Adjoint-based Broadband Noise Minimization using Stochastic Noise Generation

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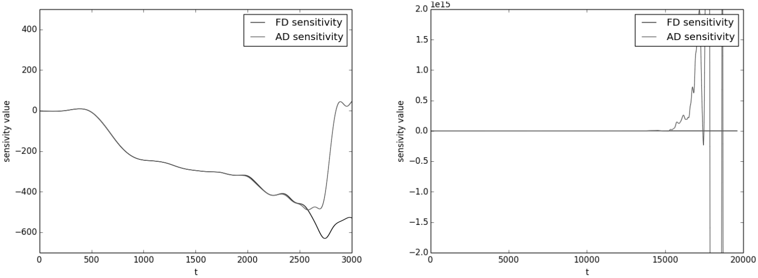
Motivation

- Broadband noise prediction: scale-resolving simulations (DNS, LES or at least DES) needed to resolve noise source + wave propagation (LEE, APE or FW-H)
- For design optimization, necessary to use adjoint-based methods
- A fundamental obstacle: regularization problem encountered in adjoint computation of scale-resolving simulations (Blonigan and Wang, 2012)



First 3000 Δt's   Full Horizon

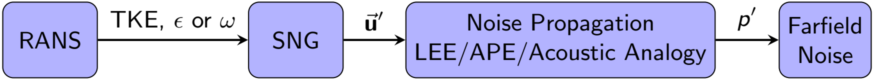
Figure: Divergence of sensitivities observed in a jet noise application by Oezkaya et al. (FD: Finite Difference; AD: Algorithmic Differentiation)

- A 'middle-ground' between RANS-based approach and scale-resolving simulations needs to be found

RANS-SNG Broadband Noise Assessment Framework

Basic Idea

Use stochastic noise generation (SNG) to reconstruct the turbulent velocity field based on turbulence kinetic energy (TKE) and dissipation rates (ϵ or ω) estimated by a preceding RANS computation.



- Pioneering work in RANS-SNG by Bechara et al. and Bailly et al. in the 1990s
- Method improved by the works of Billson et al., Casalino and Barbarino, and di Francescantonio et al. over the years.
- Similar idea to the RANS-RPM approach of Ewert et al. at DLR (circa. 2000)

What RANS-SNG Method IS and ISN'T

- Fast assessment of broadband noise source characteristics and trends for design optimization
- A method to circumvent the regularization issue plaguing adjoint solutions for scale-resolving simulations
- NOT designed to predict broadband noise to an *absolute* level

Stochastic Noise Generation

A space-time turbulent velocity field can be expressed as a sum of N_F random Fourier modes:

$$\vec{u}(\vec{x},t) = 2\sum_{n=1}^{N_F} \hat{u}_n cos\left[\vec{k}_n \cdot \left(\vec{x} - \vec{U}t\right) + \psi_n\right]\vec{\sigma}_n \tag{1}$$

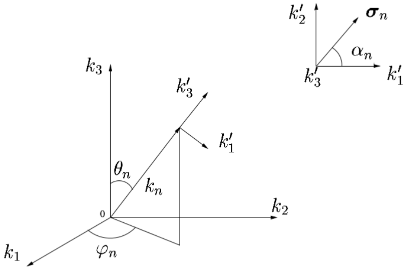
$\hat{u}_n$, $\vec{k}_n$, ψ_n and $\vec{\sigma}_n$ are statistical velocity magnitude, wave number vector, phase and direction associated with the n^{th} Fourier mode, convecting in a mean velocity $\vec{U}$

The vector $\vec{k}_n$ is generated randomly on a sphere with radius k_n, based on two polar angles φ_n and θ_n

The velocity vector $\vec{\sigma}_n$ is constrained to lie in a plane orthogonal to $\vec{k}_n$ with an angle α_n

The magnitude $\hat{u}_n$ of each mode is computed so that the turbulence energy spectrum $E(k_n)$ corresponds to the energy spectrum for isotropic turbulence, giving:

$$\hat{u}_n = \sqrt{E\left(k_n\right)\Delta k_n} \tag{2}$$



Probability distributions of the four random angles necessary for the stochastic generation of $\vec{u}(\vec{x},t)$:

$\mathcal{P}(\varphi_n) = 1/(2\pi)$	$0 \le \varphi_n \le 2\pi$
$\mathcal{P}(\theta_n) = (1/2)sin(\theta_n)$	$0 \le \theta_n \le \pi$
$\mathcal{P}(\psi_n) = 1/(2\pi)$	$0 \le \psi_n \le 2\pi$
$\mathcal{P}(\alpha_n) = 1/(2\pi)$	$0 \le \alpha_n \le 2\pi$

Stochastic Noise Generation

The energy spectrum is assumed in the form of Von Kármán-Pao isotropic turbulence spectrum as

$$E(k) = \frac{2A}{3}\frac{K}{k_e}\left(\frac{k}{k_e}\right)^4 exp\left[-2\left(\frac{k}{k_\eta}\right)^2\right]\left[1+\left(\frac{k}{k_e}\right)^2\right]^{(-17/6)} \tag{3}$$

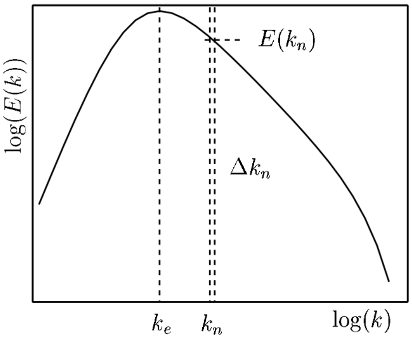
K: turbulence kinetic energy

$k_e = 0.747/L_T$: wavenumber of the maximum energy determined by the turbulent length scale L_T from RANS ($L_T = c_1 u'^3/\epsilon$, where $u' = \sqrt{2K/3}$)

$k_\eta = \epsilon^{1/4}\nu^{-3/4}$: wavenumber of the Kolmogorov scale.

ϵ: turbulence dissipation rate

Constants $A \simeq 1.453$ and $c_1 = 1.0$.



K and ϵ extracted from RANS solution

Adjoint-Based RANS-SNG Noise Reduction Framework

The optimization problem for minimizing the broadband noise can be posed as:

$$\min_{\alpha} \quad J(U(\alpha), X(\alpha)) \tag{4}$$

$$\text{subject to} \quad U(\alpha) = G(U(\alpha), X(\alpha)) \tag{5}$$

$$X(\alpha) = M(\alpha) \tag{6}$$

α: design variables (shape, actuation, or porosity parameters)

U: state variables

X: mesh variables

G: fixed-point representation of the RANS solution

M: mesh equation

J: design objective, currently defined as a function of near-field noise source $T_{i,j}$ computed by RANS-SNG

We can define the Lagrangian associated to this problem as

$$L(\alpha, U, X, \bar{U}, \bar{X}) = J(U, X) + [G(U, X) - U]^T \bar{U} + [M(\alpha) - X]^T \bar{X} \tag{7}$$

$$= N(U, \bar{U}, X) - U^T \bar{U} + [M(\alpha) - X]^T \bar{X} \tag{8}$$

where N is the shifted Lagrangian

$$N(U, \bar{U}, X) := J(U, X) + G^T(U, X)\bar{U}. \tag{9}$$

Adjoint-Based RANS-SNG Noise Reduction Framework

If we differentiate L with respect to α using the chain rule, we can choose the adjoint variables $\bar{X}$ and $\bar{U}$ in such a way, that the terms $\frac{\partial U}{\partial \alpha}$ and $\frac{\partial X}{\partial \alpha}$ can be eliminated. This leads to the following equations for $\bar{U}$ and $\bar{X}$:

$$\bar{U} = \frac{\partial}{\partial U} N(U, \bar{U}, X) = \frac{\partial}{\partial U} J^T(U, X) + \frac{\partial}{\partial U} G^T(U, X)\bar{U} \tag{10}$$

$$\bar{X} = \frac{\partial}{\partial X} N(U, \bar{U}, X) = \frac{\partial}{\partial X} J^T(U, X) + \frac{\partial}{\partial X} G^T(U, X)\bar{U} \tag{11}$$

Finally, the derivative of the Lagrangian, that is, the total derivative of J, reduces to

$$\frac{\mathrm{d}L}{\mathrm{d}\alpha}^T = \frac{\mathrm{d}J}{\mathrm{d}\alpha}^T = \frac{\mathrm{d}}{\mathrm{d}\alpha} M^T(\alpha)\bar{X}. \tag{12}$$

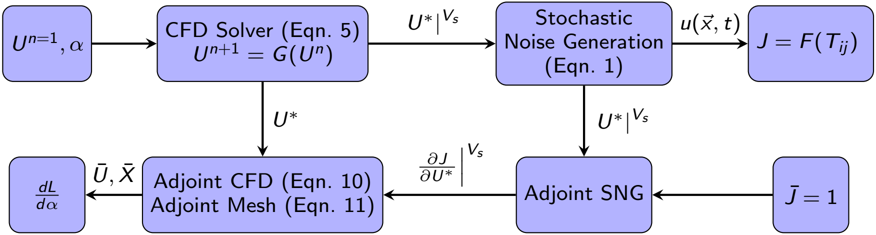
Equation (10) is a fixed-point equation in $\bar{U}$ and can be solved in the style of the flow solver using the iteration

$$\bar{U}^{n+1} = \frac{\partial}{\partial U} N(U^*, \bar{U}^n, X) \tag{13}$$

once we have found a numerical solution $U = U^*$ of Equation (5).

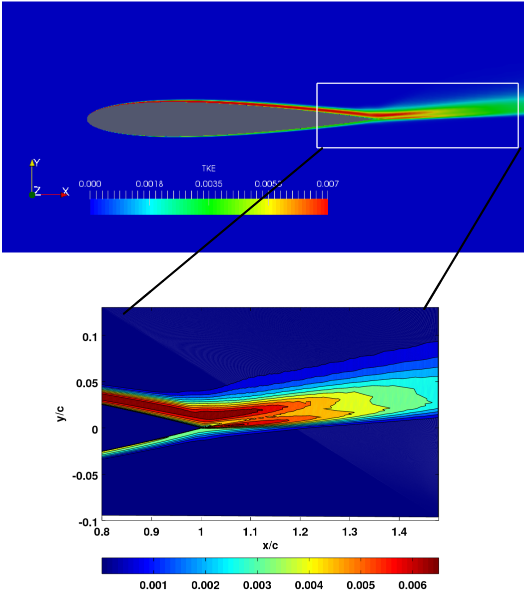
- Derivative terms in Eqn. (10) & (11) computed using Algorithmic Differentiation (AD)
- Adjoint iterator inherits the same convergence properties as primal iterator
- G includes: turbulence model, grid movement, limiters, etc.
- AD implementation details see Albring et al. AIAA-2016-3518

Adjoint-Based RANS-SNG Noise Reduction Framework



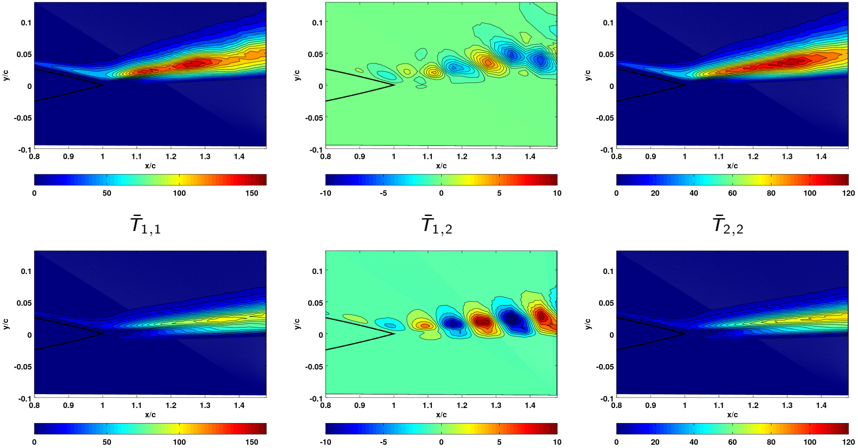
- $U^*|^{V_s}$: Turbulent flow variables extracted from the user-defined noise source region V_s.
- J: a function of stochastically generated Lighthill's stress tensor
- $\frac{\partial J}{\partial U^*}\Big|^{V_s}$: sensitivity of the broadband noise objective with respect to turbulent flow variables extracted from V_s
- Adjoint CFD: $\bar{U} = \frac{\partial}{\partial U} G^T(U, X)\bar{U} + (\frac{\partial J}{\partial U^*}\Big|^{V_s})^T$
- The effect of the turbulent flow variables (k, ϵ or ω) in the source region V_s on the broadband noise design objective J is 'transmitted' through the term $\frac{\partial J}{\partial U^*}\Big|^{V_s}$, which is accumulated to the flow adjoint iterator in evaluating the coupled adjoint of RANS-SNG

Airfoil Self-Noise and Design Sensitivities



- 2-D NACA0012 airfoil
- $M_\infty = 0.2$
  $Re_c = 6.0 \times 10^6$
  $AoA = 8^\circ$
- RANS solution computed with SST $k - \omega$ turbulence model
- Steady aerodynamic results validated against experiment
- TKE and ω extracted from RANS solution
- SNG and sensitivities computed in the focus region:
  $x \in [0.8, 1.5], y \in [-0.1, 0.15]$
- Frequency range: 1-5 KHz
- Both primal and adjoint computations implemented in open-source solver SU2, fully parallelized.

Stochastically Constructed Lighthill's Stress Tensor ($T_{i,j}$)

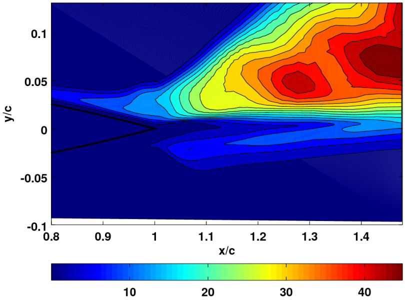


- Top row: AoA = 8 deg
- Bottom row: AoA = 4 deg
- $\bar{T}_{i,j}$: time-averaged over 1000 stochastically generated samples
- Larger AoA $\implies$ wider wake $\implies$ stronger quadrupole source

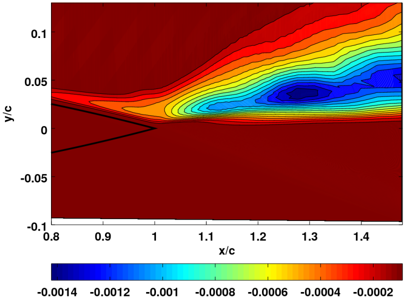
Sensitivities

$$\mathcal{J}^{BBN} = ||\frac{1}{V_s}\frac{1}{N_t}\sum_{m=1}^{N_x}\sum_{n=1}^{N_t}\mathbf{T}(\vec{x}_m, t_n)\Delta V_m||^{Frob} \quad (14)$$

where $\mathbf{T} = T_{ij} = \rho u_i u_j$ and $||\cdot||^{Frob}$ is the Frobenius norm of a tensor.



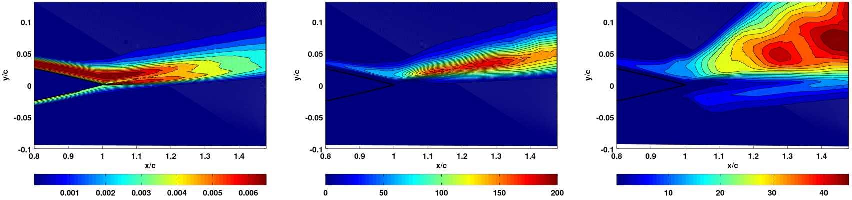
$\frac{\partial \mathcal{J}^{BBN}}{\partial K}$



$\frac{\partial \mathcal{J}^{BBN}}{\partial \omega}$

	$max(\frac{\partial \mathcal{J}^{BBN}}{\partial K})$	$max(\frac{\partial \mathcal{J}^{BBN}}{\partial \omega})$
Adjoint-Mode AD	4.785054967E+1	1.426646088E-3
Finite Difference	4.785054950E+1	1.426646095E-3

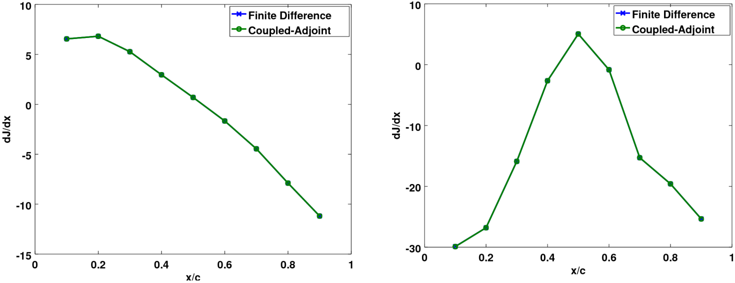
TKE, $T_{i,j}$, and Sensitivity Distributions



TKE $||\bar{T}||^{Frob}$ $\frac{\partial \mathcal{J}^{BBN}}{\partial K}$

- While the peak TKE is located in the turbulent boundary layer, the broadband noise source is actually located further down in the wake and more importantly, so is the peak sensitivity region
- It would not be effective to directly target the high-TKE regions in the boundary layer.
- Shape optimization should be conducted to morph the shape so as to reduce the TKE **in the wake**, where the strong quadrupole sources are.

Coupled-Sensitivity Validation

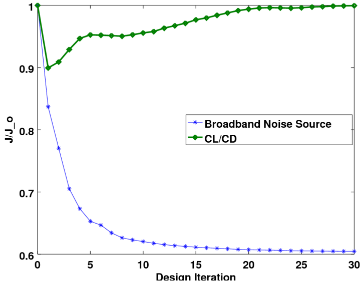


Pressure Side

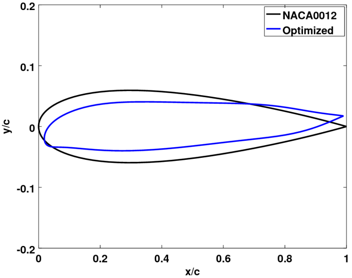
Suction Side

- Airfoil surface parameterized with 18 Hicks-Henne bump functions (9 on each surface) to enable shape deformation
- $\frac{dJ}{dx}$: design sensitivity of the broadband noise source (as predicted by RANS-SNG) with respect to the 18 shape design variables
- Coupled adjoint sensitivity validated against finite difference ($\delta = 10^{-6}$)

Broadband Noise Source Minimization



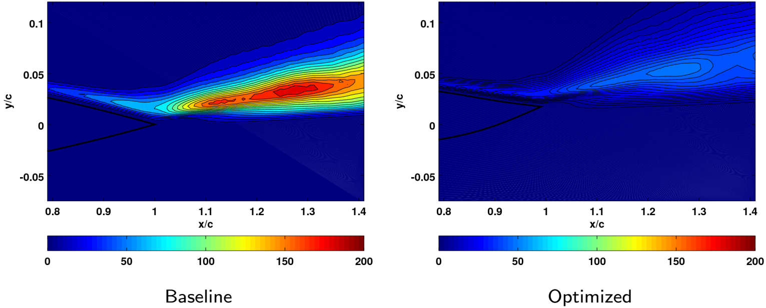
Optimization History



Design Comparison

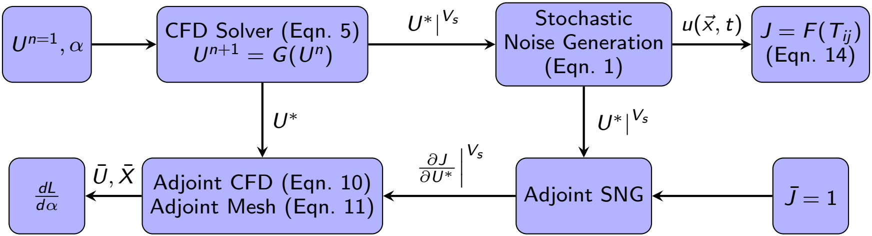
- Optimization process fully automated in SU2
- Broadband noise minimization performed for 30 design iterations, leading to $\sim 40\%$ reduction in design objective
- No apparent loss of aerodynamic efficiency, even though no aerodynamic constraints are applied.
- Can impose aerodynamic or geometric design constraints.

Broadband Noise Source Minimization



- Comparison: Frobenius norm of the time-averaged Lighthill's stress tensor in the trailing-edge region
- Shape optimization effectively removes broadband noise source
- Peak BBN source ($||\bar{T}||^{Frob}$) reduced by $\sim$ 75%
- This should be verified by a scale-resolving simulation

Next Steps



- Compare baseline and optimized results with LES solutions (quasi-2D)
  - Can use SU2 in implicit LES mode
  - LES solver from Chalmers?
- Apply to 3-D cases:
  - Optimal slat setting for a 30P30N configuration
  - Optimal shape design for flap side edge noise reduction
- Extend the formulation to unsteady: URANS-SNG framework for rotor/propeller broadband noise design