Challenges in Adjoint-Based Aerodynamic Design for Unsteady Flows

Eric J. Nielsen
NASA Langley Research Center

Patrick J. Blonigan
Massachusetts Institute of Technology

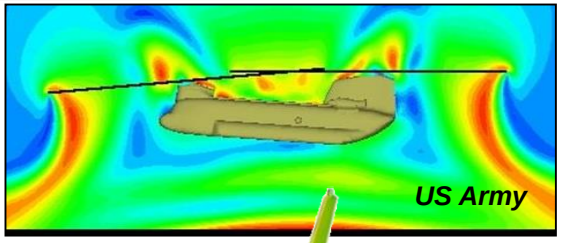
Boris Diskin
National Institute of Aerospace

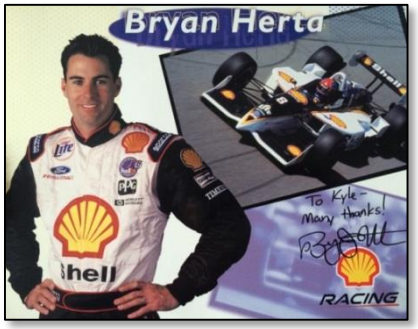
Qiqi Wang
Massachusetts Institute of Technology

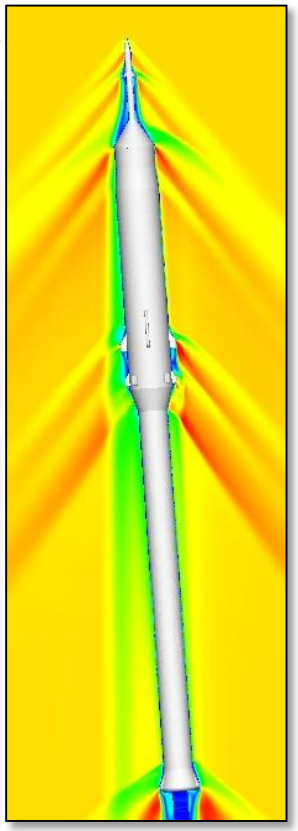
FUN3D Core Capabilities

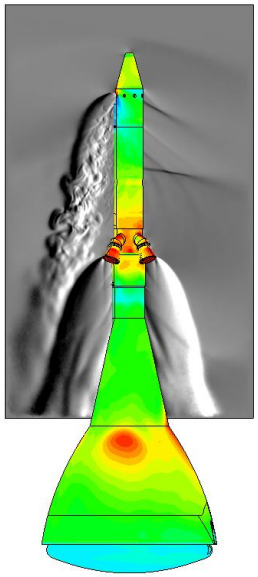
http://fun3d.larc.nasa.gov

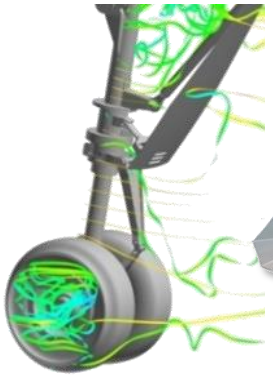
- Established as a research code in late 1980s; now supports numerous internal and external efforts across the speed range
- Solves 2D/3D steady and unsteady Euler and RANS equations on node-based mixed element grids for compressible and incompressible flows
- General dynamic mesh capability: any combination of rigid / overset / morphing grids, including 6-DOF effects
- Aeroelastic modeling using mode shapes, full FEM, CC, etc.
- Constrained / multipoint adjoint-based design and mesh adaptation
- Distributed development team using agile/extreme software practices including 24/7 regression, performance testing
- Capabilities fully integrated, online documentation, training videos, tutorials

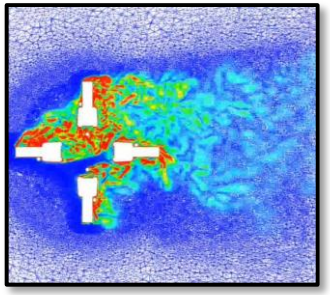


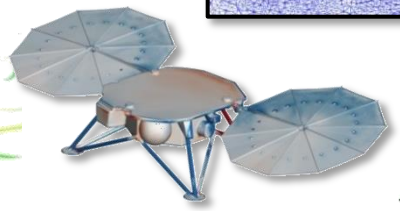


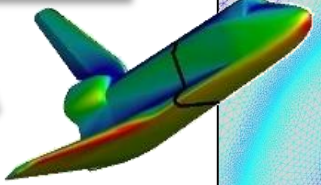


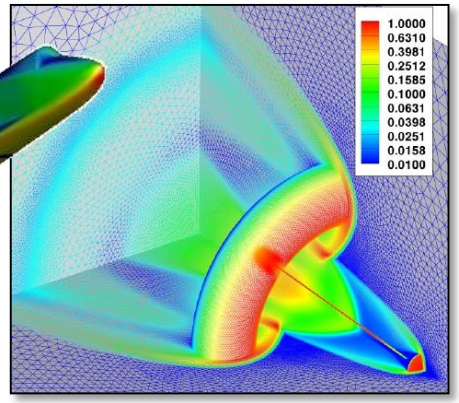












Conventional Adjoint-Based Design

$$
\begin{aligned}
L(\mathbf{D},\mathbf{Q},\mathbf{X},\boldsymbol{\Lambda},\boldsymbol{\Lambda}_g) &= f\Delta t + \sum_{n=1}^{N}[\boldsymbol{\Lambda}_g^n]^T\mathbf{G}^n\Delta t \\
&+ \sum_{n=1}^{N}\Bigg\{[\mathbf{C}_s^n \circ \boldsymbol{\Lambda}_s^n]^T\Bigg[a\frac{\mathbf{Q}_s^n - \mathbf{I}_s^n\mathbf{Q}^{n-1}}{\Delta t}\circ\mathbf{V}_s^n \\
&+ c\frac{\mathbf{I}_s^n\mathbf{Q}^{n-2} - \mathbf{I}_s^n\mathbf{Q}^{n-1}}{\Delta t}\circ(\mathbf{I}_s^n\mathbf{V}^{n-2}) \\
&+ d\frac{\mathbf{I}_s^n\mathbf{Q}^{n-3} - \mathbf{I}_s^n\mathbf{Q}^{n-1}}{\Delta t}\circ(\mathbf{I}_s^n\mathbf{V}^{n-3})\Bigg] \\
&+ [\boldsymbol{\Lambda}_s^n]^T[\mathbf{R}^n + ((\mathbf{I}_s^n\mathbf{Q}^{n-1})\circ\mathbf{C}_s^n + \beta\bar{\mathbf{C}}_s^n)\circ\mathbf{R}_{\mathrm{GCL}}^n] \\
&+ [\boldsymbol{\Lambda}_f^n]^T[\mathbf{A}^n\mathbf{Q}^n] + [\boldsymbol{\Lambda}_h^n]^T[\mathbf{P}^n\mathbf{Q}^n]\Bigg\}\Delta t \\
&+ (f^0 + [\boldsymbol{\Lambda}_g^0]^T\mathbf{G}^0 + [\boldsymbol{\Lambda}^0]^T\mathbf{R}^{\mathrm{in}})\Delta t
\end{aligned}
$$

- Flow field and grid adjoint equations derived for the time-dependent Navier-Stokes equations on arbitrary combinations of static/rigidly moving/deforming overset grids undergoing parent-child motion
- The following terms are included in the Lagrangian
  - Objective function
  - Grid terms
  - Higher-order temporal terms
  - Fluxes
  - Geometric Conservation Law term
  - Overset interpolation terms
  - Initial conditions
- Implemented by hand and verified using complex variables

$\circ$ is the Hadamard vector multiplication operator; see
Nielsen, E.J. and Diskin, B., "Discrete Adjoint-Based Design for Unsteady Turbulent Flows on Dynamic Overset Unstructured Grids," AIAA Journal, Vol. 51, No. 6, June 2013.

Conventional Adjoint-Based Design

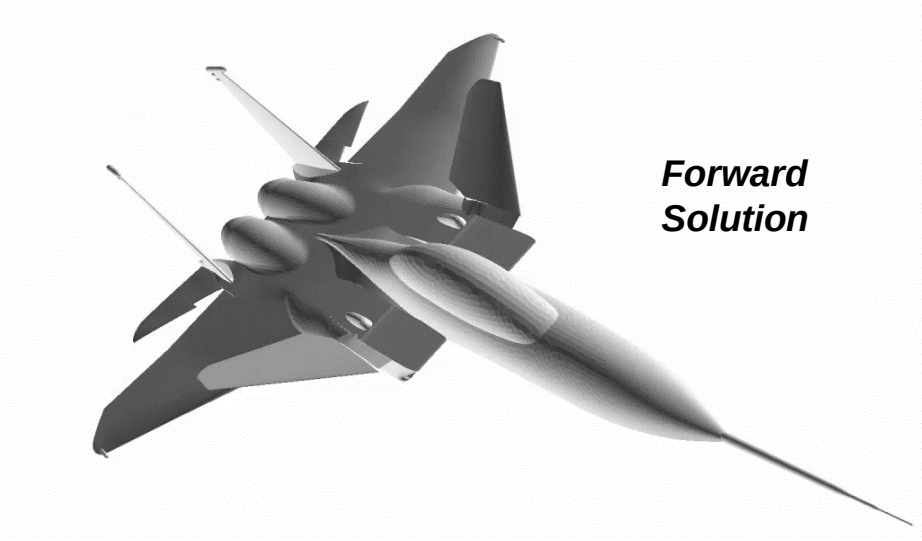
- After linearizing the Lagrangian and solving the flow and grid adjoint equations, the desired sensitivities are computed as follows

$$
\begin{aligned}
\frac{\partial L}{\partial \mathbf{D}} &= \frac{\partial f}{\partial \mathbf{D}} \Delta t + \sum_{n=1}^{N} [\mathbf{\Lambda}_g^n]^T \frac{\partial \mathbf{G}^n}{\partial \mathbf{D}} \Delta t \\
&+ \sum_{n=1}^{N} [\mathbf{\Lambda}_s^n]^T \left[\frac{\partial \mathbf{R}^n}{\partial \mathbf{D}} + ((\mathbf{I}_s^n \mathbf{Q}^{n-1}) \circ \mathbf{C}_s^n + \beta \bar{\mathbf{C}}_s^n) \odot \frac{\partial \mathbf{R}_{\mathrm{GCL}}^n}{\partial \mathbf{D}} \right] \Delta t \\
&+ \left([\mathbf{\Lambda}_g^0]^T \frac{\partial \mathbf{G}^0}{\partial \mathbf{D}} + [\mathbf{\Lambda}^0]^T \left[\frac{\partial \mathbf{R}^{\mathrm{in}}}{\partial \mathbf{D}} \right] \right) \Delta t
\end{aligned}
$$

$\odot$ is the extension of the Hadamard operator to vector-matrix multiplication where the vector on the left multiplies each column in the matrix on the right.

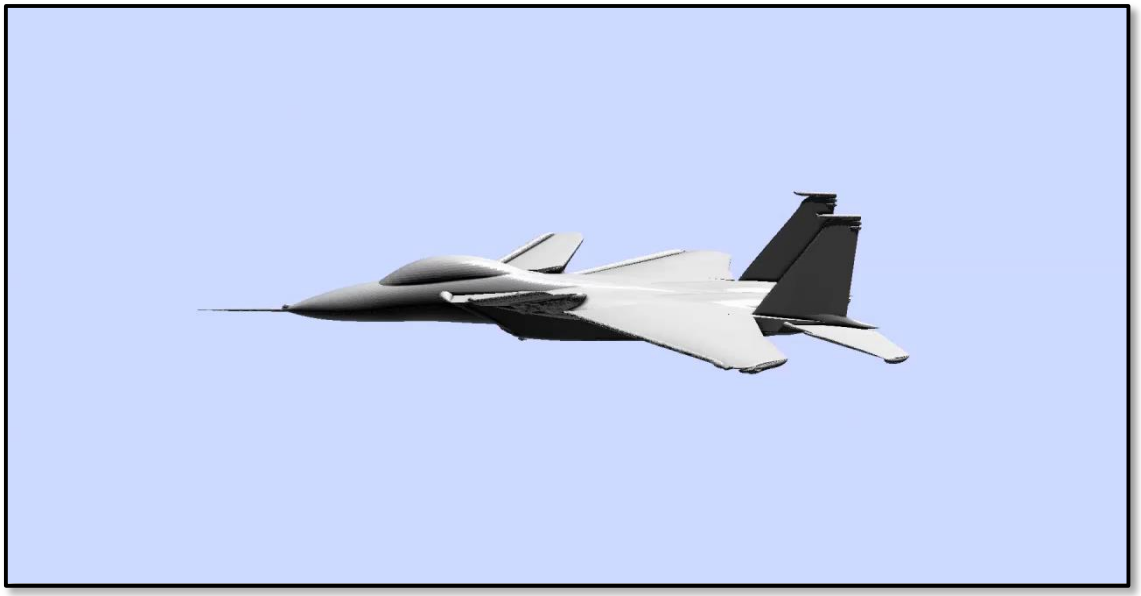
Examples

Forward / Reverse Solutions for F-15



Forward Solution

- Transonic turbulent flow over modified F-15 configuration
- Propulsion effects included as well as simulated aeroelastic deformations of canard/wing/h-tail
- Objective is lift-to-drag ratio



Reverse Solution

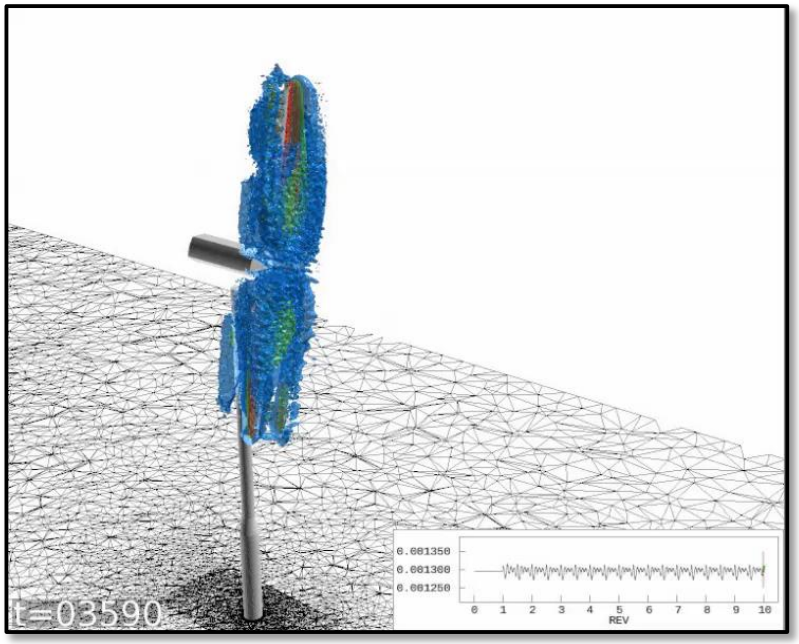
Examples

Forward / Reverse Solutions for Wind Turbine

Forward Solution

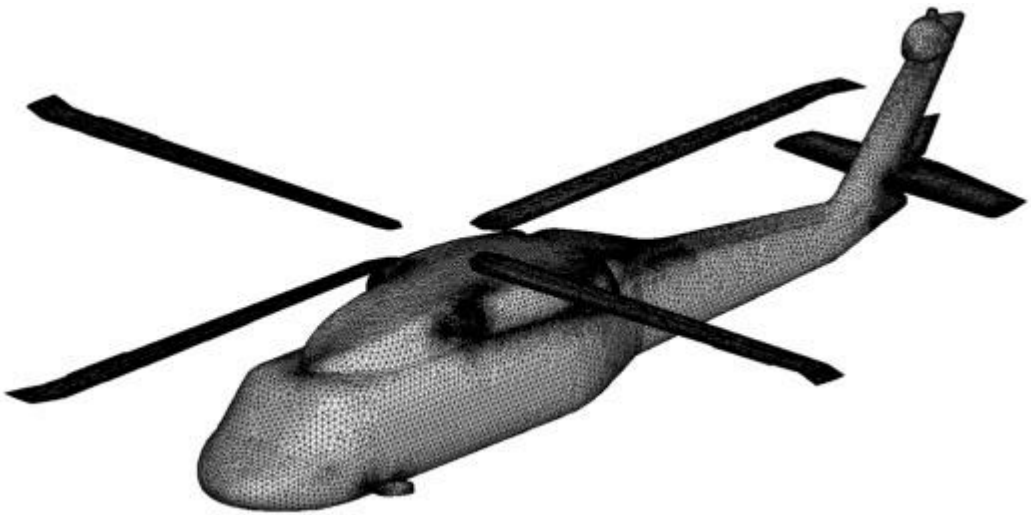
- Incompressible turbulent flow over NREL Phase VI wind turbine
- Overset grids used to model rotating blade system
- Objective function is based on the torque

Reverse Solution



UH-60A Blackhawk Helicopter

Overview



- Composite grid consists of 9,262,941 nodes / 54,642,499 tetrahedra
- Compressible RANS: M_{tip}=0.64, Re_{tip}=7.3M, μ=0.37, α=0.0º
- Blade pitch has child motion governed by collective and cyclic control inputs:

$$\theta = \theta_c + \theta_{1c}\cos\psi + \theta_{1s}\sin\psi$$

Blade pitch — Collective — Lateral cyclic — Longitudinal cyclic

- Baseline value of all control inputs is zero

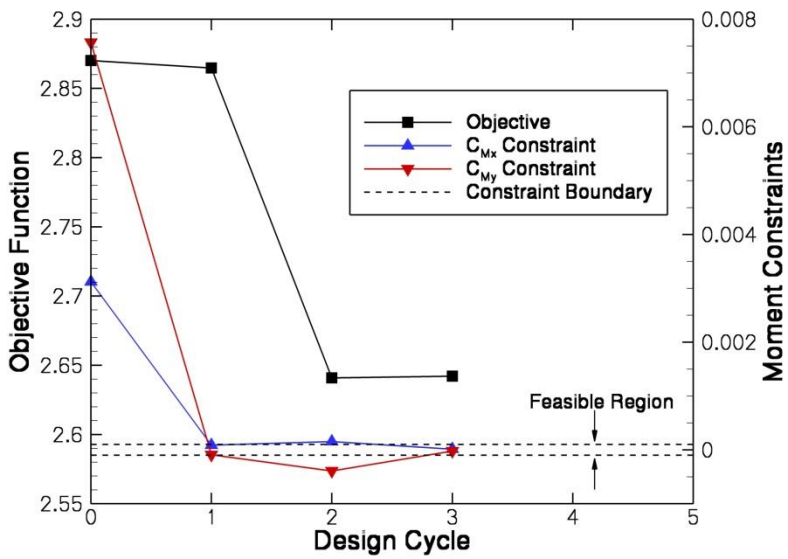
UH-60A Blackhawk Helicopter

Problem Definition and Results

- Objective is to maximize $\bar{C}_L$ while satisfying trim constraints over second rev:

$$\min f = \left[\left(\frac{1}{360}\sum_{n=361}^{720} C_L^n\right) - 2.0\right]^2 \Delta t \quad \text{such that} \quad g_1 = \frac{1}{360}\sum_{n=361}^{720} C_{M_x}^n \Delta t = 0 \qquad g_2 = \frac{1}{360}\sum_{n=361}^{720} C_{M_y}^n \Delta t = 0$$

- Separate adjoint solutions required for all three functions
- 67 design variables include 64 thickness and camber variables across the blade planform, plus collective and cyclic control inputs up to $\pm 7^o$

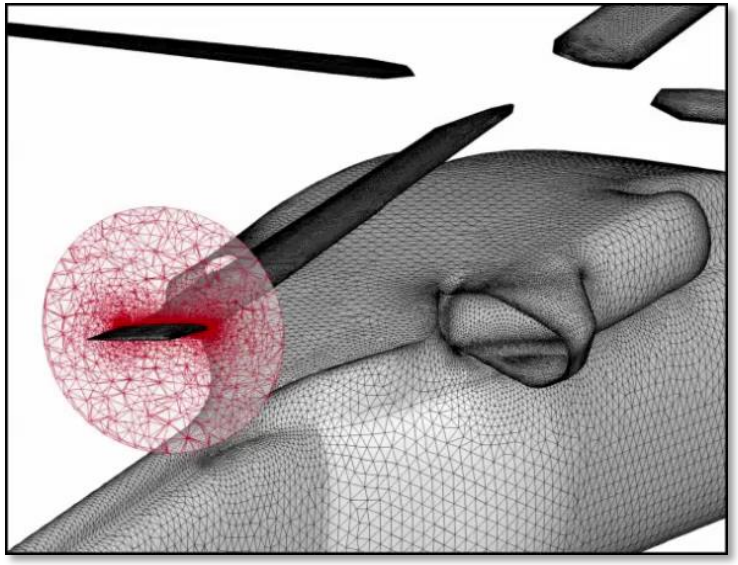


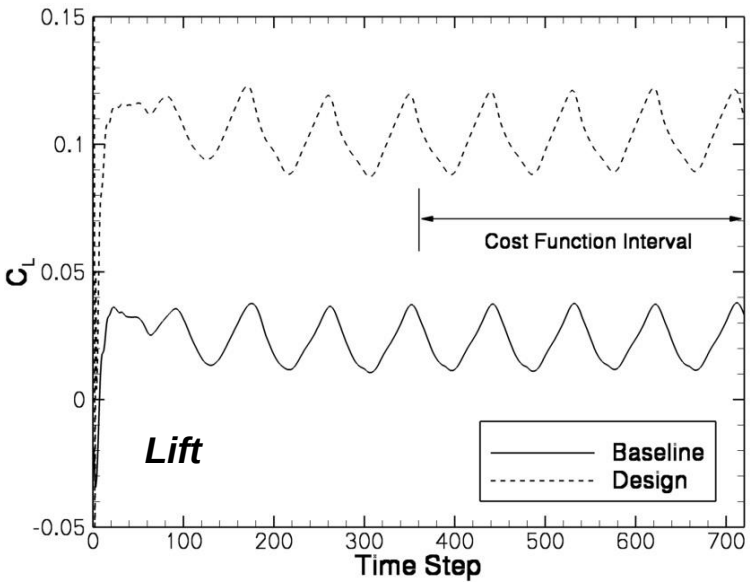
	$\bar{C}_L$	Flow Solves (2 hrs)	Adjoint Solves (3 hrs)	Total Time
Baseline	0.023	-	-	-
Design	0.103	4	4	0.8 days (38,400 CPU hrs)

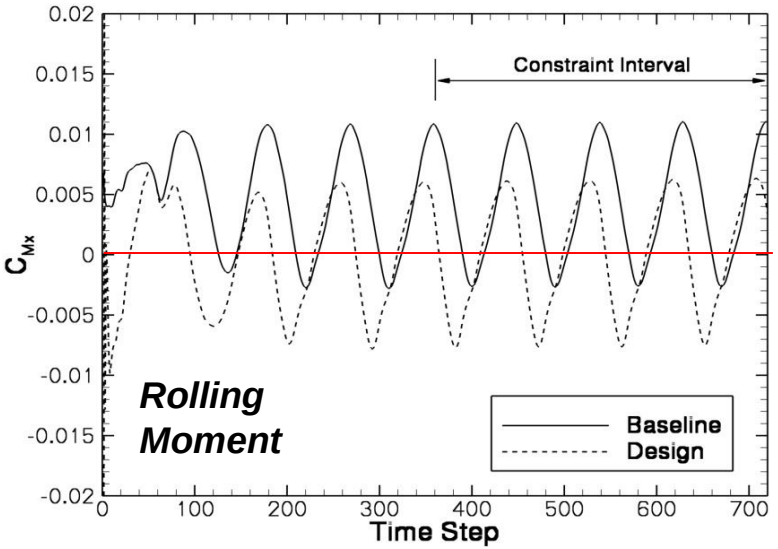
- Feasible region is quickly located
- Both moment constraints are satisfied within tolerance at the optimal solution
- Final controls: θ_c=6.71º, θ_{1c}=2.58º, θ_{1s}=-7.00º

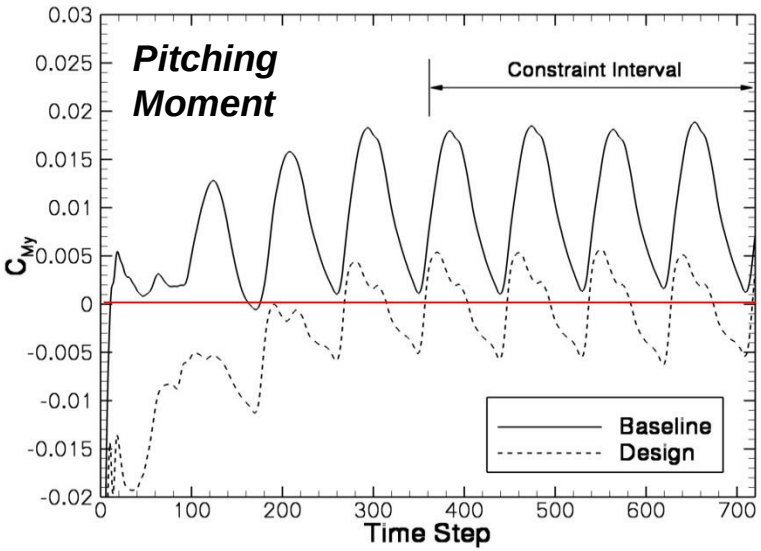
UH-60A Blackhawk Helicopter

Results





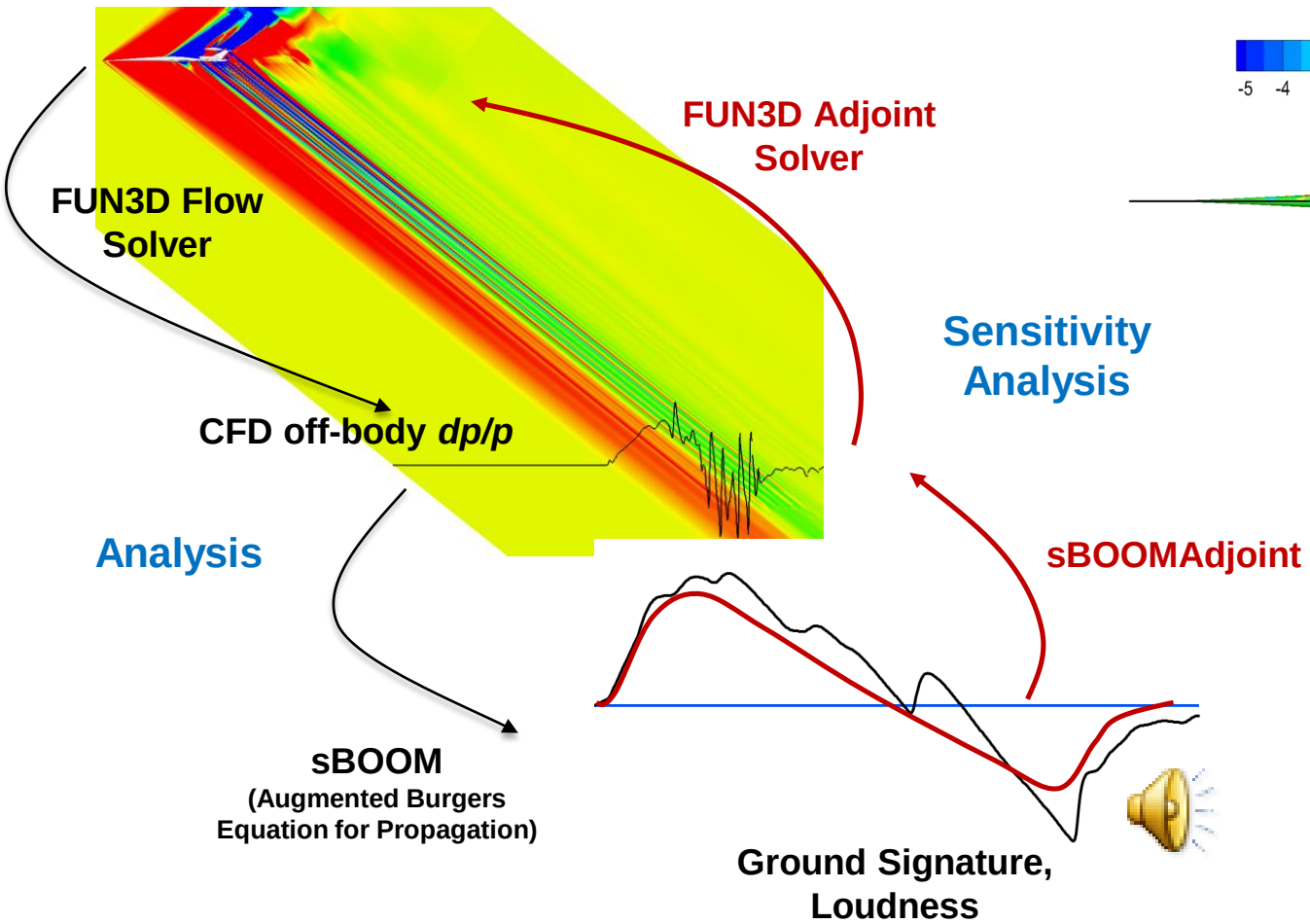


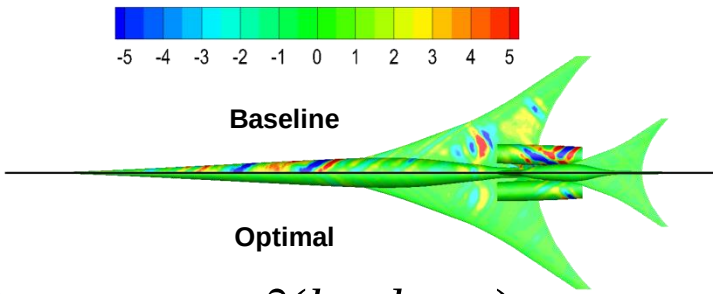


Multidisciplinary Design

Sonic Boom Mitigation

- Multidisciplinary discrete adjoint has been very successful for sonic boom mitigation - discrete derivatives of ground-based metrics with respect to OML
- Many other disciplines being considered / pursued





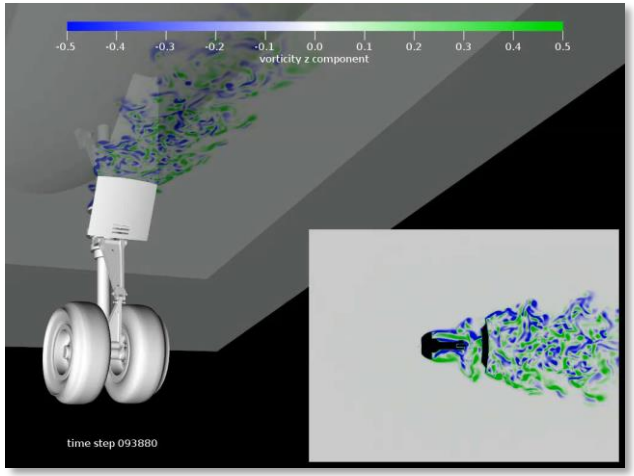
Sensitivity Analysis

$$\frac{\partial(loudness)}{\partial \hat{n}}$$

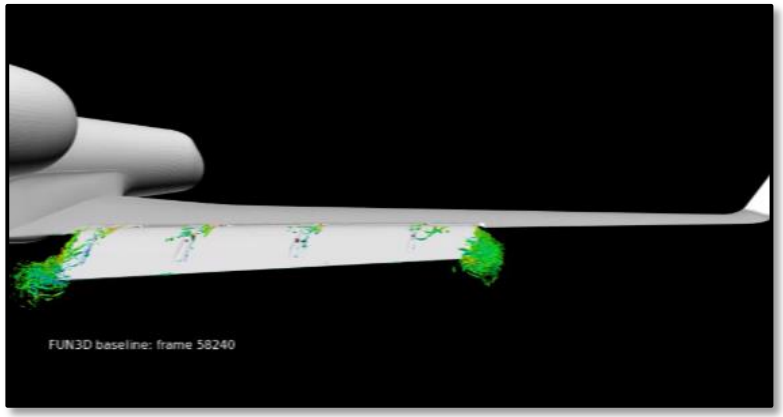
Challenges for Unsteady Problems

- Extensive linearization and infrastructure effort, particularly for dynamic and overset grids

- Sheer cost – every simulation is now a time-dependent run
  - For steady flows, terms could be computed once and stored for efficiency
  - Unsteady flows require these linearizations to be recomputed at every time step



- Need for entire forward solution
  - Brute force it: Store to disk (big data)
  - Recompute it: Store periodically, recompute intermediate steps as needed (checkpointing)
  - Approximate it: Store periodically, interpolate intermediate steps as needed



- Chaotic flows

Goal of Current Work

Compute sensitivities of infinite time averages for chaotic flows

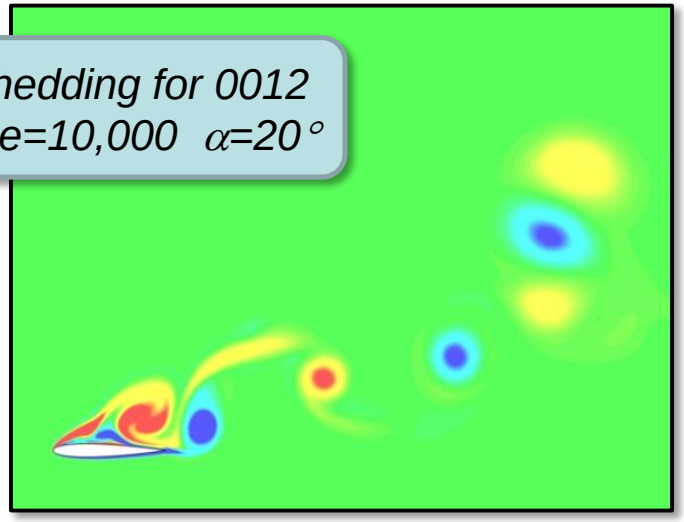
- Theory exists that states these sensitivities are well-defined and bounded

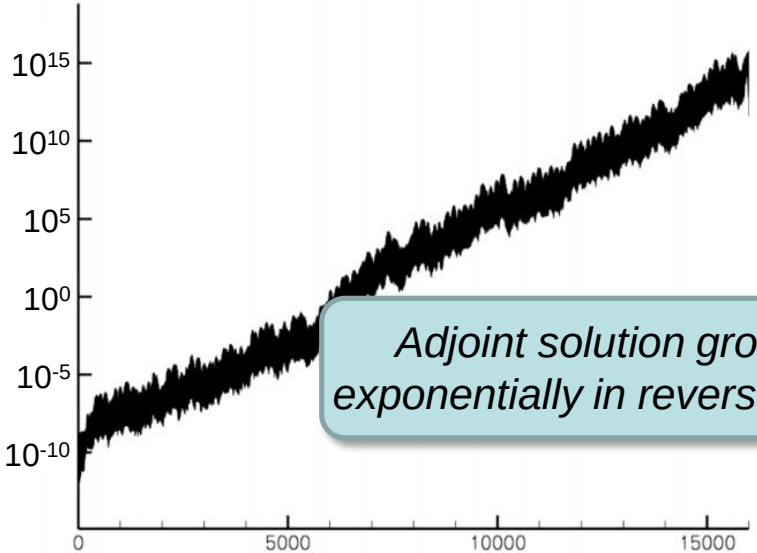
Why does conventional approach not work?

For chaotic flows:

- The finite time average approaches the infinite time average
- The sensitivity for a finite time average does not approach the sensitivity for the infinite time average

Chaotic shedding for 0012
M_∞=0.1 Re=10,000 α=20°





Adjoint solution grows exponentially in reverse time

Approach

- Least-Squares Shadowing (LSS) method proposed by Wang and Blonigan
  - Key assumption is ergodicity of the simulation: long time averages are essentially independent of the initial conditions
  - Also assumes existence of a shadowing trajectory
- The LSS formulation involves a linearly-constrained least squares optimization problem which results in a set of optimality equations
- Preliminary LSS exploration for fluids applications

Define the following quantities:

$\boldsymbol{Q}_i \equiv$ Vector of conserved variables at time level i

$\boldsymbol{R}_i \equiv$ Vector of spatial residuals at time level i

$\mathbf{V} \equiv$ Matrix of cell volumes

$t \equiv$ Time

$f_i \equiv$ Objective function at time level i

LSS System

$$
\begin{pmatrix}
\begin{pmatrix} \mathbf{V} & & & & \\ & \mathbf{V} & & & \\ & & \ddots & & \\ & & & \mathbf{V} & \\ & & & & \mathbf{V} \end{pmatrix}
&
\begin{pmatrix} -\mathbf{V}/\Delta t & & & & \\ \mathbf{G}_1^T & -\mathbf{V}/\Delta t & & & \\ & \mathbf{G}_2^T & \ddots & & \\ & & \ddots & \ddots & \\ & & & -\mathbf{V}/\Delta t & \\ & & & \mathbf{G}_m^T & \end{pmatrix}
\\
\begin{pmatrix} \alpha^2 & & & & \\ & \alpha^2 & & & \\ & & \ddots & & \\ & & & \ddots & \\ & & & & \alpha^2 \end{pmatrix}
&
\begin{pmatrix} \boldsymbol{R}_1^T & & & & \\ & \boldsymbol{R}_2^T & & & \\ & & \ddots & & \\ & & & \ddots & \\ & & & & \boldsymbol{R}_m^T \end{pmatrix}
\\
\begin{pmatrix} -\mathbf{V}/\Delta t & \mathbf{G}_1 & & & \\ & -\mathbf{V}/\Delta t & \mathbf{G}_2 & & \\ & & \ddots & \ddots & \\ & & & -\mathbf{V}/\Delta t & \mathbf{G}_m \end{pmatrix}
&
\begin{pmatrix} \boldsymbol{R}_1 & & & \\ & \boldsymbol{R}_2 & & \\ & & \ddots & \\ & & & \ddots \end{pmatrix}
\end{pmatrix}
\begin{pmatrix}
\begin{pmatrix} \Phi_0 \\ \Phi_1 \\ \vdots \\ \vdots \\ \Phi_m \end{pmatrix}
\\
\begin{pmatrix} \Psi_1 \\ \Psi_2 \\ \vdots \\ \Psi_m \end{pmatrix}
\\
\begin{pmatrix} \Lambda_1 \\ \Lambda_2 \\ \vdots \\ \Lambda_m \end{pmatrix}
\end{pmatrix}
= -
\begin{pmatrix}
\begin{pmatrix} \boldsymbol{g}_0 \\ \boldsymbol{g}_1 \\ \vdots \\ \vdots \\ \boldsymbol{g}_m \end{pmatrix}
\\
\begin{pmatrix} h_1 \\ h_2 \\ \vdots \\ h_m \end{pmatrix}
\\
\begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}
\end{pmatrix}
$$

Tangent solution

Parameters from shadowing lemma

Adjoint solution

$$\mathbf{G} = \mathbf{V}/\Delta t + \partial \boldsymbol{R}/\partial \boldsymbol{Q}$$

$$\boldsymbol{g} = \partial f / \partial \boldsymbol{Q}$$

$\boldsymbol{h}$ is related to time dilation

α is a regularization parameter

This is a globally coupled space-time problem, where each sub-row represents a time level

Reduced LSS System

- To determine sensitivities, we need the LSS adjoint solution
- Use a Schur complement approach to arrive at a reduced system for the LSS adjoint variables:

Writing the previous system as

$$\begin{pmatrix} \mathbf{V} & 0 & \mathbf{B}^T \\ 0 & \alpha^2\mathbf{I} & \mathbf{C}^T \\ \mathbf{B} & \mathbf{C} & 0 \end{pmatrix} \begin{pmatrix} \boldsymbol{\Phi} \\ \boldsymbol{\Psi} \\ \boldsymbol{\Lambda} \end{pmatrix} = -\begin{pmatrix} \boldsymbol{g} \\ \boldsymbol{h} \\ \mathbf{0} \end{pmatrix}$$

The LSS adjoint solution can be determined from

$$\left[\mathbf{B}\mathbf{V}^{-1}\mathbf{B}^T + \frac{1}{\alpha^2}\mathbf{C}\mathbf{C}^T\right]\boldsymbol{\Lambda} = -\mathbf{B}\mathbf{V}^{-1}\boldsymbol{g} - \frac{1}{\alpha^2}\mathbf{C}\boldsymbol{h}$$

- This remains a globally coupled space-time problem
- $\mathbf{B}\mathbf{B}^T$ increases the fill of the matrix
- Furthermore, the system is dense due to $\mathbf{C}\mathbf{C}^T$ term

Sensitivity Evaluation

- To determine sensitivities, we evaluate the conventional sensitivity expression using the LSS adjoint solution
- Conventional terms related to initial conditions drop out

$$\frac{\partial L}{\partial \mathbf{D}} = \frac{\partial f}{\partial \mathbf{D}} \Delta t + \sum_{n=1}^{N} [\mathbf{\Lambda}_g^n]^T \frac{\partial \mathbf{G}^n}{\partial \mathbf{D}} \Delta t$$

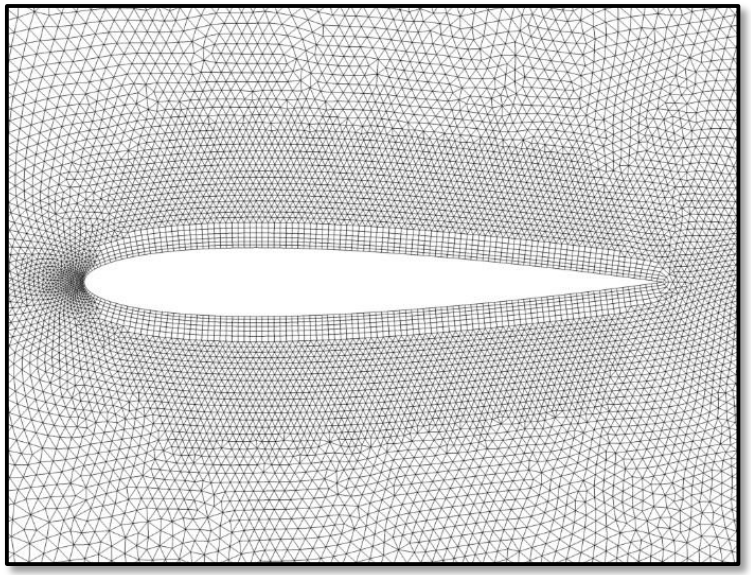
$$+ \sum_{n=1}^{N} [\mathbf{\Lambda}_s^n]^T \left[\frac{\partial \mathbf{R}^n}{\partial \mathbf{D}} + ((\mathbf{I}_s^n \mathbf{Q}^{n-1}) \circ \mathbf{C}_s^n + \beta \bar{\mathbf{C}}_s^n) \odot \frac{\partial \mathbf{R}_{\mathrm{GCL}}^n}{\partial \mathbf{D}} \right] \Delta t$$

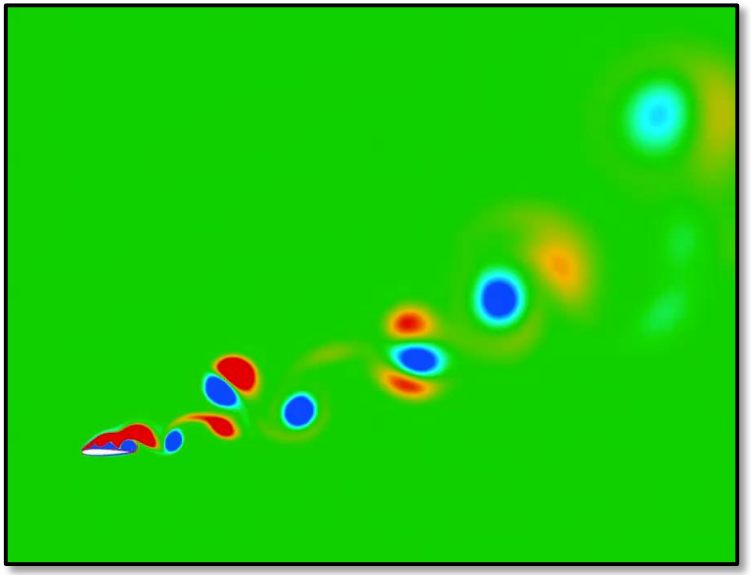
$$\cancel{+ \left([\mathbf{\Lambda}_g^0]^T \frac{\partial \mathbf{G}^0}{\partial \mathbf{D}} + [\mathbf{\Lambda}^0]^T \left[\frac{\partial \mathbf{R}^{\mathrm{in}}}{\partial \mathbf{D}} \right] \right) \Delta t}$$

Problem Definition

Shedding NACA 0012

M_∞=0.1 Re=10,000 α=20°

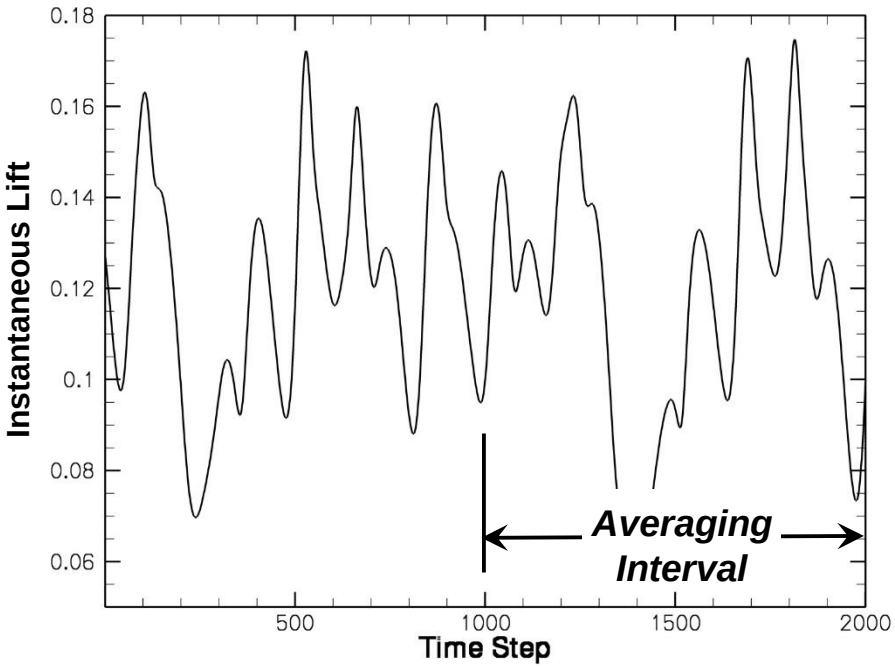




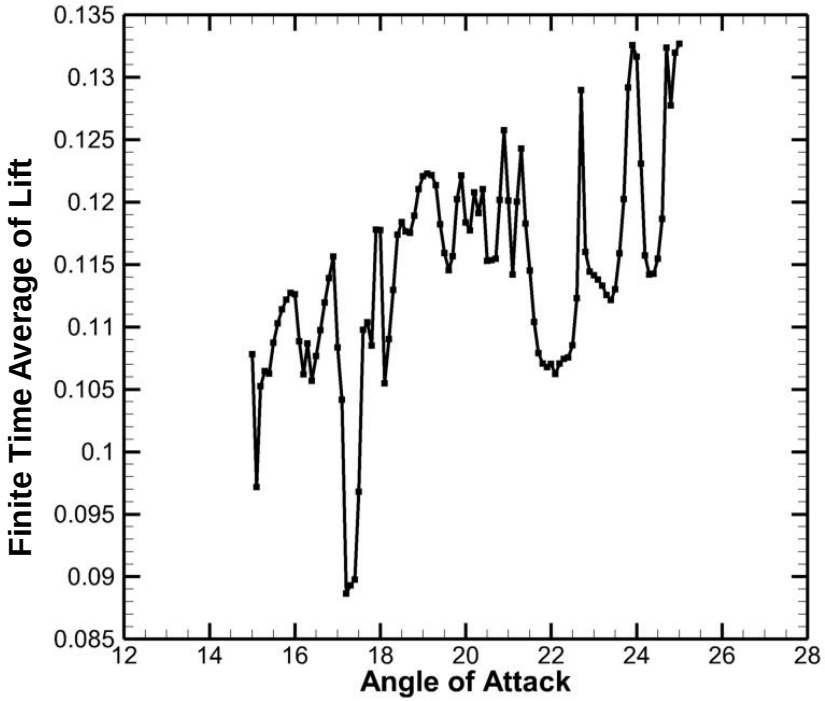
- Unstructured mesh consisting of 102,940 grid points with 100,139 prisms and 1,144 hexes in spanwise direction
- Relatively coarse wall spacing to alleviate stiffness in LSS system
- Laminar Navier-Stokes equations with second-order spatial discretization
- First-order backward differencing in time for LSS simplicity

Problem Definition

Instantaneous Lift vs Time



Finite Time Average of Lift vs Alpha



- To improve ergodicity, simulation started from chaotic initial solution (generated by 2,000 steps from free stream)
- Objective is to maximize time-averaged lift over final 1,000 time steps

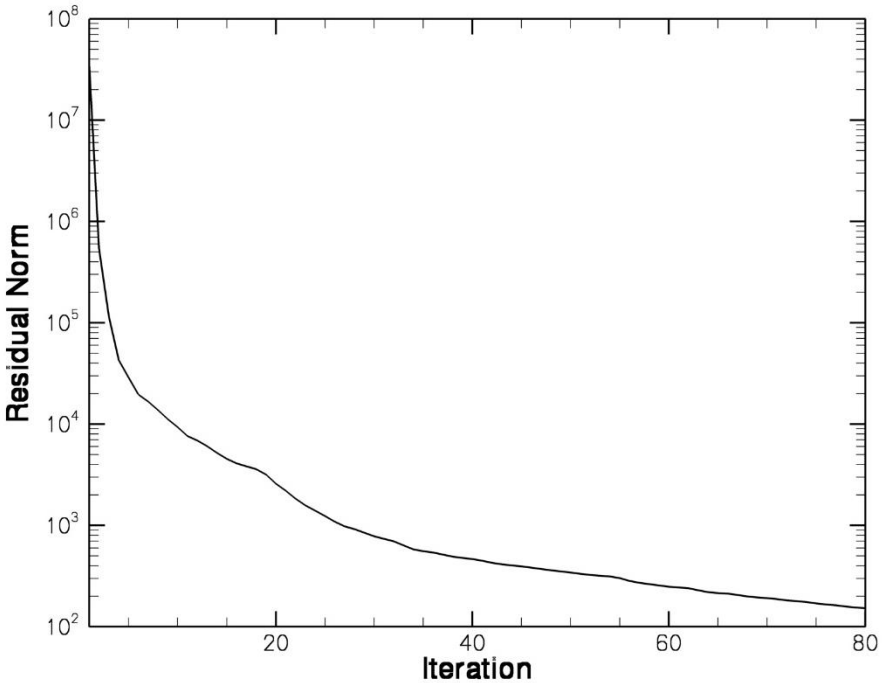
Approach

- Execute FUN3D flow/adjoint solvers to output data to disk for use in LSS: nonlinear residual vectors and Jacobians of residual and objective function
- For this tiny problem, the raw dataset is ~1.1 TB (in-core requirement much larger)
- Developed standalone LSS solver, where partitioning is performed in time with a single time plane per core
  - For simplicity, the spatial discretization fits on a single core
  - Spatial decompositions are possible but not considered here
- Global GMRES solver used with a local ILU(0) preconditioner for each time plane, with $\mathbf{C}\mathbf{C}^T$ term neglected in preconditioner
- Execution was constrained to a subset of the cores available on each 128 GB Haswell node to provide sufficient memory for solving the LSS adjoint system
- Checked discrete consistency of LSS implementation using complex variables
- This complex variable test does not provide the same rigor for LSS as for conventional adjoint implementations; additional verification approaches needed

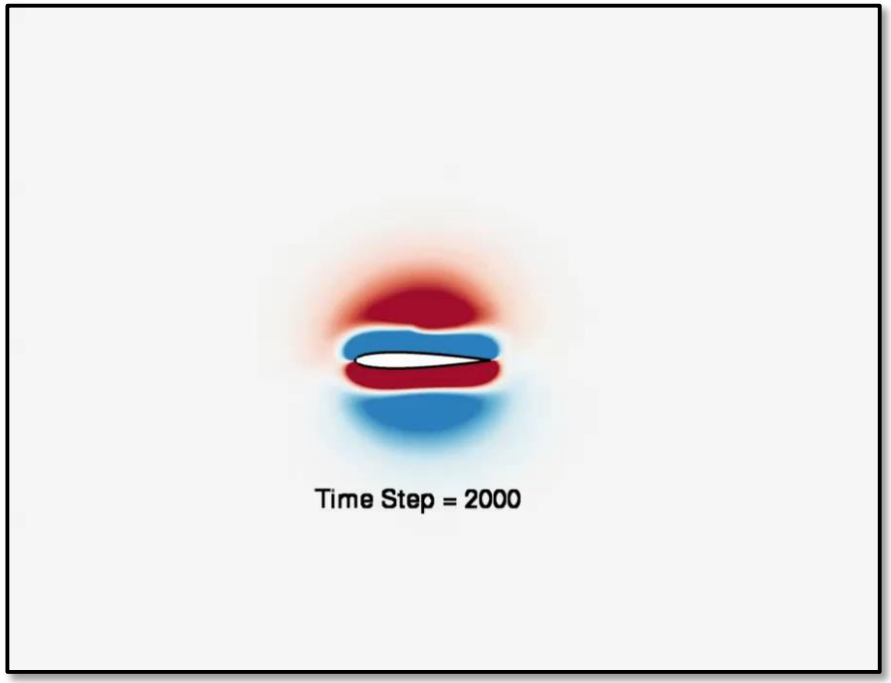
Solution of LSS Adjoint System

- After ~30 minutes for I/O, solution converges 5 orders of magnitude in ~30 mins on 2,000 cores
- Solution remains bounded

Convergence of LSS Adjoint System



LSS Adjoint Solution for Energy Equation



Current Status and Future Outlook

- Assess if (or how well) ergodicity assumption is satisfied for this problem
- Evaluate quality of computed sensitivities
- Attempt design optimization

- How to afford extension of LSS to realistic problems?

Thank you to the organizers for having us!