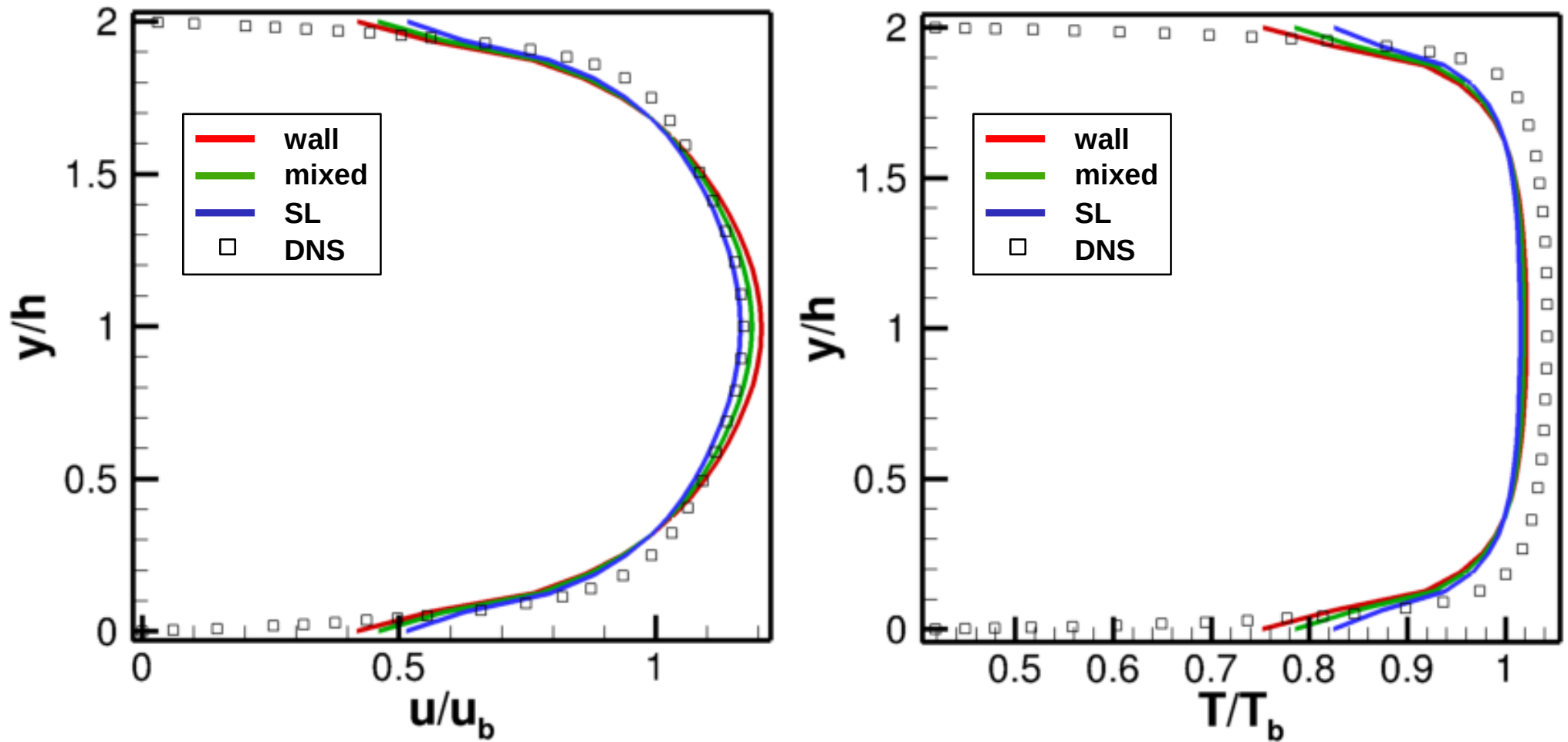
- CharLES solver from Cascade Technologies
- Compressible N-S, unstructured finite volume discretization, spatially 2nd order accurate
- Explicit RK3 time integration
- Low dissipation solver
- Vreman SGS model with equilibrium wall model
- Additional cost of solving wall model equations:
~10-30% for equilibrium WM.

Compressible Turbulent Channel Description



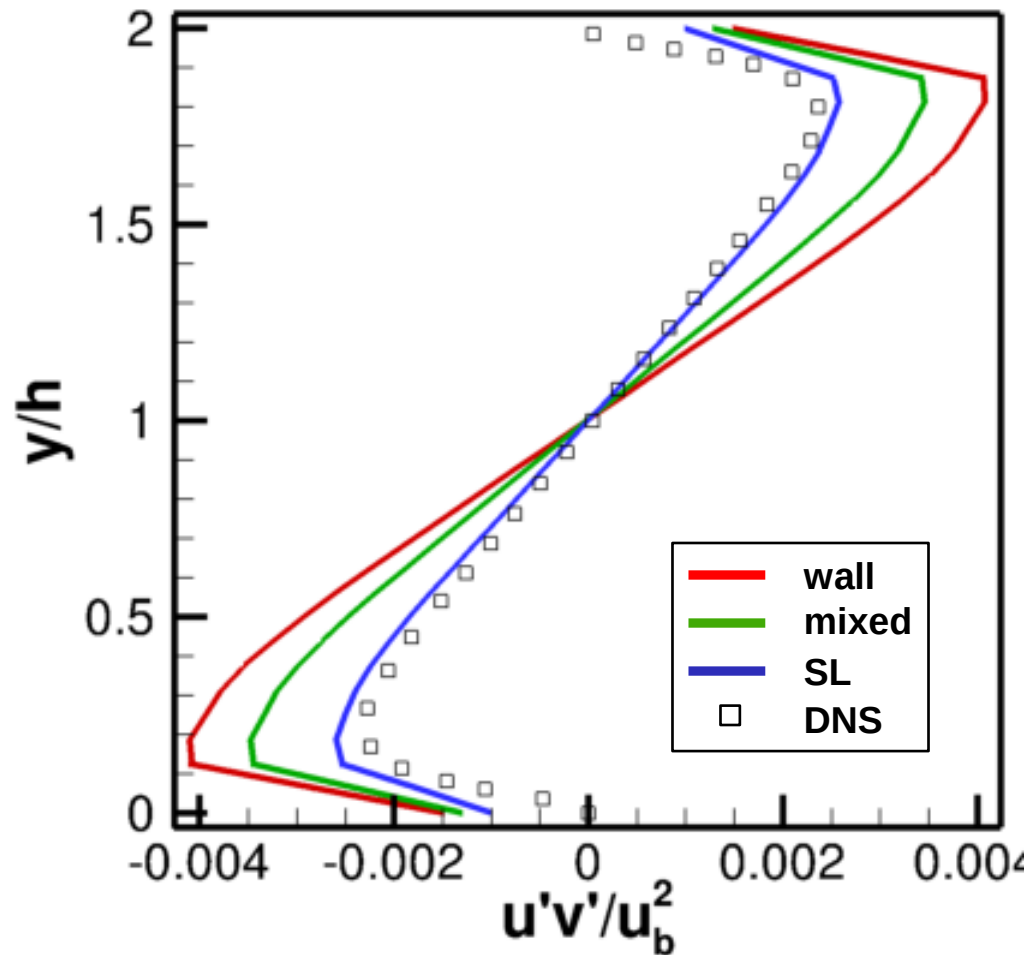
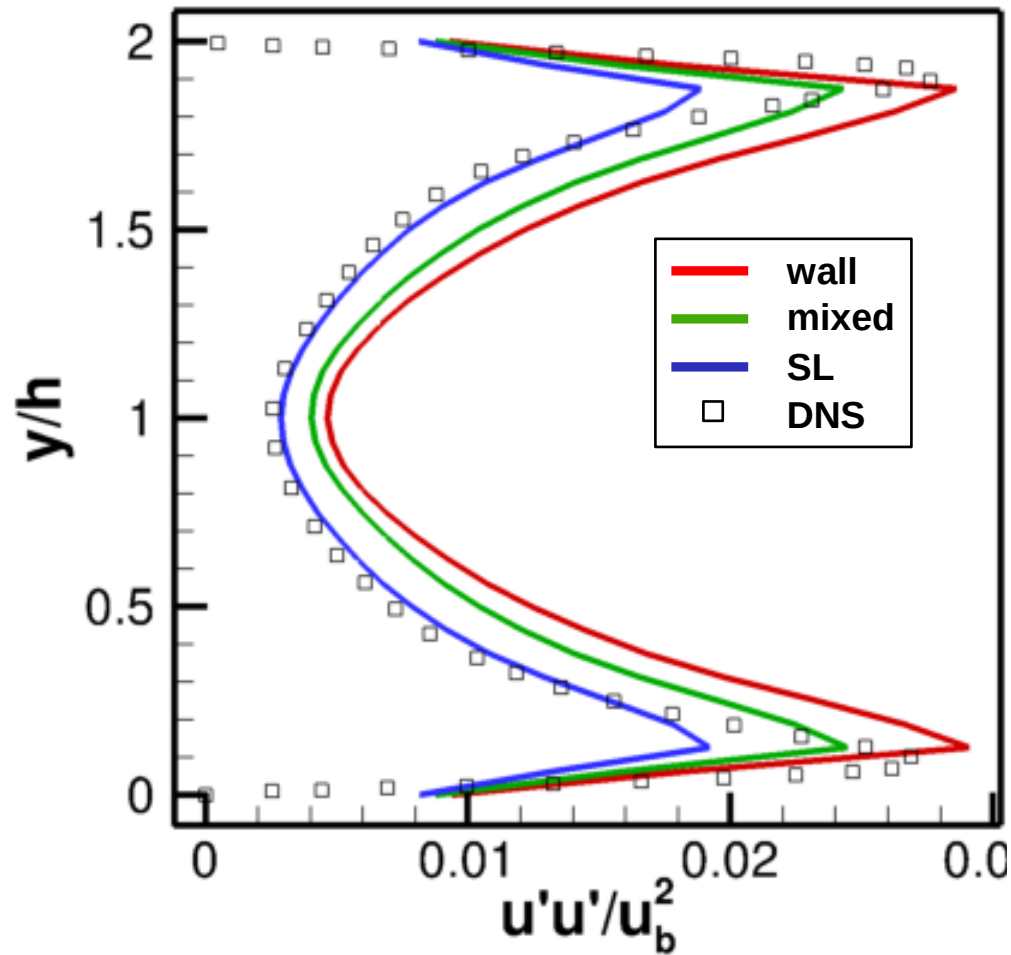
- Conditions corresponding to DNS of Coleman, Kim & Moser (1995).
- Periodic BCs in x , z . Wall-model applied at the walls.
- Source terms added to x -momentum and energy equations to maintain constant bulk velocity and temperature (values obtained from DNS).
- Uniform grid in all directions. 16 points per channel half width (coarse grid).
- Exchange location set to $0.1h$.

Mach 3 Coleman



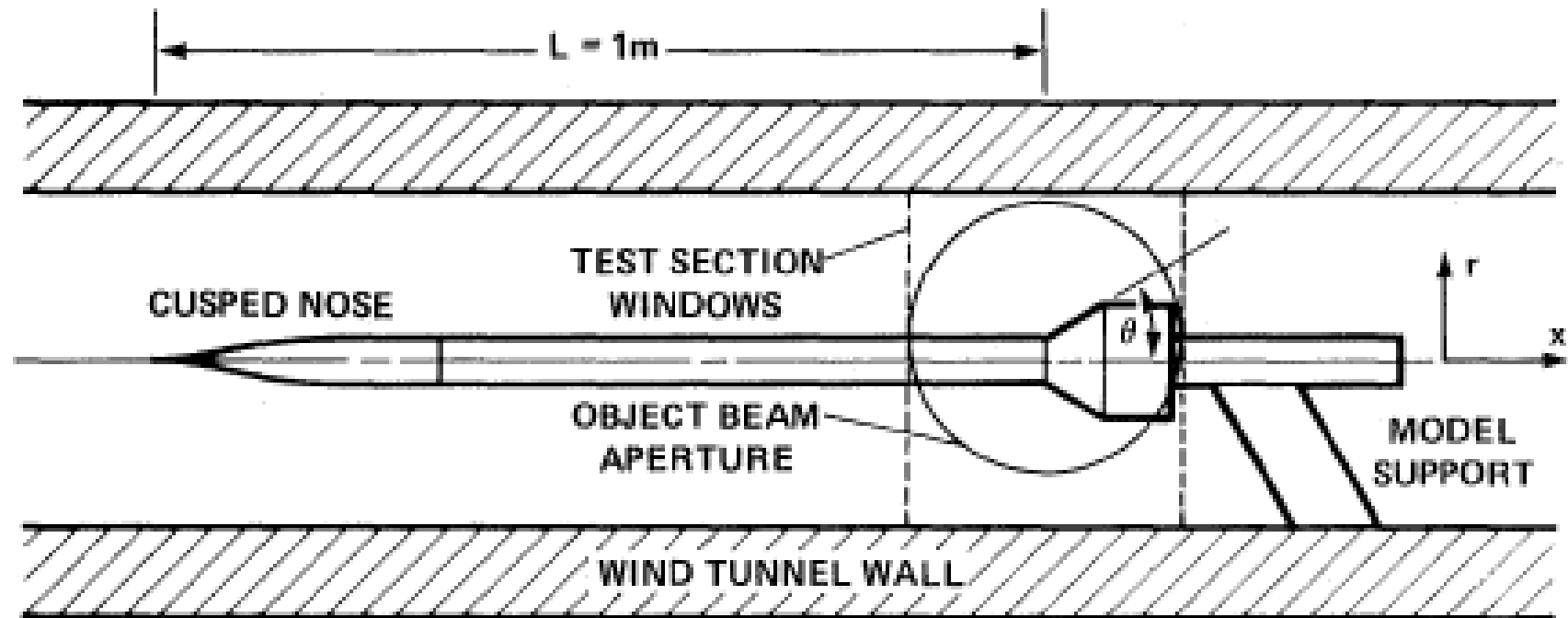
- Even though C_f predictions are way off for wall and mixed scalings, the results look similar for all scalings when scaled with outer units.
- Clearly, if we used inner scaling, there would be a large log-layer mismatch for mixed and wall scalings.
- Overall, agreement with DNS in terms of outer units quite good.

Mach 3 Coleman



- SL damping results give superior predictions due to the more accurate skin friction prediction.

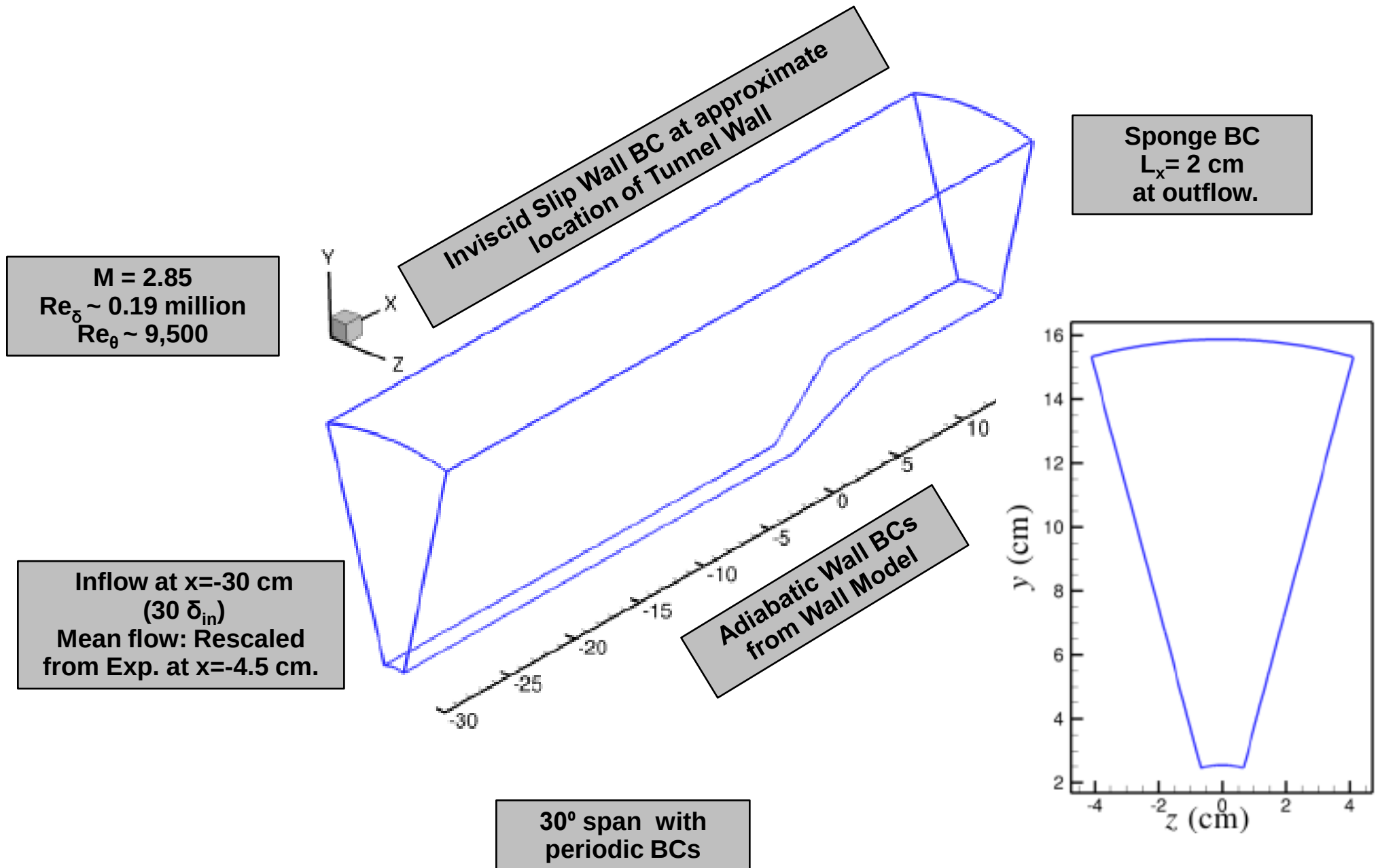
Axisymmetric Mach 2.85 TBL



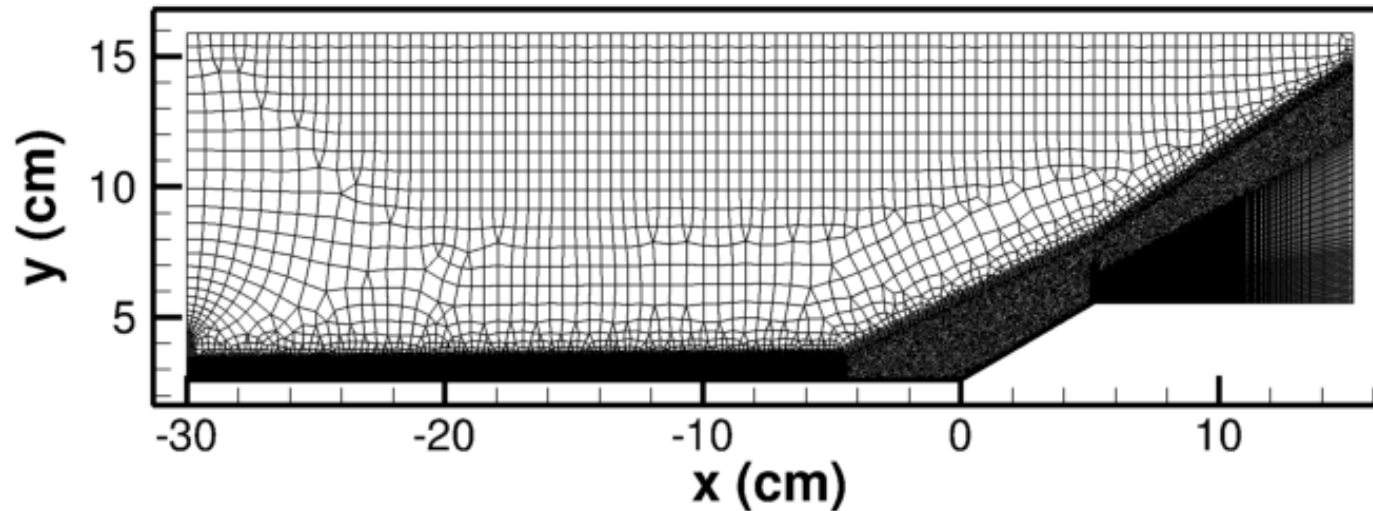
M_∞	$Re = \rho_\infty u_\infty / \mu_\infty$	Re_δ	Re_{δ^*}	Re_θ	p_0	T_0
2.85	18 million/m	1.98×10^5	59,400	11,880	1.7 atm	270K

- Experiment by [Dunagan et al. \(1986\)](#) corresponding to 30° flare angle.
- Rectangular Tunnel Wall, Asymmetric model support, Leading edge effects **neglected** in simulations.
- Hence, an [exact quantitative match is not expected](#), but above effects are expected to be small.

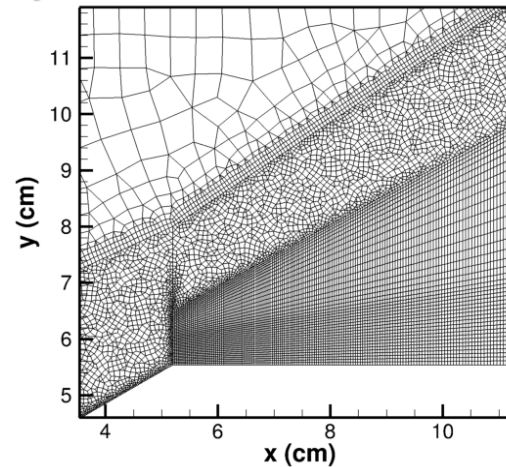
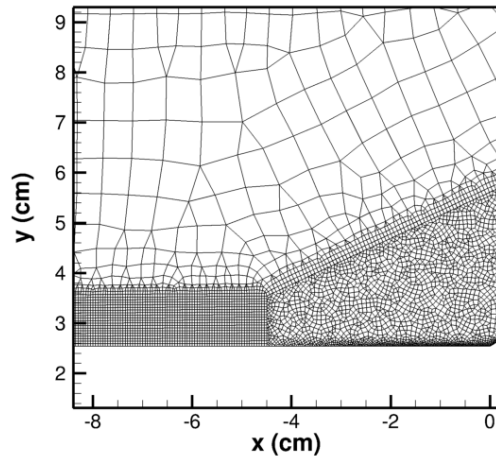
Computational Domain



Computational Grid Details



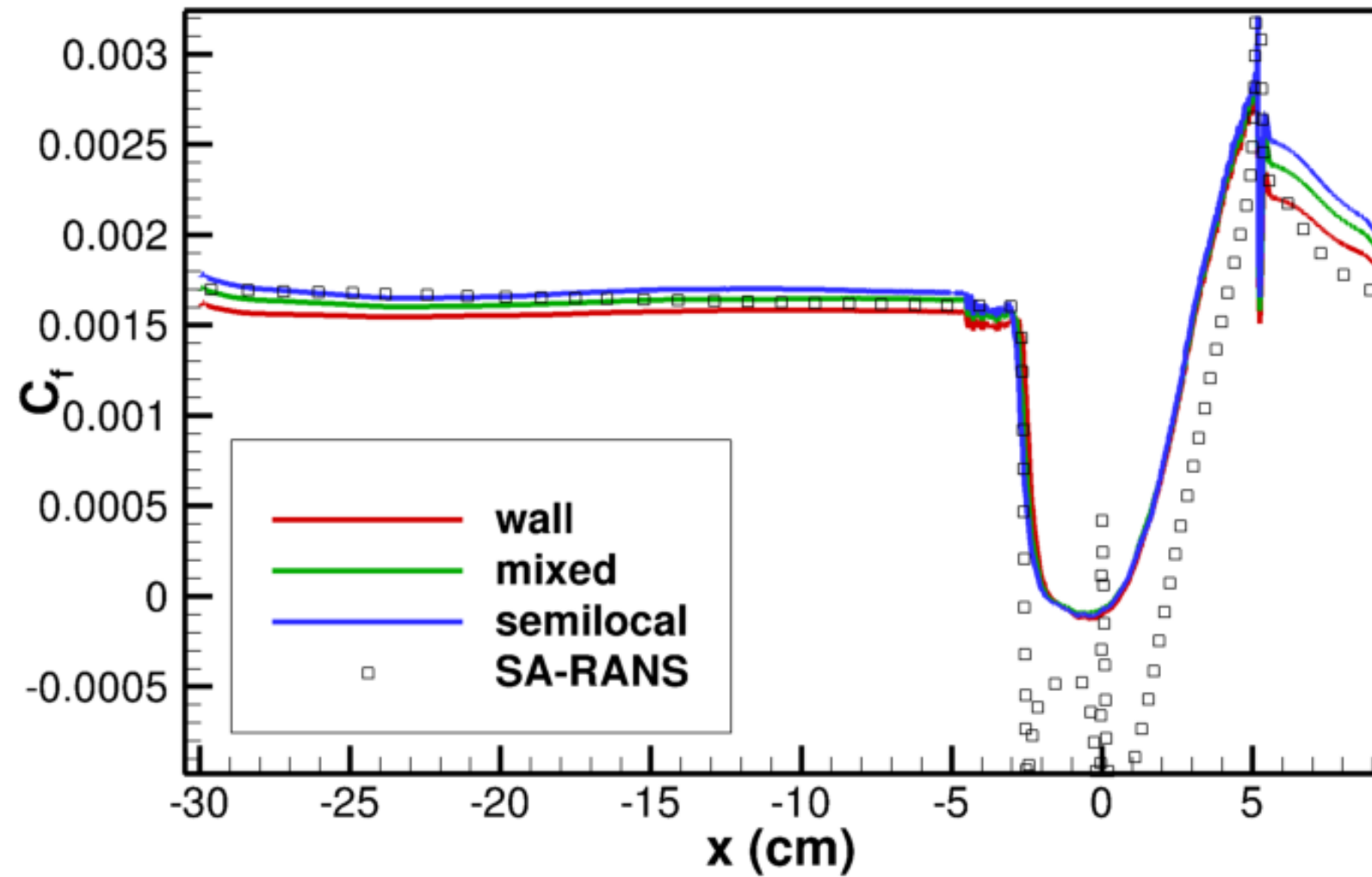
**RANS grid ~
0.9 million!**



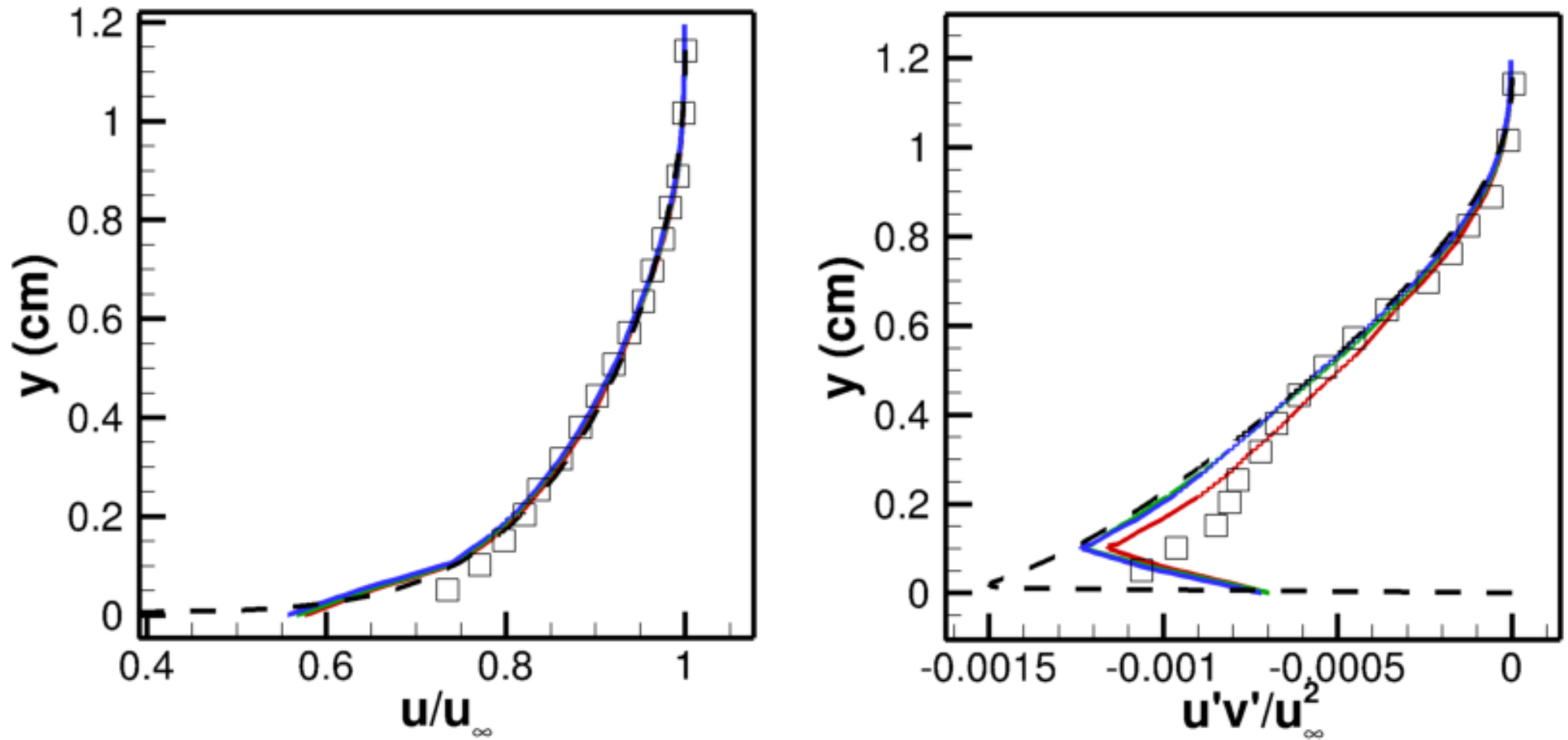
- Compression corner at $x = 0$ cm.
- ~ 1 week on 500 cores for fine grid.
- Exchange location at ~9% of BL (0.09 cm).

Grid	Region	$\Delta x, \Delta y_w, \Delta z_w$ (cm)	$\Delta x^+, \Delta y_w^+, \Delta z_w^+$	δ (cm)	$N_x/\delta, N_y/\delta, N_z/\delta$	Grid count
Coarse	$x = -5$ cm	0.05, 0.052, 0.022	86, 90, 38	1.1	22, 21, 50	2 million

Mach 2.85 Adiabatic TBL



Mach 2.85 Adiabatic TBL

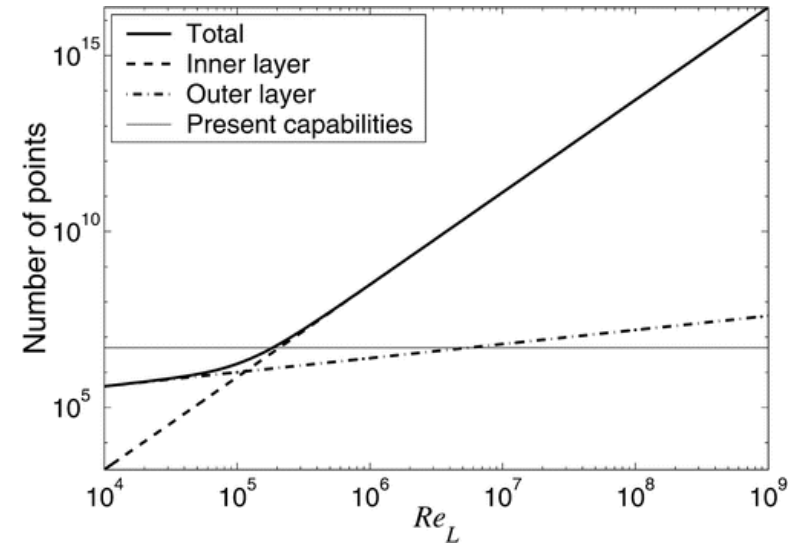


- Profiles at $x = -5$ cm. In terms of outer scaling, all curves collapse onto each other.
- Minor differences in Reynolds Shear Stress with the wall scaling underpredicting the magnitude of of the stress due to lower C_f prediction.

Motivation



Boeing 787 Dreamliner¹



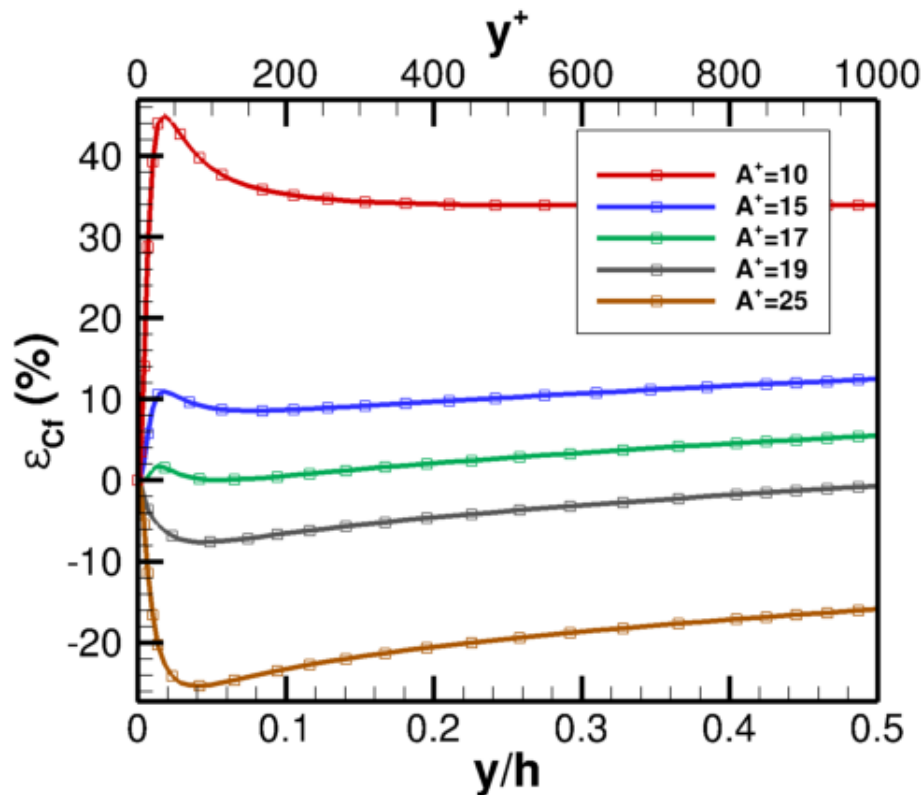
Piomelli & Balaras (2002)²

- Most RANS models work reasonably well for attached flows, **but do not accurately predict flows involving separation.**
- **High fidelity DNS/ LES too expensive/ infeasible** for realistic high Re encountered in flight.
- **Wall-Modeled LES (WMLES) and hybrid RANS-LES** methods show improved predictions when compared to the state-of-the-art RANS models for **turbulent separated flows.**
- **Further understanding/ establishment of best practices** of WMLES for compressible canonical flows necessary before application to more complex flows.

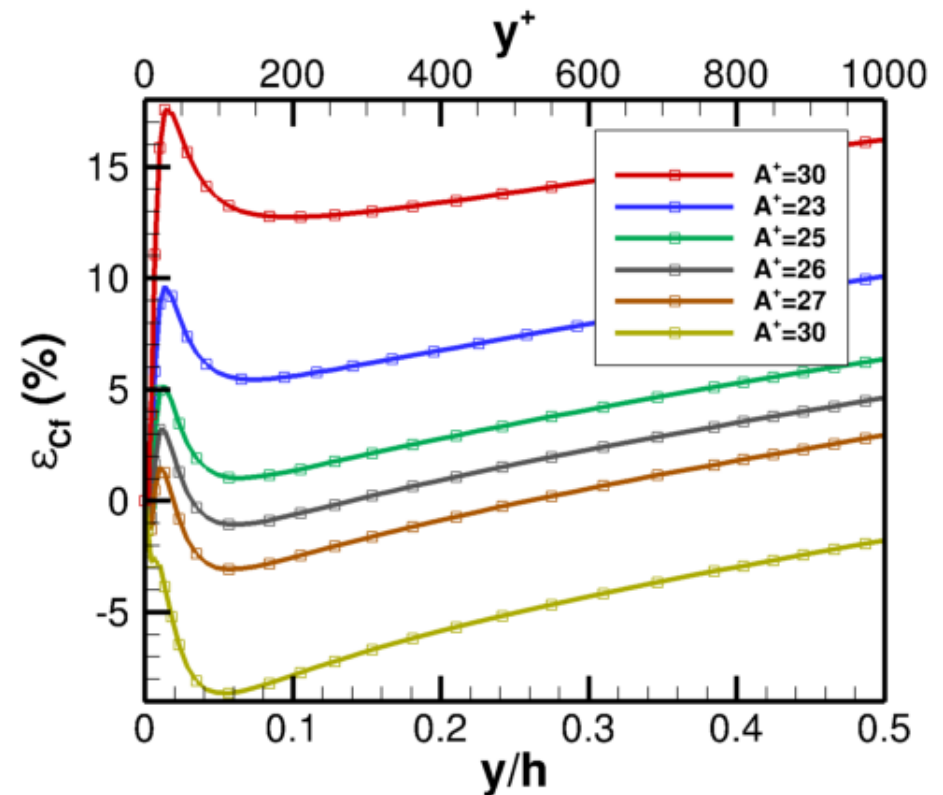
¹ Figure taken from : https://en.wikipedia.org/wiki/Boeing_787_Dreamliner

² Piomelli, U., & Balaras, E. (2002). Wall-layer models for large-eddy simulations. *Annual review of fluid mechanics*, 34(1), 349-374.

Effect of A^+ for Incompressible Channel



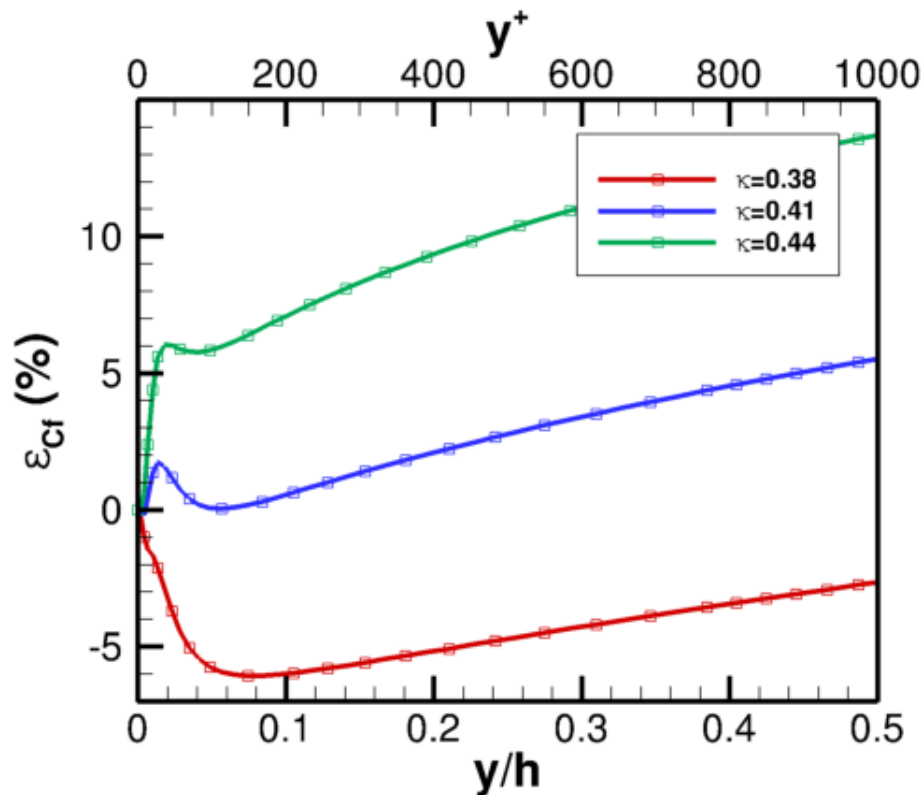
JK model (optimal $A^+=17$)



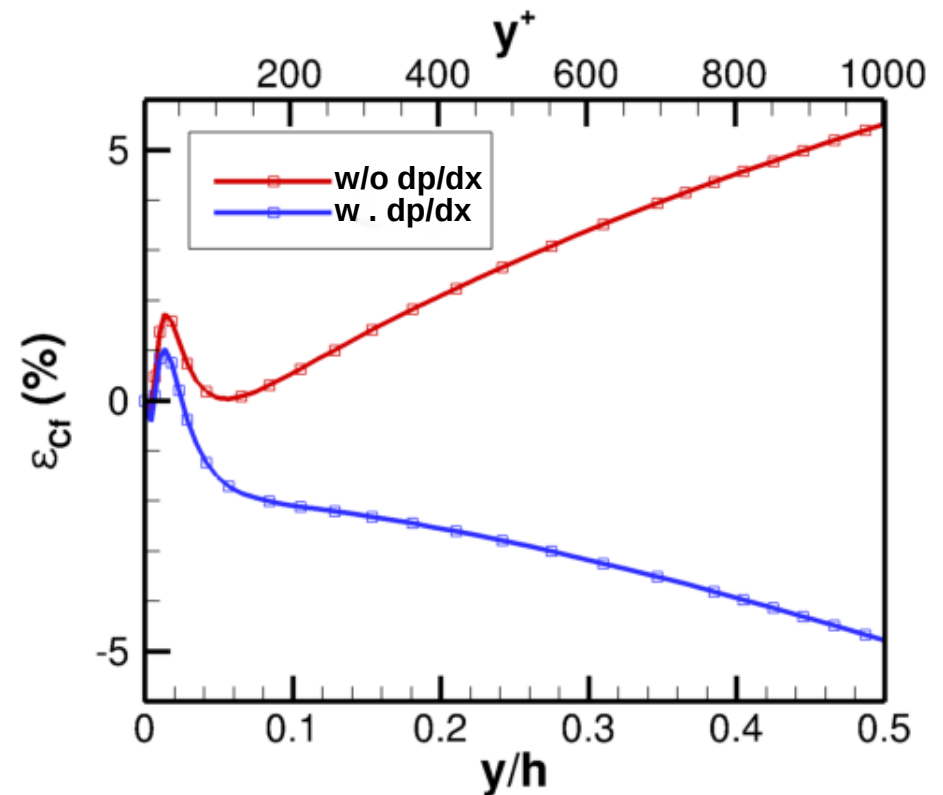
Prandtl model (optimal $A^+=26$)

- Optimal values of A^+ dependent on the mixing length model used.
- $Re_\tau = 2000$ DNS data of Moser.
- $\kappa = 0.41$ used in all the calculations.

Effect of dp/dx and κ for Incompressible Channel



JK model (optimal $\kappa = 0.41$)



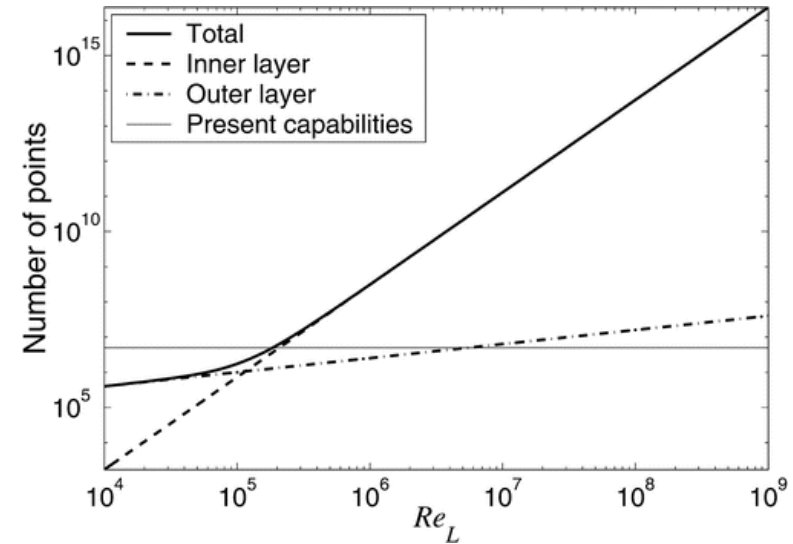
JK model with and without dp/dx

- Including dp/dx does not significantly improve the results. Likely because the eddy viscosity model is not perfect anyways!

Motivation



Boeing 787 Dreamliner¹



Piomelli & Balaras (2002)²

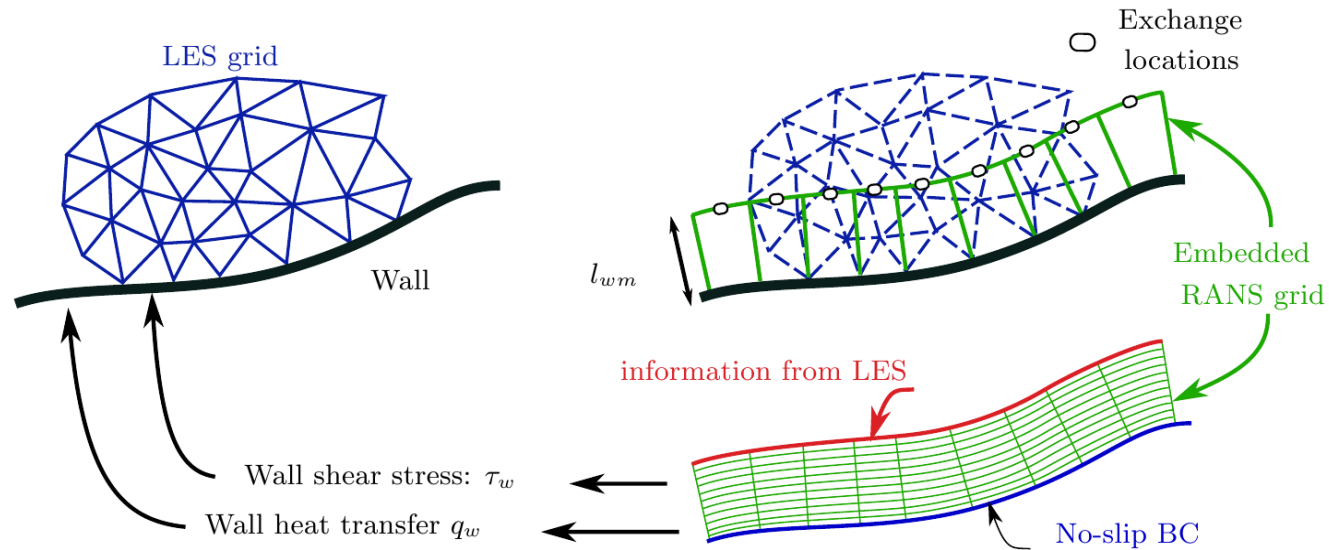
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Equilibrium Wall Model Equations



- Compressible equilibrium BL ODEs solved in WM region.

$$\frac{d}{dy} \left[(\mu + \mu_{t,wm}) \frac{du_{||}}{dy} \right] = 0$$

$$\frac{d}{dy} \left[(\mu + \mu_{t,wm}) u_{||} \frac{du_{||}}{dy} + (\lambda + \lambda_{t,wm}) \frac{dT}{dy} \right] = 0$$

- Nonequilibrium effects (pressure gradient, acceleration, unsteady terms) neglected.

Law of the Wall

- At the edge of the inner layer (log-law region), turbulent stresses dominate over viscous stresses.

$$\mu_t \gg \mu$$

$$D \approx 1$$

- WM momentum equations reduce to :

$$\mu_t \frac{du}{dy} = \tau_w = \rho_w u_\tau^2$$

$$\left. \begin{aligned} \mu_{t,JK} \frac{du}{dy} &= \rho_w u_\tau^2 & \kappa y \frac{du}{dy} &= \sqrt{\frac{\rho_w}{\rho}} u_\tau \\ \mu_{t,Pr} \frac{du}{dy} &= \rho_w u_\tau^2 & (\kappa y)^2 \left| \frac{du}{dy} \right|^2 &= \frac{\rho_w}{\rho} u_\tau^2 \end{aligned} \right\} \text{Compressible Law of the Wall}$$

- The chosen eddy viscosity relationships reduce to the compressible law of the wall (Bradshaw 1994, Trettel & Larsson 2016), and $\kappa \sim \text{constant}$, thus no “degree of freedom” to vary anything.
- Now, let us look at the damping function!

Motivation

- Most RANS models work reasonably well for attached flows, but do not accurately predict flows involving separation. Fixes available, but problem-specific.
- **Wall-Modeled LES (WMLES) and hybrid RANS-LES** methods show improved predictions when compared to the state-of-the-art RANS models for turbulent separated flows.
- High fidelity DNS/ LES too expensive/ infeasible for realistic high Re encountered in flight.
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