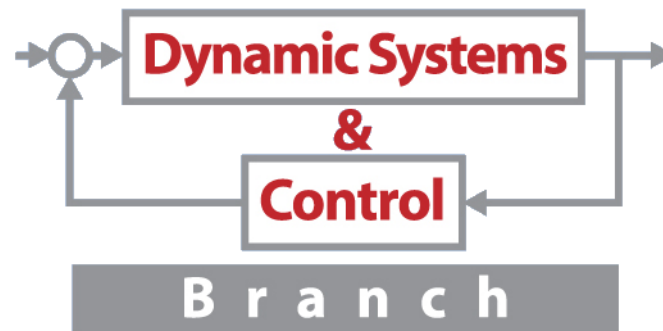


GTM Flight Test Video

The NASA Langley UQ Challenge



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Overview



- UQ Challenge: motivation
- UQ Challenge: problem statement
- Summary of responses

Motivation to NASA



- NASA missions often involve the development of new vehicles and systems that must be designed to operate in harsh domains with a wide array of operating conditions.
- These missions involve high-consequence and safety-critical systems for which quantitative data is either very sparse or prohibitively expensive to collect.
- Limited heritage data may exist, but is also usually sparse and may not be directly applicable to the system of interest, making UQ extremely challenging.
- NASA modeling and simulation standards require estimates of uncertainty and descriptions of any processes used to obtain these estimates.

UQ Challenge Objectives



- To determine suitability/limitations of existing methods
- To address UQ engineering needs
- To promote the development of new UQ technologies
- Pull vs. push

UQ Challenge: Physical System



- The challenge problem is based upon a model of the NASA Langley Generic Transport Model (GTM). The GTM is a 5.5% dynamically scaled, remotely piloted, twin-turbine, research aircraft used to conduct experiments for the NASA Aviation Safety Program
- Although a discipline-specific application is the focus of this challenge problem, **the problem was specifically structured so that specialized aircraft knowledge is not required**
- We consider aircraft motion that extends outside the normal operating flight envelope, not predictably altered by pilot inputs, driven by nonlinearities and coupling, possibly having oscillatory and divergent behavior

UQ Challenge: Physical System



- Dynamically scaled, highly instrumented, flight test article

UQ Challenge: Physical System



- Dynamically scaled, highly instrumented, flight test article

UQ Challenge: Physical System



- AIRSTAR, pilot commands

UQ Challenge: Computational Model



- A high-fidelity mathematical model matching closely the dynamics of the test article was developed
- The aerodynamic model is based on system identification experiments and wind tunnel data
- Features: nonlinear aerodynamics, avionics, engine and sensor dynamics, atmospheric & turbulence models, telemetry, time delays, wash-out filters, etc. (278 states)
- Mathematical model has a flight envelope extending beyond normal operating conditions

UQ Challenge: Computational Model



Flight Control System

ADAPTIVE CONTROLLER

Adaptive law

Reference model

Saturation modification

Anti-wind up modification

Dead zone

Projection algorithm

NOMINAL CONTROLLER

Longitudinal controller

Lateral/dir controller

Auto-throttle

Anti-wind up

Pilot

Generic Transport Model

Failures

Damages

Time delay

Uncertainties

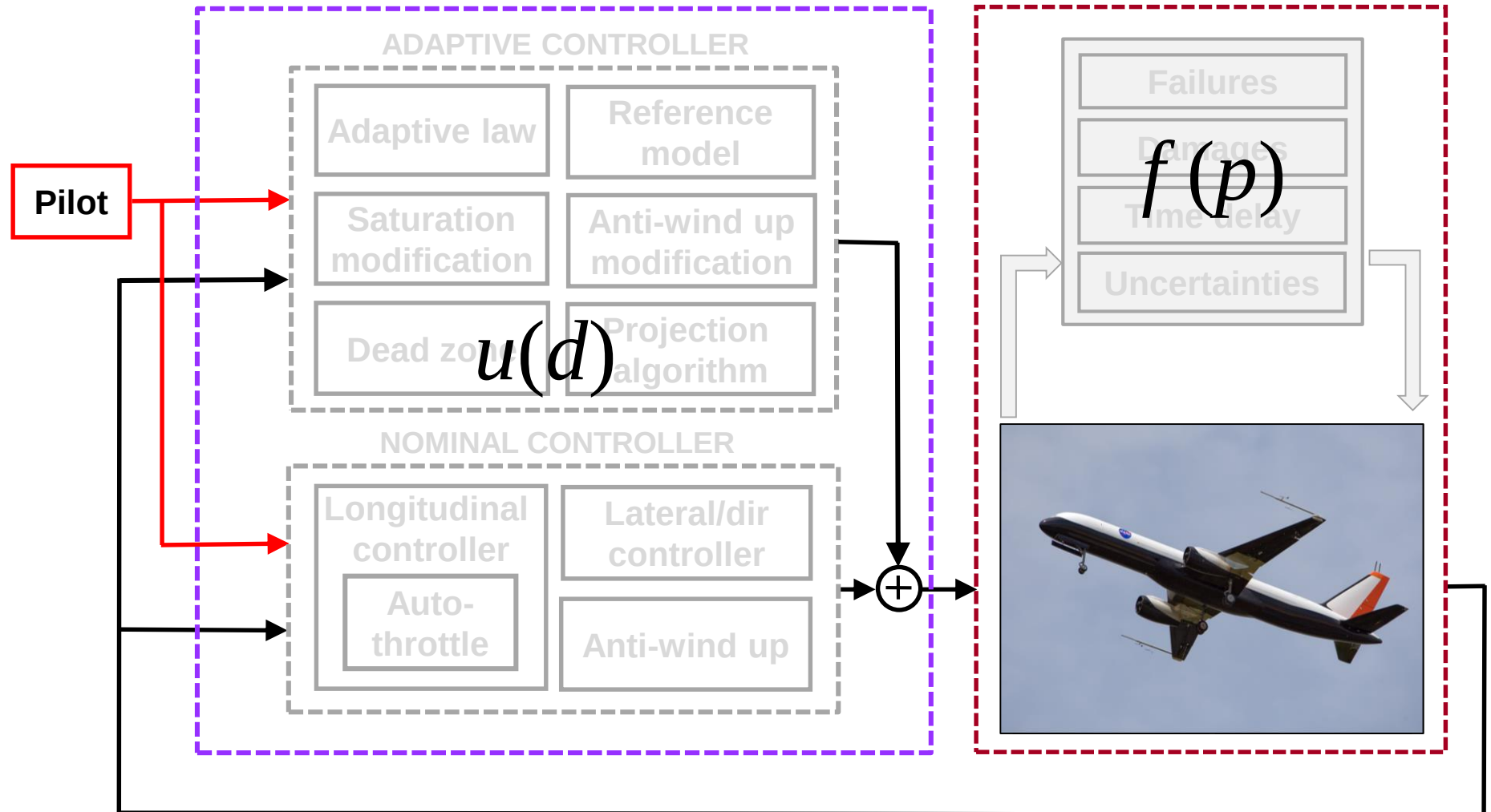


UQ Challenge: Computational Model

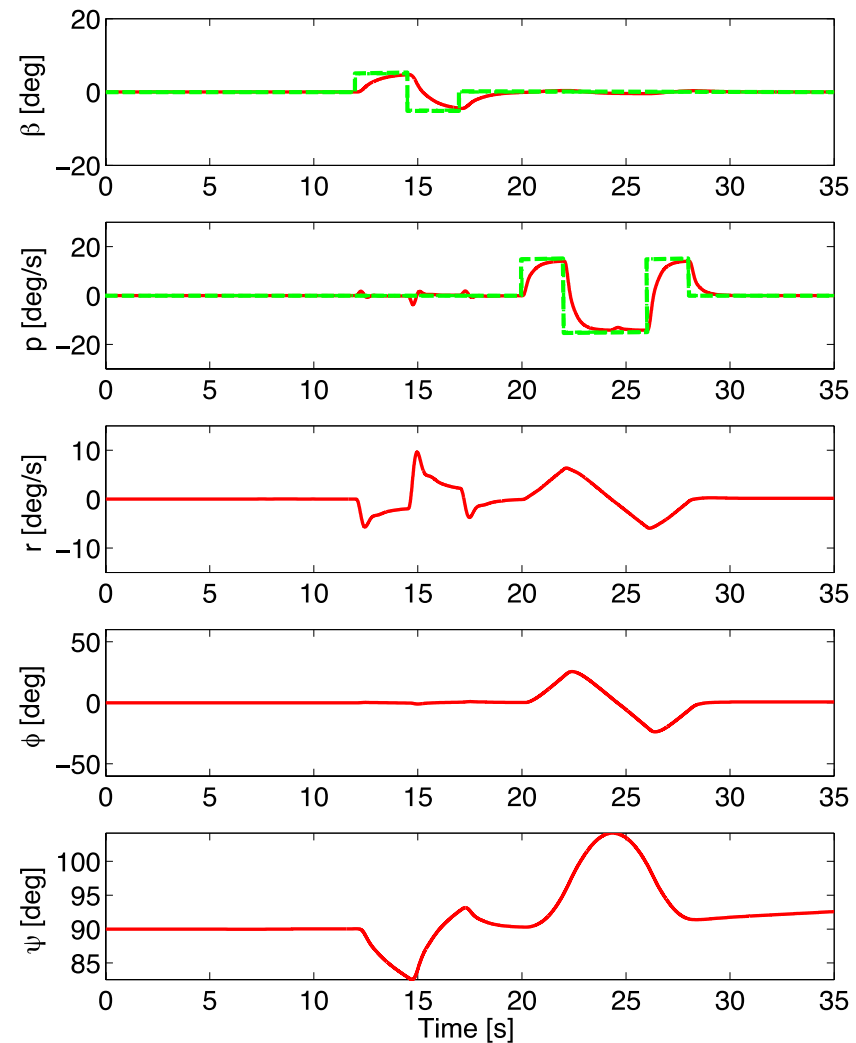
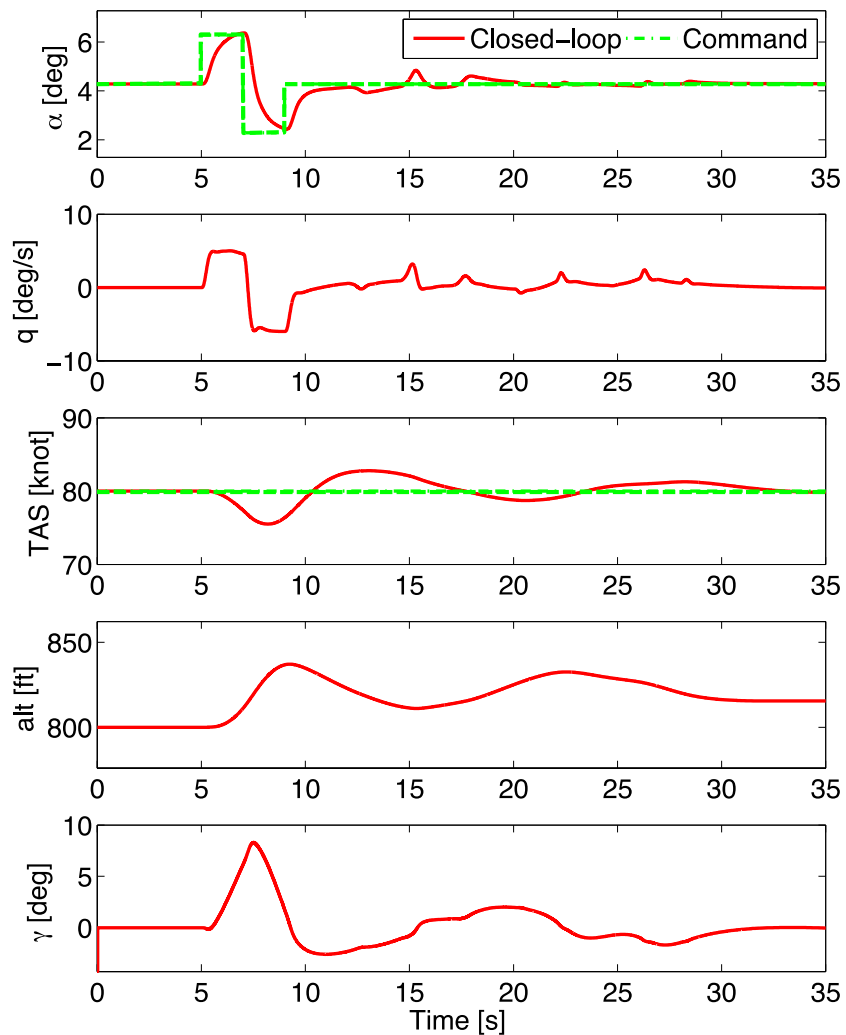


Flight Control System

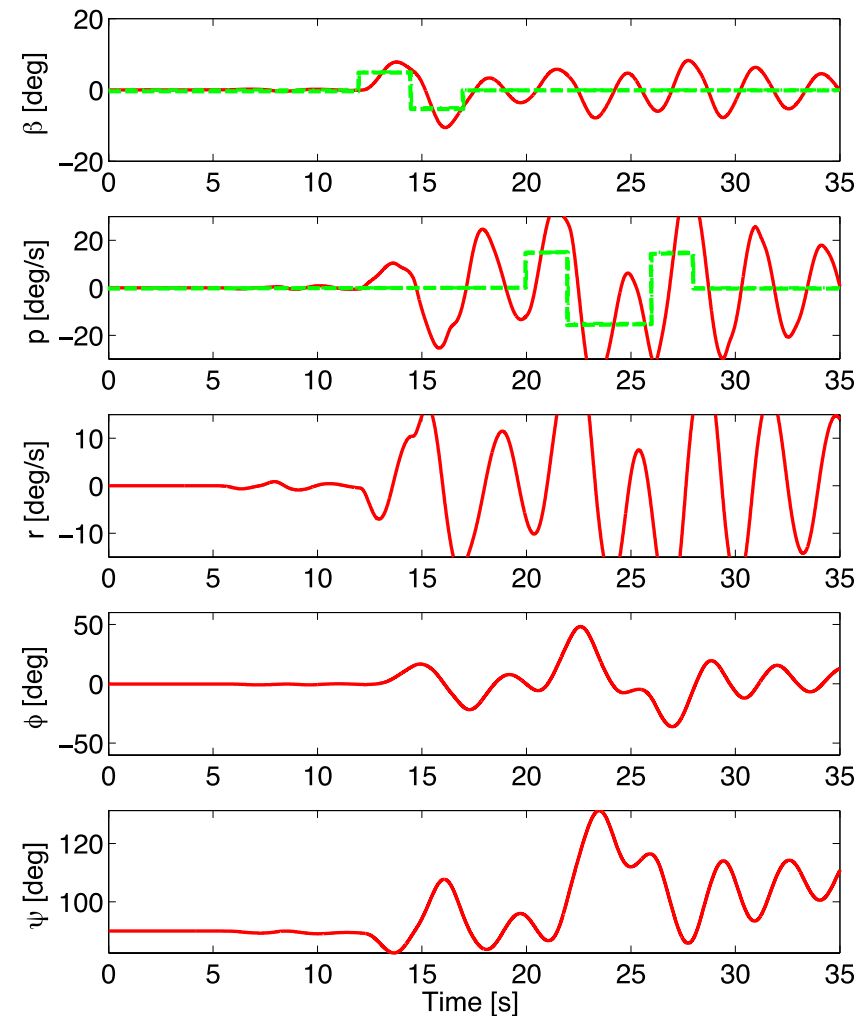
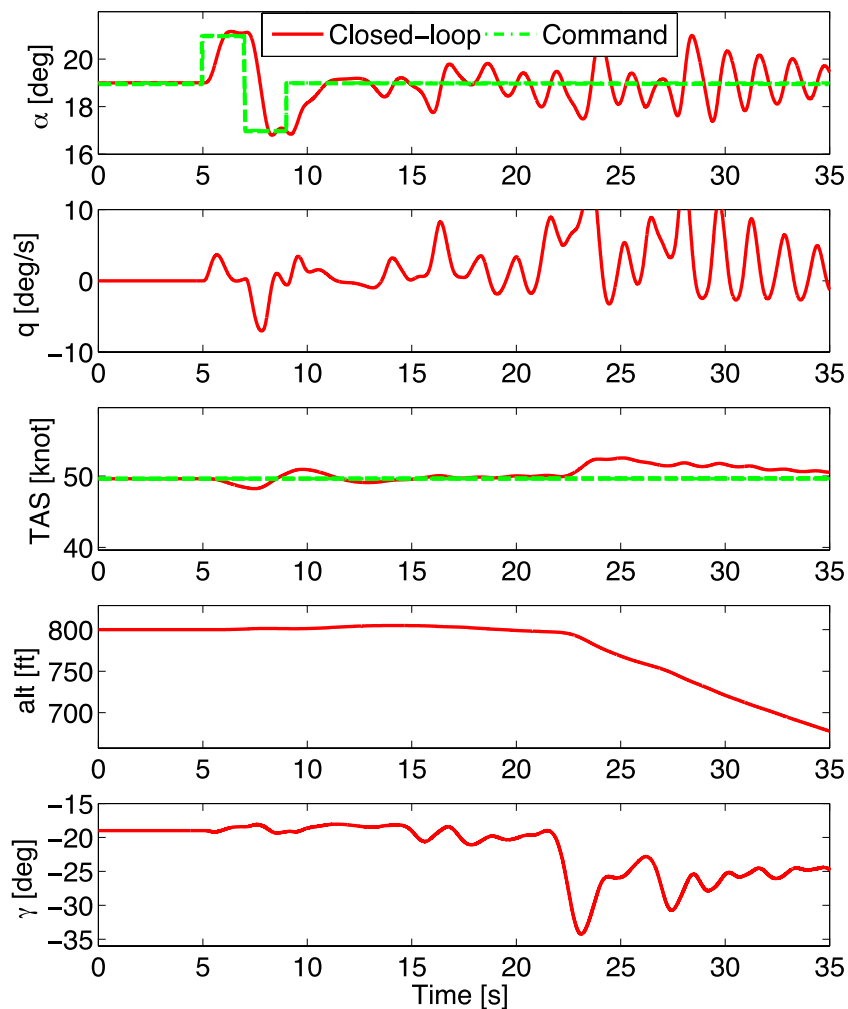
Generic Transport Model



Time Response: $\{p_{\text{nominal}}, d_1\}$



Time Response: $\{p_{\text{off-nominal}}, d_1\}$



Simulation @ $\{p_{\text{off-nominal}}, d_2\}$

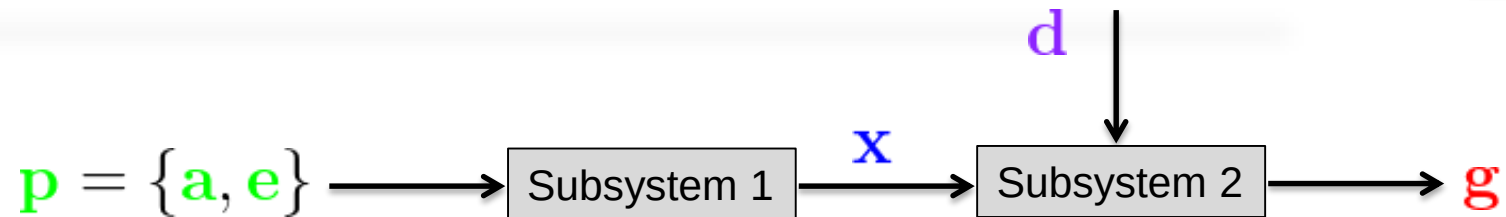
Simulation @ $\{p_{\text{off-nominal}}, d_3\}$

Overview

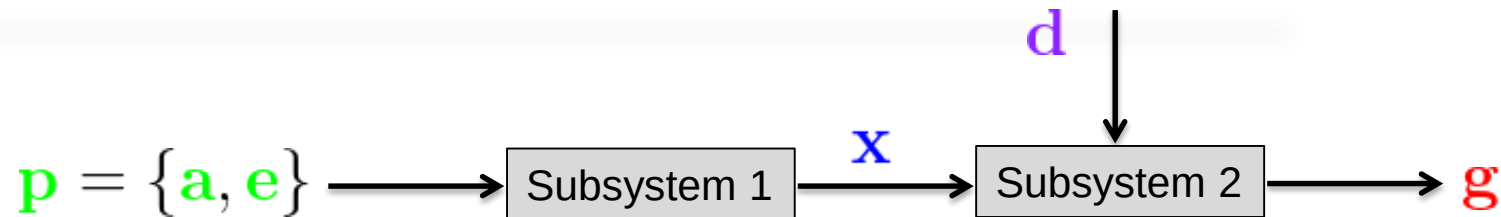


- UQ Challenge: motivation
- UQ Challenge: problem statement
- Summary of responses

Framework

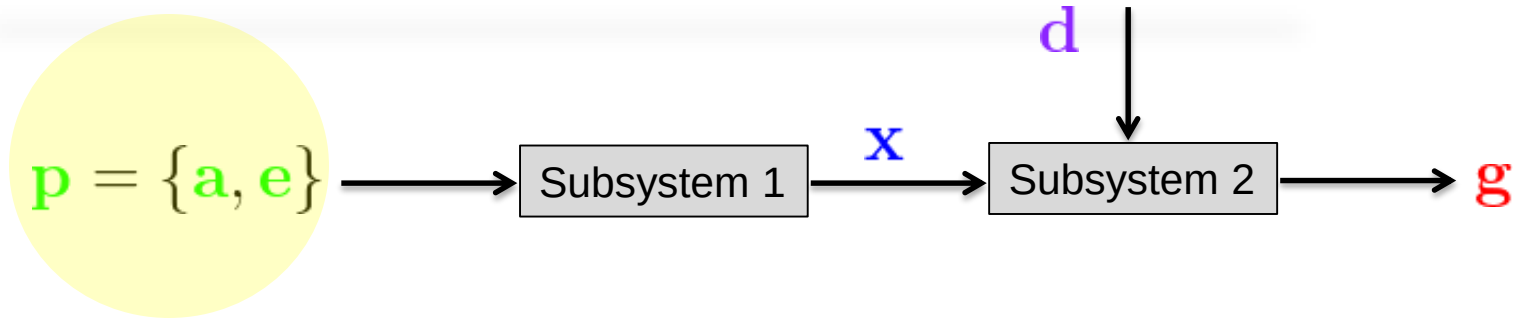


Framework



- **Uncertain parameter**, $\mathbf{p} \in \mathbb{R}^{21}$: accounts for the effects of loss in control effectiveness, actuator failure, dead-zone, and desired range of operating conditions
- **Intermediate variable**, $\mathbf{x} \in \mathbb{R}^5$: `aoa_cmd`, control effectiveness of elevator, aileron and rudder, time delay due telemetry and communications
- **Design variable**, $\mathbf{d} \in \mathbb{R}^{14}$: controller gains
- **Performance metrics**, $\mathbf{g} \in \mathbb{R}^8$: stability, `aoa`, `p` and sideslip tracking; elevator, aileron and rudder actuation

Uncertain Parameters



- Uncertainty model (UM) for $\mathbf{p}$ is given
- Aleatory, epistemic and mixed uncertainties

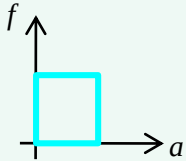
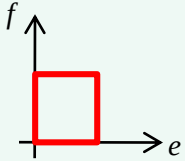
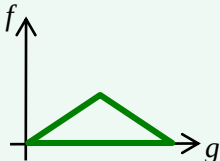
{ Uncertainty Modeling



	$a \text{ in } [0,1]$	$e \text{ in } [0,1]$	$g = a+e$	$E[g]$	$V[g]$	$P[g > 3/2]$
Approach 1						
Approach 2						
Approach 3						

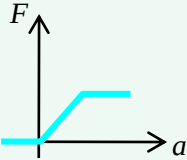
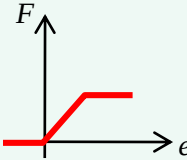
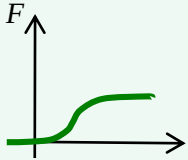
{ Uncertainty Modeling



	$a \text{ in } [0,1]$	$e \text{ in } [0,1]$	$g = a+e$	$E[g]$	$V[g]$	$P[g > 3/2]$
Prob/Prob				1	3/18	1/16

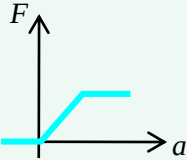
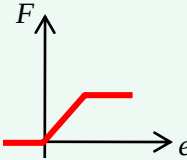
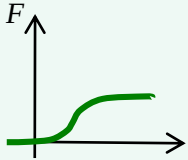
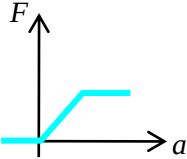
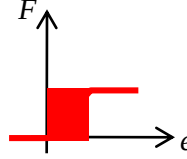
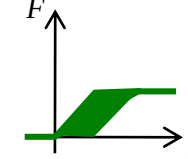
{ Uncertainty Modeling



	$a \text{ in } [0,1]$	$e \text{ in } [0,1]$	$g = a+e$	$E[g]$	$V[g]$	$P[g > 3/2]$
Prob/Prob				1	3/18	1/16

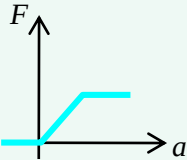
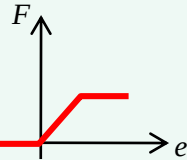
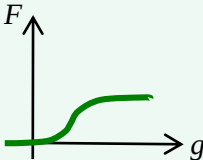
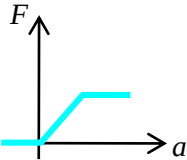
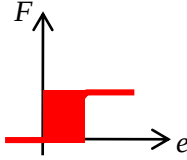
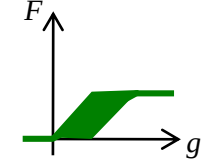
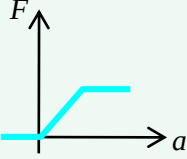
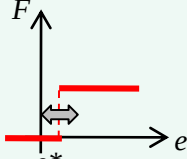
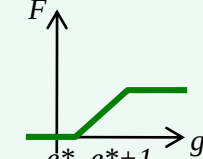
{ Uncertainty Modeling



	$a \text{ in } [0,1]$	$e \text{ in } [0,1]$	$g = a+e$	$E[g]$	$V[g]$	$P[g > 3/2]$
Prob/Prob				1	3/18	1/16
Prob/P-Box				$[1/2, 3/2]$	$[0, 13/16]$	$[0, 1/2]$

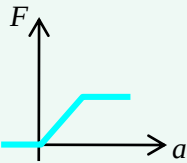
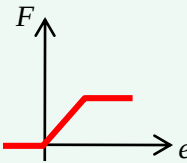
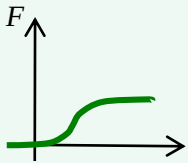
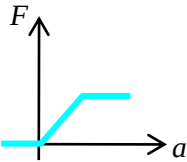
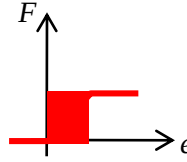
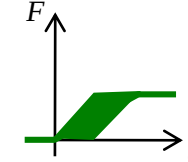
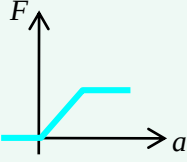
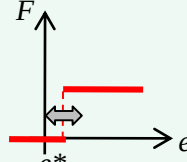
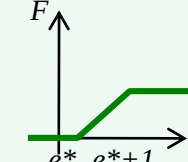
{ Uncertainty Modeling



	a in $[0,1]$	e in $[0,1]$	$g = a+e$	$E[g]$	$V[g]$	$P[g > 3/2]$
Prob/Prob				1	3/18	1/16
Prob/P-Box				$[1/2, 3/2]$	$[0, 13/16]$	$[0, 1/2]$
Prob/UnkC				$[1/2, 3/2]$	1/12	$[0, 1/2]$

{ Uncertainty Modeling



	a in $[0,1]$	e in $[0,1]$	$g = a+e$ <i>Rand variable</i>	$E[g]$	$V[g]$	$P[g > 3/2]$
Prob/Prob				1	3/18	1/16
				Not even a conservative estimate		
Prob/P-Box				$[1/2, 3/2]$	$[0, 13/16]$	$[0, 1/2]$
Prob/UnkC				$[1/2, 3/2]$	1/12	$[0, 1/2]$

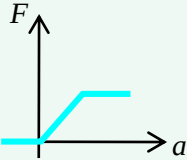
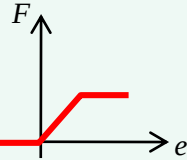
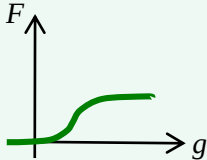
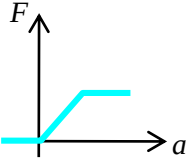
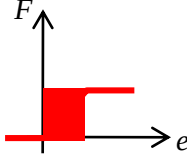
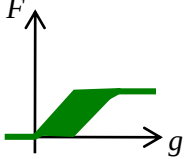
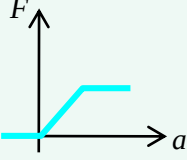
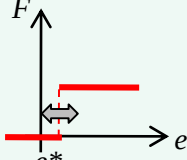
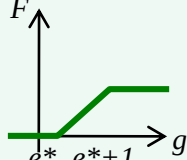
{ Uncertainty Modeling



	a in $[0,1]$	e in $[0,1]$	$g = a+e$ <i>P-Box</i>	$E[g]$	$V[g]$	$P[g > 3/2]$
Prob/Prob				1	3/18	1/16
Prob/P-Box				$[1/2, 3/2]$	$[0, 13/16]$	$[0, 1/2]$
				Irreducible conservatism		
Prob/UnkC				$[1/2, 3/2]$	1/12	$[0, 1/2]$

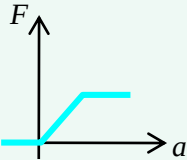
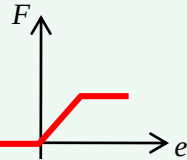
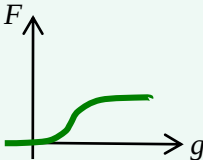
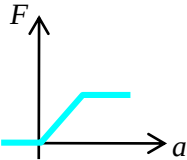
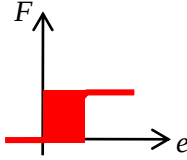
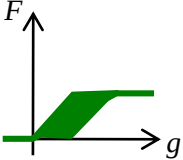
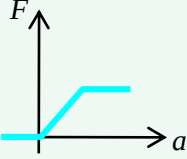
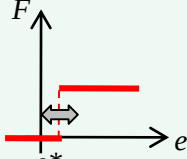
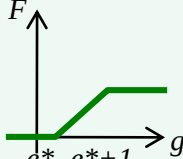
{ Uncertainty Modeling



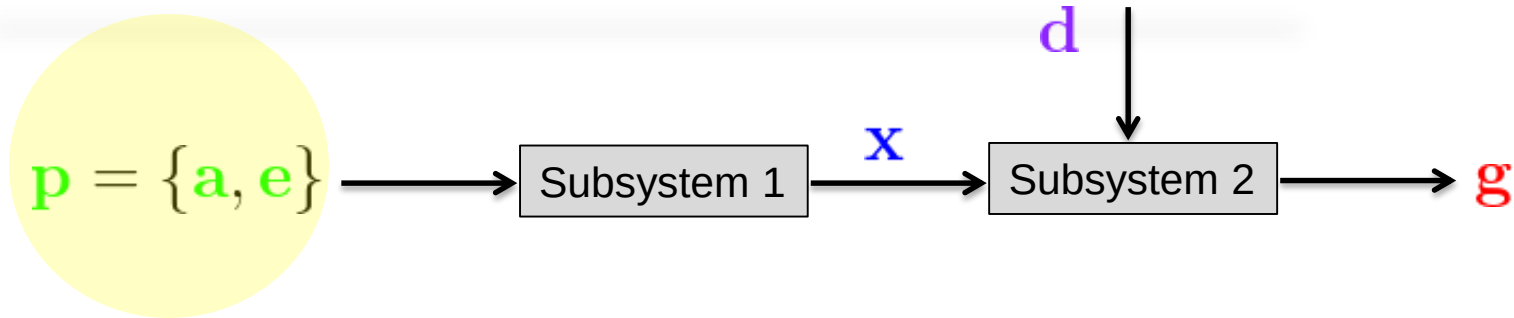
	a in $[0,1]$	e in $[0,1]$	$g = a+e$	$E[g]$	$V[g]$	$P[g > 3/2]$
Prob/Prob				1	3/18	1/16
Prob/P-Box				Values are exact but formulations are wrong		
				$[1/2, 3/2]$	$[0, 13/16]$	$[0, 1/2]$
Prob/UnkC				$[1/2, 3/2]$	1/12	$[0, 1/2]$

{ Uncertainty Modeling }



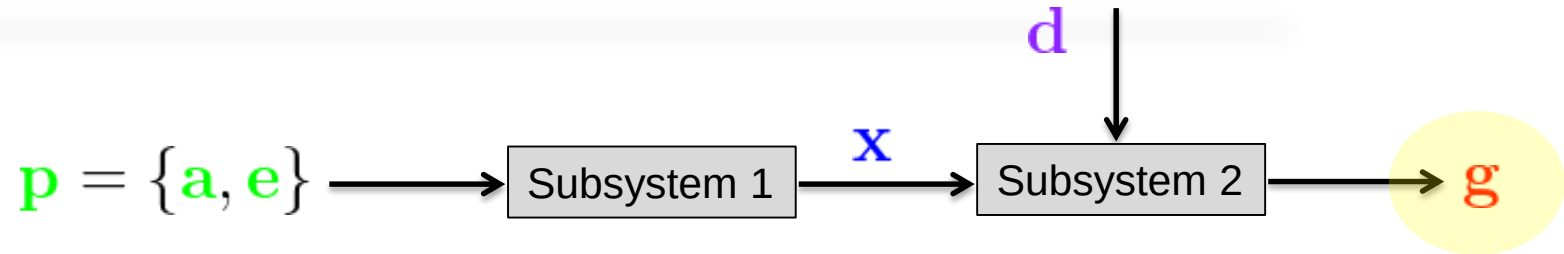
	a in $[0,1]$	e in $[0,1]$	$g = a+e$ <i>Random process</i>	$E[g]$	$V[g]$	$P[g > 3/2]$
Prob/Prob				1	3/18	1/16
Prob/P-Box				$[1/2, 3/2]$	$[0, 13/16]$	$[0, 1/2]$
Prob/UnkC				Q: How to do this systematically? $[1/2, 3/2]$ $1/12$ $[0, 1/2]$ A: 2-level analysis		

Uncertain Parameters

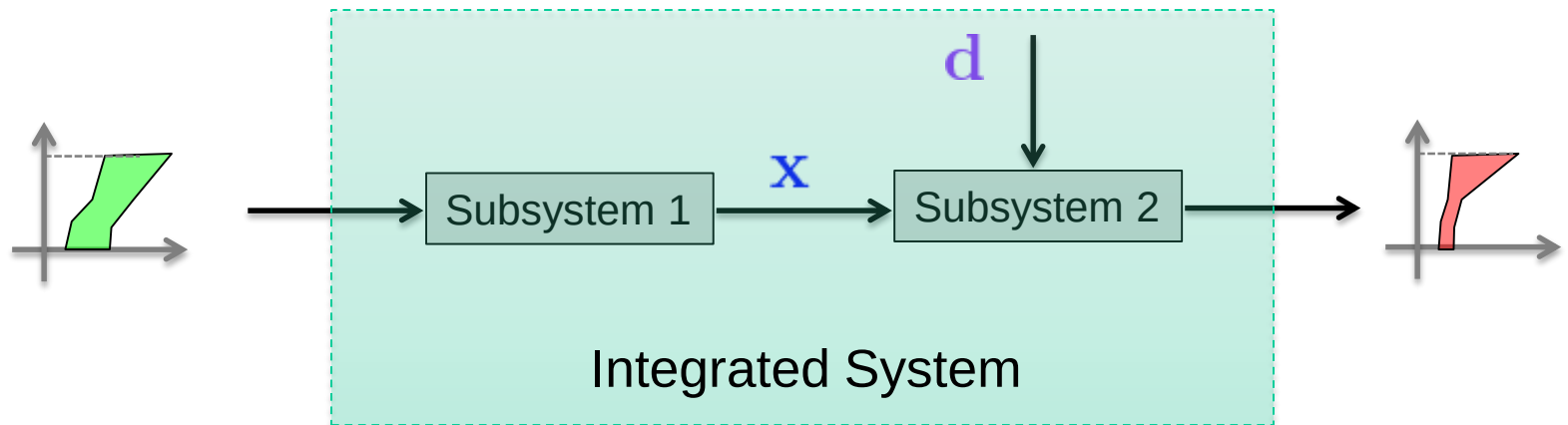
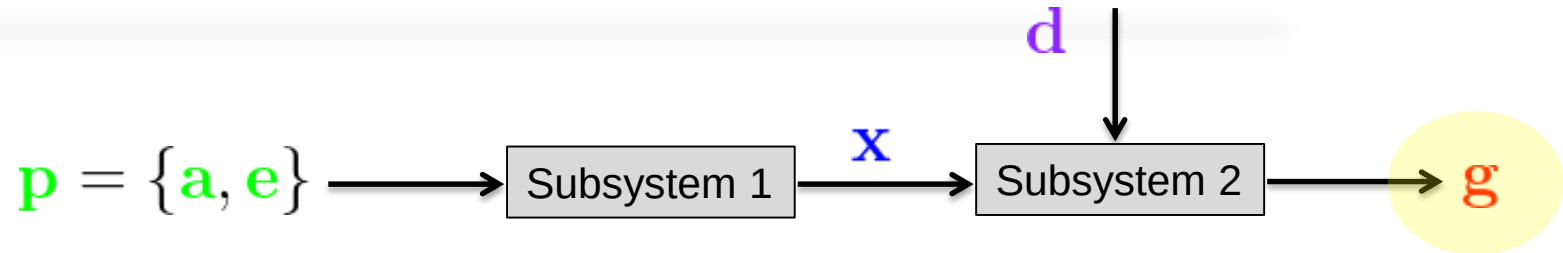


- An uncertainty model (UM) for $\mathbf{p}$ is given
- Aleatory, epistemic and mixed uncertainties
- The UM contains three types of parameters:
 1. Aleatory: random variable
 2. Epistemic: unknown constant in an interval (reducible)
 3. Nested: p-box (reducible)

Performance Metrics

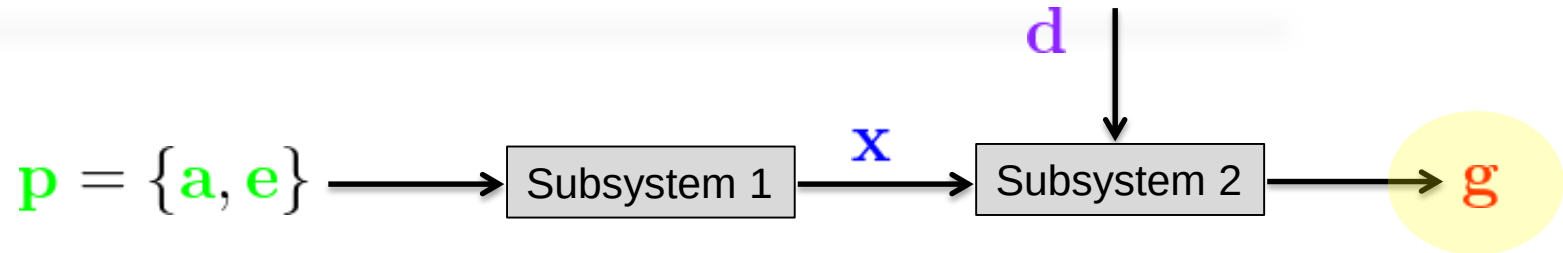


Performance Metrics



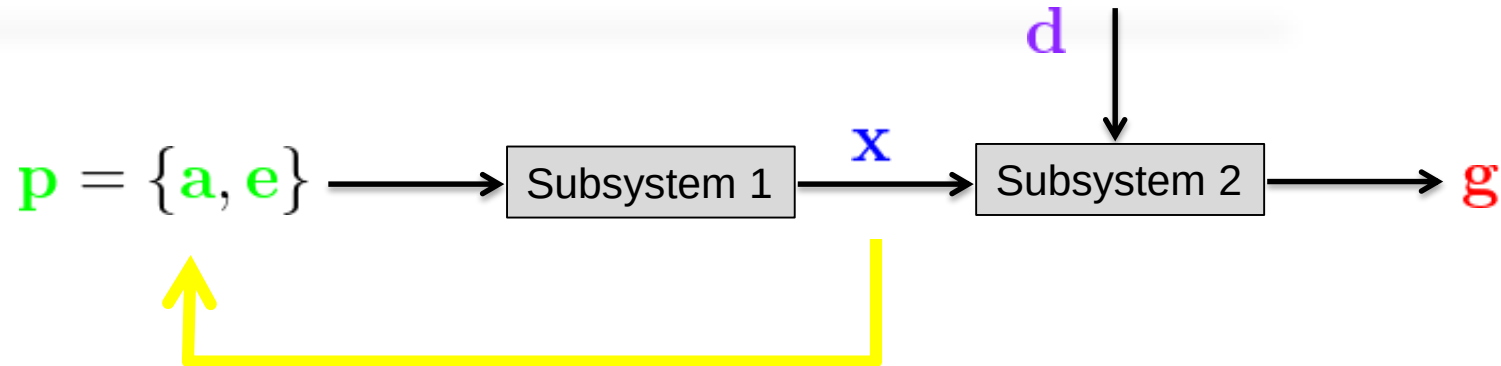
➤ *Challenge: 2-level search in a high-dimensional space*

Performance Metrics



- We consider the metrics
 - The mean $J_1 = \mathbb{E}[\max(\mathbf{g}) > 0]$
 - The probability of failure $J_2 = \text{Prob}[\max(\mathbf{g}) > 0]$
- Epistemic uncertainty makes such metrics vary in a range

Task 1: Model Calibration

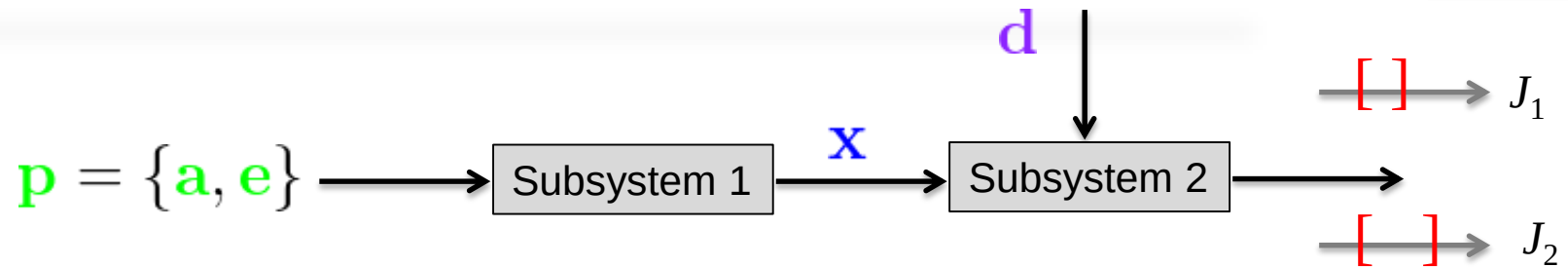


- Given an UM of $\mathbf{p}$ and observations of $\mathbf{x}$ refine the UM

➤ Challenges

- *Where and what is the prior in Bayesian calibration?*
- *What is the actual benefit of having more data?*
- *Computational complexity*

Task 2: Global Sensitivities

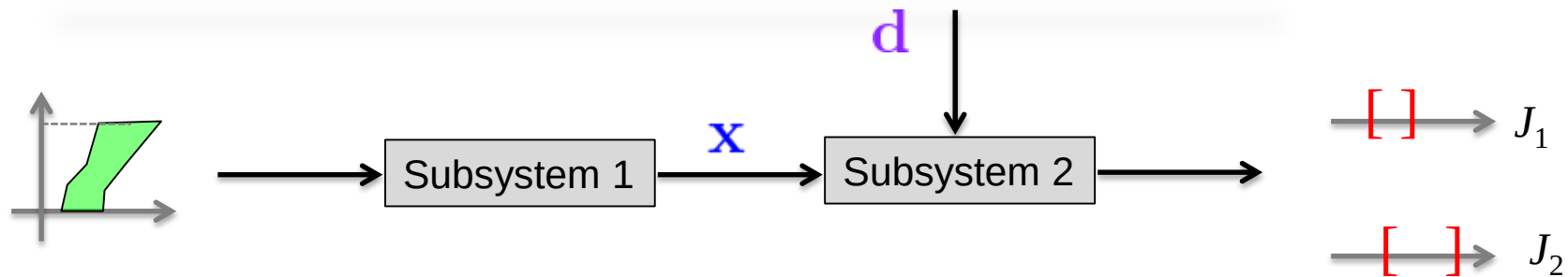


- For a fixed $\mathbf{d}$, rank the parameters in $\mathbf{p}$ according to
 - the degree of refinement in the p-box of $\mathbf{x}$
 - the reduction in the range of J_1
 - the reduction in the range of J_2which one could hope to obtain by refining the UM

➤ Challenges

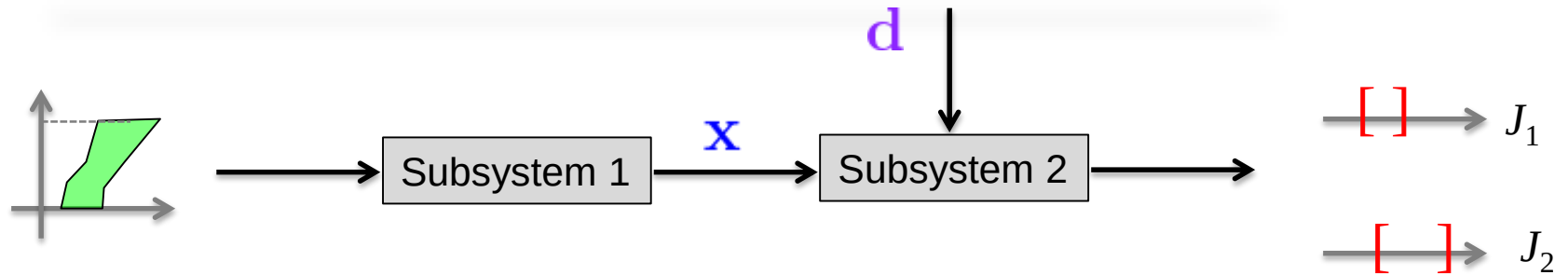
- Presence of epistemic uncertainty
- Relevance at the subsystem level vs. at the integrated level

Task 3: Uncertainty Propagation



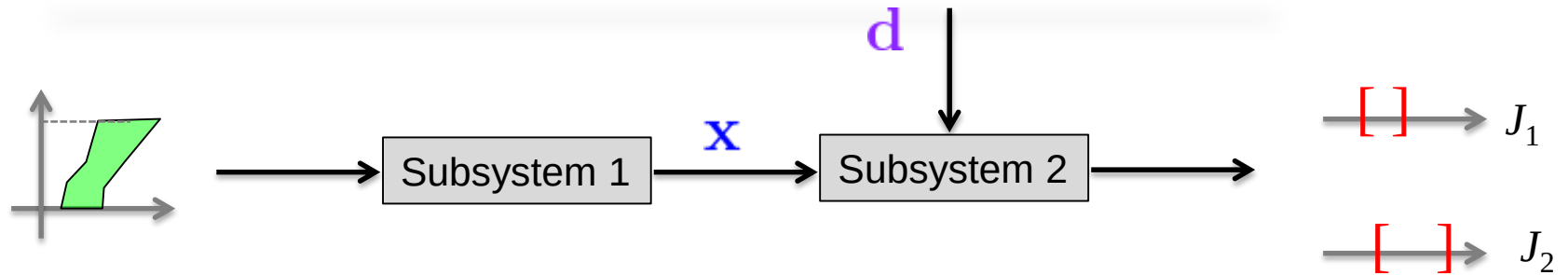
- For a fixed d , find range of J_1 and J_2 for original UM

Task 3: Uncertainty Propagation



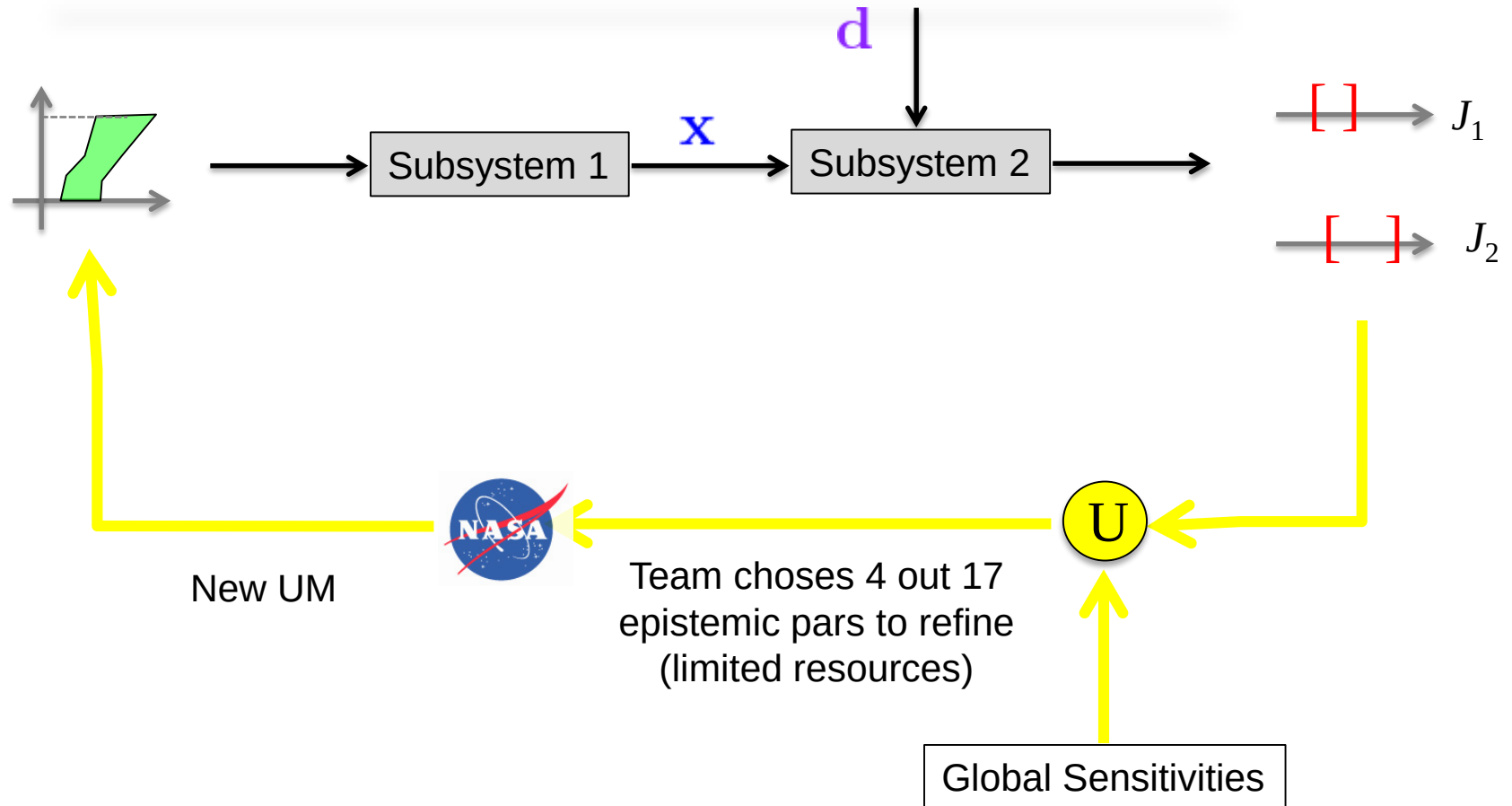
- For a fixed d , find range of J_1 and J_2 for original UM
- What shall we do if a range is inadmissibly wide?

Task 3: Uncertainty Propagation

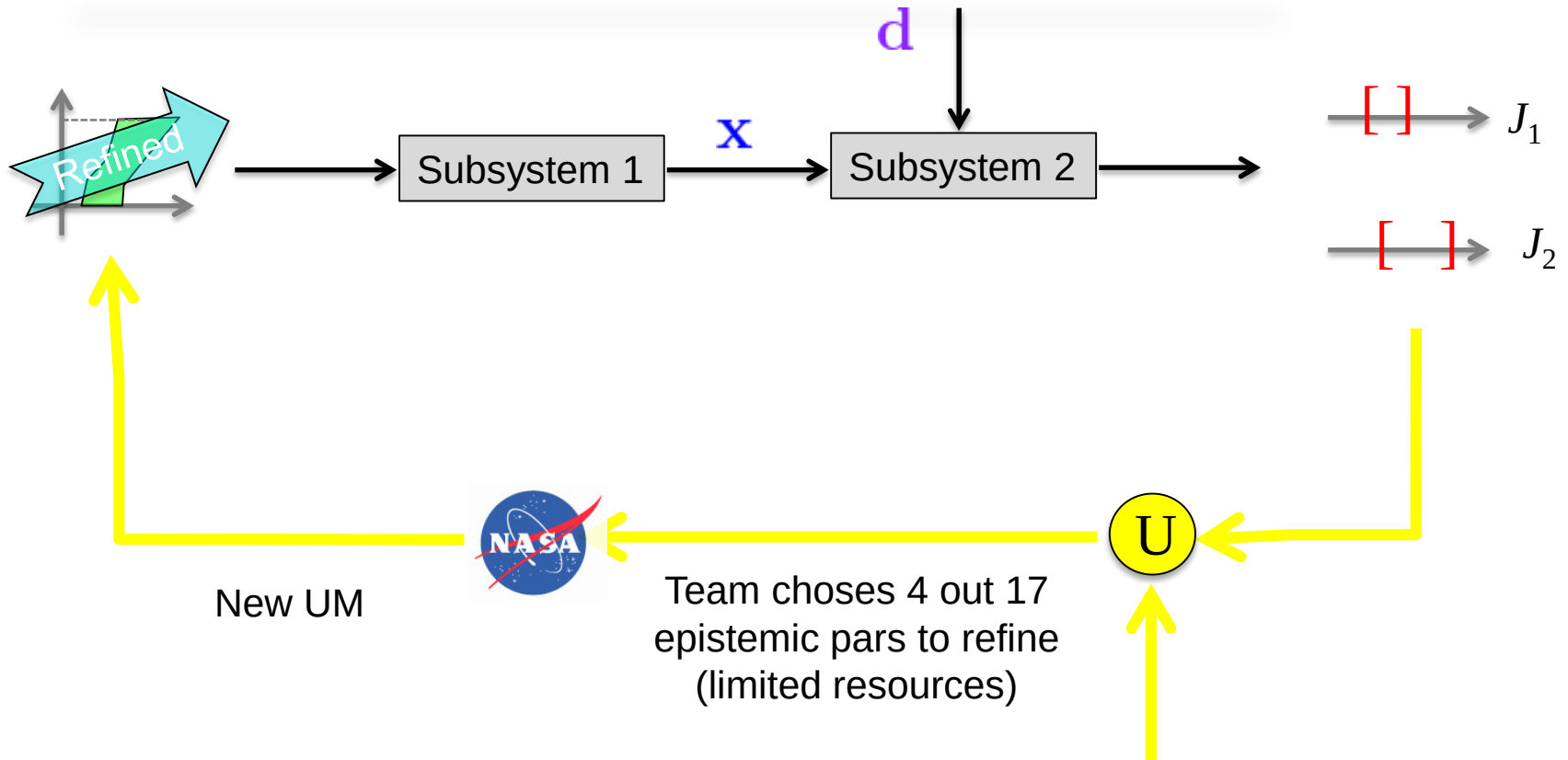


- For a fixed $\mathbf{d}$, find range of J_1 and J_2 for original UM
- What shall we do if a range is inadmissibly wide?
- Possible culprits
 - UQ method is overly conservative
 - Uncertainty Model encompasses too much variation
 - The design point is not sufficiently robust

Task 3: Uncertainty Propagation



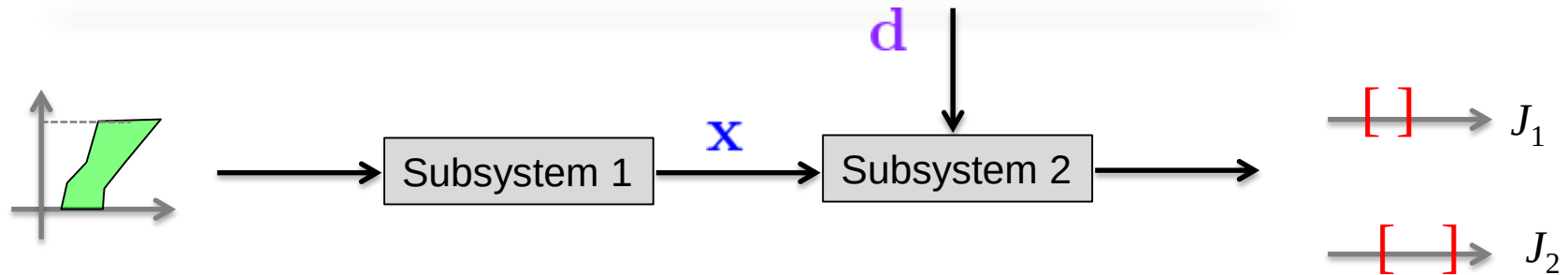
Task 3: Uncertainty Propagation



➤ Challenges

- Which parameter to refine so the spread is reduced the most
- Educated gambling
- Dominant at the subsystem level vs. at the integrated level

Task 3: Uncertainty Propagation

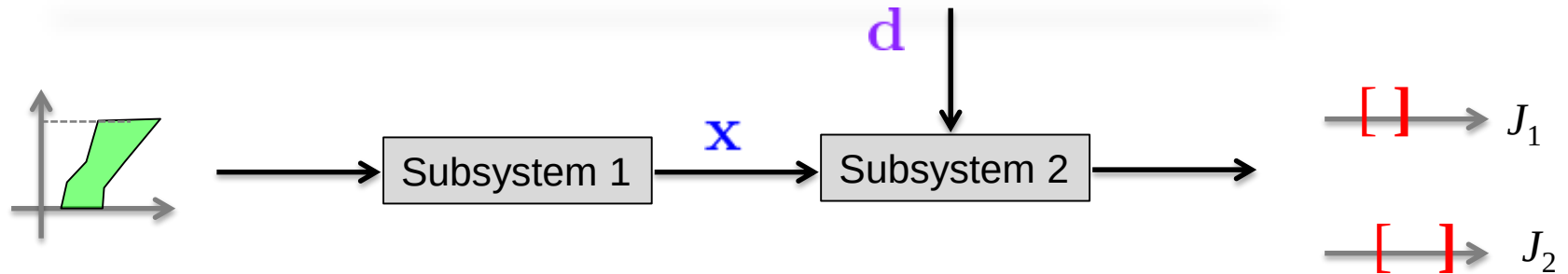


- For a fixed d , find range of J_1 and J_2 for original UM
- Based on the ranking done earlier, request refined UM for 4 of the epistemic parameters in p from NASA
- Find new range of J_1 and J_2

➤ Challenges

- Underprediction vs. overprediction
- Refined UM might not be much different from original UM

Task 4: Extreme-case Analysis

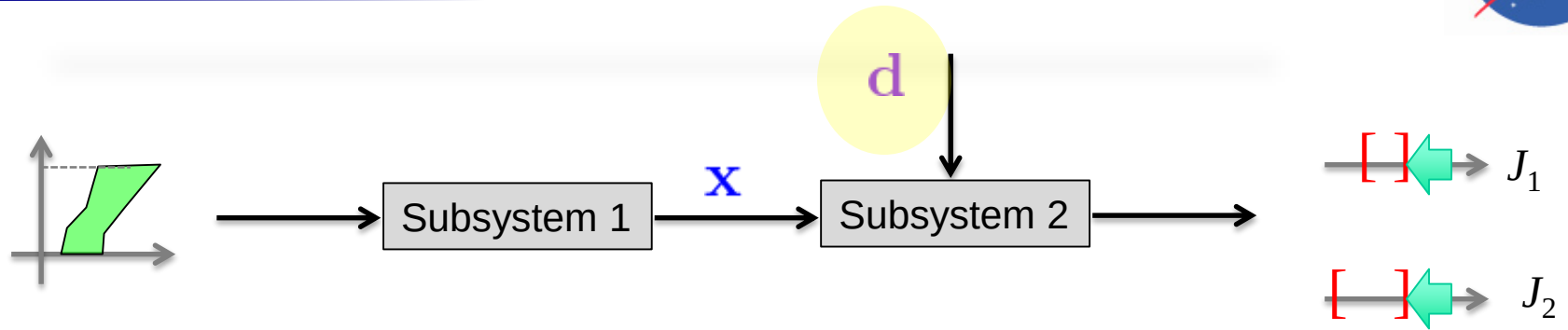


- Find the epistemic realizations in the UM of p leading to extreme values of J_1 and J_2 : causality

➤ Challenges

- Optimization based: local optima yields to under-prediction
- Sampling based: leads to under-prediction

Task 5: Robust Design



- For a fixed UM, find the design point d that minimizes
 - The largest value of J_1
 - The largest value of J_2

➤ Challenges

- Computational complexity of 3-level search
- Global sensitivities change with d

Overview



- UQ Challenge: motivation
- UQ Challenge: problem statement
- Summary of responses

Agenda



- 01/2013: 100+ UQ experts were invited to participate. Eleven groups from the US government, industry and academia accepted the invitation
- 01/2014: Papers presented in the AIAA Scitech
- 10/2014: Papers compiled in an special edition of the AIAA Journal of Aerospace Information Systems

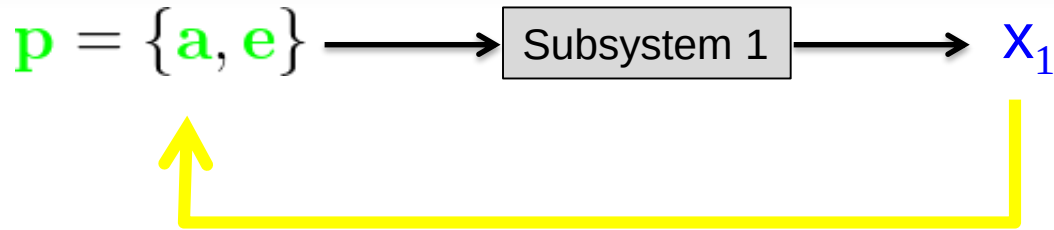
	Organization	Title	Authors
1	Sandia National Lab	UQ methods for model calibration, validation and risk analysis	C. Safta, H. Najm, B. Debusschere, K. Sargsyan, K. Chowdhary, M. Eldred, L. Swiler
2	Los Alamos National Lab	Robust design applied to the NASA Langley UQ challenge	Kendra Van Buren, Francois Hemez
3	Ecole Centrale Paris/Supelec	Uncertainty and sensitivity analysis of the mathematical model of a..	Nicola Pedroni, Enrico Zio
4	Swiss Federal Institute of Technology	The Bayesian multilevel framework for the NASA multidisciplinary..	Joseph Nagel
5	Stinger and Ghaffarian Technologies	Subjective approach to UQ: solution to the NASA UQ challenge	Shankar Sankaraman
6	Institute for Risk and Uncertainty, U. Liverpool	An integrated and efficient numerical framework for UQ:..	Edoardo Patelli, Mateo Broggi, Marco de Angelis
7	University of Florida	Prioritized information based UQ: the NASA UQ challenge...	A. Chaudhuri, G. Waycaster, N. Price, T. Matsumara, C. Park, R. Haftka
8	Vanderbilt University	Bayesian method framework for multidisciplinary UQ and optimization	Chen Liang, Snakaran Mahadevan
9	University of Southern California & Sandia National Labs	A probabilistic approach to the NASA Langley multidisciplinary UQ...	R. Ghanem, H. Meidani, E. Kalligiannaki, C. Thimmisetty, V. Keshavarzzadeh, I. Yadegaran, et. al
10	General Electric Global Research	A hybrid Bayesian solution to the NASA Langley multidisciplinary UQ	Ankur Srivastava, Arun Subramanayan
11	Southwest Research Institute	A Bayesian probabilistic treatment of multiple uncertainty types	John McFarland, Barron Bichon, David Riha

Task 0: Mathematical Framework



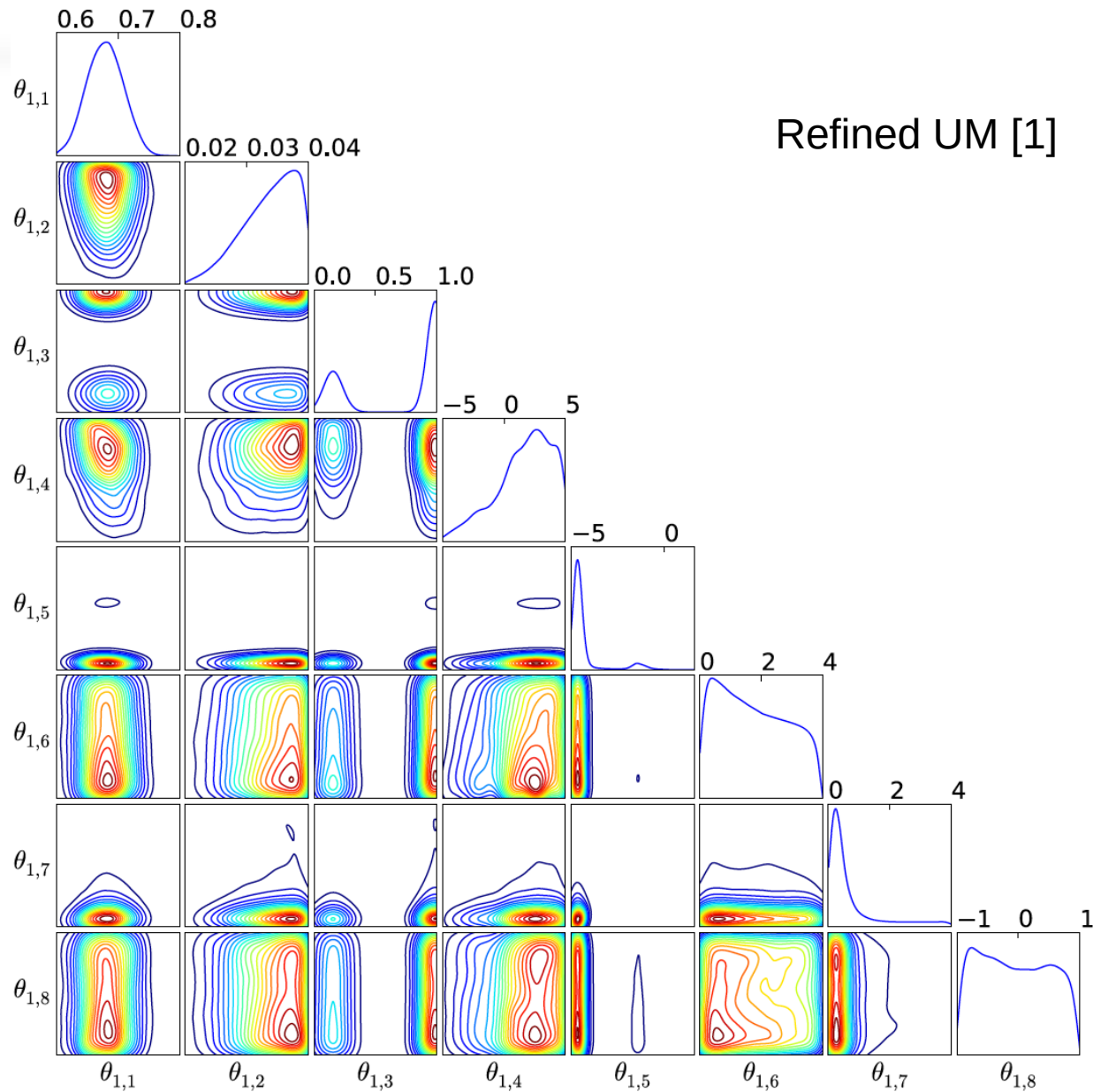
- Pure probabilistic
- 2-level epistemic (set) – aleatory (random)
- Random set theory
 - Probabilistic
 - Intervals
 - Distribution-free pboxes
 - Distribution-fixed pboxes

Task 1: Model Calibration

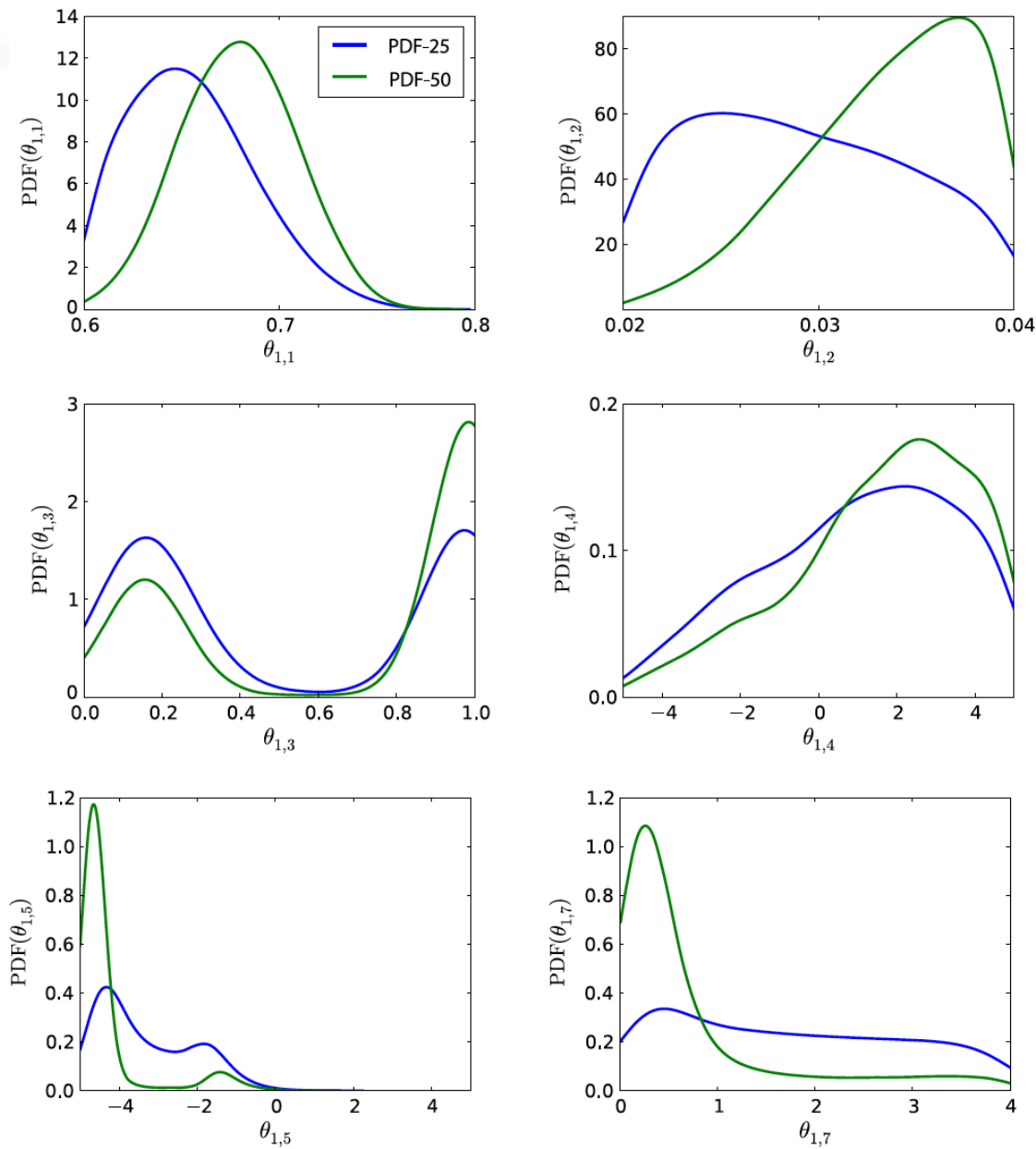


- $\dim(\mathbf{e})=8$, $\dim(x_1)=1$ output, CPU time=1s
 - **Bayesian calibration** with MCMC [1,6]
 - KDE or full likelihood and binning
 - Approximate Bayesian Computation (2 orders of mag better)
- Epistemic pars as uniform random variables: which space?
- Refined UM as interval or random vector: UM “upgrade”
- Setup: $n_{\text{steps}}:[5\text{K}, 500\text{K}]$, $n_a:[5\text{K}, 100\text{K}]$, CPU time:[5h, 96h]
- **KS-based approaches** for comparing empirical CDFs [3,7]

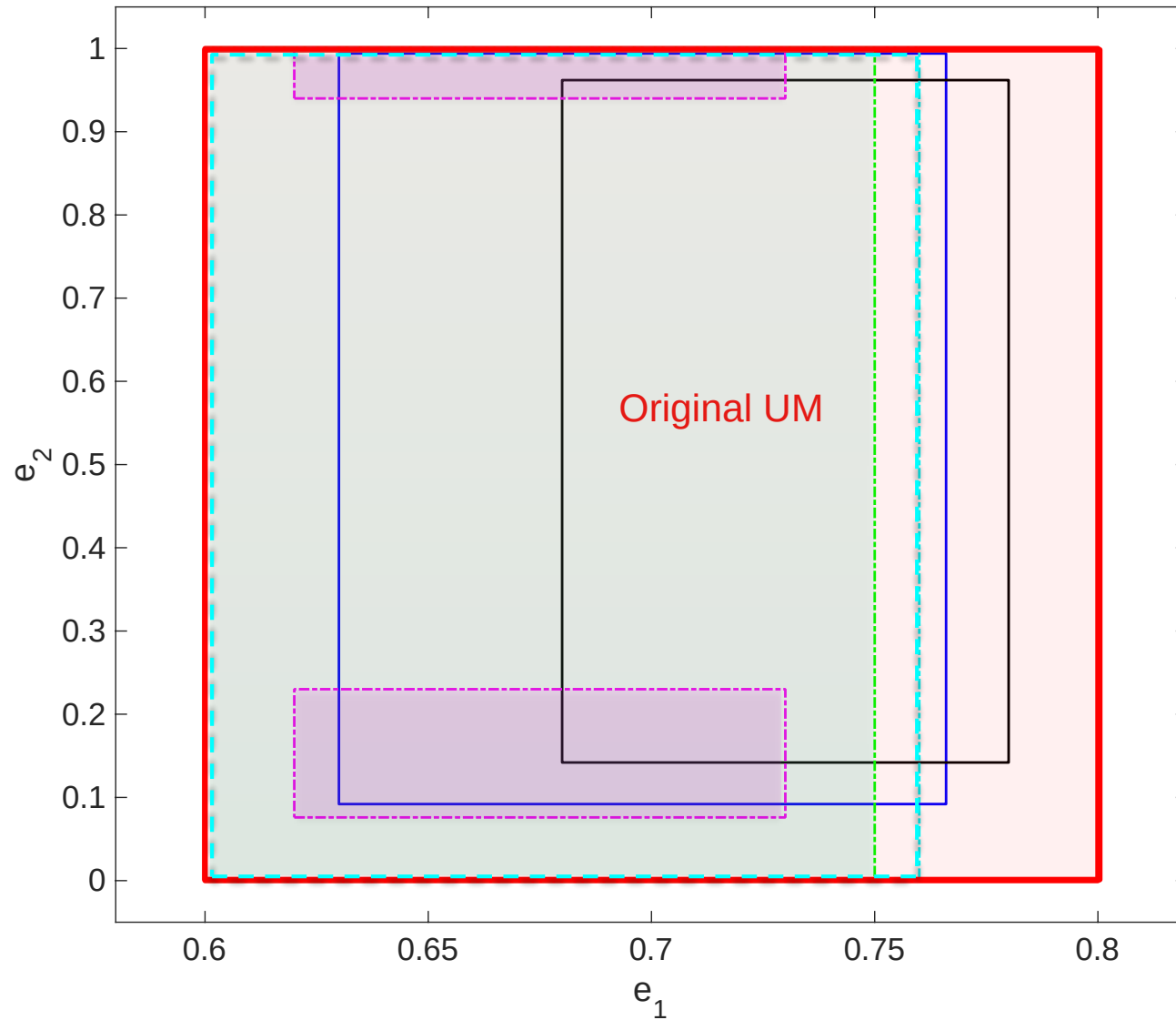
Task 1: Model Calibration



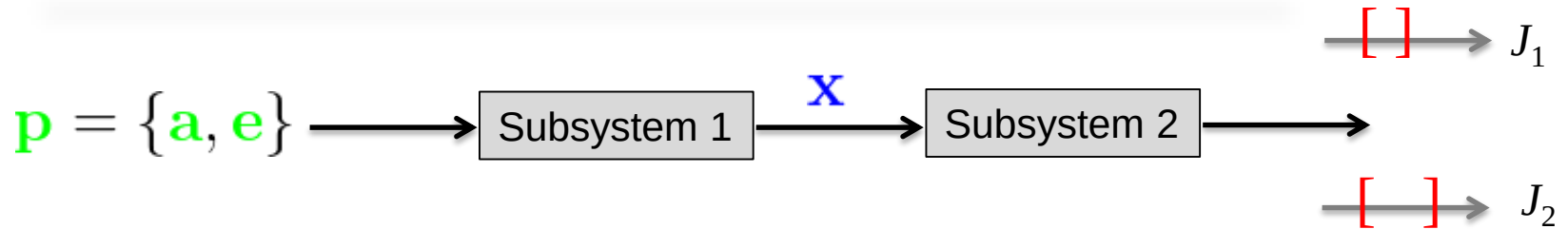
Task 1: Model Calibration



Task 1: Model Calibration



Task 2: Global Sensitivities



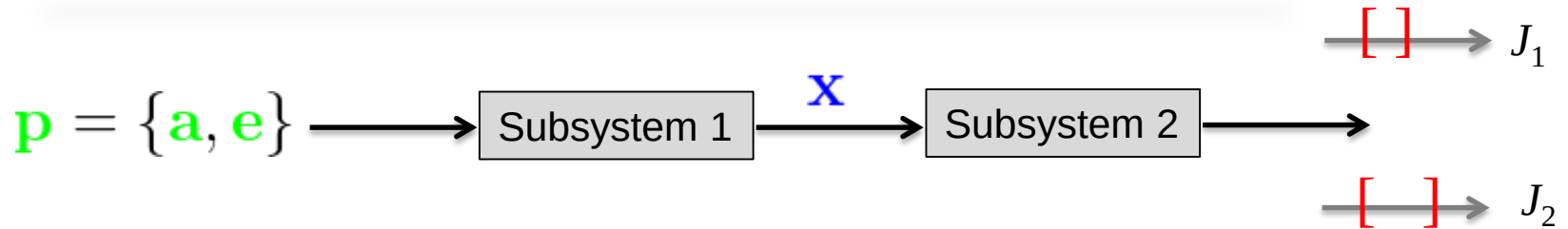
- **Reduction** in epistemic uncertainty of dependent variable (area of pbox, interval length) caused by fixing e_i [3,6]
- **ANOVA**: first-order sensitivity index and total-effect index

$$S_i = \frac{V_{x_i} [E_{\sim x_i} [y | x_i]]}{V[y]}$$

$$St_i = \frac{E_{\sim x_i} [V_{x_i} [y | x \sim x_i]]}{V[y]}$$

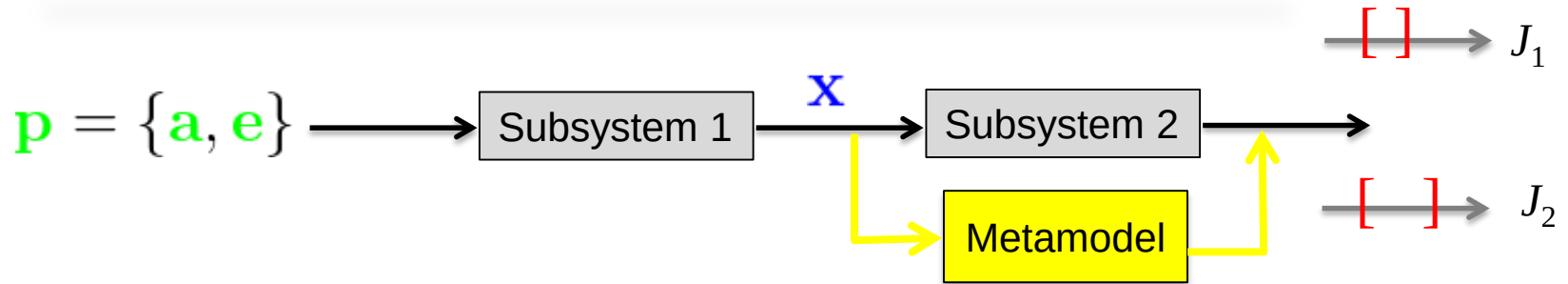
What are the dependent and independent variables?

Task 2: Global Sensitivities

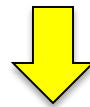


- Independent variable
 - Epistemic variable
 - Limit of the range of variation of epistemic variable
- Dependent variable
 - Area of the $\mathbf{x}1$ -pbox
 - Average over selected quantiles of the $\mathbf{x}1$ -pbox [11]

Task 2: Global Sensitivities

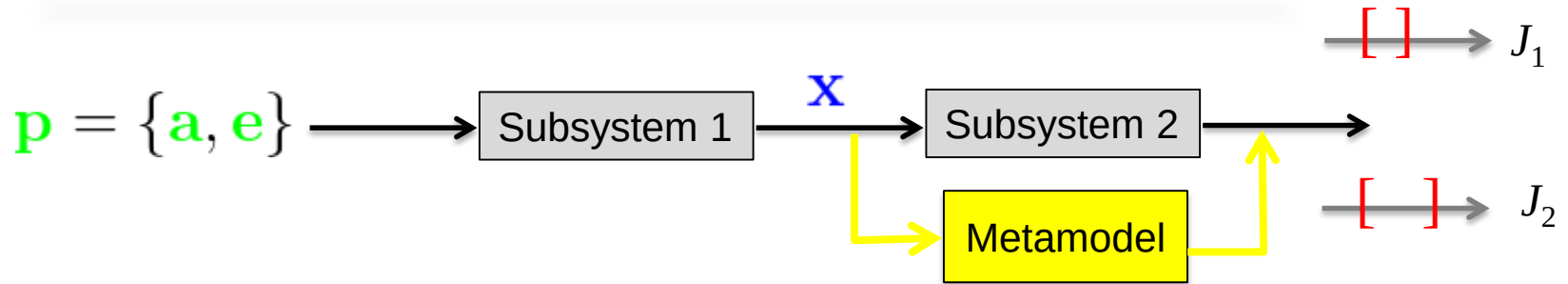


- Independent variable
 - Epistemic variable
 - Limit of the range of variation of epistemic variable
- Dependent variable
 - Area of the x_1 -pbox
 - Average over selected quantiles of the x_1 -pbox [11]
 - J_1 and J_2 after using a metamodel (PC, GP, NN) for Subsystem 2



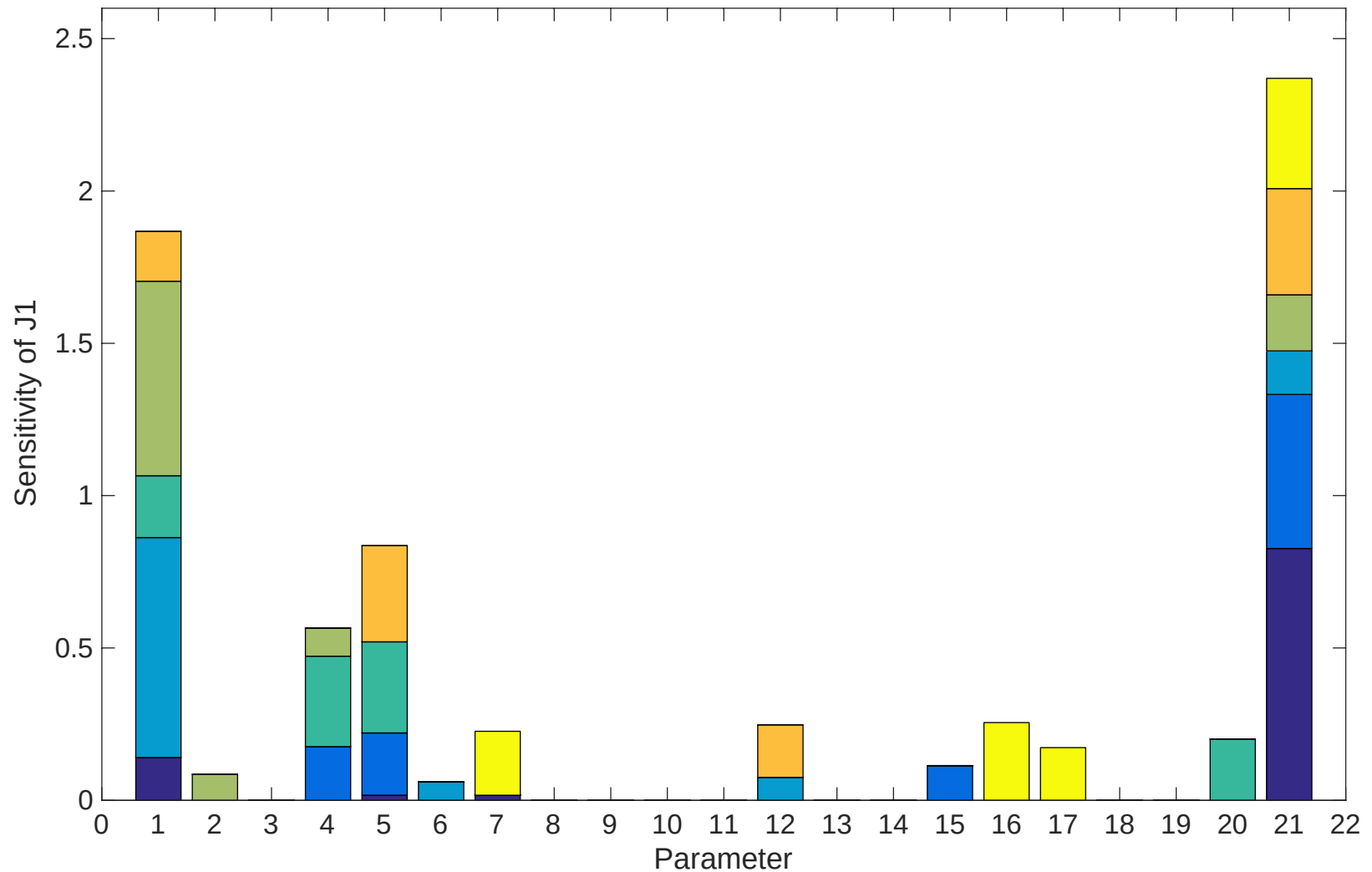
Source or approximation error

Task 2: Global Sensitivities



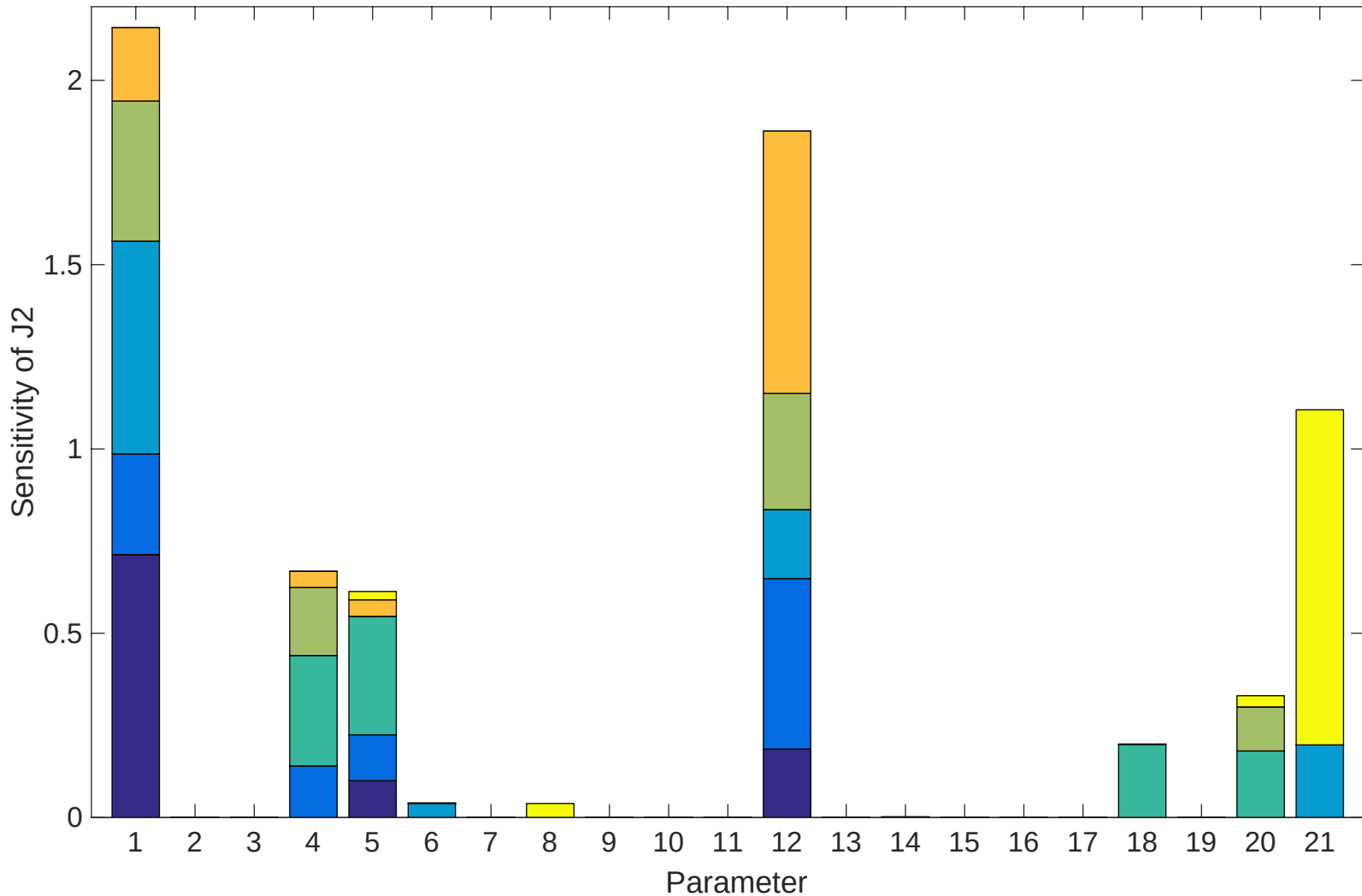
- Independent variable
 - Epistemic variable
 - Limit of the range of variation of epistemic variable
- Dependent variable
 - Area of the x -pbox
 - Average over selected quantiles of the x -pbox [11]
 - J_1 and J_2 after using a metamodel (PC, GP, NN) for Subsystem 2
- Setup: n_e : [0.5K, 5K], n_a : [1K, 5000K]

Task 2: Global Sensitivities



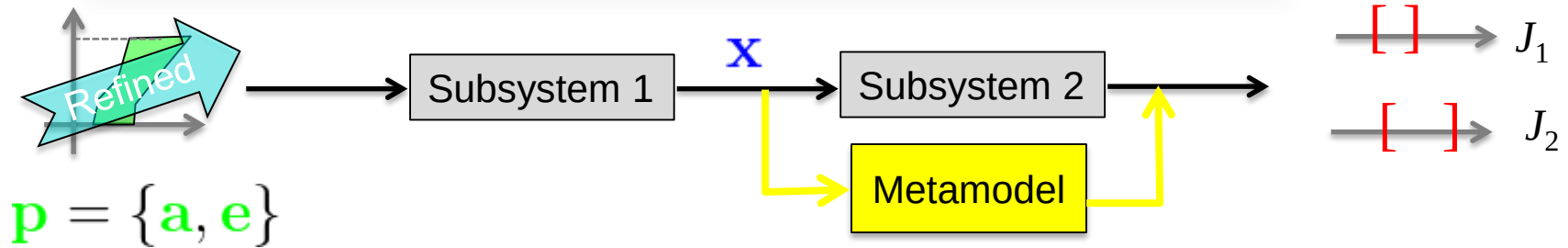
Considerable variation in FOM thus in ranking

Task 2: Global Sensitivities



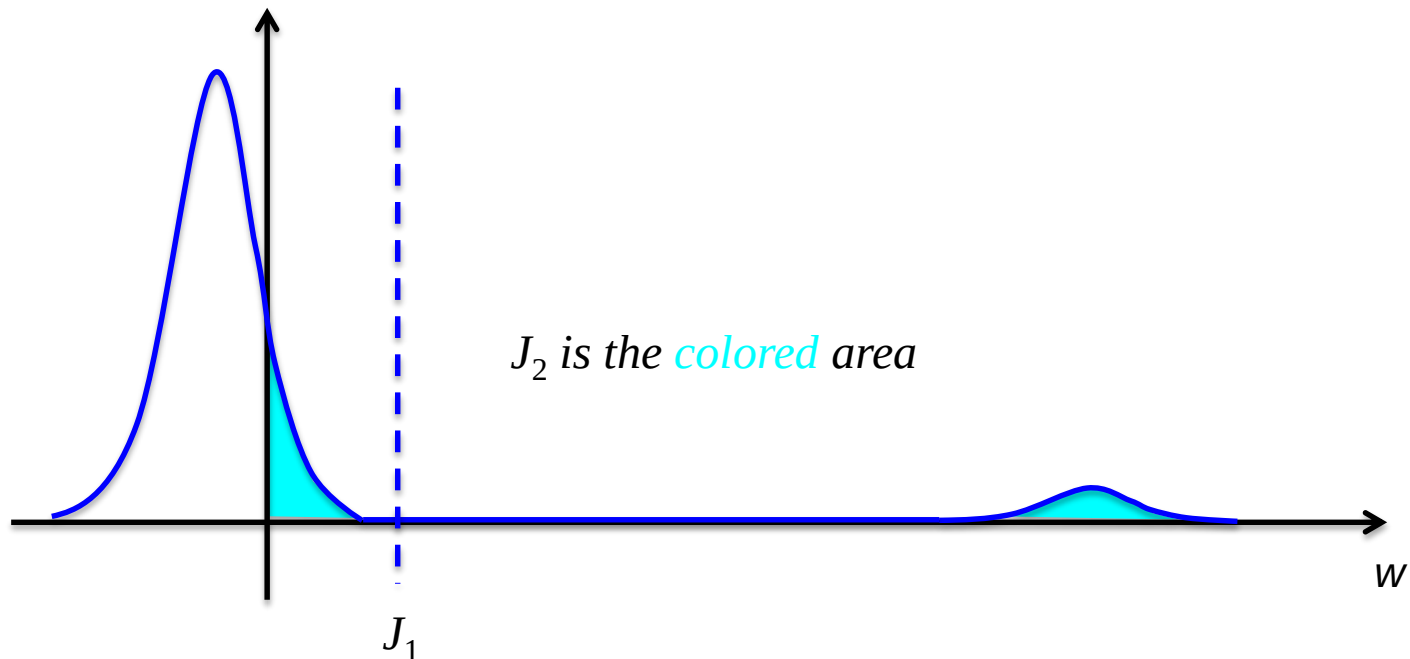
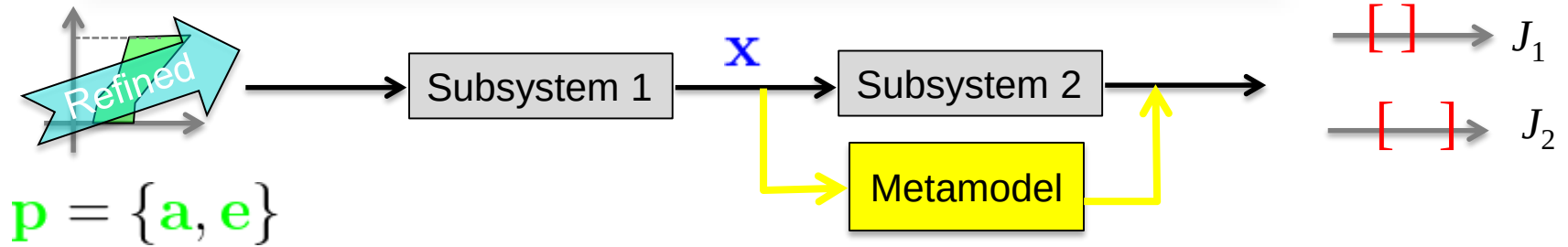
Considerable variation in FOM thus in ranking

Task 3: Uncertainty Propagation

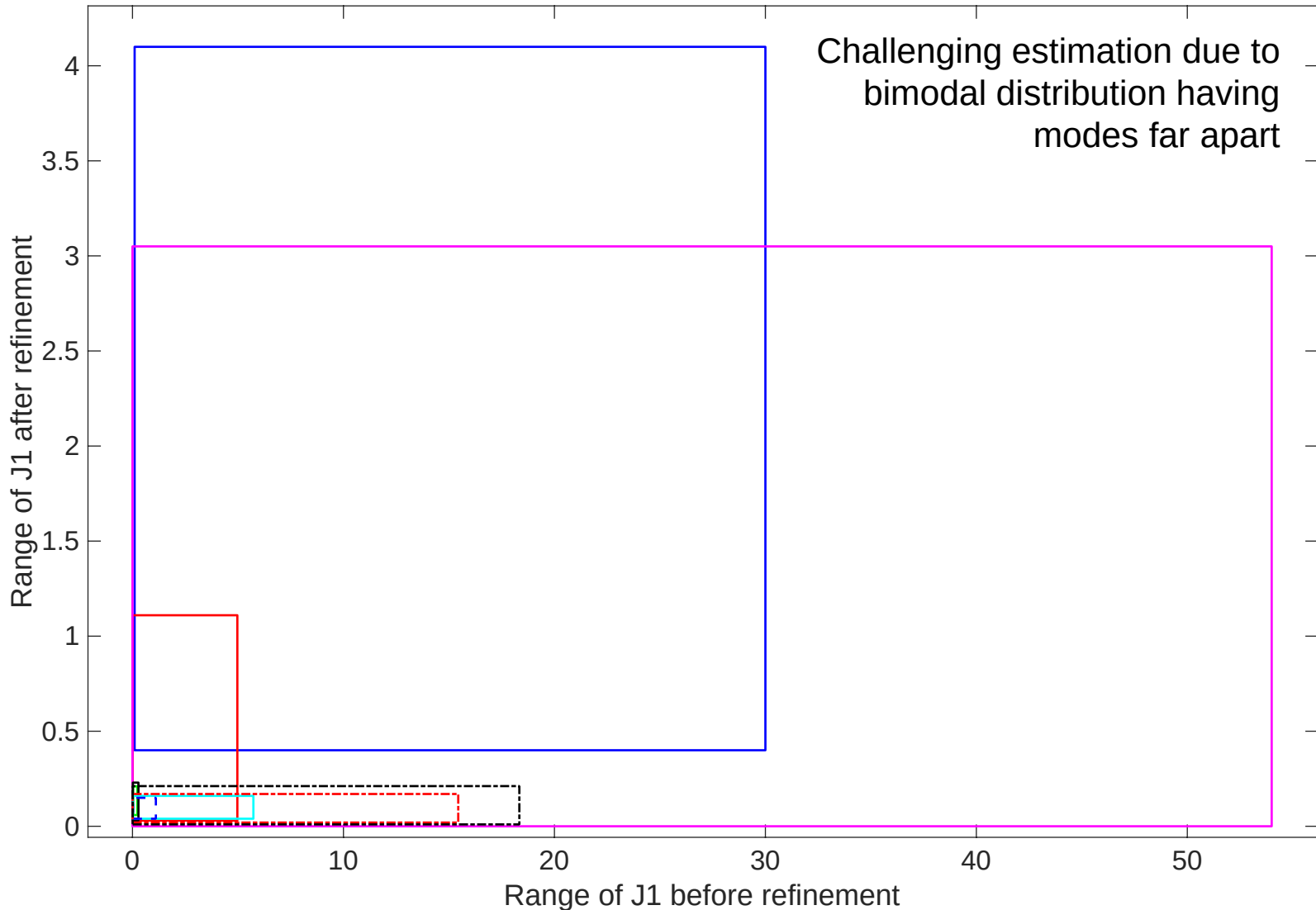


- **2-level epistemic (set) - aleatory (random)**
 - Aleatory loop: $n_a : [0.1K, 10K]$
 - Epistemic loop by sampling ($n_e : [0.5K, 5K]$) or optimization (GA)
- **2-level aleatory (random) - epistemic (focal element)**
- Results affected by methods and numerical setup used by the calibration strategy, global sensitivities, surrogate modeling, and uncertainty propagation

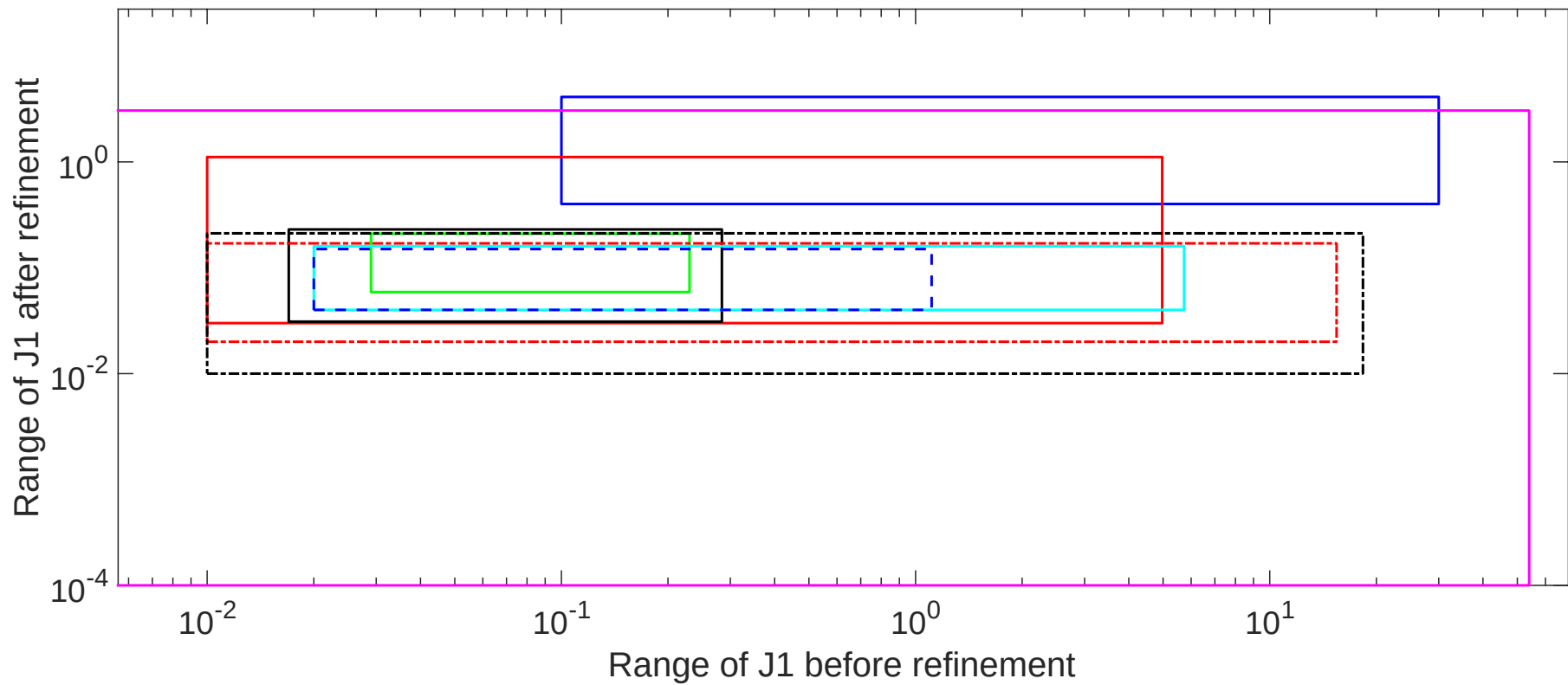
Task 3: Uncertainty Propagation



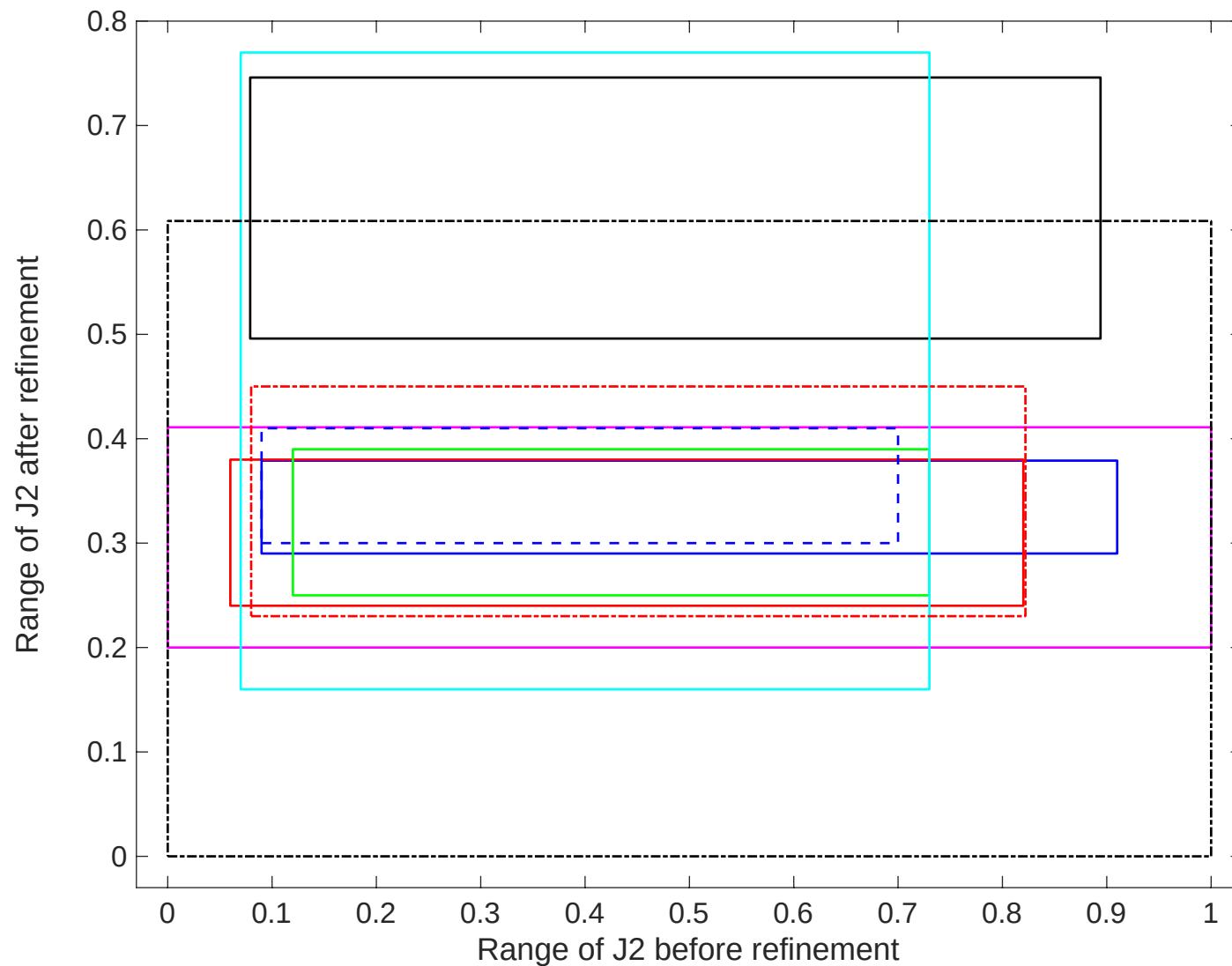
Task 3: Mean Value J_1



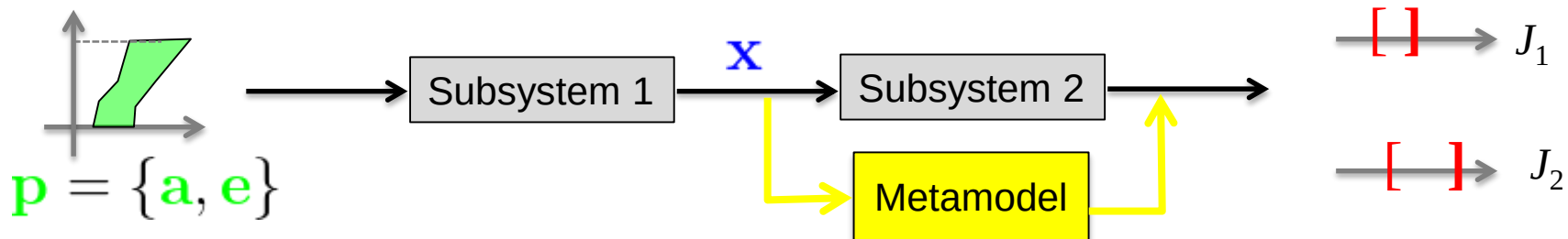
Task 3: Mean Value J_1



Task 3: Failure Probability J_2

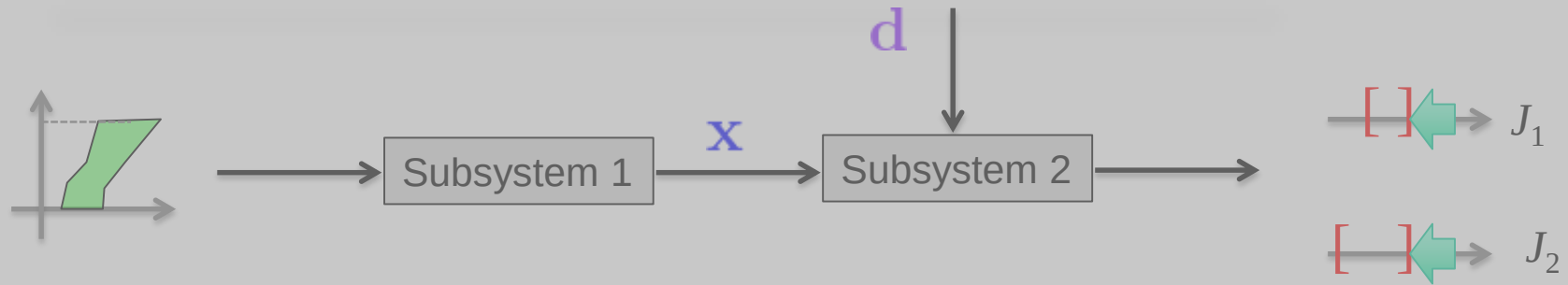


Task 4: Extreme-case Analysis



- $\dim(\mathbf{a})=21$, $\dim(\mathbf{e})=30$
- **Random sampling in epistemic space**
- **Optimization (GA, Particle swarm)**
- Byproduct of the 2-level epistemic-aleatory approach
- Both approaches often yield under-predictions of the extreme epistemic realizations
- Both approaches performances degrade with $\dim(\mathbf{e})$

Task 5: Robust Design



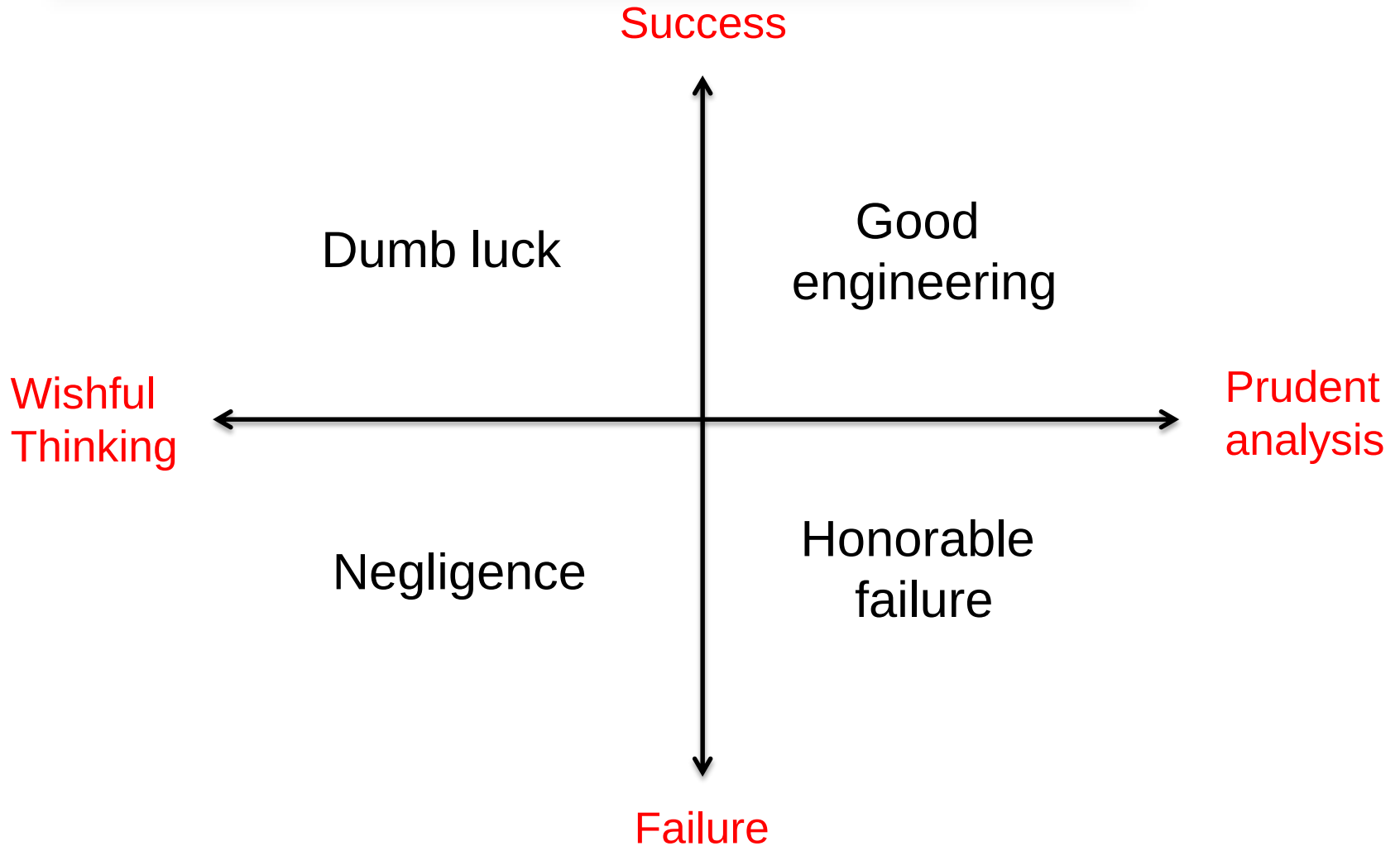
- $\dim(a)=21$, $\dim(e)=30$, $\dim(d)=14$

Technical Observations

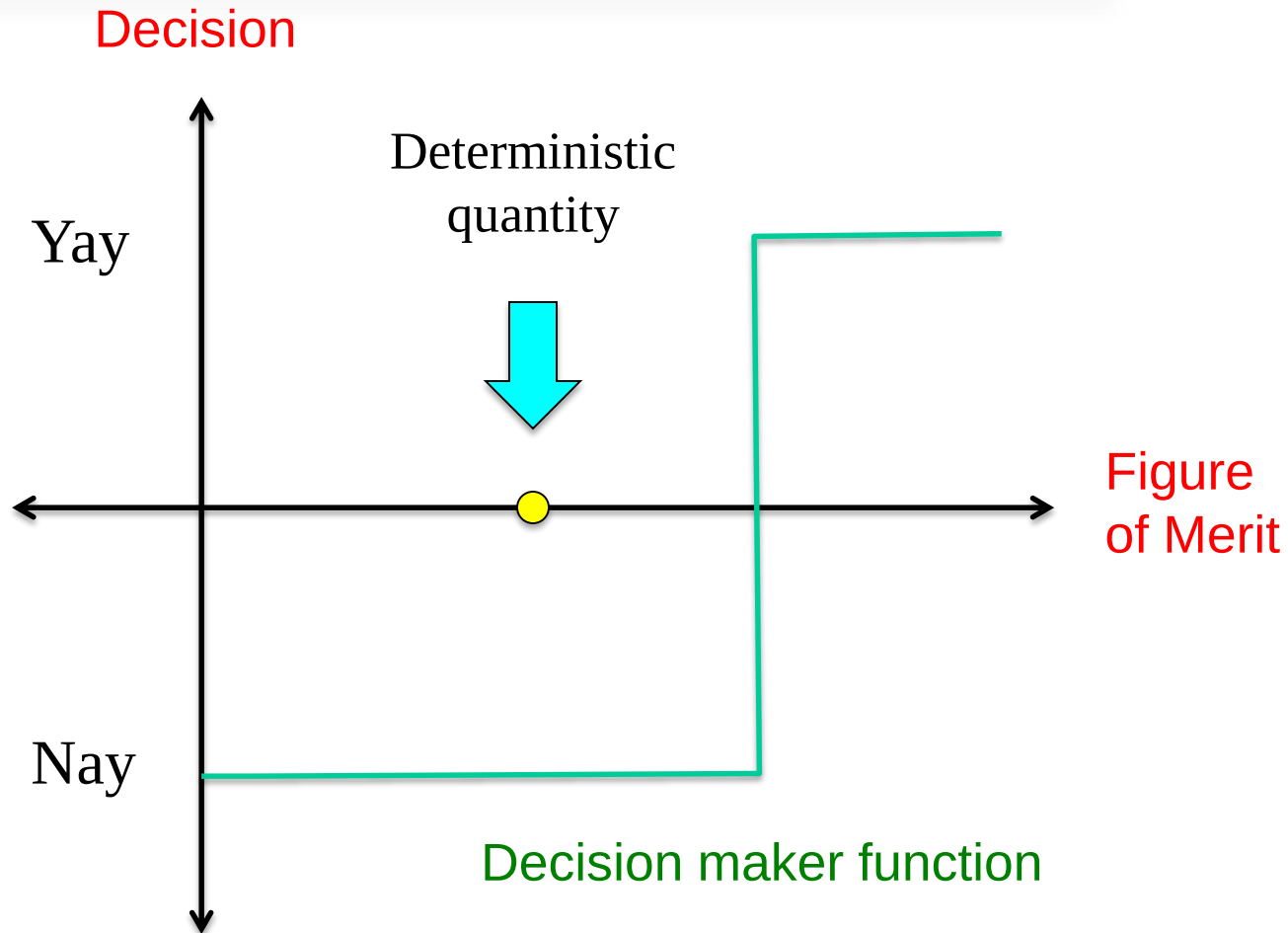


- Model Calibration
 - Do we need to account for parameter dependencies resulting from Bayesian calibration? We go from a set of intervals to a fully correlated JPDF requiring MCMC
 - Refined UM for $n=50$ $\not\subset$ the refined UM for $n=25$
- Refine epistemic uncertainties in series: do full loop
- What is the approximation error of a surrogate model?
- **Method verification**: perform convergence studies supporting the chosen numerical setup
- Multi-modality, non-linearity and large dimensions will complicate the usage of UQ methods
- Contribution to variance vs. contribution to other metric
- Are we under-predicting/over-predicting and by how much?

Conclusions

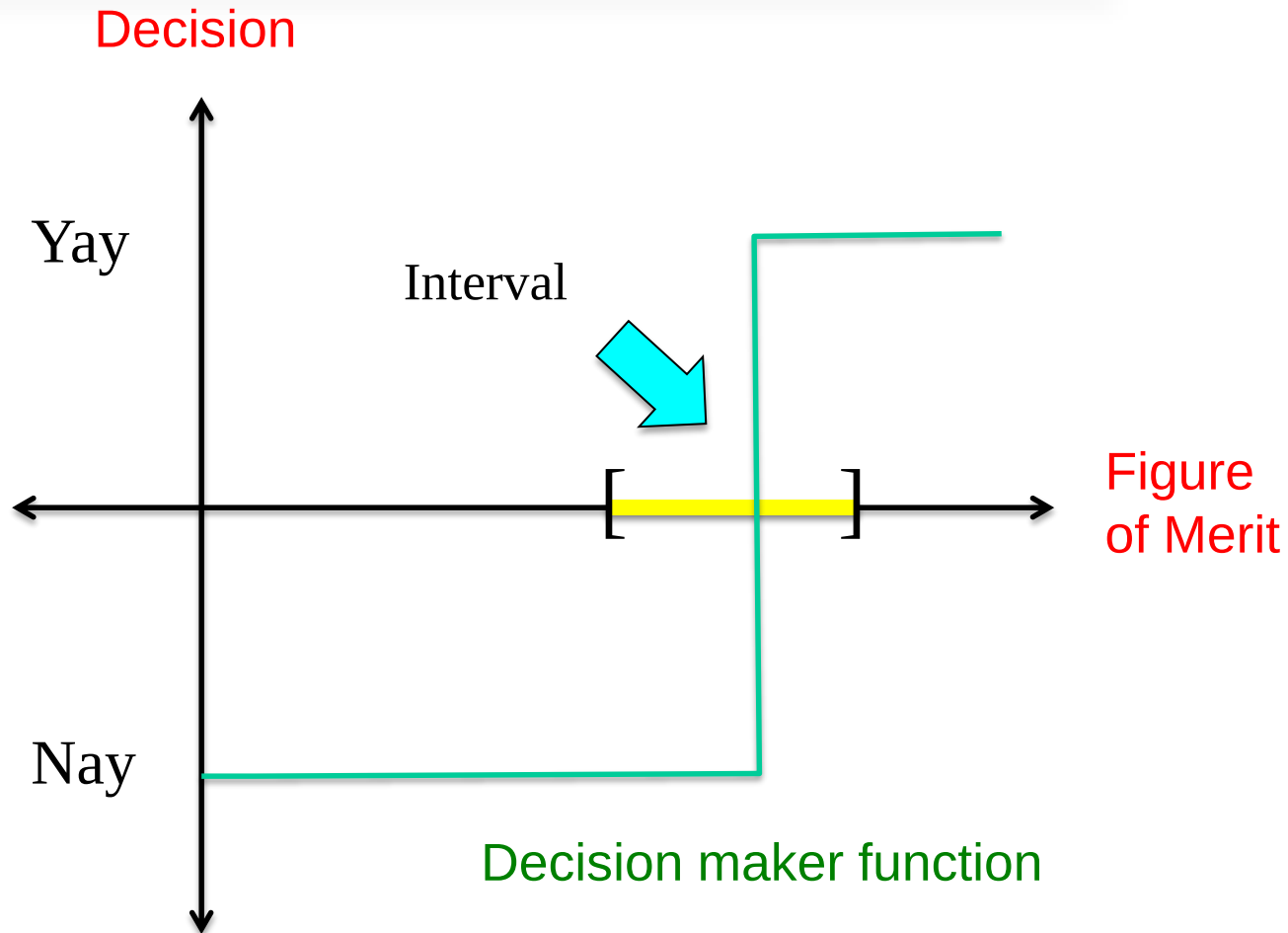


Decision Making w/o Epistemic Uncertainty

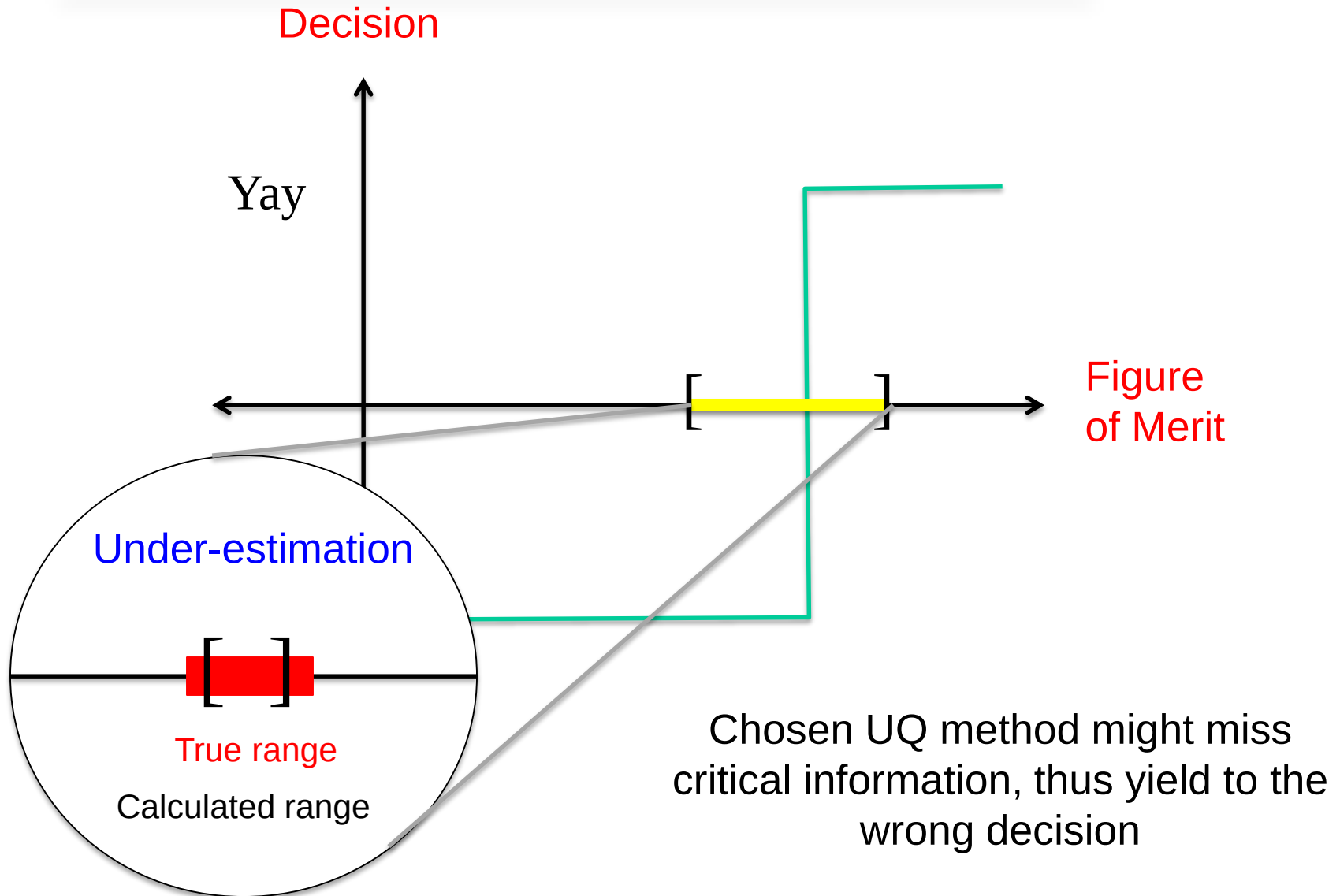


Decision maker objective vs. UQ analyst objective

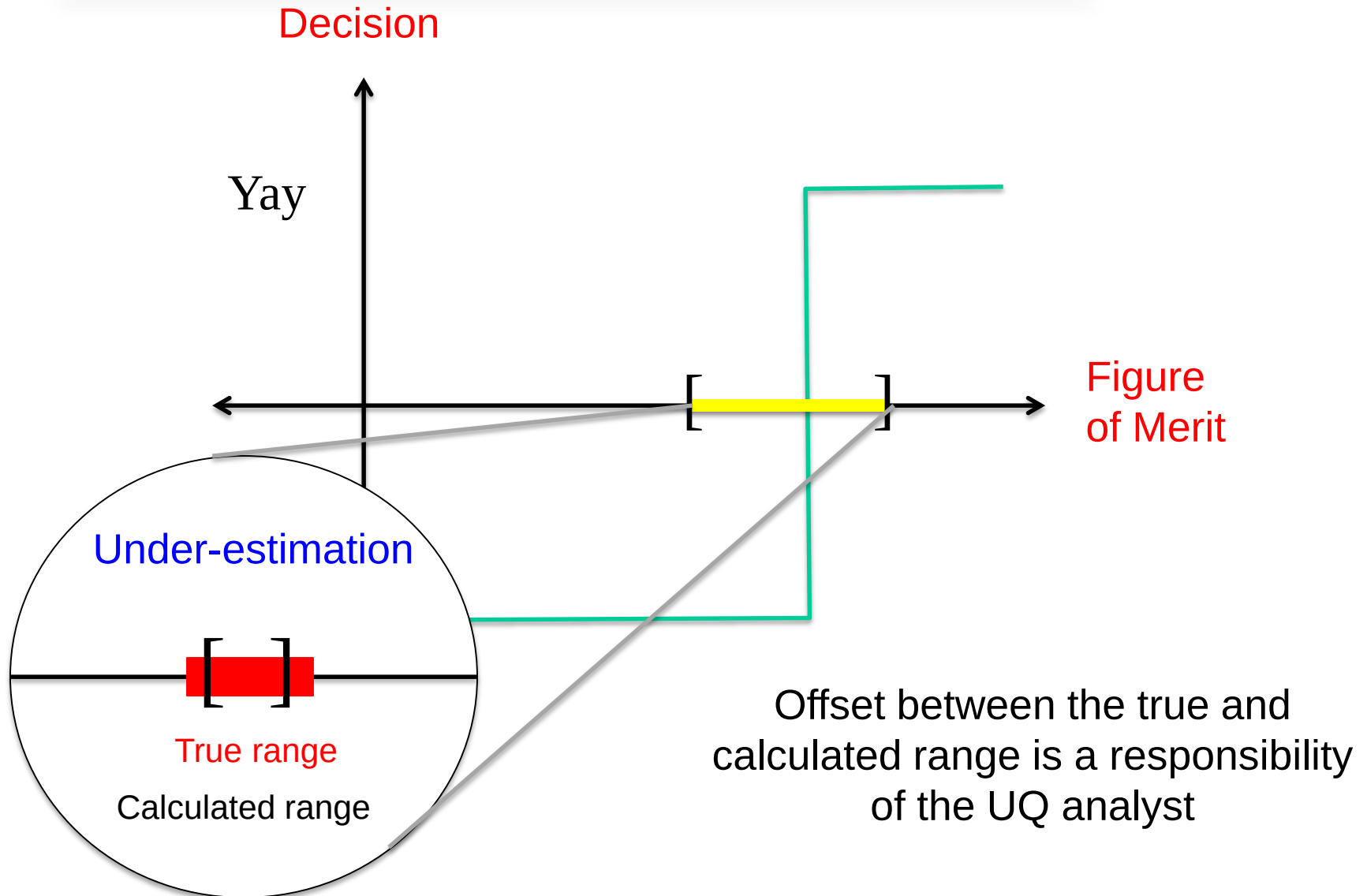
Decision Making with Epistemic Uncertainty



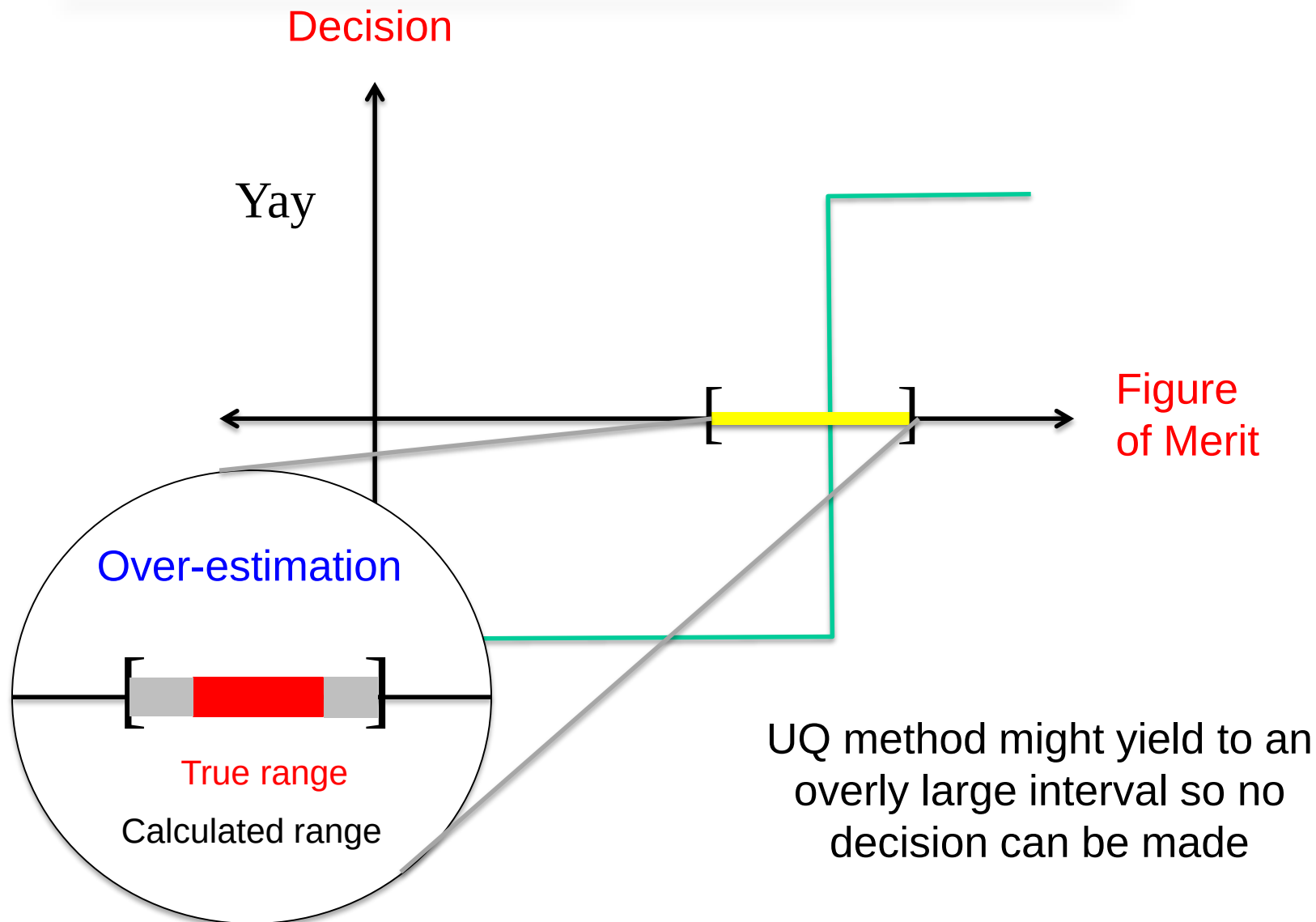
Decision Making with Epistemic Uncertainty



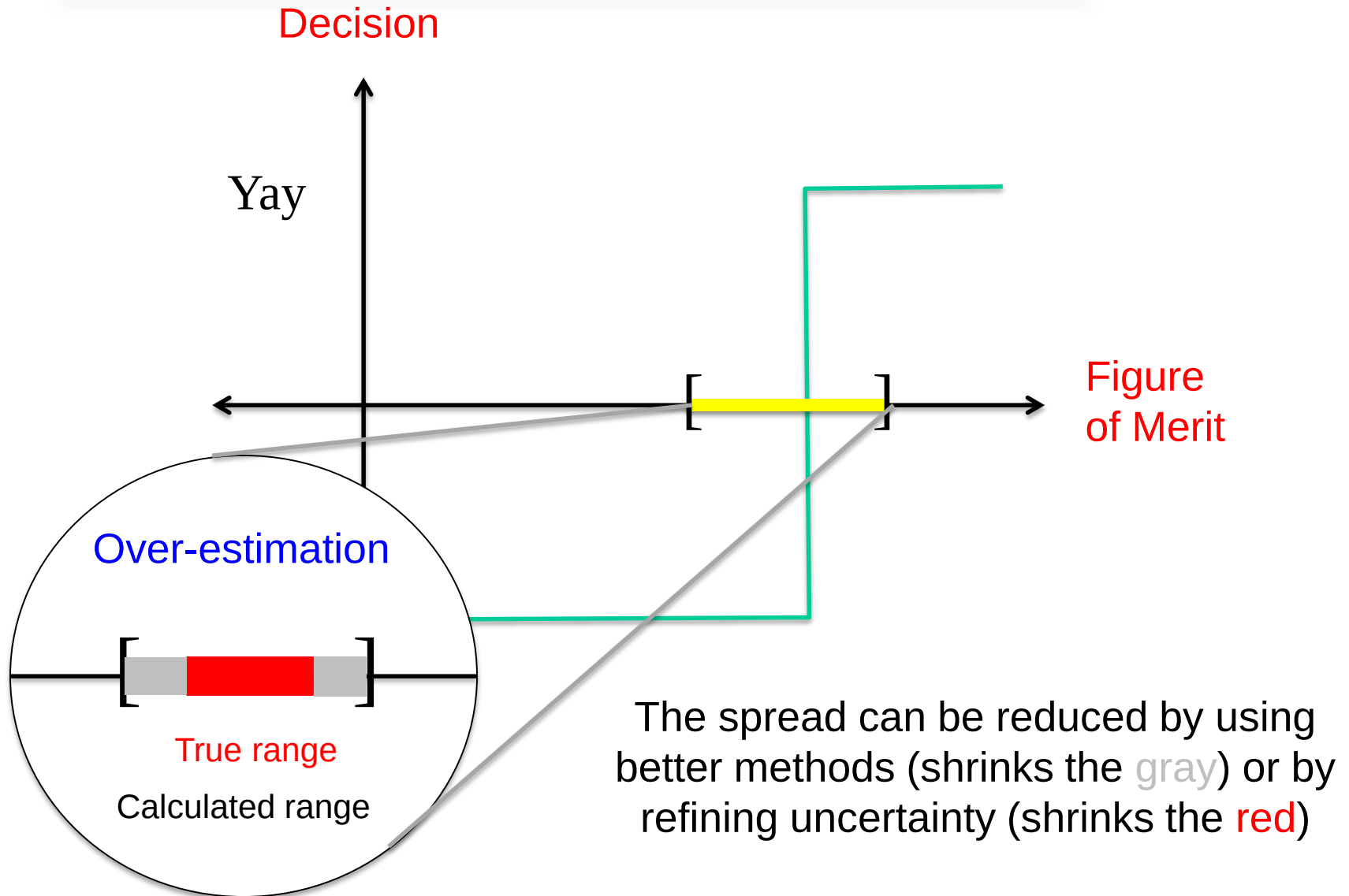
Decision Making with Epistemic Uncertainty



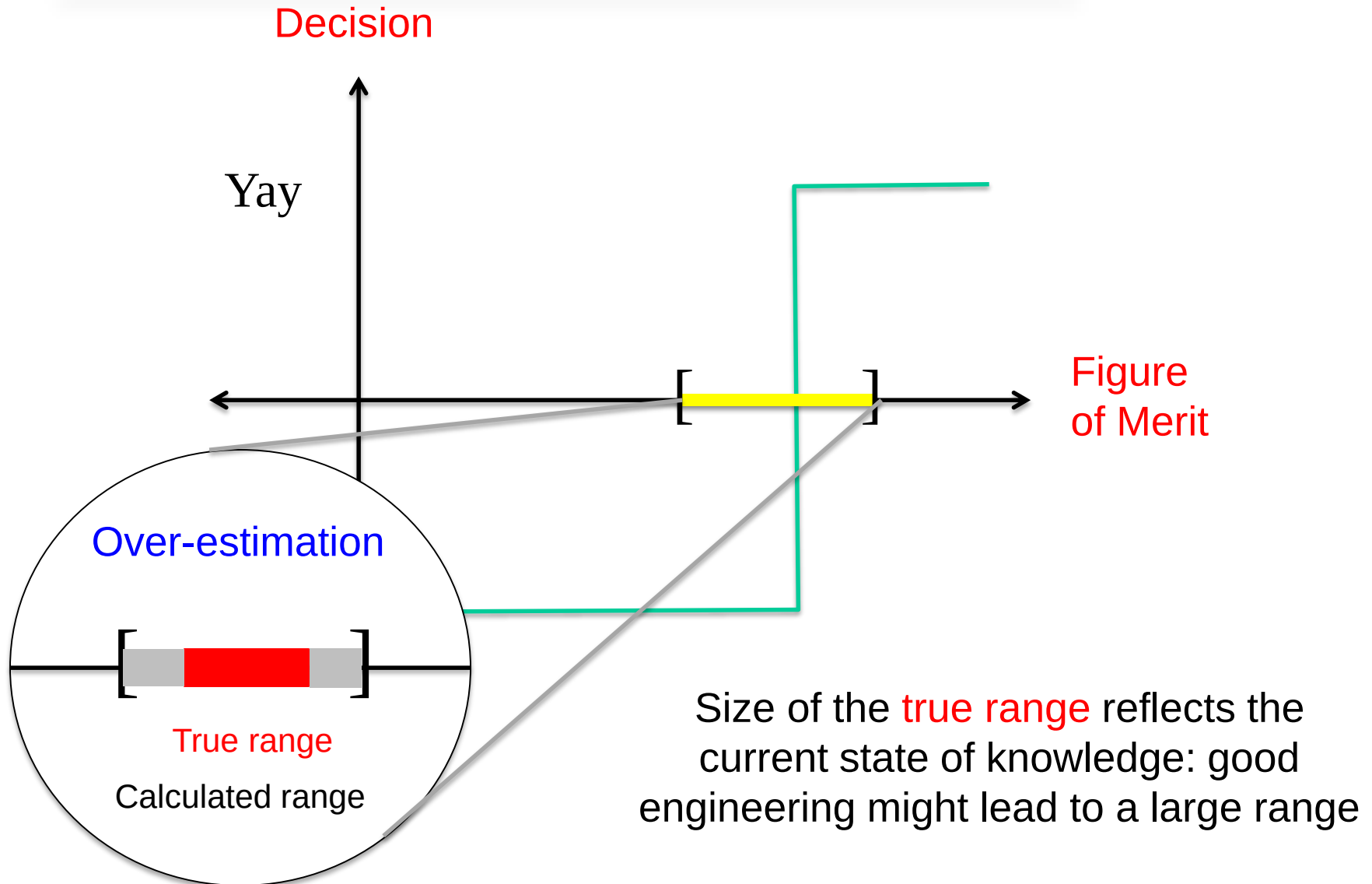
Decision Making with Epistemic Uncertainty



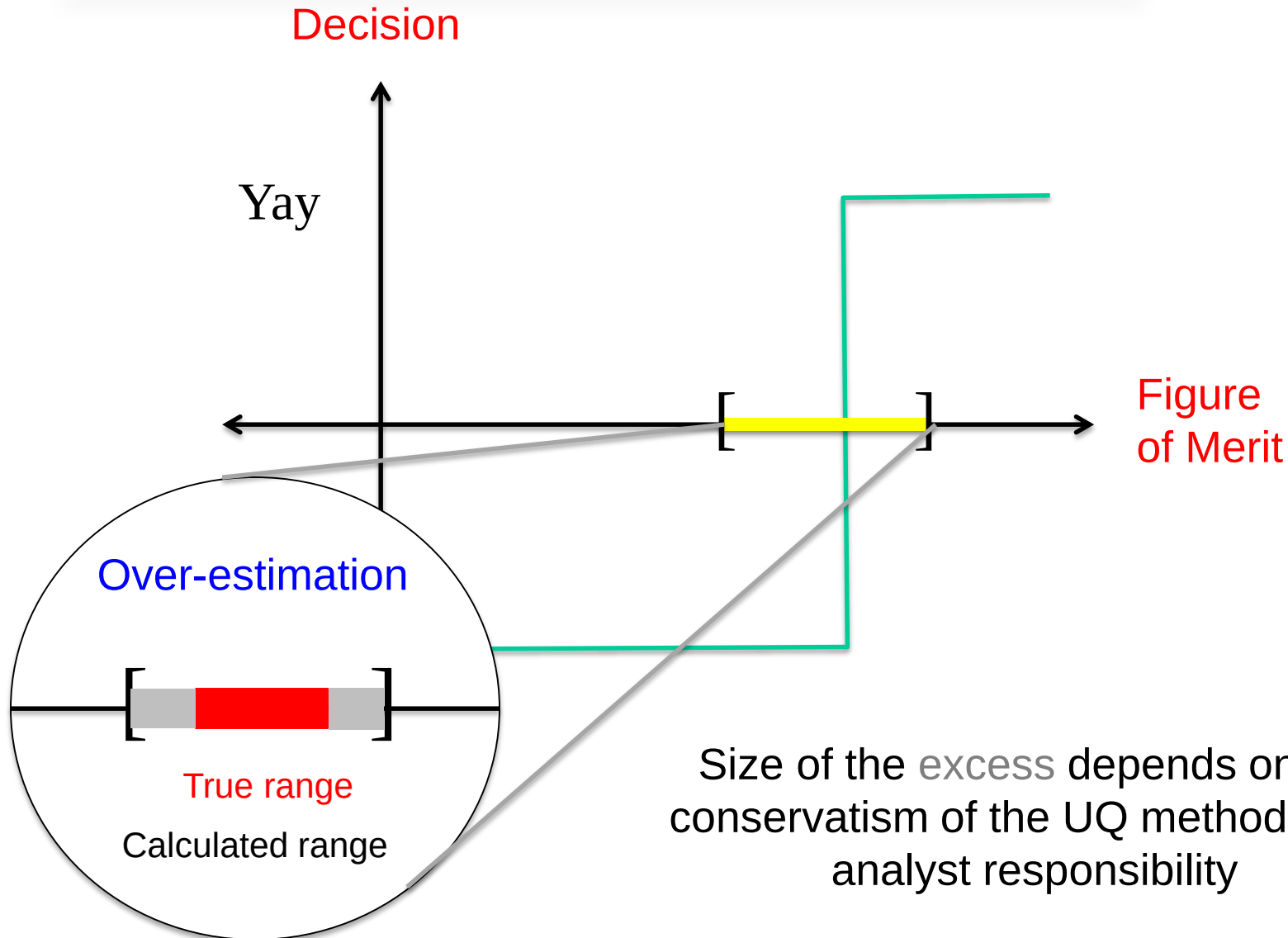
Decision Making with Epistemic Uncertainty



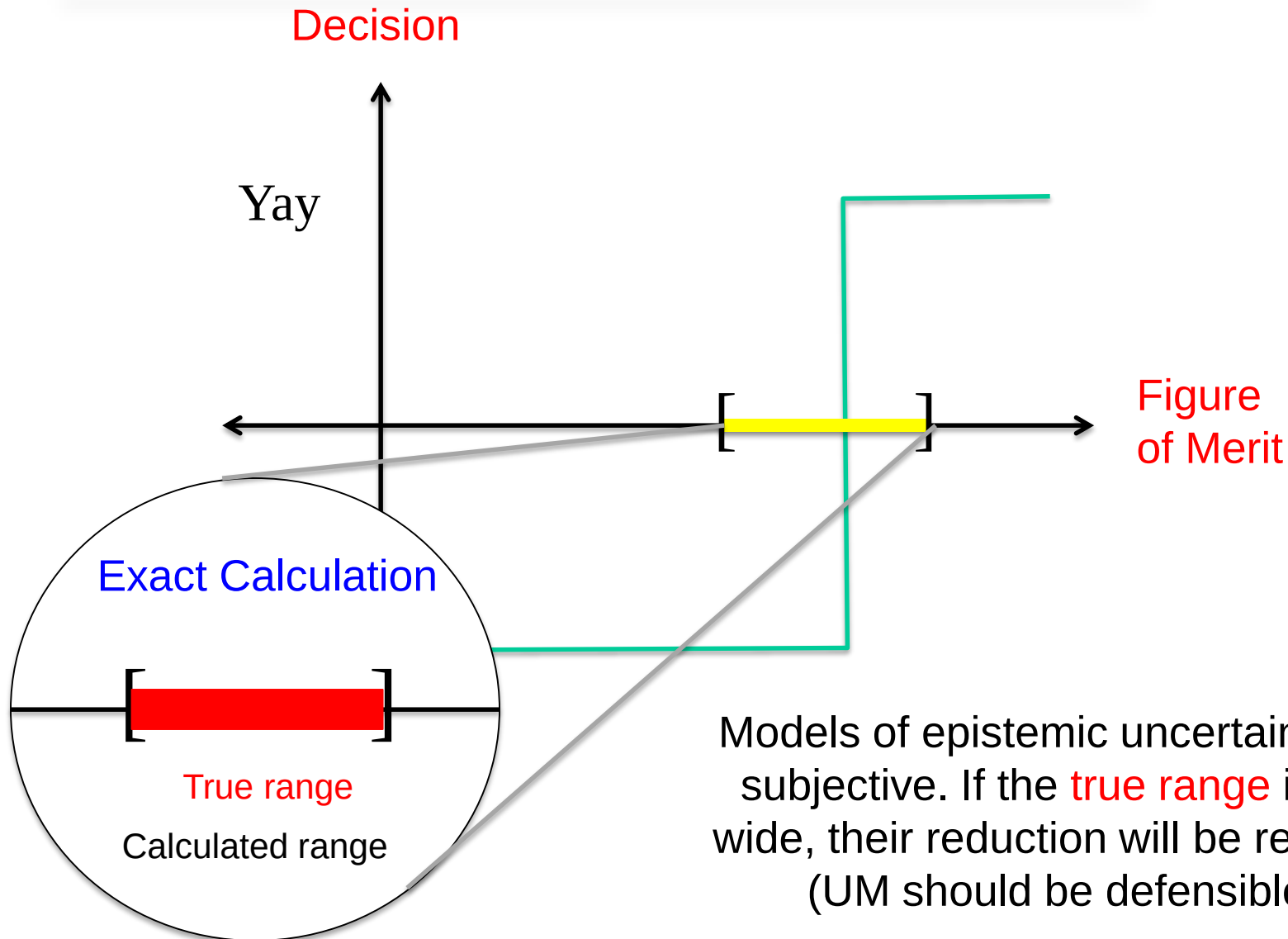
Decision Making with Epistemic Uncertainty



Decision Making with Epistemic Uncertainty



Decision Making with Epistemic Uncertainty



Conclusions



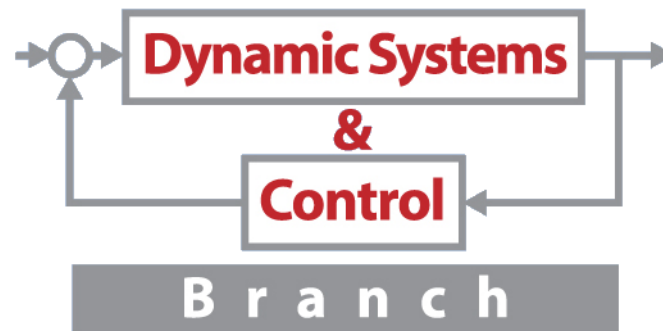
- Thanks to respondents
- Challenge was an end-to-end UQ problem replicating situations commonly found in practice
- Challenge aimed at breaking the gap between philosophy and mathematics when dealing with epistemic uncertainties
- Responses illustrate the state of the practice
- A considerable spread in the results observed
- Key UQ needs are not fully addressed by existing methods
- Further R&D needed

Conclusions



- Under-prediction and over-prediction might lead to the wrong decision/no decision
- How does UQ support the decision process?
 - State of knowledge might lead to wide predictions
 - Decision maker should accept a state of indefiniteness
 - Refinement of epistemic uncertainty & robust design enable leaving such a state
- Need for a crisp decision setting should encompass good engineering: don't claim/appear to know what you don't
- Who is liable?

The NASA Langley UQ Challenge



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NASA Langley Research Center
UQ&M SIG, London, UK

Definitions



Verification

- The process of determining that a computational model accurately represents the underlying mathematical model and its solution from the perspective of the intended uses of modeling and simulation: mathematics

Validation

- The process of determining the degree to which a model or a simulation is an accurate representation of the real world from the perspective of the intended uses of the model or simulation: physics, experiments

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	Approved: 07-11-2008 Expiration Date: 07-10-2013 Superseding NASA-STD-(I)-7009