

Tail Sizing Considerations for Wingtip Propulsor Driven Aircraft Applied to the Parallel Electric-Gas Architecture with Synergistic Utilization Scheme (PEGASUS) Concept

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This research focuses on the vertical tail sizing implications of a single propulsion system fault when using wingtip propulsors as applied to the PEGASUS concept. PEGASUS is a conceptual regional aircraft featuring wingtip propulsors, an aft boundary layer ingestion propulsor, and inboard propulsors. Using four propulsors on the wing, rather than the typical two, enables a reduction in the vertical tail size needed in the event of a single fault in the propulsion system. Achieving a reduction in vertical tail size, and its associated reduction in drag, is dependent on splitting the takeoff thrust appropriately between the inboard and wingtip propulsors. At takeoff, the wingtip propulsors can only provide a small portion of the takeoff thrust. Compared with a conventional baseline vehicle employing only inboard propulsors, the increased wetted area from the wingtip nacelles outweighs the cruise drag benefits obtained from the reduced tail size and the reduction in wingtip vortices due to the wingtip propellers. The research highlights the need for aggressive cruise drag reductions on PEGASUS, possibly through increased use of the boundary layer ingestion propulsor. By lowering the wingtip propulsor power at cruise, the propulsors would be better sized for takeoff and have smaller nacelles with lower drag.

I. Introduction

NASA's Aeronautics Systems Analysis Branch (ASAB) created the Parallel Electric-Gas Architecture with Synergistic Utilization Scheme (PEGASUS) aircraft concept in order to explore opportunities created by electrified aircraft propulsion (EAP). Antcliff et al. documented the genesis for PEGASUS in an earlier work, where the authors studied the implications of electrification on a regional aircraft [1]. The authors made several observations that would shape much of the PEGASUS concept development in the future. First, the work suggested that a smaller aircraft like the ATR 42-500 could succeed in implementing EAP given substantial improvements in battery technology. Second, Antcliff's work highlighted challenges that aircraft with multiple energy sources create for our current mission analysis tools. Though Antcliff's work showed that EAP can be competitive with advanced, conventionally fueled aircraft over short distances, EAP may be leveraged to provide additional benefits.

One of EAP's defining features is that it is largely scale invariant with regard to specific power, which allows designers to place propulsors in strategic locations to complement the airframe's aerodynamics. Initial work on other NASA programs, such as the X-57 Maxwell and Single-aisle Turbo-electric Aircraft with an Aft Boundary Layer propulsor (STARC-ABL), has indicated that wingtip and boundary layer ingestion (BLI) propulsors can be advantageous for an aircraft [2, 3]. Researchers at Delft University of Technology have shown that wingtip propellers can reduce induced drag on the order of 15% [4]. Eviation is developing Alice, an all-electric, nine passenger aircraft featuring wingtip propulsors and an aft BLI propulsor [5]. However, moving the location of thrust generation further from the centerline of the aircraft requires careful thrust balancing at takeoff to avoid increasing the yawing moment associated with a one engine inoperative (OEI) or critical loss of thrust scenario, and in turn, the vertical tail size. This paper investigates the overall effect of implementing wingtip propulsors, factoring in the reduction in induced drag coupled with the changes in viscous drag due to resizing the tail.

Antcliff and Capristan performed the first analysis on PEGASUS, shown in Fig. 1, and described the rationale behind its features and mission [6]. The PEGASUS concept features three propulsor classes: electric inboard propulsors, parallel hybrid-electric wingtip propulsors, and an electrically powered, tail-mounted BLI propulsor. Figure 2 shows these propulsor classes and their power sources graphically. The inboard propulsors operate during a small segment

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Fig. 1 Artist's rendering of PEGASUS.

of the mission but enable the wingtip propulsors to be sized for the cruise power requirements, which should further reduce energy consumption [7]. Limiting the thrust at the wingtips through utilizing inboard propulsors also reduces concerns over critical loss of thrust conditions during takeoff and landing. The inboard propulsors are intended to be used during takeoff and climb, and they fold during cruise in order to maintain a more efficient wing lift distribution. The BLI propulsor could operate at all phases of flight, but tail strike concerns may limit its application during takeoff and landing. Antcliff and Capristan's mission requirements state that PEGASUS should have a 200 nmi range when powered solely by battery and a 400 nmi range when using both fuel and battery [6]. Like the ATR 42-500, PEGASUS has a cruise speed near Mach 0.5 and a service ceiling of 25,000 ft. Operating a shorter mission on electricity alone would eliminate the generation of emissions en route.

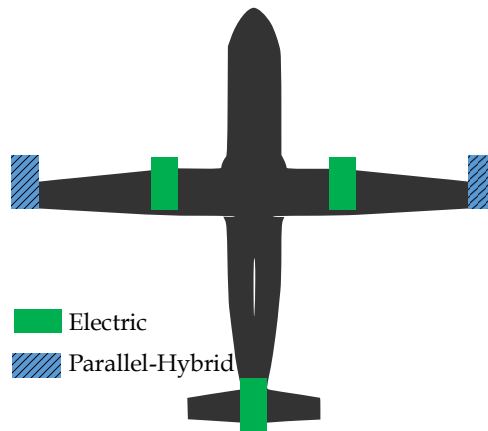


Fig. 2 PEGASUS propulsion architecture.

The Flight Optimization System (FLOPS) [8] has been the main aircraft analysis tool used in ASAB. FLOPS was designed to analyze conventional aircraft and it lacks the capability to analyze a configuration like PEGASUS that includes multiple propulsor classes and energy sources. For this reason, PEGASUS has been analyzed with the prototype of a new aircraft analysis tool called the Layered and Extensible Aircraft Performance System (LEAPS) [9]. Capristan and Blaesser previously used LEAPS to analyze PEGASUS and found that the use of multiple propulsor classes could yield a 20% savings in energy consumed per nautical mile compared with a conventional baseline [10]. That study indicated that the battery weight would be approximately 15,000 lb. Considering the conventional baseline vehicle had a maximum takeoff weight (MTOW) of 40,000 lb, the airframe would need to be larger to accommodate added volume required for such a large battery weight. In a separate work, Blaesser investigated the propulsion-airframe integration (PAI) effects of the PEGASUS propellers along the wing [11]. The key result from that effort was that

operating the inboard propulsors at cruise increased drag, but the disk loading was reduced on each propulsor class, which increased the propulsive efficiency. The net effect was a reduction in total energy consumed during cruise.

The goals of this paper are to understand the requirements for allocating thrust between the inboard and wingtip propulsors at takeoff and to understand how the takeoff thrust requirement for the wingtip propulsors compares with the thrust available when sized for cruise. Section II highlights the need for drag reduction by performing a simplified analysis of aircraft power consumption during cruise and its associated battery weight. Section III describes the requirements for sizing vertical tails in the event of an OEI scenario. The section also describes the definition of OEI in this study and the yawing moment implications for PEGASUS at takeoff. Section IV looks at the effect of hybridization on the wingtip propulsors' takeoff power generation. Using assumed power lapses for the electric and gas turbine portions of the propulsor, the takeoff power is derived from the required cruise power. Section V analyzes the cruise drag for three different configurations: a conventional airframe, a PEGASUS concept using only wingtip propulsors, and PEGASUS using inboard and wingtip propulsors at takeoff, but folding the inboard propulsors at cruise. To close the paper, Section VI summarizes the work and describes the contributions of this paper, and Section VII describes additional avenues of research on this topic.

II. Motivation

The power expended by an aircraft during level flight is the product of the vehicle's drag and velocity. In steady level flight, when the thrust equals the drag, the power produced by the propulsor is equivalently the product of thrust and velocity. Every component involved in producing thrust, from the propeller to the engine/motor to the wires and cables that carry electricity, has some inefficiency that increases the energy required to be stored compared with the energy required for flight. Within this analysis, we do not consider these losses, though they would increase the power consumption from an energy source during flight.

To understand the impact of drag reduction on battery weight, we can consider a simplified analysis looking at the impact of a single drag count (one drag count is equivalent to a C_D of 0.0001) in steady level flight. Assume that PEGASUS has a wing reference area, S_{ref} , of 590 ft² and cruises in a standard atmosphere at 20,000 ft and Mach 0.48 (a freestream velocity, V_∞ , of 295 kts). Equation 1 shows the power required to maintain steady level flight for a given drag coefficient and vehicle configuration, where q_∞ is the freestream dynamic pressure and P is power. Similarly, we can perturb Eq. 1 to find the change in power associated with a change in C_D by one drag count.

$$P = q_\infty S_{ref} C_D V_\infty \quad (1)$$

The total energy consumed is the integral of the power over the time of the mission, and the minimum required battery weight is the energy divided by the battery specific energy. For a mission range of 200 nmi (assuming the full electric mission) at 295 kts, the mission duration is 41 minutes and the total energy associated with one drag count is 4,200 W·h. This assumes that no range is covered in climb or descent, though it also ignores the energy spent climbing and in other mission phases. If the battery specific energy is 500 W·h/kg, then we can conclude each drag count would require roughly 18.5 lb of battery. The estimated cruise drag coefficient for PEGASUS is 280 drag counts, resulting in a 5,200 lb battery weight. These estimates are rough order of magnitude calculations that ignore PAI effects, sizing non-linearities, and inefficiencies in the propulsion system; however, the estimates highlight the need to reduce drag and ensure that the mission is chosen properly.

Separating thrust and drag on aircraft with tightly coupled PAI can be difficult, which is why analyzing vehicles with PAI is better accomplished through calculating power input and output [12]. However, we can still investigate the fundamental means of reducing vehicle drag, independent of PAI. Equation 2 describes the fundamental drag relationship for subsonic aircraft, where the total drag coefficient, C_D , is the sum of the viscous drag coefficient, C_{D0} — a function of Reynolds number, Re , and wetted area, S — and the induced drag coefficient, where e is the Oswald efficiency factor and C_L is the three-dimensional lift coefficient. The basic means of reducing viscous drag are to decrease the wetted area, increase the Reynolds number, or increase the amount of laminar flow. To decrease induced drag, the most common approach is to decrease the span loading, which for a fixed wing area means increasing aspect ratio, $\mathcal{R}$.

$$C_D = C_{D0}(Re, S) + \frac{C_L^2}{\pi e \mathcal{R}} \quad (2)$$

Strategic use of EAP can provide the ability to reduce the vertical tail size and drag. Throughout this analysis, PEGASUS was compared against a conventional baseline aircraft, which is modeled after the ATR 42-500. FLOPS

estimated the baseline vehicle has 232 counts of viscous drag ($C_{D0} = 0.0232$) with Fig. 3 providing the breakdown by component. Eliminating the tail would reduce the viscous drag by 12%, and though a complete elimination of the tail may not be possible due to weathervane stability issues, it still represents an attractive target. Using the above simplified analysis, eliminating the tail would provide a savings on the order of 540 lb of battery, or the ability to add three passengers to the flight. These estimates are presented as a motivation for studying the unique benefits of PEGASUS and EAP.

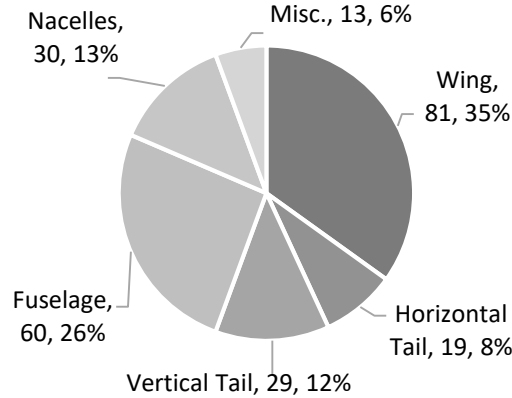


Fig. 3 C_{D0} breakdown for major components of the conventional baseline, expressed in counts and percents.

The induced drag generated by a wing is inversely related to the wing's span loading. In many commercial aircraft, the wingspan is constrained by the International Civil Aviation Organization (ICAO) and Federal Aviation Administration (FAA) airport design specifications. Boeing is using foldable wingtips on its 777X in order to achieve a longer span while conforming to existing airport infrastructure [13]. For the ATR 42-500, the wingspan is 80.4 ft, which is just larger than the Class II/B span limit of 78.8 ft, but well short of the 118 ft limit for Class III/C airports [14]. As a result, the ATR 42-500 wing has considerable room to grow before reaching the Class III/C span limit. Of course, increasing span and aspect ratio does not always mean that the entire aircraft system will benefit. Increasing aspect ratio may have an adverse effect on the vehicle in terms of wing weight, bending, and flutter.

III. Vertical Tail Sizing

Generally, the most important scenario for determining vertical tail size for multi-engine transport aircraft is the asymmetric thrust condition due to engine out at a low airspeed, where there is minimal dynamic pressure available to generate a side force with the rudder. Other major factors in tail sizing include crosswind takeoff and landing, maximum roll rate, and spin recovery [15]. These conditions were ignored for this design space exploration but could be included in future studies. The critical scenario for sizing the vertical tail was assumed to be the failure of one critical propulsion system at takeoff with the remaining operating propulsors at takeoff power. This study assumed the windmilling drag to be 25% of the takeoff thrust generated by the failed propulsor [16]. A critical propulsion system was defined as either a single propulsor, as is traditionally the case, or a collection of propulsors. For example, if a common battery bus powers multiple propulsors, the loss of that battery bus would shut down all linked propulsors and would therefore be the critical failure mode. Similarly, Federal Aviation Regulation (FAR) 14 Code of Federal Regulations (CFR) Part 25 uses the terminology of OEI and Part 23 uses "critical loss of thrust." For simplicity, this paper refers to propulsion system failures as an OEI event for any single fault, whether that be an engine, motor, or bus. FAR 14 CFR §25.149 specifies the conditions for the minimum control speed, V_{MC} , and defines the relationship between V_{MC} and reference stall speed, V_{SR} , as $V_{MC} \leq 1.13 \times V_{SR}$. In this analysis, we set $V_{MC} = 1.13 \times V_{SR}$ to calculate the dynamic pressure during an OEI event.

The traditional failure mode for propulsion systems is losing a single propulsor, as can occur with bird strikes or propulsor malfunction. For PEGASUS there are two possible scenarios: losing a wingtip propulsor or losing an inboard propulsor. Losing the aft BLI propulsor is also possible, but since it does not contribute asymmetric thrust its failure

modes were neglected in this study. If a pair of propulsors on the wing are coupled to a common power source, there are an additional four failure modes depending on the coupling. Figure 4 shows the six total failure modes that result from a single system failure on the PEGASUS concept for the two possible OEI conditions and four different common bus architectures that PEGASUS could employ. Each of these possibilities warrants further investigation into the impacts on tail sizing and thrust distribution for safety and certification.

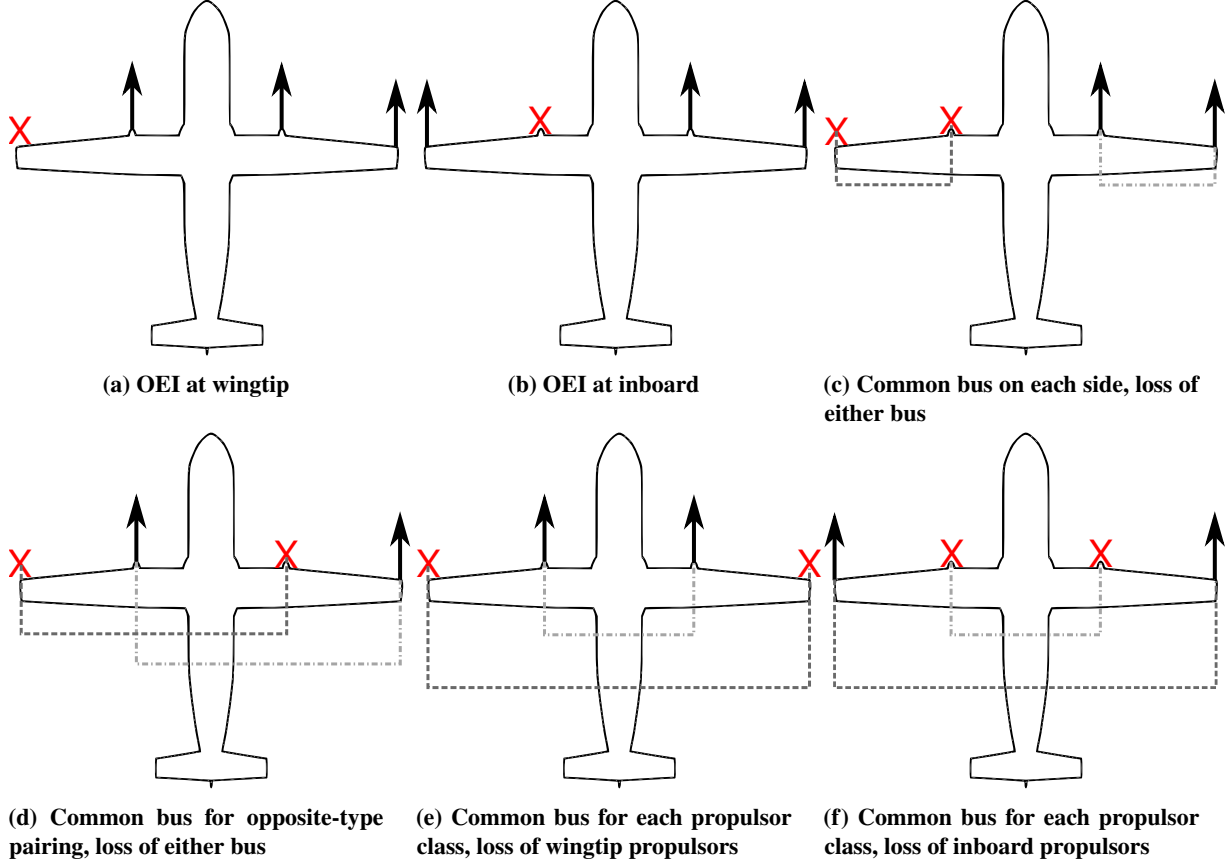


Fig. 4 Failure modes for a single loss of a propulsion system.

Vertical (and horizontal) tail sizing in the early design stages is frequently accomplished through the use of volume coefficients. Equation 3 shows the equation for the vertical tail volume coefficient, where c_{VT} is the vertical tail volume coefficient, S_{VT} is the reference area of the vertical tail, L_{VT} is the length between the quarter-chord of the wing and quarter-chord of the vertical tail, b_W is the wingspan, and S_W is the reference wing area. The conventional baseline has an estimated vertical tail volume coefficient of 0.102. As a point of comparison, Raymer notes that a typical c_{VT} for a jet transport is approximately 0.09 [15].

$$c_{VT} = \frac{S_{VT} L_{VT}}{b_W S_W} \quad (3)$$

A. Thrust Generation

Turboshaft engines and electric motors are rated in terms of power output rather than thrust. In order for these systems to develop thrust, they must be attached to a propeller, which absorbs the shaft power and generates thrust. A propeller's efficiency is a measure of how well it converts shaft power into flow power, which is the product of the freestream velocity and thrust generated. Propeller design aims to maximize propeller efficiency across the operating map of the aircraft and engine. Performing detailed propeller design is out of the scope of this research, thus the propellers for the conventional baseline and PEGASUS use a constant propeller efficiency of 80% for all phases of flight. This is a justifiable assumption because an 80% efficient propeller represents a balanced design for takeoff, top of climb,

and cruise performance. The conventional baseline and PEGASUS employ constant speed propellers, which enables a near constant propeller efficiency across a range of advance ratios [17]. The assumption may be to the detriment of PEGASUS, as the wingtip propellers could be designed for cruise without regard for adequate low speed thrust generation [18]. The result would be a higher cruise propeller efficiency and a lower takeoff propeller efficiency. The primary benefit of assuming a constant propeller efficiency across the vehicles and propellers is that thrust and power can be easily interchanged.

The conventional baseline vehicle has two 2400 shp engines, which equates to approximately 8900 lb of total thrust at takeoff. For simplicity, we assumed that changes in thrust distribution do not affect the takeoff performance. This assumption implies no PAI effects during takeoff, which is counterintuitive for a vehicle that relies on PAI for its design. However, at takeoff, the drag coefficient varies considerably due to the characteristics of the high lift system. Changes in flap and takeoff settings such as roll rate and takeoff speed have a stronger impact on the lift and drag than changes in thrust distribution. Using this assumption provided us with the option of selecting the thrust split across the inboard and wingtip propulsors and evaluating the impact on the vertical tail size.

B. Approach to Estimate the Vertical Tail Area

Detailed vertical tail sizing requires accurately computing an aircraft's stability and control derivatives and then solving the yaw, roll, and pitch equations of motion. In the current study, the yawing moment required from the tail was calculated as the resultant moment due to the thrust of the operating propulsors and the drag of the windmilling propeller of the failed propulsor. The tail was sized such that the yawing moment generated by the fully deflected rudder is equal to the yawing moment required. This analysis used a simplified method by performing a static analysis assuming zero sideslip and bank angle. The analysis employed the approximations by Nicolosi et al. to determine the change in yawing moment coefficient per rudder deflection, $C_{N_{\delta_r}}$ [19]. Equation 4 shows Nicolosi's equation to estimate $C_{N_{\delta_r}}$, where $C_{L_{a_v}}$ is the lift curve slope of the vertical tail, K_{δ_r} is a scaling factor based on tail geometry (body-mounted or T-tail), and τ is the rudder effectiveness. We estimated the lift curve slope of the vertical tail using values presented in Roskam [20] for very low Mach number. Nicolosi performed computational fluid dynamics (CFD) and wind tunnel tests on a generic commuter aircraft to estimate a relationship between rudder effectiveness and rudder deflection angle. Based on Nicolosi's results, τ was estimated at a constant value of 0.75.

$$C_{N_{\delta_r}} = C_{L_{a_v}} K_{\delta_r} \tau c_{VT} \quad (4)$$

The maximum deflection of the rudder is set to 25° , as this is common for this class of aircraft. The conventional baseline thrust was estimated at 4450 lb for each engine and located 13.5 ft from the aircraft centerline. This approach predicted a vertical tail area of 149 ft² whereas scaled drawings of the ATR 42-500 have a reference area of 137 ft². Given the different methodologies (calculated versus measured), the vertical tail areas agree well.

C. Single Propulsor Failure

Beginning with the single propulsor failure cases, Fig. 4a and Fig. 4b, the trade of thrust split versus the resulting OEI moment is shown in Fig. 5. Having all of the takeoff thrust coming from the inboard propulsor, which is the far right option in Fig. 5, is the conventional baseline configuration and results in the same vertical tail size. Conversely, the farthest left position corresponds to using only the wingtip propulsors for takeoff. The slope of each line is a function of the distance between the propulsor and the aircraft centerline. As such, if the wingspan of the vehicle were increased, the wingtip propulsor's OEI moment chart would have an even steeper line.

A distinct feature of Fig. 5 is the intersection of the two lines. This represents where the OEI yawing moment is the same for both propulsors and is the smallest OEI yawing moment possible. An inboard thrust fraction greater than the intersection implies the inboard propulsor is the critical propulsor, and any inboard thrust fraction less than the intersection leads to the wingtip propulsor being the critical propulsor. By solving a simple engineering statics problem, the thrust split that leads to the smallest OEI moment is shown in Eq. 5, where $\%T_{IB}$ is the percentage of thrust carried on the inboard propulsor, η_T is the dimensionless spanwise position of the outboard (wingtip) propulsor, and η_{IB} is the dimensionless spanwise position of the inboard propulsor. Solving Eq. 5 for the PEGASUS configuration results in 75% of the thrust coming from the inboard propulsors and the remaining 25% coming from the wingtip propulsors.

$$\%T_{IB} = \frac{\eta_T}{\eta_T + \eta_{IB}} \quad (5)$$

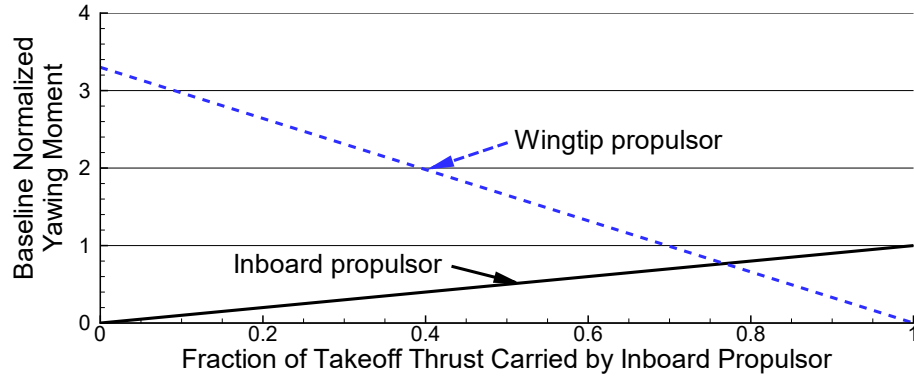


Fig. 5 Moment caused by an OEI scenario involving a single propulsor at various thrust splits.

If the propulsors are equally critical and the vertical tail's maximum yawing moment is linearly related to tail area, then the tail size can be reduced from the conventional baseline by a percentage equal to the percent thrust carried by the wingtips. For this scenario, the smallest tail that could be achieved is 25% smaller than the conventional baseline's tail. One should also note that if the wingspan were to increase, the intersection point would be at a larger yawing moment value, implying an increase in wingspan leads to a larger tail required. Figure 6 shows the normalized vertical tail area for three separate configurations: the conventional baseline, a configuration where the takeoff thrust is ideally split between inboard and wingtip propulsors, and a configuration that only has wingtip propulsors. The conventional baseline has the same vertical tail area, regardless of wingspan. The ideal thrust split configuration has a smaller tail than the conventional baseline, regardless of wingspan, though the tail approaches the conventional baseline as the wingspan increases. The wingtip propulsor only configuration requires a very large tail to counter to considerable moments generated from losing a single propulsor at takeoff. We should note that throughout this investigation, the tail size was used to calculate viscous drag, but increases in weight, and associated induced drag, are not included.

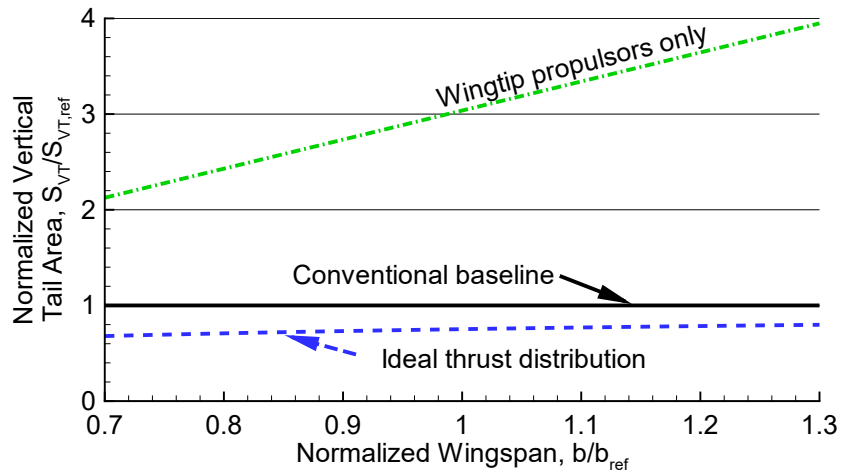


Fig. 6 Vertical tail area to balance the yawing moment generated at takeoff using the ideal thrust fraction.

D. Common Bus Failure

If the propulsors share a battery bus or some other common system, then the possibility of losing two propulsors via a single failure exists. As noted earlier, this scenario is called an OEI condition despite two “engines” being inoperative. The four combinations of connecting two propulsors were shown in Fig. 4c–4f. In determining the moment generated by a system failure, it is apparent that Fig. 4c would result in the largest moment of all the combinations and this moment would be larger than that of the conventional baseline. For this reason, pairing propulsors on the same side of the vehicle

to a common bus should be avoided. Pairing propulsors of the same type, such as in Fig. 4e and 4f, is attractive because it would result in no asymmetric thrust and thus no yawing moment for the tail to counter. Having no yawing moment to counter does not necessarily mean that the vertical tail size could be reduced. If the tail size were reduced, then in the event of a single propulsor failure, as described in Sec. III.C, the pilot would have to throttle back, or perhaps completely turn off, the opposite propulsor. Typically in OEI scenarios, aircraft are constrained by minimum climb gradients and decreasing thrust from the operating propulsor, or propulsors, is not an option. However, with the number of propulsors on PEGASUS, it is feasible that advanced software architecture and fault detection could enable safe operation with like-paired propulsors, provided the remaining propulsors provide sufficient thrust.

Pairing one inboard propulsor to the opposite tip propulsor requires a more detailed investigation. Figure 7 shows the yawing moment contribution from losing an inboard propulsor and the opposite wingtip propulsor. Within Fig. 7, the sign convention of the yawing moment is such that the loss of the tip propulsor produces a positive moment and the loss of the opposite side inboard produces a negative moment. The tail size is dictated by the absolute value of the combined moment, which is also shown. Using this architecture means that PEGASUS could have a wide range of thrust fractions between propulsors and still maintain a smaller vertical tail size than the conventional baseline. Whereas for a single OEI the conventional baseline tail size supports an inboard thrust fraction range from 0.67 to 1.0, the opposite-type pairing supports an inboard thrust fraction from 0.51 to 1.0. If PEGASUS used an inboard thrust fraction between 0.51 and 0.67, the loss of a wingtip propulsor would require advanced software to adjust the thrust distribution across the working propulsors in order to limit the yawing moment. The most significant advantage is that when the moment contribution from the wingtip and inboard propulsor are equal in magnitude, the aircraft would have no net yawing moment during an OEI event. Similar to the discussion on losing a single propulsor, the pair of propulsors generate an equal moment when the thrust is split per Eq. 5.

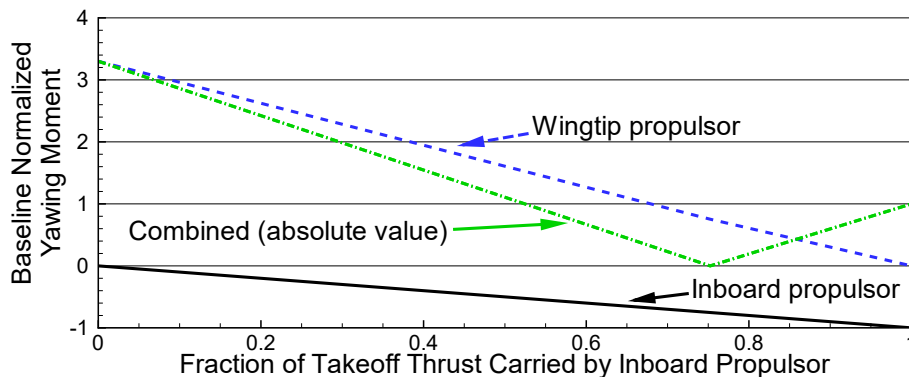


Fig. 7 Moment caused by losing a wingtip and inboard propulsor at various thrust splits.

Given this analysis, there are two options for pairing propulsors that result in no OEI yawing moment to the aircraft, and one of those shares the same takeoff thrust split as is ideal for losing a single propulsor. We noted earlier that splitting the thrust per Eq. 5 would result in a 25% smaller tail for PEGASUS (with certain assumptions); however, aircraft still require a vertical tail without the risk of OEI yawing moments. Vertical tails are generally sized by OEI conditions, but still must be sufficiently sized to enable cross wind landings and provide yaw dampening during flight. Raymer indicates the typical vertical tail volume coefficients for a glider and single engine, general aviation aircraft are 0.02 and 0.04, respectively, despite these aircraft not having OEI yawing moment constraints [15]. These vertical tail volume coefficients are roughly half to one quarter the size of the conventional baseline ($c_{VT} = 0.102$). Thus the PEGASUS architecture could reduce the vertical tail size from 25% of the conventional baseline if the thrust split is ideal, to up to 75% if the yawing moment is eliminated entirely and the tail is sized using general aviation guidelines.

IV. Available Thrust from the Wingtip Propulsors

The previous section noted that the ideal thrust split at takeoff occurs when the majority of the takeoff thrust is generated by the inboard propulsors. The original design goal of PEGASUS was to enable “right sizing” the wingtip propulsors for cruise and using the inboard propulsors to augment thrust at takeoff and top of climb. Given this idea, one must determine how much thrust the wingtip propulsors are able to generate at takeoff given cruise thrust requirements.

At cruise, the aircraft requires a certain power to maintain steady level flight, P_{cr} . Using the PEGASUS concept of operations (ConOps), the parallel hybrid-electric wingtip propulsors and fully electric aft BLI propulsor provide all the cruise power required. Within the wingtip propulsion system, the fraction of power provided by the gas turbine portion of the engine is κ . Thus Eq. 6 shows the power provided by the two propulsion sources within the wingtip propulsors, where P_{wt} is the total power delivered by the wingtip propulsors. Both the gas turbine and electric motor will have a lapse rate with altitude, α_{GT} and α_E , respectively, where the lapse rate α is defined in Eq. 7 [21]. The lapse for an electric motor is negligible, but between takeoff conditions and cruise conditions for PEGASUS, the gas turbine lapse is approximately 0.5 [22, 23]. By combining Eqs. 6 and 7, we can create an equation for the takeoff power generated, P_{TO} , in terms of wingtip propulsor cruise power in Eq. 8.

$$P_{wt} = \kappa P_{cr} + (1 - \kappa) P_{cr} \quad (6)$$

$$\alpha = \frac{P_{cr}}{P_{SLs}} \quad (7)$$

$$P_{TO} = P_{wt} \left(\frac{\kappa}{\alpha_{GT}} + \frac{(1 - \kappa)}{\alpha_E} \right) \quad (8)$$

Using 4,800 shp as the baseline, we can compare the power available from the wingtip propulsors against what is required at the ideal thrust split. Figure 8 plots Eq. 8 at three separate cruise drag coefficients, $C_D = 0.0280, 0.0240$, and 0.0200 . These drag coefficients serve to show a baseline, as well as the impact of drag reductions on the vehicle through aerodynamic improvements or increased aft BLI contribution. The left side of Fig. 8 shows when the wingtip propulsor is only powered by an electric motor. The right side shows the performance if the wingtip were powered solely by a gas turbine. The solid lines show the full takeoff power available if the electric motor and gas turbine are used. When using the maximum available power, the propulsor generates more power at sea level static conditions than can be safely used. The dashed lines correspond to the available power when only the gas turbine portion of the propulsor is used, and the dash-dot lines show the available power if the electric motor is used while the gas turbine is idle.

Within Fig. 8, the solid, horizontal line shows the power setting for the wingtip propulsors that results in the smallest vertical tail size. Looking at the full power takeoff lines, the available power at any gas turbine power fraction is greater than the ideal power level. The vertical offset between the ideal power level line and a point on another line represents the overcapacity at takeoff for that gas turbine power fraction. Previous work by Capristan and Blaesser [10] suggested that 80% of the power should come from the gas turbine to balance weight and energy use. At this power split, the wingtips would be able to provide nearly 80% of the baseline takeoff power, though only a fraction could be used with the smallest tail size. Based on this gas turbine power fraction both the inboard and the wingtip propulsors could each provide 75–80% of the takeoff power. Using a propulsor this large has weight and drag implications, as the inboard and wingtip propulsors would both be nearly as large as the engines on the conventional baseline.

Another challenge related to the overcapacity at the wingtips is how to operate the propulsor. Ideally, the gas turbine would be operated at a constant power setting that exhibits a high efficiency on its operating map, with additional power added by the electric motor. At the 80% gas turbine split, the gas turbine is oversized, implying it would have to be throttled down to an off-design condition, even with an idle electric motor. Similarly, idling the gas turbine is not an option because at the 80% gas turbine split, the electric motor is not powerful enough to provide the needed takeoff power. One means of lowering the thrust at takeoff for a given cruise power level is to have a less efficient propeller at takeoff. Whereas most propellers are designed to meet thrust requirements at takeoff or top of climb and maximize efficiency at cruise, the PEGASUS wingtip propellers could be designed for cruise without regard to off-design performance. Designing for cruise would lower the power requirements at cruise and lessen the excess thrust at takeoff.

The difference between the power level that provides the smallest tail and the full takeoff power available shows that the cruise drag sizes the propulsion system and extenuates the motivation for reducing drag at cruise. Power overcapacity at takeoff will add to the weight of the vehicle and hurt its performance through the need for two sets of large propulsors. Past studies that have looked at a simpler propulsion architecture have found that resizing the core of a gas turbine for cruise by supplementing takeoff and climb performance with an electric motor would perform poorly compared to a baseline gas turbine engine [24, 25]. PEGASUS is a more extreme version of these concepts, though it is considerably smaller and has a much shorter mission. In addition, the conventional baseline has two nacelles on the vehicle, whereas PEGASUS has four. The additional nacelles increase the wetted area and viscous drag, offsetting some of the drag gains obtained through the reduced vertical tail size and wingtip propulsors. Regardless, understanding the system drag at cruise, and working to reduce drag as much as possible, is essential to PEGASUS's success.

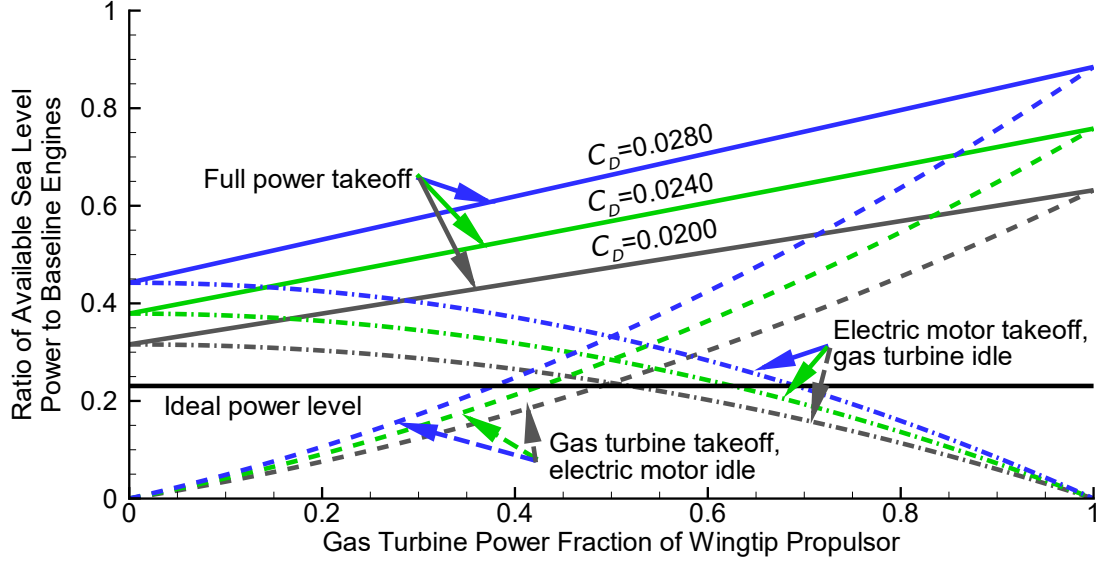


Fig. 8 Wingtip propulsor performance for various hybridization levels and takeoff power sources.

A. Vertical Tail Sizing at Cruise

The low dynamic pressure and high thrust levels required at takeoff typically size the vertical tail, but PEGASUS requires investigation across all phases of flight. At cruise, the ConOps calls for folding the inboard propellers and relying on the wingtip propulsors alone. Within the cruise phase, an OEI event is defined as losing a single wingtip propulsor, each of which provide half of the cruise thrust. Given that cruise thrust is equal to the drag, one can relate the tail volume coefficient required to counter the cruise OEI moment via Eq. 9. Within Eq. 9, c_{VT} is the vertical tail volume coefficient defined in Eq. 3, C_D is the cruise drag coefficient, η_T is the non-dimensional spanwise position of the wingtip propulsor (with a value of unity if the propulsor is at the tip), and $C_{N,VT}$ is the maximum normal force coefficient generated by the vertical tail. Given the baseline values for PEGASUS, the vertical tail volume coefficient must be larger than 0.009 at cruise, which is an order of magnitude smaller than the takeoff criterion. The conclusion is that even with wingtip propulsors, takeoff and low speed operation sizes the tail.

$$c_{VT} \geq \frac{C_D \eta_T}{4C_{N,VT}} \quad (9)$$

Though Eq. 9 includes the cruise drag coefficient, this is the C_D that determines the cruise thrust that the wingtip propulsors must provide in order to maintain steady level flight. Therefore it is the aircraft C_D , incorporating any benefit from the BLI propulsor, whether bookkept as a drag reduction or a thrust contribution. Another observation is that the tail size required to counteract the OEI yawing moment in steady level flight is not dependent on dynamic pressure except as C_D is dependent on dynamic pressure.

An OEI event in the cruise phase is generally less critical than at low altitudes because the pilot has more time to react and respond to the situation. This simple analysis did not investigate the dynamic response to asymmetric thrust, nor pilot responses such as reducing thrust on the operating propulsor or re-engaging the inboard propulsors to provide additional thrust or reduce the yawing moment. Similarly, this analysis did not consider the asymmetric increase in induced drag that would result from losing the induced drag benefit associated with the wingtip propulsor. Given the extra margin provided in the tail size, PEGASUS would likely remain safe in the event of an OEI event at cruise.

B. Vertical Tail Sizing at Climb

The preceding sections demonstrated that PEGASUS's wingtip thrust must be below the maximum available at takeoff in order to reduce the vertical tail size compared with a conventional baseline, and that an OEI event at cruise does not size the vertical tail. Once the aircraft reaches V_{MC} , it will continue to accelerate to its best climb speed, which occurs near the minimum drag speed for the vehicle [26]. Estimates for PEGASUS show the minimum drag speed is approximately 60% higher than the V_{MC} speed, thus in typical climb conditions, the tail has almost 2.5 times more

dynamic pressure than at V_{MC} . Increasing flight speed also decreases the propulsor thrust for a constant shaft power. We calculated the yawing moment due to an OEI event using Eq. 8 to estimate the available power, then calculating the thrust given the flight velocity and propeller efficiency. The increase in dynamic pressure, coupled with a decrease in wingtip propulsor thrust enables safe operation as soon as the climb phase is established. Once the climb speed is established, the aircraft maintains the flight speed at a constant equivalent airspeed (EAS), thus the dynamic pressure is constant throughout the climb.

V. Cruise Analysis

In the previous section, we parametrically varied the cruise drag to understand the trade space involved in wingtip propulsor sizing. This section uses an analysis framework modified from a previous study [11] to determine the thrust required to maintain steady level flight at cruise. The framework used FlightStream [27] as the aerodynamic solver to measure the induced drag and viscous drag of the wing and propulsor system. Figure 9 shows the setup, wherein the wing and propellers are modeled. FlightStream models propellers as uniformly loaded actuator disks based on Conway's method [28] to include swirl. The parametric study varied the wingspan and the gas turbine power fraction independently to understand both parameter's impacts on PEGASUS's cruise drag performance.

Changing the thrust split between the inboard and wingtip propulsors at takeoff impacts the required vertical tail sizing in the event of an OEI event or critical loss of thrust. For each takeoff thrust split and wingspan, the analysis framework varied angle of attack and total thrust input into FlightStream until the thrust equaled drag and lift matched the target cruise C_L . The framework held diameter and RPM constant regardless of the takeoff thrust split. Holding the propeller parameters constant is a simplifying assumption that is reasonable for this analysis. The analysis did not include the use of the BLI propulsor for drag reduction.

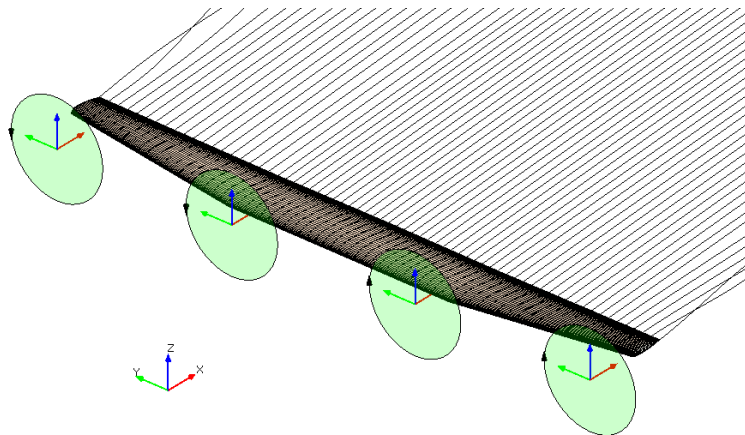


Fig. 9 FlightStream solver setup.

The vertical tail area required for a specific takeoff thrust split and wingspan was calculated using the method described in Section III. The drag contributions are grouped into four categories: the vertical tail, nacelles, wing (both viscous and induced drag), and "other." The "other" category includes the fuselage, horizontal tail, and any excrescence drag. The viscous drag corresponding to the vertical tail, nacelles, and "other" were estimated using FLOPS. Nacelle drag was taken from the conventional baseline nacelles, which are designed for gas turbines and not electric motors. The drag from these components was added to the drag estimate for the wing from FlightStream. The total C_D determined the drag that the thrust of the propulsors must match for steady level flight. The target C_L of the vehicle was 0.5, as this represents a typical cruise condition. Given this framework, trades were performed varying thrust split, wingspan, and ConOps methods for reducing yawing moment required with an OEI event.

A. Wingtip Propulsor Benefit

Three configurations were evaluated using the FlightStream framework. The first was a conventional baseline aircraft using only inboard propulsors. The second was a vehicle with a vertical tail sized using the ideal takeoff thrust split for that given wingspan, though at cruise all the thrust was generated by the wingtip propulsors. The third was

a vehicle employing only wingtip propulsors without the aide of inboard propulsors. The three configurations were evaluated at the baseline wingspan of 80.4 ft, and Fig. 10 shows the results.

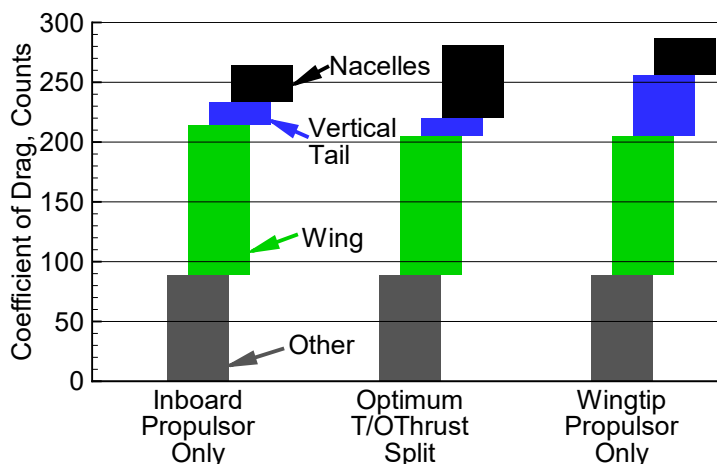


Fig. 10 Component drag, in counts, for three different propulsor configurations.

The drag contribution from the “other” components are constant regardless of propulsor configuration. In regards to the wing contribution, the drag benefit of using wingtip propulsors at cruise compared to inboard propulsors was 7.5%, with all the benefit coming from reduced induced drag. For the wingtip only configuration, the OEI tail sizing effects made the configuration untenable due to the drastic increase in tail area required. The baseline inboard only propulsor configuration resulted in a tail area of 145 ft², compared with 108 ft² for the ideal takeoff thrust split and 421 ft² for the wingtip only configuration. For the wingtip only configuration, the increase in viscous drag due to increased tail area is greater than the calculated drag benefit of the wingtip propulsors, resulting in a 10% increase in total drag coefficient at cruise compared to the inboard only configuration. The PEGASUS configuration, making use of four propulsors of approximately equal size, enabled a 25% reduction in vertical tail area and less induced drag at cruise. However, the increase in viscous drag due to two additional nacelles canceled out these benefits, resulting in a 6.2% increase in total drag compared to the inboard only configuration. This result again highlights the need to reduce the size and power of the wingtip propulsors in order to achieve a drag benefit for the PEGASUS concept.

Although the preceding discussion shows that PEGASUS has additional challenges to overcome, it is important to note that this analysis focuses only on a simplified PAI investigation of vertical tail area that neglected structural or weight implications for configuration changes. Aircraft performance metrics include more than simply energy or fuel consumption and must also consider emissions and noise to determine areas of improvement. A full mission analysis is required to understand the true implications of vehicle performance and metrics. EAP still holds promise in regards to reducing total mission energy and emissions. Lastly, this analysis did not consider detailed takeoff performance for PEGASUS and assumed it would be similar to that of a conventional baseline aircraft. The additional dynamic pressure over the wing due to the wingtip propulsors, combined with other system-level changes, may provide additional improvements that have yet to be considered.

B. Wingspan Trade Study

We performed a trade study to investigate the effect of wingspan on the potential benefit of wingtip propulsors. The wing maintained the general shape of the conventional baseline planform, namely a constant chord inboard section with a constant taper outboard section. The wing breakpoint was held at 37% of the wingspan as the span varied. This allows the aspect ratio to vary but fully defines the wing for a given span. For each wingspan considered, one propulsor was placed at the wingtip, and the inboard propulsor was placed the same distance from the centerline as the baseline. Thus the non-dimensional spanwise placement of the inboard propulsor decreased as wingspan increased. Figure 11 shows the results of the trade, namely that the inboard only configuration has the lowest total drag of the three configurations for all wingspans considered.

One position to note is at approximately 87% of the baseline span, the wingtip only and ideal thrust split configurations have equal drag. The wingtip propulsors only configuration has a larger tail but fewer nacelles, thus balancing out. The

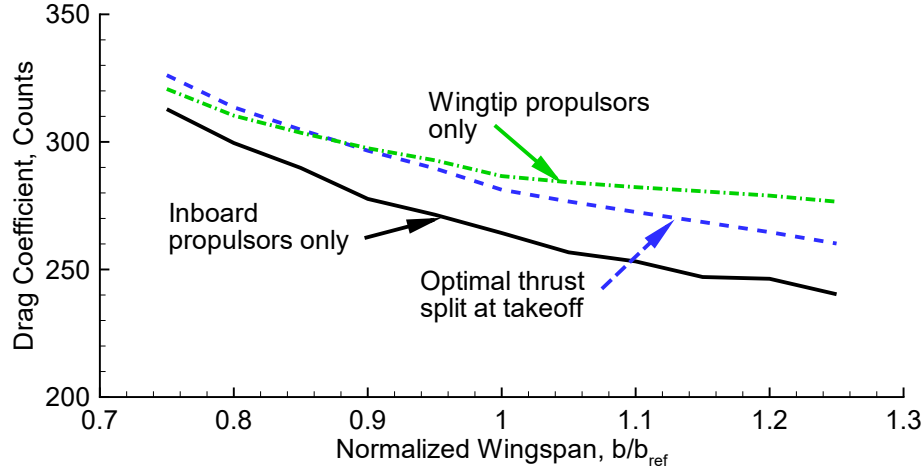


Fig. 11 Impact of wingspan on cruise drag coefficient for three different propulsor configurations.

two configurations are aerodynamically equivalent at cruise, but further studies may show benefits in trading tail size for extra wing-mounted propulsors. Regardless, both of these configurations showed higher drag across every wingspan when compared to the conventional baseline with inboard propulsors only. Across all configurations, the main trend shown in the wingspan trade is the decrease in induced drag with increasing wingspan, which is due to the decrease in span loading. For the wingtip propulsors only configuration, this effect is offset by the rapidly increasing viscous drag of the vertical tail. Increasing the span does not improve the PEGASUS concept against the baseline, inboard only configuration, implying PEGASUS must reduce drag through other options that are specific to its unique capabilities, such as the BLI propulsor.

From Eq. 5, as wingspan increases, the ideal ratio of wingtip thrust to total thrust at takeoff decreases. Even at the optimum takeoff thrust split between the inboard and wingtip propulsors, increasing wingspan requires that the vertical tail increase in size, though it remains smaller than the conventional baseline's tail. The ordinate axis of Fig. 12 plots the wingtip propulsor power available at sea level conditions, estimated with Eq. 8, normalized by the baseline aircraft's power. The upper three lines show the available power when the wingtip propulsors are sized for cruise power at varying levels of hybridization. The solid black line is the ideal ratio based on Eq. 5. Figure 12 shows that increasing wingspan decreases the cruise power required and available takeoff power. Regardless of hybridization factor or wingspan, the wingtip propulsors are always oversized for takeoff. The gap between the ideal power ratio line and any other line represents the excess takeoff thrust available. Figure 12 also shows that the gap between the ideal and available power widens as wingspan increases, which demonstrates that increasing wingspan does not lessen the disparity between the ideal takeoff power and takeoff power available.

VI. Concluding Remarks

This research explored the trades associated with vertical tail sizing given the loss of a single propulsion system on the PEGASUS concept. Within this study, a single fault was considered to be the loss of a propulsor, such as from a bird strike, or a pair of electrically linked propulsors, such as if a battery bus fails. A static analysis showed that distributing the thrust at takeoff between the inboard and wingtip propulsors can result in a smaller vertical tail size than the conventional baseline, reducing the drag on the vehicle. When the wingtip propulsor is sized to provide all of the thrust required at cruise, as in the PEGASUS ConOps, the ideal takeoff thrust split requires throttling down the wingtips during takeoff. The throttling down results in the inboard propulsors being larger than would be necessary if the wingtip propulsors' full capability could be used. In evaluating the steady, level cruise condition for three vehicles, the additional drag due to having four nacelles required for the PEGASUS concept outweighed the benefit that came from a smaller tail and the reduction in induced drag due to wingtip propulsors. Increasing the wingspan in an attempt to further reduce induced drag did not make the PEGASUS concept more competitive compared with a baseline aircraft of the same wingspan.

There are several means to making PEGASUS more competitive with a conventional baseline. One approach is to

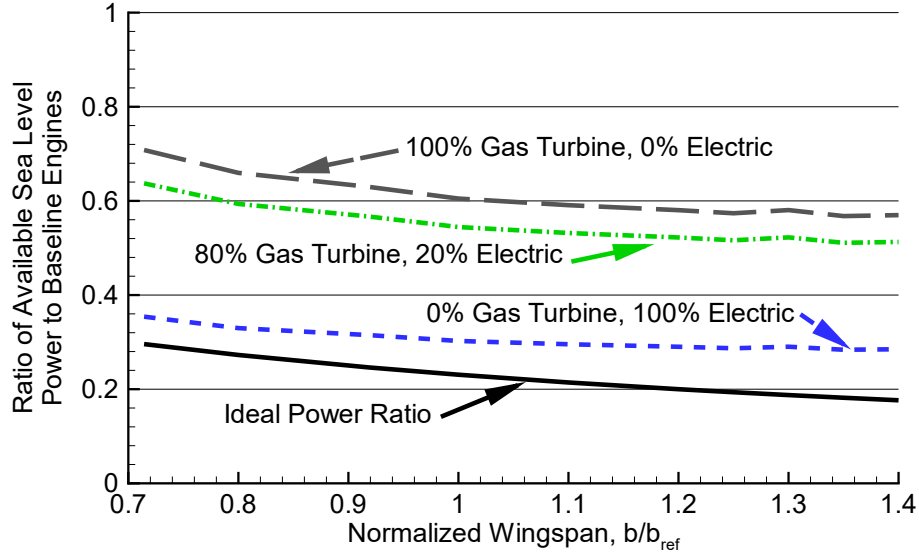


Fig. 12 Effect of wingspan and hybridization on the ideal and available takeoff power at the wingtips.

alter the ConOps to use the inboard propulsors at cruise in addition to takeoff, another is to reduce the wingtip propulsor power at cruise through increased use of the BLI propulsor or additional drag improvements. Many drag reduction efforts should benefit both PEGASUS and a conventional baseline; however, the wingtip propulsors on PEGASUS are sized for cruise, whereas the engines on the baseline are typically sized for top of climb. Therefore drag reductions on PEGASUS would have a larger impact on propulsor sizing. Increasing the wingspan as a means of reducing drag is ineffective, as increased span lowers the wingtip thrust contribution at takeoff, exacerbating the disparity in available takeoff power and the ideal takeoff power.

This work did not consider the contribution of the aft BLI propulsor, which lowers the cruise thrust required by the wingtip propulsors. The analysis did look at a parametric study of BLI contribution and found that the aft propulsor would have to reduce the fuselage drag dramatically in order to bring the wingtip propulsor takeoff thrust available and ideal for minimum tail size into alignment. At high levels of hybridization, where the electric motor is contributing most of the wingtip propulsor power, the disparity is small, but past PEGASUS research has shown that high levels of hybridization are difficult to achieve from a mission analysis perspective. The electrical energy required translates into a large battery and increased vehicle weight.

VII. Future Work

PEGASUS development remains active with multiple branches for analysis. In regards to this study, the static OEI analysis should be replaced with a dynamic analysis that considers the transient response of losing a propulsor. Alternative means of mitigating OEI conditions should also be explored. The NASA X-57 Maxwell uses multiple power controllers to mitigate the impacts of an OEI event on mission operations [29]. A failure modes and effects analysis (FMEA) on PEGASUS would be required to fully understand the system and potentially elucidate ConOps that enable higher thrust levels by the wingtip propulsors without a corresponding increase in vertical tail size. An example of this would be system logic that cuts power to certain propulsors should one propulsor fail in an effort to reduce OEI yawing moments. Analysis would be required to ensure safe climb gradients can be maintained, but such a system would enable smaller inboard motors, reducing nacelle drag and making PEGASUS more competitive.

Many of the PEGASUS ConOps need to be reevaluated. To start, allowing the inboard propulsors to operate during cruise was previously shown to reduce the PEGASUS's power consumption [11] and this analysis showed that decreasing the cruise thrust generated by the wingtip propulsor would provide system level benefits. As currently designed, with 80% of the wingtip power provided by a gas turbine, PEGASUS has considerably more power at takeoff than can safely be used. This study assumed that operating the wingtip propulsor at an off-design condition would result in poor performance, especially for the gas turbine. Performing a study of the gas turbine, electric motor, and propeller

to evaluate the motor and propeller maps is essential to quantifying this challenge. Aside from the challenges at takeoff, once PEGASUS enters climb the full power available can be used. Another avenue for research is to explore different takeoff and climb scenarios to reduce noise or achieve other benefits, akin to Berton and Nark's study [30].

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