

Streak Instability Analysis for BOLT Configuration

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The BOLT configuration designed for an upcoming flight experiment exemplifies the complexities of hypersonic boundary layer transition in the presence of multiple and potentially interacting instability mechanisms. A numerical study of the boundary layer instabilities over this configuration has been performed with the goal of applying state-of-the-art tools in multidimensional stability analysis that complement the other ongoing research related to the BOLT flight campaign. Specifically, the paper outlines modal instability characteristics of the boundary layer streaks adjacent to the minor-axis symmetry plane of a 1/3rd scale BOLT configuration that was tested in the low-disturbance hypersonic wind tunnel at Purdue University and both quiet and conventional tunnels at Texas A&M University (TAMU). The basic state computations agree well with the heat flux measurements in the Purdue quiet tunnel as well as with the off-body velocity field described in a previous DNS study at the University of Minnesota. The encouraging agreement between the predicted frequency spectra of the streak instability in the region of increased boundary layer thickness and the surface pressure spectra measured during the TAMU experiments suggests that these streak instabilities may play an important role during the transition process near the symmetry plane. The modest value of the predicted peak N-factor ($N_{\max} \approx 5$) for the streak instabilities provides a potential explanation for why transition is not observed under quiet flow conditions, but may occur in a noisy wind tunnel. Similar instability analyses are performed for the flight configuration at two selected flow conditions from the ascent portion of the planned flight trajectory. The predicted values of the peak N-factor suggest that transition onset in the vicinity of the center plane is possible at the ascent time of $t = 26.98$ seconds ($N_{\max} \approx 12.5$ at $X/L = 0.77$), but the streak instabilities weaken sufficiently at the later time of $t = 28.88$ seconds ($N_{\max} \approx 11.5$ at $X/L \approx 1.00$) and are less likely to cause transition at that time.

Nomenclature

c_{ph}	=	phase velocity along boundary layer edge streamline at the center plane [m/s]
f	=	frequency of instability waves [kHz]
h	=	specific enthalpy [J/kg]
L	=	axial body length [m]
M	=	Mach number (nondimensional)
M_{∞}	=	freestream Mach number (nondimensional)
M_e	=	Mach number at the edge of the boundary layer (nondimensional)
N	=	logarithmic amplification ratio of a fixed frequency secondary instability mode (nondimensional)
p'	=	pressure fluctuations [Pa]
P_{∞}	=	freestream pressure [Pa]
Pr	=	Prandtl number (nondimensional)
$\dot{q}$	=	surface heat flux [W/m ²]
Re_L	=	Reynolds number based on body length (nondimensional)
Re_{∞}	=	freestream unit Reynolds number [m ⁻¹]
s	=	specific entropy [J/(K kg)]
St	=	Stanton number (nondimensional) $= (\dot{q}/\rho_{\infty} U_{\infty})/(h_{0,\infty} - h_{0,w})$

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t	=	time [s]
T_0	=	stagnation temperature [K]
T_{ad}	=	adiabatic wall temperature [K]
T_w	=	wall temperature [K]
T_∞	=	freestream temperature [K]
u	=	streamwise velocity [m/s]
u'	=	streamwise velocity fluctuation normalized to a peak value of unity at given location (nondimensional)
U_∞	=	freestream velocity [m/s]
X	=	axial coordinate [m]
Y	=	coordinate along major axis of model cross section [m]
Z	=	coordinate along minor axis of model cross section [m]
α	=	angle of attack [deg]
β	=	yaw angle [deg]
$\delta_{0.995}$	=	boundary layer thickness based on $h_0/h_\infty = 0.995$
ρ	=	fluid density [kg/m ³]

Subscripts

0	=	stagnation condition
e	=	edge of the boundary layer based on $h_0/h_\infty = 0.995$
w	=	wall
∞	=	freestream

Abbreviations

BOLT	=	Boundary Layer Transition
DNS	=	direct numerical simulations
HIFiRE	=	Hypersonic International Flight Research Experimentation
PSE	=	parabolized stability equations
WENO	=	weighted essentially nonoscillatory

I. Introduction

Laminar-turbulent boundary layer transition has a major impact on the design and performance of hypersonic flight vehicles because of the increased skin friction and heat transfer rate of a turbulent boundary layer in comparison with laminar flow. In the absence of surface roughness, transition during high-altitude flight tends to be dominated by linear instabilities of the boundary layer flow. While conventional wind tunnel facilities are generally too noisy to provide reliable information concerning the transition characteristics in flight, the Reynolds number limitations of the existing quiet facilities often restricts their application to fundamental studies of disturbance growth over subscale configurations. Therefore, in-flight measurements of transition over canonical model configurations are critical to the development of reliable physics-based models for the typical transition scenarios that are likely to be encountered on realistic hypersonic configurations. In that regard, the HIFiRE-1 and HIFiRE-5b flight experiments involving circular cone and elliptic cone configurations, respectively, have provided valuable data toward the calibration of stability-based models for the transition mechanisms associated with the Mack mode and crossflow instabilities, respectively [1–4]. The HIFiRE-5b configuration also exhibited an additional, complex flow feature in the form of a centerline streak near the minor-axis symmetry plane that is likely to have transitioned via shear layer instabilities that are concentrated within one or more localized regions of high shear within the streak [5–7]. Building upon the success of the HIFiRE-1 and HIFiRE-5b flight campaigns, the BOLT flight experiment [8, 9] will focus on a waverider-like geometric configuration. The BOLT configuration (Fig. 1) shares certain geometric and flowfield features with the HIFiRE-5b elliptic cone but also exhibits a few key differences in the model geometry that result in a more complex basic state than over HIFiRE-5b. For overviews of the BOLT flight campaign, the reader is referred to the papers by Wheaton et al. [8, 9].

The BOLT geometry includes a cylindrical nose tip with four swept leading edges emanating from the corners of the tip. These four leading edges are joined by four laterally concave surfaces, each of which has a single plane of symmetry. The top side of the BOLT configuration corresponds to the main test surface, denoted as side A, whereas the bottom surface (side B) will be used to acquire additional measurements related to the effects of a backward facing step. The strongly concave gutter regions on the sides were designed to isolate the flows on sides A and B,

respectively. One of the rationales behind this design was that the decoupling of the flow behavior over these four surfaces would allow multiple independent experiments to be carried out at the same time. Since the conception of the BOLT configuration, considerable efforts have been devoted to both wind tunnel testing and numerical computations. Among these, Berridge et al. [10] conducted experiments at Purdue University in the Boeing/AFOSR Mach 6 Quiet Tunnel (BAM6QT) and found spectral peaks at sufficiently high Reynolds numbers and a nonzero angle of attack. A likely explanation of the spectral peaks was a boundary layer instability of some type. They also carried out wind tunnel tests in the LENS II facility at CUBRC and, again, found evidence of a high-frequency instability in the spectral measurements based on one of the surface pressure sensors. In both experiments, however, the nature of the flow instabilities could not be ascertained because of the limited detail in the measurements.

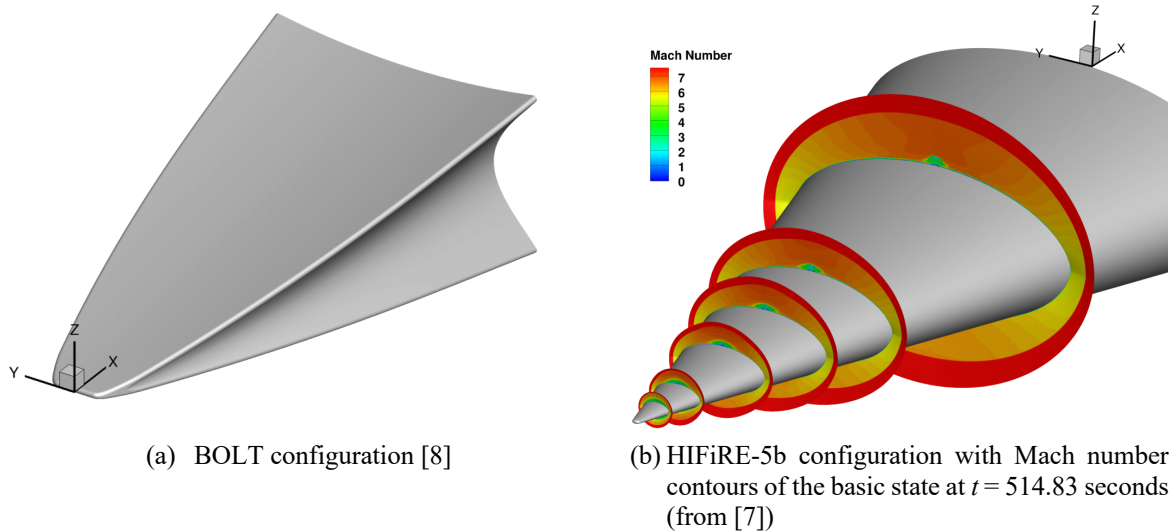


Figure 1. Schematic of the BOLT geometry. For reference, the HIFiRE-5b configuration is also shown in part (b) of the figure.

Kostak et al. [11] investigated the effects of wind tunnel disturbances on the process of boundary-layer transition on the BOLT configuration over a range of Reynolds numbers and in four different wind tunnels. They found a spectral hump between the frequencies of approximately 15 kHz and 50 kHz in the surface pressure measurements near the center plane, i.e., near the minor-axis plane of symmetry. The spectral amplitudes within the aforementioned hump increased with an increasing unit Reynolds number, clearly pointing to the existence of a disturbance amplification mechanism within the region. Subsequent measurements in the TAMU facilities [12] confirmed the existence of this spectral hump, and the accompanying hot-wire measurements yielded additional, but limited, information about the cross-plane structure of the disturbance field near the center plane.

During experiments in the BAM6QT, Berridge et al. [13] noted spectral peaks centered near the frequencies of 40 kHz and 75 kHz, respectively. They suspected that the latter peak was related to a traveling crossflow instability; however, they were unable to discern the origin of the former peak, which was observed in the unsteady pressure signals along the edge of the thick vortical region associated with the change in surface curvature where the nose transitions from a 2D wedge to a swept leading edge. Berry et al. [14] tested a 0.3048 m long model in the NASA Langley 20-Inch Mach 6 Air Tunnel and found a two-lobed transition pattern. The apex of each lobe was away from the centerline, i.e., closer to the attachment line on the side of the model. Transition did occur along the center plane as well, even though it was not immediately obvious from the images of the surface heat transfer. The axial locations of both the apex and the overall transition front were strongly dependent on the flow Reynolds number. While the transition pattern was not very sensitive to the angle of attack, a nonzero sideslip led to a loss of symmetry between the transition fronts on either side of the symmetry plane.

Due to the similarities in the basic states over the BOLT configuration [8] and the HIFiRE-5 elliptic cone [4, 5], one may expect that the stability characteristics of these two flows would also exhibit common features. Preflight stability computations for the HIFiRE-5 cone [5, 6] had shown that the elliptic cone model could support a variety of instabilities that dominate in different portions of the model surface, including Mack mode instabilities along the attachment line, a combination of Mack modes and stationary as well as traveling crossflow instabilities in the shoulder region beyond the attachment line, and more complex, localized instabilities in the bulge region associated with the

convergence of the secondary flow near the center plane. The post-flight analysis at a representative point from the descent part of the HIFiRE-5b trajectory [7] revealed that the most amplified flow instabilities within the bulge region were associated with a “stem” mode that was concentrated within a pair of nearly vertical shear layers below the cap of the mushroom-shaped bulge in the mean velocity contours. The analysis by Choudhari et al. [7] found a peak N-factor of approximately 15 at the transition location estimated from the thermocouple measurements along the leeward centerline. They also noted that the correlating N-factor was close to the transition N-factor for Mack mode transition during the HIFiRE-1 circular cone flight experiment with nearly zero degrees angle of attack. The preflight computations of instability waves over the BOLT flight article [8] have indicated analogous features to those on the HIFiRE-5 model, with the exception of some key differences. Unlike the HIFiRE-5 flight configuration, the attachment line flow along the BOLT model did not reveal a strong Mack mode instability, and furthermore, the N-factor contours away from the attachment lines displayed at least one additional lobe in comparison to those noted for the elliptic cone.

Thome et al. [15] carried out direct numerical simulations of BOLT by imposing distributed roughness on the model surface, which was found to amplify the surface heat flux due to the prominent vortex structures, i.e., the streaks in the boundary layer. The DNS also indicated significant growth in a nonstationary perturbation that was localized in the azimuthal direction and resembled Mack’s second mode instability in form. Knutson et al. [16] showed that, unlike HIFiRE-5, the boundary layer streaks near the center plane of the BOLT model always occur at a finite distance from the plane of symmetry. The response of the BOLT boundary layer to a sustained stochastic forcing was studied via DNS in Refs. [16, 17]. For the 1/3rd scale wind tunnel configuration, the DNS solutions identified four different types of nonstationary disturbances that amplified in different regions of the boundary layer flow, namely, a so-called vortical mode associated with longitudinal streaks adjacent to the minor-axis symmetry plane (interchangeably referred to as the center plane or, simply, as the symmetry plane in this paper), traveling crossflow disturbances inboard of the attachment line, a Mack mode instability that amplified in two different azimuthally localized regions, and the so-called mixed modes with a broad range of frequencies within the midspan region. The DNS methodology was further extended in Ref. [18] to account for the effects of conjugate heat transfer.

Moyes et al. [19] analyzed the stability characteristics of the BOLT boundary layer and found significantly large N-factors for the Mack mode instabilities, especially in the region close to the leading edge where an adverse pressure gradient existed along with an acoustic impedance well. They also identified a strong amplification of crossflow instability modes within a wedge-shaped region on either side of the symmetry plane. Moyes et al. [20] found that some of the stationary streaks included in the DNS solutions from Refs. [16, 17] were not captured in the basic state computations with a second-order accurate, upwind-biased CFD scheme in the DPLR flow solver. The physical origin of the latter streaks has been a matter of some debate within the community. Whereas Thome et al. [17] had suggested that the streaks within the crossflow region originate from the inviscid flowfield, Moyes et al. [20] sought to determine whether those streaks might represent the crossflow vortices resulting from a receptivity process in the upstream region. They showed that the streak evolution within the DNS solution could also be captured in the computations based on the nonlinear parabolized stability equations (PSE) via suitable tuning of the initial disturbance amplitudes.

While the aforementioned DNS computations had indicated the existence of relatively strong fluctuations within the streak region adjacent to the center plane, a formal analysis of those fluctuations and their connection, if any, with a potential modal instability of the streak(s) has not been carried out to date. Whether transition first occurs near the centerline or in the crossflow instability dominated midspan region is an important consideration in practice. However, the existing wind tunnel data does not provide sufficient information on how to extrapolate the centerline transition data from ground tests to flight [13]. Measurements on a subscale BOLT configuration in the 20-Inch Mach 6 wind tunnel at the NASA Langley Research Center had indicated a significantly aft onset of transition along the centerline, suggesting the centerline transition to be caused by the spreading of turbulence from the upstream onset of transition in the crossflow region. Data obtained on a full-scale model in the CUBRC LENS II facility had indicated that the location of earliest transition could be either in the vicinity of the centerline or in the crossflow region, depending on the flow conditions. These findings led Berridge et al. [13] to highlight the lack of understanding of the transition mechanisms in the vicinity of the centerline as an important need. This paper seeks to address that gap by targeting a computational analysis of the streak instabilities adjacent to the centerline of the BOLT configuration. Specifically, we employ state-of-the-art tools in multidimensional stability analysis to compute the modal instability characteristics of these streak instabilities and to evaluate their variation with flow conditions. Such analysis can shed significant light into the nature of flow disturbances identified on the basis of both dynamic mode decomposition in previous DNS [16, 17] and a spectral analysis of surface pressure measurements in wind tunnel experiments [12]. Furthermore, this analysis would extend the recently reported results for the HIFiRE-5b flight article [7] to the BOLT configuration, allowing for an improved understanding of how the differences in the nose tip geometry can influence the nature of flow instability in the vicinity of the center plane. A description of the flow configuration and the

computational methodology is given in Section II. The laminar basic states at the flow conditions of interest are described in Section III, and the results of the instability analysis are presented in Section IV. Conclusions are presented in Section V.

II. Flow Configuration and Numerical Methodology

As mentioned in the Introduction, the flow configuration of interest corresponds to the BOLT configuration from Fig. 1. The subscale wind tunnel model used during the experiments in the Purdue quiet tunnel and both the Mach 6 Quiet Tunnel and the ACE Tunnel at the Texas A&M University corresponds to a 1/3rd-scale version of the flight test article. The flight article is 0.866 meters long, with a base width and base height of 0.438 meters and 0.256 meters, respectively. Thus, the base of the BOLT model has an aspect ratio of 1.71, which is comparable to the aspect ratio of two for the elliptic cross-section of the HIFiRE-5 flight configuration. Similar to the HIFiRE-5 model, the BOLT geometry is symmetric about two mutually orthogonal planes and we denote the normal directions to these planes as the Y and Z axes of a Cartesian coordinate system, with the X axis along the axis of the model. The BOLT configuration differs from the HIFiRE-5 elliptic cone by virtue of its 2D nose tip and a cross-section with concave transverse curvature on all four sides. Whereas the HIFiRE-5 model had a nose radius of 2.5 mm within the minor-axis plane of symmetry, BOLT has a larger bluntness with a leading edge radius of 5 mm along the entire length of the model. Downstream of the nose tip, however, the centerline geometry (i.e., within the $Y = 0$ plane) of both the HIFiRE-5 and the BOLT configurations corresponds to a straight line with an inclination of 7 degrees with respect to the major-axis plane of symmetry (i.e., $Z = 0$).

The basic state computations employ a hybrid strategy similar to that employed by Laible et al. [21] and Mayer et al. [22] in the context of supersonic transition over a circular cone, by Sivasubramanian and Fasel [23] for wavepacket evolution over a Mach 6 cone, and by Li et al. [24] and Choudhari et al. [25] to study the nonlinear breakdown of stationary crossflow instability via secondary instabilities. Following the latter two references, a precursor computation including the bow shock around the BOLT configuration is performed by using the second-order, shock capturing code VULCAN (Viscous Upwind ALgorithm for Complex flow ANALysis) [26]. The VULCAN solution provides the boundary conditions for a higher-order DNS code focused on the flow downstream of the nose region and underneath the bow shock. The reader may consult references [24, 25] for a description of the methodology and how it was applied toward the analysis of crossflow instability, which was the primary focus of those papers. However, for completeness, a brief account of both flow solvers is given below. All of the basic state computations assume the fluid to be a perfect gas (air) and the usual constitutive relations for a Newtonian fluid are used, namely, the viscous stress tensor is linearly related to the rate-of-strain tensor and the heat flux vector scales linearly with the temperature gradient through Fourier's law. The coefficient of viscosity is computed from Sutherlands's law and the coefficient of thermal conductivity is computed by assuming a constant Prandtl number ($Pr = 0.72$).

The VULCAN code solves the unsteady, conservation equations appropriate for a laminar or turbulent flow of calorically or thermally perfect gases with a spatially second-order accurate cell-centered finite volume scheme. A variety of upwind schemes are available for fine-tuning of numerical dissipation and shock capturing in order to ensure solution accuracy for stability computations. In the present computations, the inviscid fluxes were constructed using the MUSCL $\kappa=0$ scheme, the van Albada gradient limiter [27] and the Low Dissipation Flux Split Scheme (LDFSS) of Edwards [28]. The cell face gradients required to construct the viscous fluxes were obtained using an auxiliary control volume approach that results in a compact viscous stencil that produces a second-order accurate approximation of the full Navier-Stokes viscous fluxes. The solutions were relaxed in pseudotime to steady state by using the 3-D ILU(0) scheme [29] with a constant CFL number of 20. Due care was taken to ensure that the grid was aligned with the shock surface so as to avoid the generation of spurious flow features downstream of the shock.

The basic states used for instability analysis were obtained by using higher-order computations based on the DNS-WENO code that derive the inflow and freestream boundary conditions from the VULCAN solutions. A detailed description of the governing equations and the numerical algorithms used in the DNS-WENO code is given by Wu and Martin [30]. For brevity, this code is henceforth referred to as simply the DNS code. The inviscid fluxes from the governing equations are computed using a seventh-order weighted essentially nonoscillatory (WENO) finite-difference as introduced by Jiang and Shu [31], and the DNS-WENO code also includes an optimized scheme that reduces the numerical dissipation by means of limiters [32]. Both an absolute limiter on the WENO smoothness measurement and a relative limiter on the total variation are employed simultaneously during the simulation. The viscous fluxes are discretized by using a fourth-order central difference scheme and the time integration is based on a third-order low-storage Runge-Kutta scheme [33]. The DNS code has been previously applied to a variety of hypersonic flows including the simulation of turbulence in hypersonic boundary layers [34], laminar-turbulent transition due to crossflow instability over yawed cones [24, 25, 35], Görtler instability over a concave cone in a Mach

6 wind tunnel [36, 37], instability amplification over a distributed patch of roughness elements [38], and the boundary layer breakdown following the transient growth of traveling disturbances within the entropy layer over a blunt cone [39]. In the context of the present work, we also note that the same DNS code was also used in the study of centerline transition on the HIFiRE-5b model and the evolution of the streak instabilities based on the DNS was in excellent agreement with the predictions of the plane marching PSE [7].

The specific cases investigated thus far are outlined in Table 1, and the sizes of the computational grids used in each case are listed in Tables 2 and 3 for the VULCAN and the DNS computations, respectively. These cases include a wind tunnel configuration and two points from the ascent segment of the anticipated flight trajectory, denoted as FAL (i.e., Flight-Ascent-Low Reynolds number) and FAH (Flight-Ascent-Higher Reynolds number), respectively. Exploiting the symmetry of the BOLT geometry, only one quarter of the test article was modeled in the basic computations at $\alpha = \beta = 0$ degrees and the appropriate half of the model was used for the cases with a nonzero α or nonzero β . Given the extensive prior work for similar flow configurations using the VULCAN and the DNS flow solvers, only a modest amount of effort has been devoted to confirming the lack of grid sensitivity in the computed basic states.

Table 1 Flow configurations and associated freestream conditions

<i>Case Designation</i>	<i>BOLT configuration</i>	α (deg)	β (deg)	M_∞	Re_∞ (m ⁻¹)	Re_L	U_∞ (m/s)	P_∞ (Pa)	T_∞ (K)	T_w (K)
WT	Purdue Wind Tunnel (1/3 rd scale)	0	0	6	9.88M	2.85M	864.0	564.3	51.6	300.00
FAL	Ascent ($t = 28.8767$ s)	0	0	5.53	4.25M	3.68M	1650.9	2380.2	222.0	400.00
FAL $_\alpha$	Ascent ($t = 28.8767$ s)	4	0							
FAL $_\beta$	Ascent ($t = 28.8767$ s)	0	4							
FAH	Ascent ($t = 26.9762$ s)	0	0	5.37	6.60M	5.72M	1594	3735.4	219.1	400.00

Table 2 Mesh parameters for VULCAN computations

<i>Case Number</i>	<i>Mesh</i>	<i>Leading Edge Block</i>	<i>Main Part of BOLT Model</i>
WT	V-WT _b	289×129×301	1121×417×301
FAL	V-FAL _b	289×129×301	1121×417×301
FAL $_\alpha$	V-FAL _{2Z,b}	289×257×301	1121×833×301
FAL $_\beta$	V-FAL _{2Y,b}	577×129×301	1121×833×301
FAH	V-FAH _b	289×129×301	1121×417×301

Table 3 Mesh parameters for DNS-WENO computations

<i>Case Number</i>	<i>Mesh</i>	<i>Leading Edge Block</i>	<i>Main Part of BOLT Model</i>
WT	WT _b	N/A	526×417×256
FAL	FAL _c	N/A	394×456×192
	FAL _b	N/A	526×609×256
FAL $_\alpha$	FAL _{2Z,b}	N/A	526×1138×246
FAL $_\beta$	FAL _{2Y,b}	N/A	526×1217×246
FAH	FAH _b	N/A	426×609×256 ($x/L < 0.77$)

The stability of the basic states computed in this manner is investigated by using a combination of local, multidimensional eigenvalue analysis and plane marching parabolized stability equation (PSE) implementation. The analysis of instability modes within the boundary layer streaks must account for the enhanced azimuthal gradients associated with the streaks within the region of interest, which correspond to relatively compact azimuthal regions (relative to the overall width of the body) on either side of the inviscid streamline from the center of the nose tip (i.e., $X = Y = 0$). As seen later in Section III, this region corresponds to a narrow subset of the test surface in the vicinity of the center plane (i.e., $Y = 0$) for all cases with a zero yaw angle. For a nonzero yaw angle, the above region is inclined with respect to the $Y = 0$ plane at approximately the same angle as the yaw direction. For $\beta = 0$ degrees, the basic state is symmetric about the center plane, and hence, it is more convenient to restrict the stability analysis to the $Y > 0$ side, with the flow disturbances assumed to be either symmetric or antisymmetric with respect to the $Y = 0$ plane [7, 38]. The symmetric modes have zero azimuthal gradient in the axial velocity fluctuation component u' (as well as in the fluctuations in the surface normal velocity and all thermodynamic variables) at $Y = 0$, whereas the antisymmetric modes are associated with homogeneous Dirichlet boundary conditions in those quantities.

The overall strategy for the presentation of the instability results is similar to that employed in Refs. [7, 24, 25, 38]. To illustrate the several instability modes in the region of interest, the instability characteristics based on local eigenvalue solutions at selected axial stations are described first. Analogous to the secondary instability analysis of crossflow vortex modes on a yawed cone [24–25], the local instability analysis is performed in two different ways, namely, by invoking the quasiparallel approximation for the basic state and by retaining the axial gradients of the basic state. The latter option is denoted herein as a partially nonparallel (PNP) eigenvalue analysis. To characterize the axial evolution of the unstable disturbances along the length of the BOLT configuration, and thereby, establish a topological connection between the mode shapes at different axial stations, plane marching PSE [40] computations were performed across a range of disturbance frequencies and mode types, starting with the mode shapes at the axial locations that correspond to nearly neutral conditions. Unlike the PNP eigenvalue analysis, the PSE computations incorporate all of the important nonparallel effects on the disturbance evolution within the region of streaks. The analysis of similar instabilities in the vicinity of the center plane of the HIFiRE-5b model [7] has shown that the plane marching PSE can provide accurate predictions for the development of flow instabilities in streak-like flow features over fully three-dimensional flow configurations.

A typical grid size for the eigenvalue computations as well as the plane marching PSE corresponds to 121 to 161 points in the wall-normal direction and between 151 to 251 points in the azimuthal direction. The azimuthal domain is truncated a short distance beyond the region of strong streaks adjacent to the center plane.

III. Basic State

Computational results pertaining to basic state computations for all three flow configurations of interest are presented in this section. The laminar boundary layer flow over the Purdue wind tunnel configuration (case WT from Table 1) is described first in Section III.A. The flow characteristics for the two flight cases are investigated in Sections III.B and III.C, respectively.

A. Wind Tunnel Configuration (Case WT)

Figure 2(a) shows the heat flux distribution over the test surface of the subscale BOLT configuration for Case WT. The skin friction lines, along with the streamlines based on the velocity field along the edge of the boundary layer (determined as the isosurface $h_0/h_\infty = 0.995$), are displayed in Fig. 2(b).

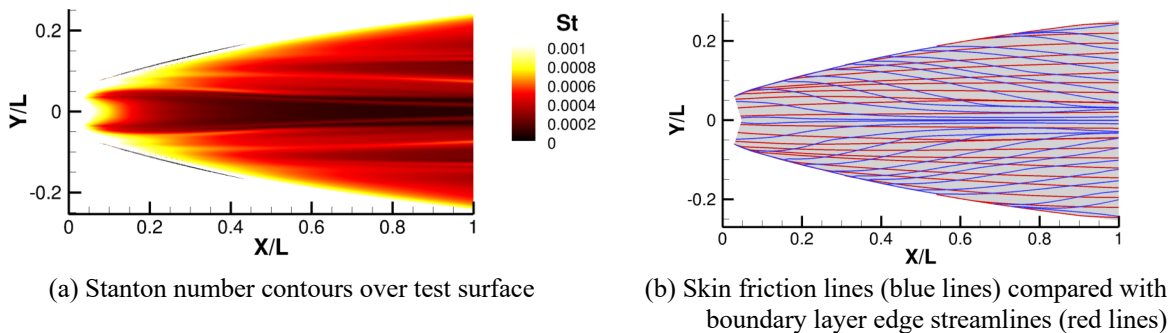


Fig. 2. Visualization of the surface flow behavior associated with the basic state for Case WT (mesh WT_b).

The heat flux contours in Fig. 2(a) indicate the presence of several longitudinal streaks that are approximately aligned with the local streamwise direction. Subsequent results pertaining to the off-body flowfield will indicate that the heat flux streaks are accompanied by short-scale variations in the axial velocity field in the boundary layer. However, the nature of the off-body variations can be different between the near-wall region and the main part of the boundary layer. Not surprisingly, the azimuthal variations in the near-wall velocity contours correlate better with the heat flux variations depicted in Fig. 2(a). The significant differences between the inclination angles of the skin friction lines and the inviscid surface streamlines in Fig. 2(b) highlight the appreciable magnitudes of crossflow velocity in the background flow in between the streaks. The fact that the streaks are approximately aligned with the inviscid streamlines suggests that their evolution is likely to be governed by perturbation dynamics similar to those of stationary crossflow vortices in a three-dimensional boundary layer. In the limit of large Reynolds numbers, the orientation of the stationary crossflow vortices in the vicinity of the upper (i.e., higher wavenumber) neutral branch is determined by the orientation of the tangential velocity vector at the generalized inflexion point of the angular momentum profile, and a more thorough analysis of the streak orientations by Moyes et al. [20] has indicated that many of the streak trajectories are better aligned with the velocity field along the surface of the generalized inflexion points across the boundary layer profiles. This finding tends to reinforce the notion of a close connection between at least some of these streaks and the stationary crossflow instability.

Mach number contours of the basic state near the symmetry plane at selected axial stations are shown in Figs. 3(a)-3(d). The sequence of images in Fig. 3 highlights the progressive rollup of the Mach number contours, which is associated with a similar rollup in the axial velocity contours. The formation begins with a relatively sharp azimuthal gradient near $Y/L \approx 0.05$ at $X/L = 0.2$ and shifts further inboard at $X/L = 0.50$, yielding a structure with a rollup across the range of $0.03 < Y/L < 0.05$ and, subsequently, leads to a fully rolled-up vorticity structure between $0.02 < Y/L < 0.04$ at $X/L = 0.98$. Since the typical streamlines at the edge of the boundary layer are inclined slightly away from the center plane, the cumulative convergence of the secondary flow inside the boundary layer is likely to be a significant contributor to the progressive intensification of the streaks from Fig. 3(a) to Fig. 3(d). One may also observe that, once the Mach number contours have rolled up into a prominent structure, the azimuthal variation along the individual contours is not the same across the entire thickness of the boundary layer. Hence, while the surface heat flux distribution in Fig. 2(a) provides useful clues into the nature of the flowfield, a reliable understanding of the latter, especially in reference to the anticipated flow instabilities, requires the knowledge of the off-surface flow characteristics.

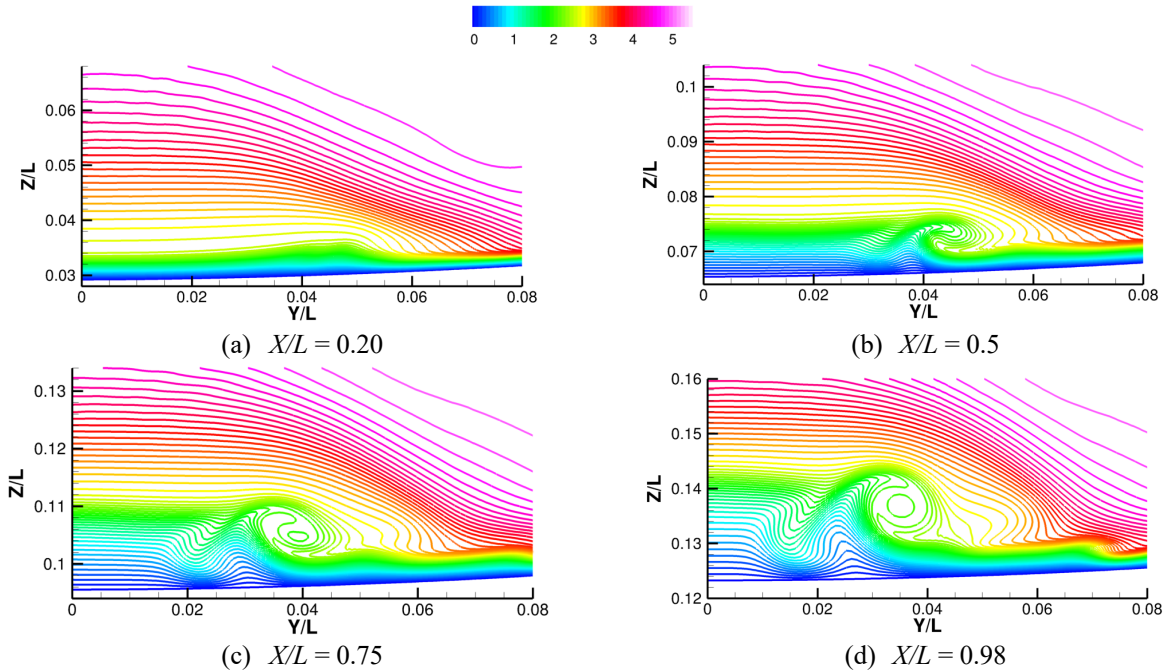


Fig. 3. Mach number contours of the basic state near the symmetry plane for case WT. Results are based on DNS computations for mesh WT_b from Table 3.

The streaks associated with the roll-up regions in Fig. 3 remain at a finite distance from the center plane across the entire length of the BOLT configuration. Indeed, a closer inspection of the overall flowfield, including the heat flux distribution in Fig. 2(a), suggests that the above streaks actually originate near the edge of the 2D nose region before eventually moving closer to the symmetry plane at increasing axial locations. The azimuthal separation between the streaks and the center plane makes the flow behavior near the BOLT center plane somewhat different from that observed on the HIFiRE-5b model at a representative trajectory location during the descent portion of the flight trajectory (see, for instance, the HIFiRE-5b basic state contours in Figs. 2 and 7 from Ref. [7], which indicate strong vortical structures in the immediate vicinity of the center plane). In Section III, we will examine if and how the instability characteristics are also modified as a result of this change.

An additional difference between the BOLT and the HIFiRE-5b flowfields corresponds to the considerably stronger entropy layer over BOLT, which may be attributed to both its larger nose radius (5 mm vs. 2.5 mm for HIFiRE-5b) and the 2D nature of the nose geometry. The entropy layer is known to persist for considerably longer distances over 2D, blunt nose configurations compared to over an axisymmetric (i.e., slender 3D) configurations with the same nose radius [41]. This may be seen from the entropy layer contours along the center plane (Fig. 4(a)). As a result of the thick entropy layer in Fig. 4(a), the Mach number near the edge of the boundary layer remains below three over nearly the entire model length. Indeed, the Mach contours in Fig. 3 indicate that the Mach contour near the edge of the boundary layer remains nearly parallel to the surface throughout the azimuthal region of interest, eventually dipping closer to the surface at an azimuthal distance from the center plane that appears to scale with the half-width of the 2D portion of the nose tip. The isosurface of specific entropy shown in Fig. 4(b) is consistent with the azimuthal variation of the Mach number within the inviscid region. The choice of the entropy value, $s = 100$, selected for this isosurface is somewhat arbitrary and was chosen to be sufficiently small to be consistent with the demarcation between the green shaded entropy layer in Fig. 4(a) and the blue shaded region corresponding to a nearly potential flow.

The computed DNS solution for the WT configuration was validated via comparisons with the measured heat flux distribution in the Purdue tunnel and also with the DNS results presented in Ref. [16]. The azimuthal heat flux distribution at two selected streamwise stations ($X/L = 0.72$ and $X/L = 0.91$, respectively) is shown in Figs. 5(a) and 5(b), respectively. The quantitative comparison between the two computational predictions is excellent, and furthermore, both predictions agree quite well with the heat flux distribution inferred from the infrared (IR) thermography during the wind tunnel experiment (Fig. 5 from Ref. [16]). The line plots in Fig. 5(a) further illustrate the nature of streaks seen from the visualization of the surface heat flux in Fig. 2(a) and from the off-body flowfield in Fig. 3. A similarly good comparison between the axial velocity contours from the present DNS and the DNS from Ref. [16] may be observed in Fig. 5(c).

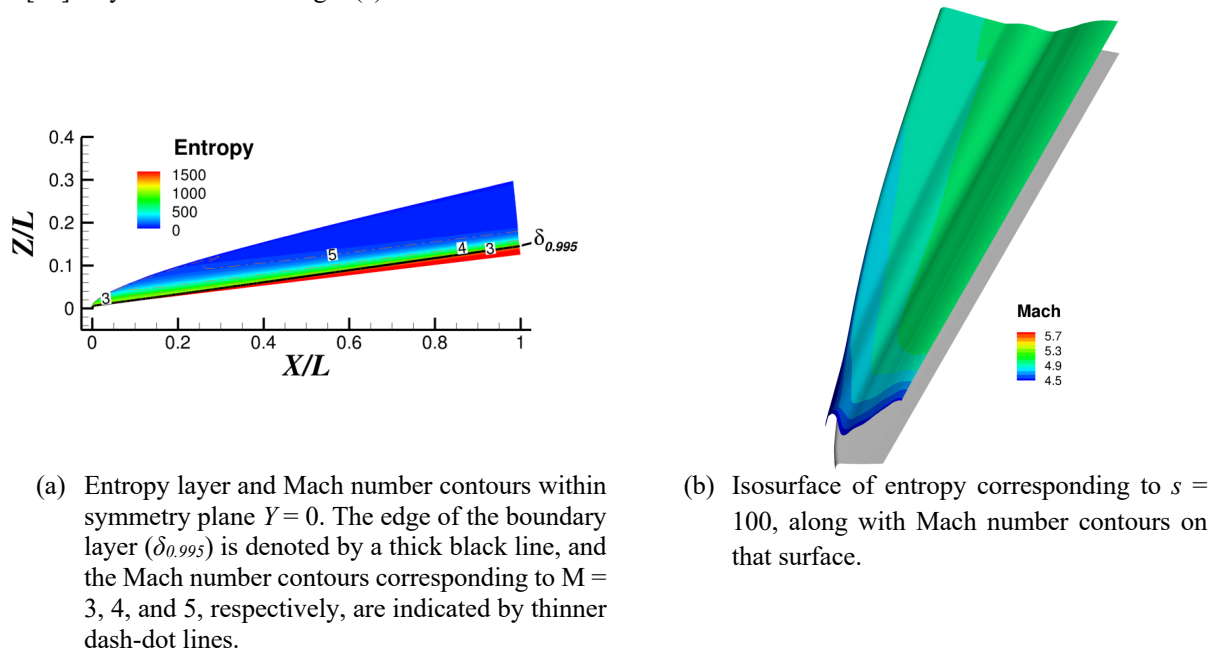


Fig. 4. Entropy layer characteristics for case WT. Results are based on DNS computations for mesh WT_b from Table 3.

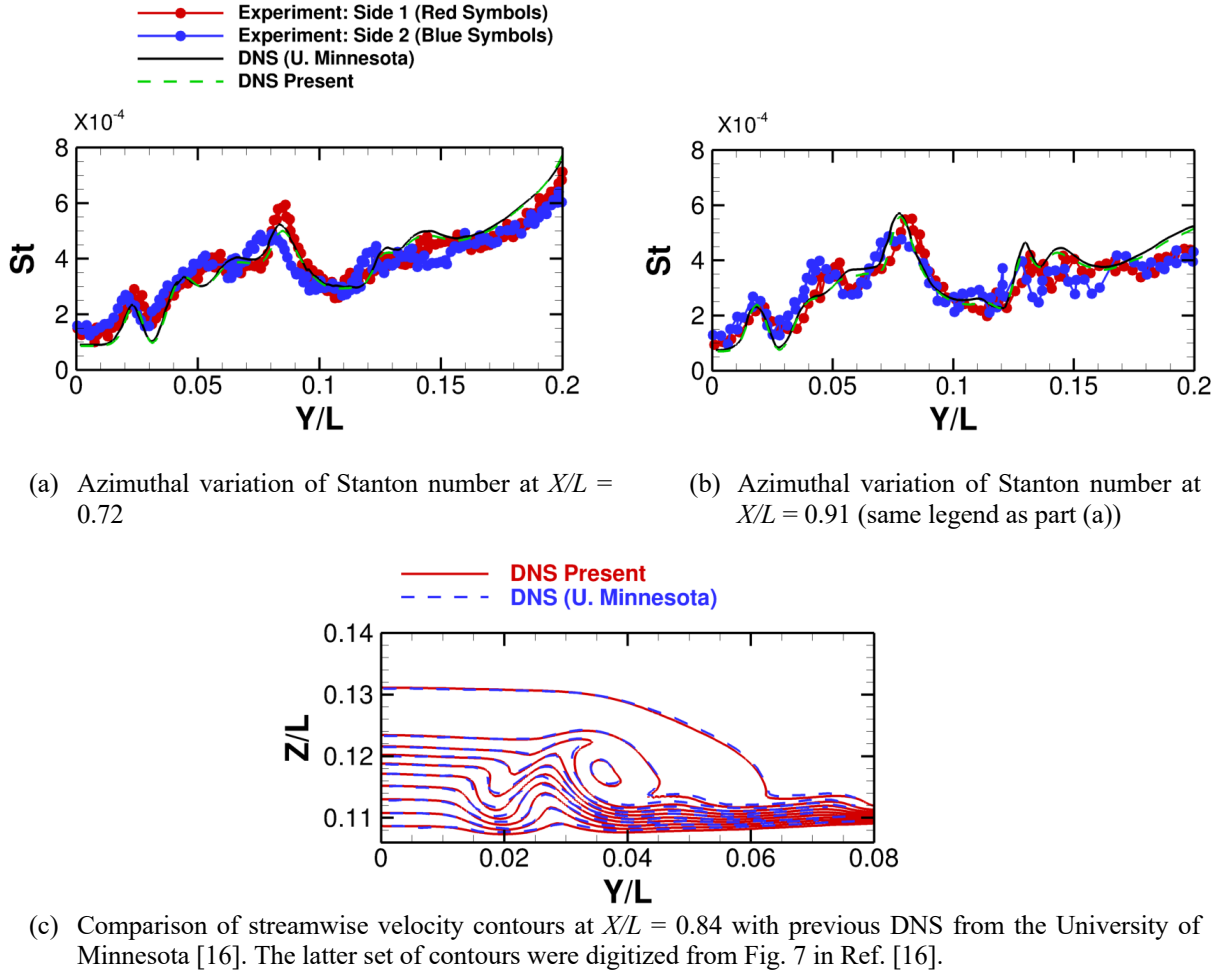


Fig. 5. Comparison of DNS-WENO based basic state for case WT with quiet tunnel measurements [11] and DNS computations from the University of Minnesota [16, 17]. The reference data for Stanton number distributions in parts (a) and (b) was digitized from Fig. 5 in Ref. [16].

B. Cases FAL , FAL_{α} , and FAL_{β}

The surface distributions of the Stanton number for the low Reynolds number flight configurations FAL , FAL_{α} , and FAL_{β} are plotted in Figs. 6(a) through 6(c), respectively. The number of clearly visible thermal streaks, their locations, and strengths are appreciably influenced by the angle of attack and the yaw. Significant differences are seen between the streak patterns along the windward and leeward segments of the test surface in Figs. 6(b) and 6(c). In particular, a prominent set of hot and cold streaks is observed in close proximity to the attachment line along the leeward surface for case FAL_{α} . The FAL_{β} case also shows a prominently asymmetric streak pattern over the test surface, with multiple strong roll-ups on the leeward side. Further computations are currently under way to confirm the lack of grid sensitivity of the streak pattern modifications at nonzero α and/or β and to gain an understanding of how these modifications may affect the amplification of instability waves in the boundary layer flow.

Cross-plane contours of the Mach number distributions at $X/L = 0.50$ and $X/L = 0.75$ for the FAL case are shown in Figs. 7(a) and 7(b), respectively. These figures indicate a qualitatively different streak evolution from the WT case considered in Subsection III.A. While the case WT showed a single rolled-up vorticity structure in the vicinity of the centerline, the flowfield in case FAL indicates two distinct rollups at $X/L = 0.50$ (Fig. 7(a)), one slightly inboard of $Y/L = 0.03$ and the other near $Y/L = 0.04$. These two structures appear to merge in the downstream region, yielding a single prominent structure at $X/L = 0.75$ (Fig. 7(b)). Figure 7 also displays the comparison between the Mach number contours obtained using the baseline grid FAL_b and the coarser grid FAL_c , respectively. The agreement between both

solutions is excellent, with the two sets of contours being virtually indistinguishable from each other. A similar agreement was also noted between the corresponding predictions of the heat flux distribution on the test surface.

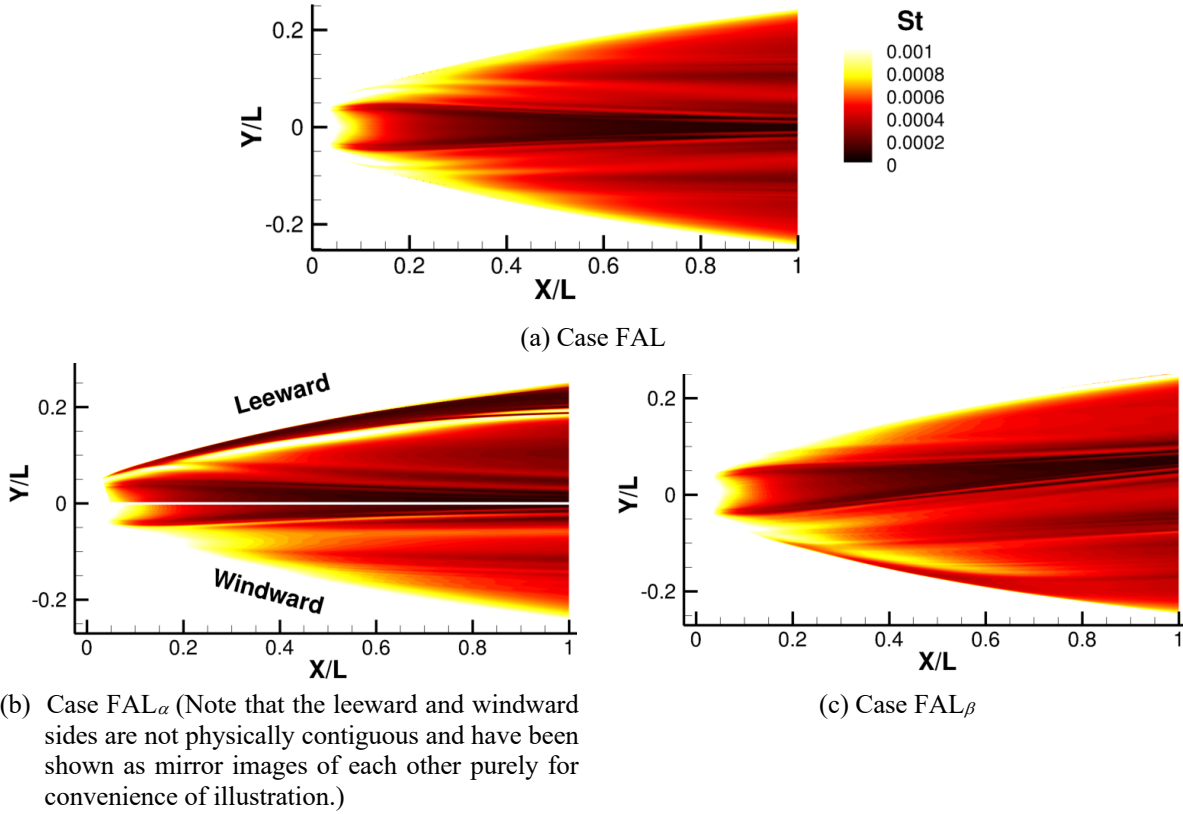


Fig. 6. Stanton number contours for Cases FAL (mesh FAL_b), FAL_α (mesh $FAL_{2Z,b}$), and FAL_β (mesh $FAL_{2Y,b}$).

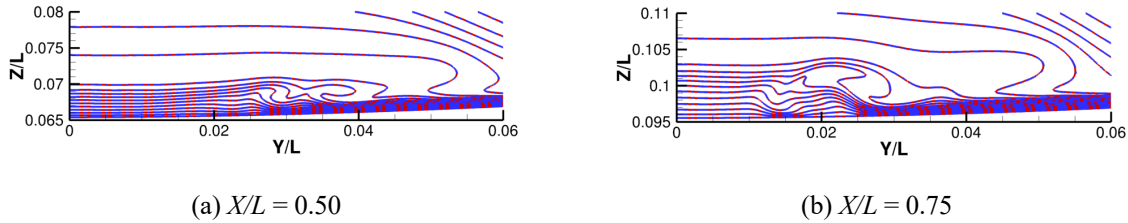


Fig. 7. Mach number contours near the center plane for Case FAL. Red lines indicate the solution on mesh FAL_b , whereas the blue lines denote the contours based on the computation using the coarser mesh (mesh FAL_c). Contour values increment in intervals of 0.25, starting from $M = 0$.

A visualization of the axial velocity contours at $X/L = 0.94$ is shown in Fig. 8(a). Figure 8(b) shows the cross-plane contours of $\rho|\nabla_{YZ}(u)|$ where ∇_{YZ} denotes the cross-plane gradient within the Y - Z plane. The highest gradients are observed in the near-wall flow, in the stem region of a mushroom-shaped structure on either side of the symmetry plane, as well as the cap region of the mushroom, particularly along the inboard side. Intermediate gradients are seen along the outboard part of the mushroom cap and the majority of the boundary layer region in between the two mushrooms. The weaker gradients across the semielliptical region above the boundary layer result from the vorticity generated by the curved shock surface associated with the nose tip, i.e., the entropy layer region previously indicated in Fig. 4.

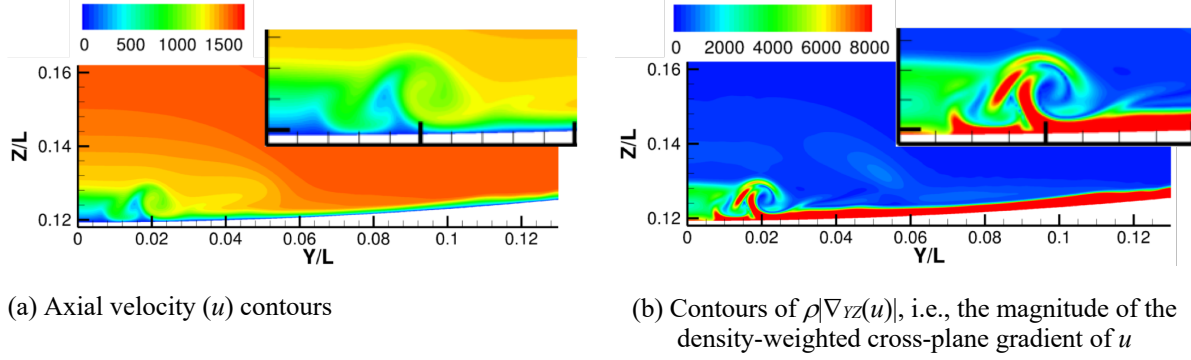


Fig. 8. Axial velocity related contours of basic state near the symmetry plane ($X/L = 0.94$) for case FAL. Insets in each part of the figure display a magnified view of the region close to the center plane.

C. Case FAH

Finally, we consider the basic state characteristics for the higher Reynolds number flight condition, i.e., case FAH. The Stanton number distribution for this case is plotted in Fig. 9(a), whereas the Mach number contours near the end of the computational domain ($X/L = 0.75$) are displayed in Fig. 9(b). Unlike the FAL case, where more than one rolled-up structure was only observed within a narrow range of axial stations near $X/L = 0.50$, the case FAH exhibits multiple structures of this type over a substantially larger range of the model length. In particular, one may observe that the Mach number contours for $X/L = 0.75$ in Fig. 9(b) indicate two fully rolled-up mushroom structures with inclined (and asymmetric) caps, along with an additional, partially-formed structure.

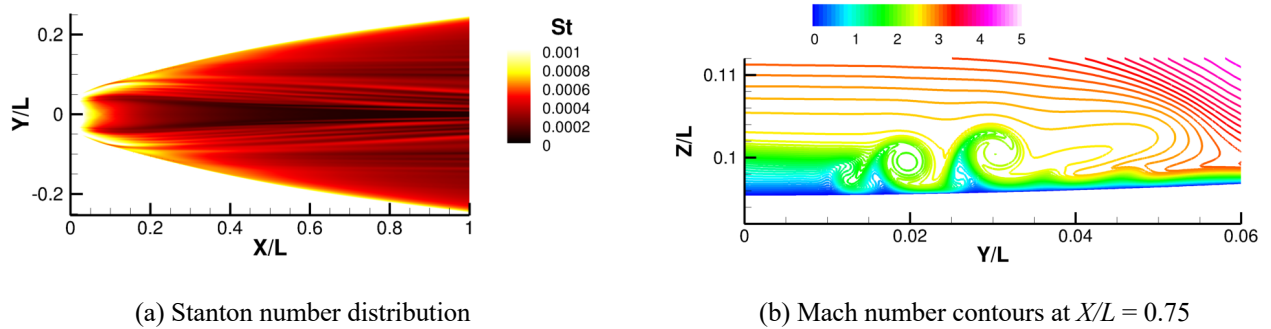


Fig. 9. Basic state behavior near the center plane for Case FAH (mesh FAH_b).

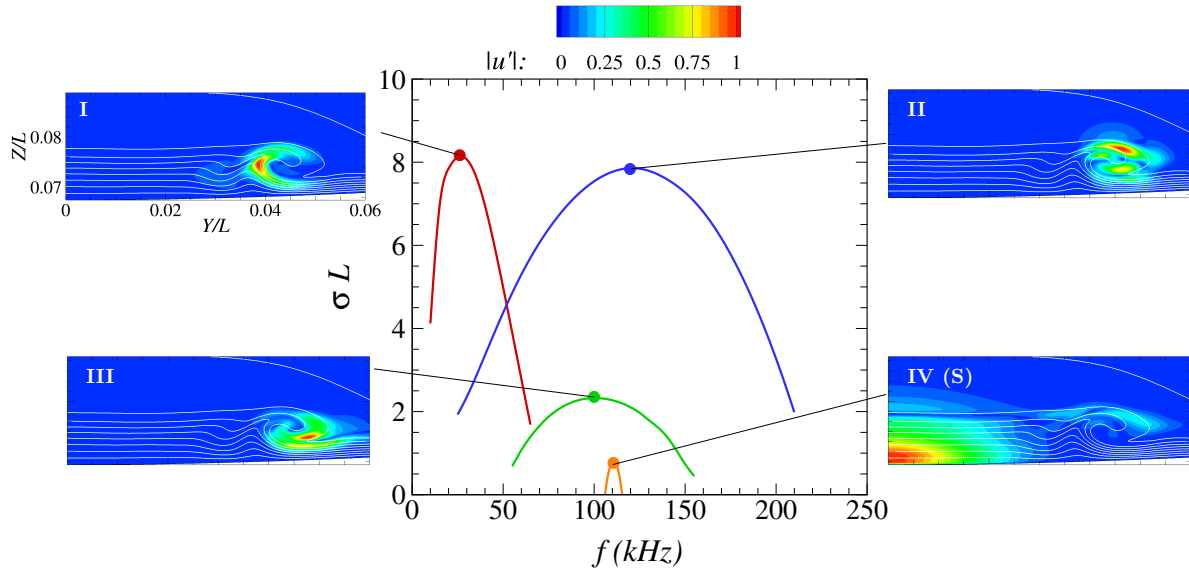
IV. Streak Instability Characteristics

Computational findings based on the stability analyses of the basic states outlined in Section III are described in this section. Some significant observations are contained below; however, more complete findings will be captured within a separate paper.

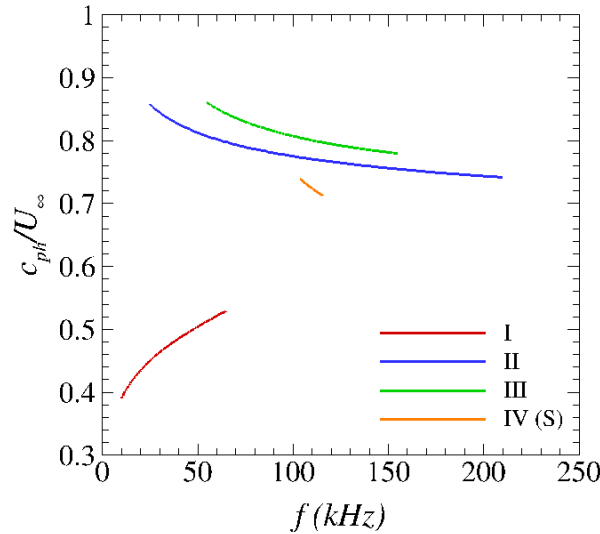
A. Wind Tunnel Configuration (Case WT)

The growth rate spectra for selected families of unstable modes obtained via the PNP eigenvalue analysis of the basic state in the vicinity of the center plane at $X/L = 0.515$ are shown in Fig. 10(a), along with the mode shapes of the $|u'|$ velocity perturbation for the disturbance frequencies that approximately correspond to the most unstable mode from each family. The white lines in the mode shape plots represent the streamwise velocity contours of the base flow, and the colored contours indicate the magnitude of the axial velocity fluctuations. The associated phase speeds of the modes is plotted in Fig. 10(b). Four different families of unstable modes are included in both figures. For ease of referencing, these mode families are numbered as I through IV, respectively, but the ordering of the modes does not have any physical significance. There are additional unstable modes, but they were deemed to be unimportant either because of low overall amplification factors or because their peak fluctuations occur outboard of the prominent

streak adjacent to the center plane. For example, disturbances resembling traveling crossflow instabilities were also identified in the outboard region, but they are considered to be beyond the scope of the present analysis, which is focused on the streak instabilities near the center plane.



(a) Growth rate spectra and associated mode shapes



(b) Phase speed spectra (line colors identical to mode families indicated in part (a))

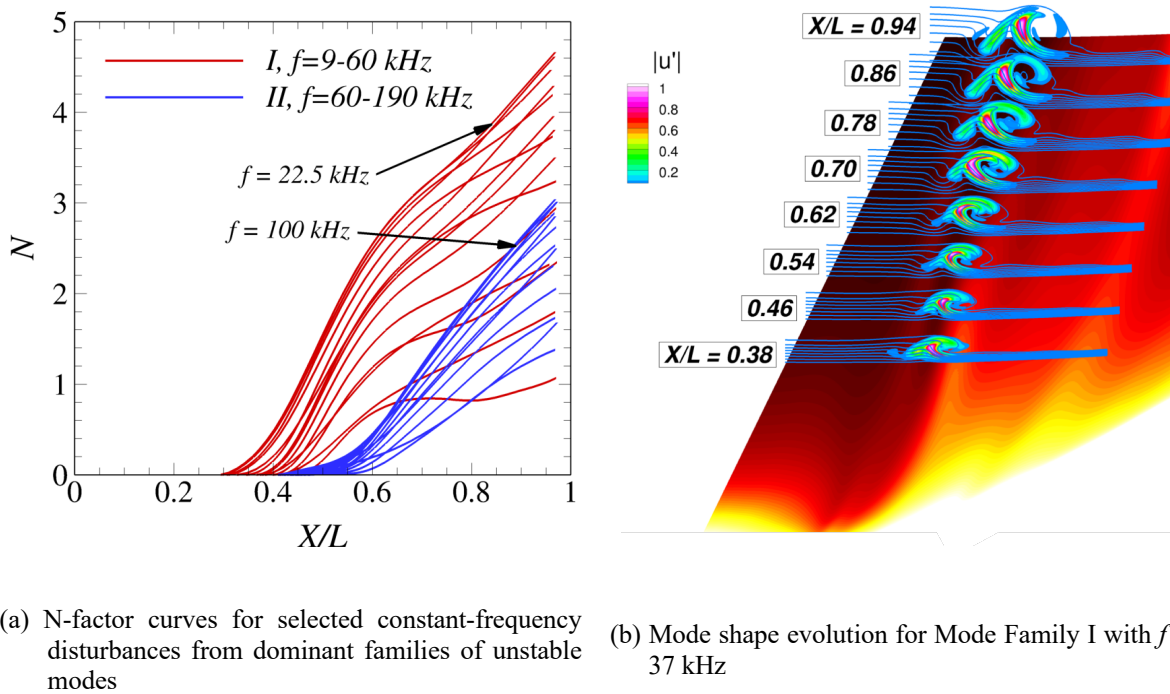
Fig. 10. Growth rate and phase speed as functions of frequency for the relevant instability modes at $X/L = 0.515$. The label “(S)” next to the mode number in part (a) of the figure indicates that the mode shape is symmetric with respect to the center plane. The representative mode shapes of each family indicate the $|u'|$ fluctuations associated with the locally most unstable disturbance frequency from that family. The $|u'|$ values are normalized to a peak value of unity.

The line contours in white in the mode shape plots of Fig. 10(a) indicate the vorticity structure or the streak region of interest adjacent to the center plane, specifically, near the outboard edge of the thickened boundary layer in the vicinity of $Z/L = 0.045$. Modes I through III, respectively denote the shear layer modes that are concentrated in the regions of high basic state shear on the left, top, and bottom of this streak. On the other hand, the wall-normal distribution of the velocity fluctuations associated with Mode IV resembles the typical structure of the Mack mode instabilities. The peak growth rates as well as the frequency bandwidth of the shear layer modes is much larger than

the narrow bandwidth of the Mack modes. These modes are expected to account for the disturbance amplification observed in the stochastically excited perturbation field within the DNS by Thome et al. [17].

The axial phase speeds of Mode Families II, III, and IV are subsonic with respect to the basic state velocity at the edge of the boundary layer. On the other hand, Mode Family I appears to be comprised of unstable disturbances that are supersonic with respect to the velocity at the edge of the boundary layer based on the total enthalpy profile. The existence of supersonic disturbances in the vicinity of the center plane was also noted for the HIFiRE-5b configuration by Choudhari et al. [7]. Similar to the findings for the HIFiRE-5b, the phase speeds of the subsonic modes decrease with an increasing frequency, whereas the supersonic modes appear to exhibit exactly the reverse behavior.

As described in Section II, the next step in our instability analysis involves predicting the spatial evolution of selected disturbance modes via plane marching PSE. The results of the eigenvalue computations are utilized to initiate the plane marching PSE calculations just upstream of the neutral station where any given unstable mode begins to amplify. Figure 11(a) displays the predicted evolution of the N-factor values (i.e., the logarithmic amplification ratios relative to the neutral station) for selected frequencies from the two dominant mode families from Fig. 10, namely, Families I and II. The N-factor values are based on the surface integral of Mack's energy norm over the cross-section of the computational domain at any given axial station [39, 40]. Observe that the disturbances from Mode Family I begin to amplify significantly further upstream in comparison with the most amplified disturbances from Mode Family II. The latter group of disturbances is seen to include substantially higher frequencies (60 kHz to 190 kHz) than those associated with Mode Family I (approximately 9 kHz to 60 kHz). At $X/L = 0.6$, the peak N-factor of the high-frequency Mode II disturbances is approximately 0.5, whereas the most amplified disturbance from Mode Family I has already achieved an N-factor of greater than 2.5. The disturbances from Mode Family II exhibit somewhat higher amplification rates downstream of $X/L = 0.6$; however, the difference in growth rates is too small to offset the initial difference in N-factors. As a result, disturbances belonging to Mode Family I represent the most amplified disturbances at any axial station over the wind tunnel model.



(a) N-factor curves for selected constant-frequency disturbances from dominant families of unstable modes (b) Mode shape evolution for Mode Family I with $f = 37$ kHz

Fig. 11. Amplification characteristics and mode shape evolution predicted by the plane marching PSE for case WT.

The most amplified disturbance from Mode Family I has a frequency of approximately 22.5 kHz. On the other hand, the DNS data from Ref. [16] appeared to indicate $f = 37$ kHz as the dominant frequency on the basis of the dynamic mode decomposition (DMD) of the computed snapshots. A possible explanation for this discrepancy in the most amplified frequency is the location of the stochastic forcing in the DNS from Ref. [16] at $X/L = 0.35$, which is downstream of the neutral location for the 22.5 kHz disturbance ($X/L = 0.30$). The mode shapes of the $f = 37$ kHz

disturbance as shown in Fig. 11(b) are very similar to those extracted from the DNS data in Refs. [16, 17] as plotted in Fig. 14(b) from Kostak and Bowersox [12]. The background contours from Fig. 11(b) correspond to the Stanton number distribution along the surface, with the regions of higher heat transfer indicated in yellow. The left edge of these contours corresponds to the symmetry plane, and a portion of the swept leading edge is visible in the lower right corner. The line contours corresponding to the basic state velocity highlight the streak seen from the mode shape plots shown earlier in Fig. 10(a). The peak velocity fluctuations associated with the $f = 37$ kHz disturbance are primarily concentrated in the stem region of the τ -shaped, mushroom-like structure within the mean velocity contours, rising upward along the stem from $X/L = 0.38$ to $X/L = 0.54$. Over the intermediate portion of the model length ($X/L = 0.54, 0.62, \text{ and } 0.70$ in Fig. 11(b)), however, the peak fluctuations (indicated by the white and pink shades) also extend partially into the outboard segment of the mushroom cap. Thus, the region of peak fluctuations resembles a C-shaped structure at those locations. The dominance of a stem mode adjacent to the center plane was also noted in the HIFiRE-5b case in Ref. [7].

Although not shown in this paper, the dominant high-frequency mode from Mode Family II ($f = 100$ kHz) corresponds to an outboard-cap mode that is primarily concentrated along an elongated shear layer within the outer part of the mushroom cap and a narrower section of another shear layer below the mushroom cap, i.e., closer to the model surface.

Experimentally measured surface pressure spectra near the center plane during the wind tunnel experiments in the Mach 6 Quiet Tunnel (M6QT) at Texas A&M University (TAMU) are shown in Fig. 12(a). The Kulite pressure transducer, sensor K1, is located at $(X/L, Y/L) = (0.736, 0.022)$, whereas the downstream sensor K27 is located on the opposite side of the center plane at $(X/L, Y/L) = (0.912, -0.022)$. Assuming azimuthal symmetry of the disturbance statistics with respect to the center plane, Fig. 12(a) indicates a clear amplification of disturbances at the lower frequencies up to approximately 60 kHz. This range of frequencies overlaps with the frequency range of Mode I disturbances from Fig. 11(a); however, no significant disturbance amplification is observed at the higher frequencies corresponding to the dominant Mode II disturbances. The amplification factor associated with the increase in spectral amplitude from the upstream sensor K1 to the downstream sensor K27 was evaluated and those values are compared with the difference in N-factors associated with disturbances from Mode Family I (Fig 12(b)). The experimental measurements appear to indicate a significantly higher frequency bandwidth as well as a larger amplification in comparison with the Mode I disturbances; however, the peak frequencies in both cases are quite similar. To our knowledge, this is the first comparison of computed amplification characteristics with actually measured disturbances on a complex configuration such as BOLT. As such, this initial comparison between the predicted and the measured spectra is deemed rather encouraging, especially if one considers that the two sensors were not located on the same side of the center plane, and therefore, could have been exposed to somewhat different flow disturbances.

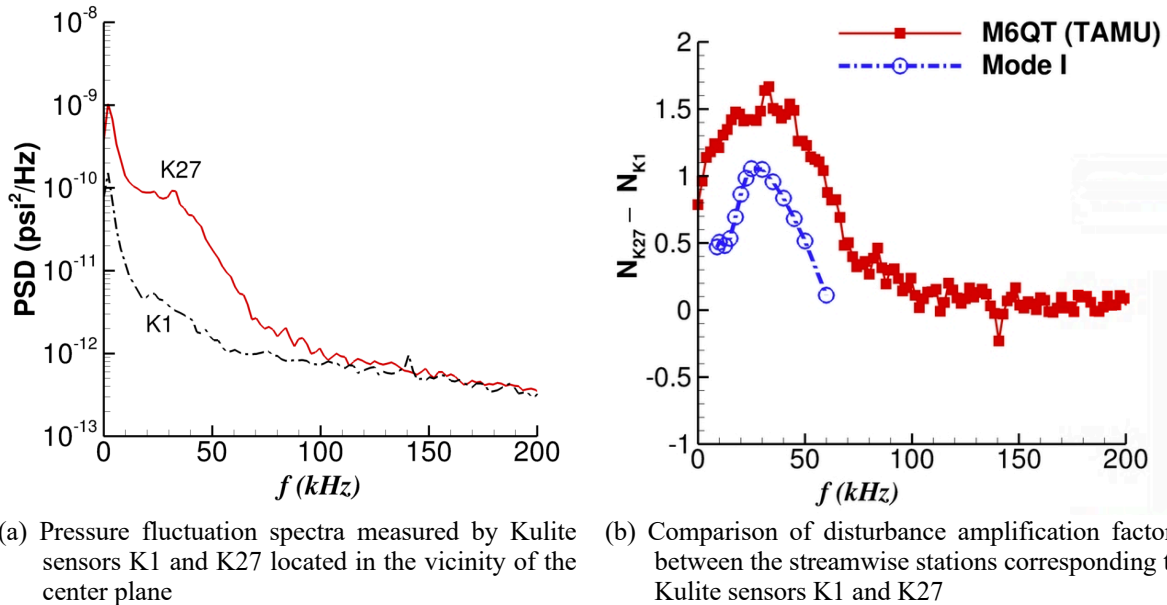


Fig. 12. Surface pressure measurements based on Kulite sensors K1 and K27 from the wind tunnel experiment in the Mach 6 Quiet Tunnel at Texas A&M University (TAMU) [12]. (Experimental data provided by H. Kostak from TAMU.)

B. Case FAL

The growth rate spectra based on the PNP eigenvalue analysis at $X/L = 0.573$ on the low Reynolds number flight configuration FAL are shown in Fig. 13, along with the representative mode shapes for the selected families of modal instabilities. The highest growth rates at this station correspond to Mode Family I, which also has the largest frequency bandwidth of unstable disturbances. The velocity fluctuations associated with this mode are concentrated in the top portion of the shear layer associated with the streak structure, approximately at the azimuthal location where the basic state Mach number contours (shown in white) indicate a slight bulge. Mode II, on the other hand, has its peak signature farther outboard in comparison with the most unstable Mode I disturbance. The Mode II disturbance ($f = 115$ kHz) also exhibits dual peaks in the wall-normal direction, with peak velocity fluctuations both near the edge of the boundary layer and within an internal layer that is located approximately midway between the model surface and the edge of the boundary layer.

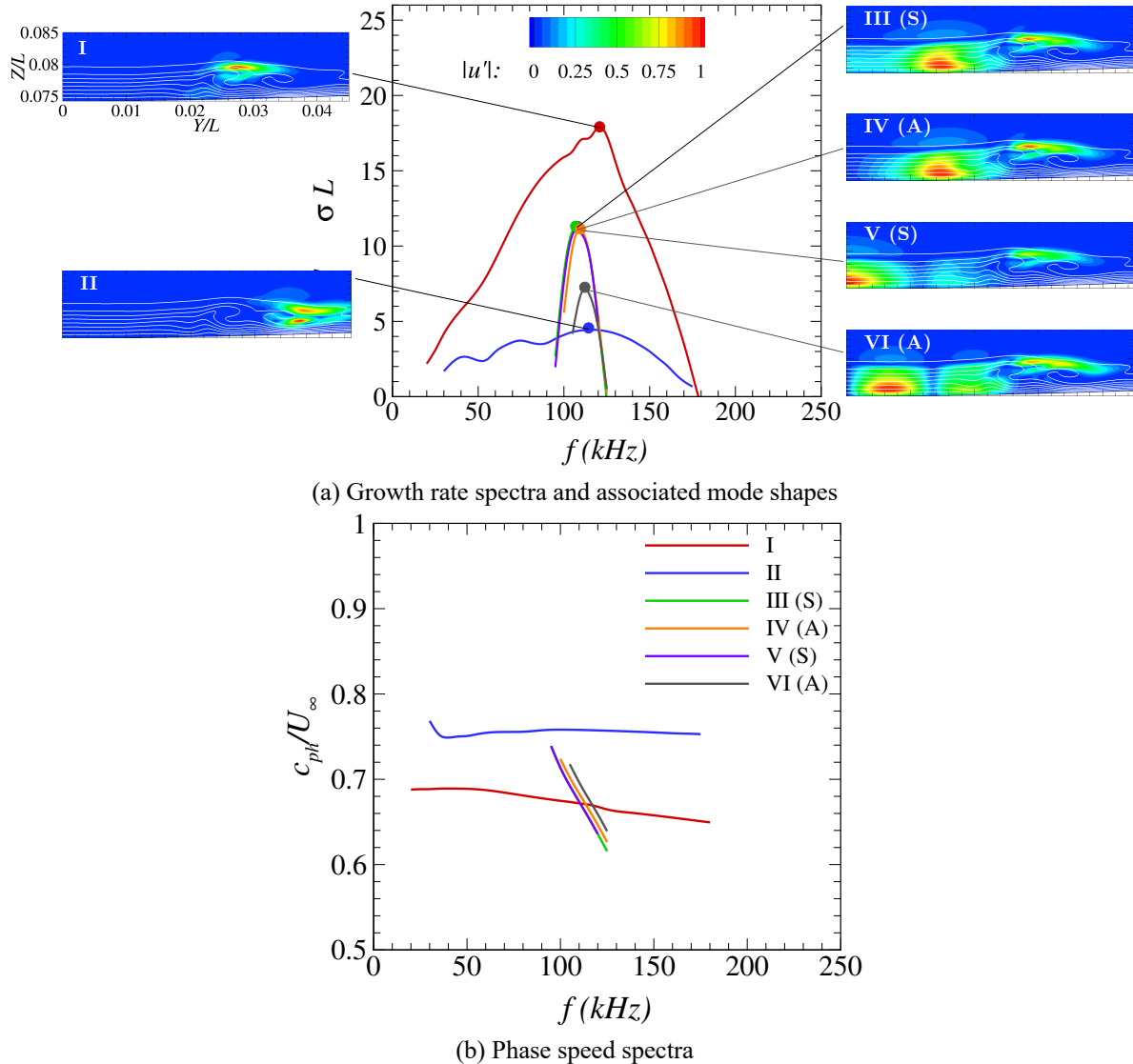
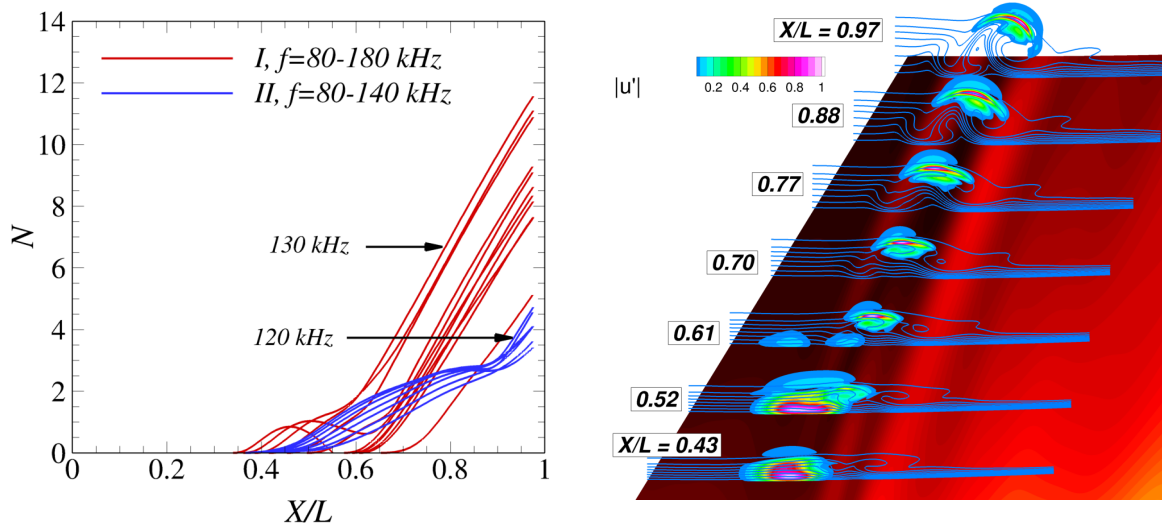


Fig. 13. Growth rate and phase speed as functions of frequency for the relevant instability modes at $X/L = 0.573$. The label within the parentheses next to the mode number indicates whether the mode corresponds to a symmetric (S) or an antisymmetric (A) mode. The representative mode shapes of each family indicate the $|u'|$ fluctuations associated with the locally most unstable disturbance frequency from that family.

Modes III through VI in Fig. 10 are believed to be Mack mode disturbances. Specifically, each of those modes corresponds to a rather narrow range of frequencies and has its primary peak in velocity fluctuations close to the surface, i.e., deep within the boundary layer. While the primary peak occurs inboard of the streak related roll-up, these modes also display a secondary region of fluctuations near the top of the streak. The azimuthal location of the Mode V primary peak is along the center plane itself, whereas the azimuthal peaks of the symmetric Mode III and antisymmetric Mode IV are located away from the center plane. Consequently, the symmetric Mode III has a relatively weak signature at the center plane itself, and therefore, the $|\mu'|$ mode shapes as well as the growth rate spectra for Modes III and IV are nearly the same. Because of the relatively uniform thickness of the boundary layer in the vicinity of the symmetry plane, the frequency range of Mode V, which is concentrated very close to the center plane, and Modes III and IV, which peak away from the center plane, nearly overlap each other. The phase speed plot in Fig. 13(b) highlights the differences between the dispersion characteristics of the shear layer Modes I and II and the Mack mode disturbances from Mode Families III to VI.

The evolution of PSE based N-factors for the shear layer modes for the FAL case is shown in Fig. 14(a), indicating the significant overlap between the frequency range for Mode Families I and II. The Mode Family I corresponds to substantially higher N-factors and, therefore, is likely to dominate the disturbance amplification at this flight condition. The dominant Mode I disturbance corresponds to a frequency of $f = 130$ kHz, which achieves a peak N-factor of $N_{\max} \approx 12$. For lack of sufficient data concerning the transition process associated the streak instabilities of interest, it is difficult to say whether or not the above N-factor is sufficient to cause transition. However, we note that analogous analysis for a single trajectory point of the HIFiRE-5b flight suggested that the N-factor at the measured transition location may be between 14 to 15 [7]. Furthermore, if one considers that the onset of transition due to the streak instabilities of interest is most likely to be at a finite distance from the center plane, the transition front would have to spread laterally in order to produce transition via turbulence contamination along the symmetry plane. Thus, a plausible conclusion would be that the streak instabilities would not cause center plane transition at the flight condition of interest even if transition were to occur slightly away from the center plane.



(a) N-factor curves for selected constant-frequency disturbances from dominant families of unstable modes (b) Mode shape evolution for Mode Family 1 with $f = 130$ kHz

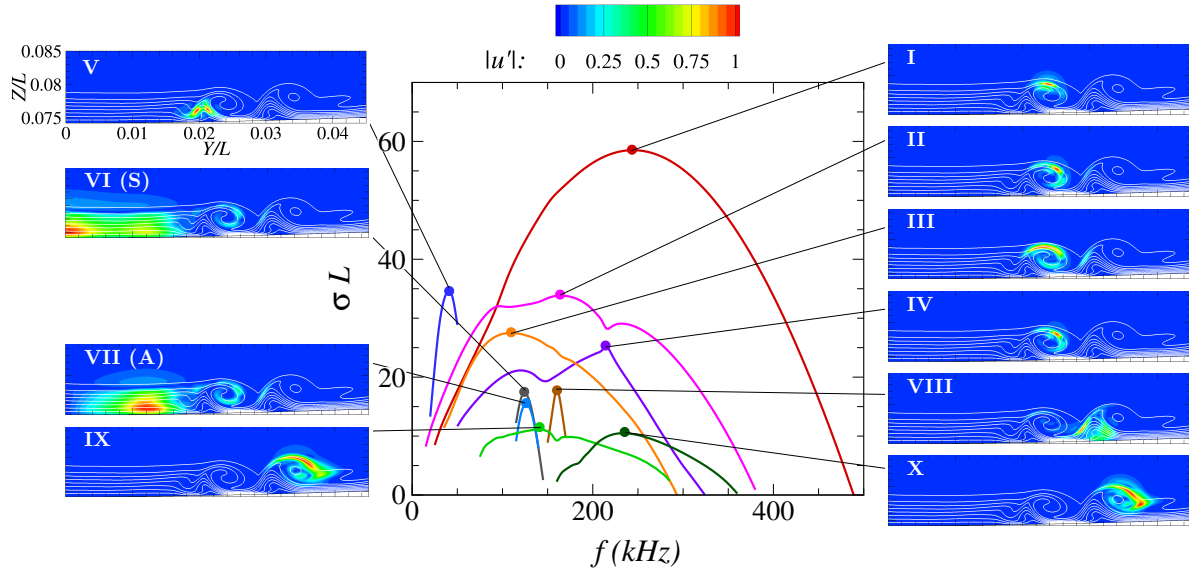
Fig. 14. Amplification characteristics and mode shape evolution predicted by the plane marching PSE for case FAL.

The mode shape evolution of the dominant Mode I disturbance at $f = 130$ kHz is shown in Fig. 14(b), where again the PSE predictions for the $|\mu'|$ mode shapes at selected streamwise locations are superimposed on the heat flux contours along the surface of the BOLT flight article. The dominant Mode I disturbance appears to originate near $X/L = 0.43$ as a Mack mode, with a disturbance peak that is close to the model surface and with significant fluctuations

across the entire thickness of the boundary layer. As the streak structure becomes stronger in the downstream direction ($X/L = 0.61$), the peak fluctuations shift upward to the shear layer near the edge of the boundary layer. At the same time, its near-wall signature appears to develop two distinct secondary peaks in the azimuthal direction. These secondary peaks in the near-wall region disappear at the downstream stations and are not observed at any of the locations starting at $X/L = 0.70$. The Mode I disturbance at 130 kHz appears to be purely a mushroom-cap mode in the downstream region. Although not shown, we note that the most amplified mode from the other, less important family of modes, namely the Family II from Fig. 13(a), is concentrated in the outboard side the streak structure, somewhat analogous to the Mode family II from the wind tunnel configuration (not shown herein). We also observe that the stem mode does not appear to dominate the disturbance amplification process at this flight condition.

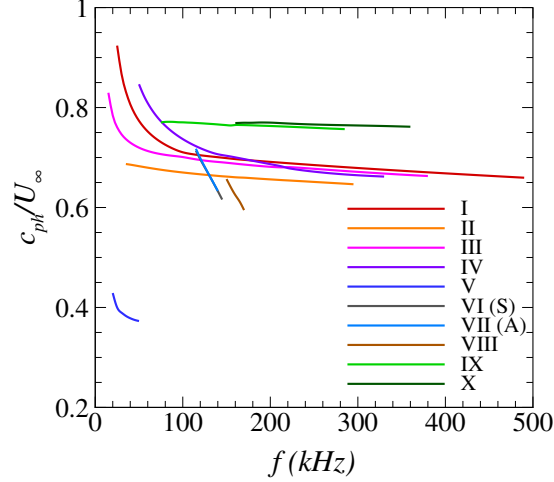
C. Case FAH

Due to the increase in Reynolds number relative to the FAL case and the increased complexity of the basic state with two distinct rolled-up structures, the growth rate spectra for case FAH (Fig. 15) include a significantly larger number of unstable modes corresponding to a greater variety of mode shapes. Besides the mushroom-cap disturbances concentrated within the inner streak (Modes I to IV), and disturbances with Mack mode behavior (Modes VI(S) and VII(A)), the other significant modes include Mode V with an inverted-V-shaped structure, the outboard mushroom-cap Mode X associated with the outer streak, and the shear layer Mode VIII that peaks relatively close to the surface within the outer streak. Mode V is concentrated within the lower part of the inboard mushroom stem as well as the inboard shear layer that feeds the rollup associated with the inner streak. Because the dominant fluctuations induced by the Mode V disturbances occur within the region of low basic state velocity, the phase speeds of these disturbances are relatively low, i.e., supersonic with respect to the basic state beyond the edge of the boundary layer. The Mode I disturbances (with a mode shape at the dominant frequency that is concentrated within the top shear layer on the inboard streak) correspond to both the highest amplification rate at this station and also the largest range of frequencies extending up to nearly 500 kHz.



(a) Growth rate spectra and associated mode shapes

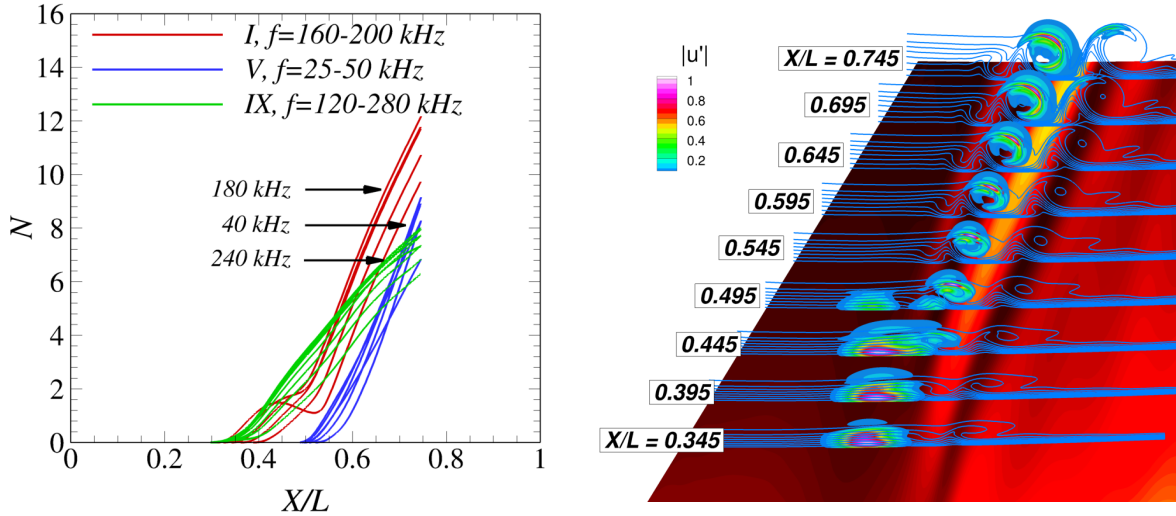
Fig. 15. Growth rate and phase speed as functions of frequency for the relevant instability modes at $X/L = 0.573$. The label within the parentheses next to the mode number indicates whether the mode corresponds to a symmetric (S) or an antisymmetric (A) mode. The representative mode shapes of each family indicate the $|u'|$ fluctuations associated with the locally most unstable disturbance frequency from that family. (Continued on next page.)



(b) Phase speed spectra

Fig. 15. Continued from previous page.

The plane marching PSE analysis for the FAH case showed that the most amplified mode across all families is a member of Mode Family I, with a disturbance frequency of $f = 180$ kHz (Fig. 16(a)). This mode is predicted to achieve an N-factor of slightly over 12 even at $X/L = 0.77$, which represents the outflow boundary of the present computations for case FAH. A linear extrapolation of the N-factor envelope to $X/L = 1.0$ would suggest a peak N-factor of well over 15 in this case, indicating that the onset of transition due to the streak instabilities would be much more likely at the FAH flight condition.



(a) N-factor curves for selected constant-frequency disturbances from dominant families of unstable modes (b) Mode shape evolution for Mode Family I with $f = 180$ kHz

Fig. 16. Amplification characteristics and mode shape evolution predicted by the plane marching PSE for case FAH.

As indicated in Fig. 16(b), the dominant Mode I disturbance in the FAH case appears to originate as a Mack mode ($X/L = 0.345$ to $X/L = 0.445$), similar to the dominant disturbance mode in the FAL case. We note that the Mack mode disturbances correspond to resonant acoustic modes that are trapped within the locally supersonic region near the wall, and hence, they are tuned to a fixed value of the boundary layer thickness. During the early phase of its growth ($X/L = 0.345$), the $f = 180$ kHz mode is confined to a narrow azimuthal region that is appreciably inboard of

the second (i.e., the outboard) streak. The above azimuthal region is located slightly outboard of the nearly horizontal portion of the basic state u -velocity contours, i.e., corresponds to the azimuthal location where the basic state contours begin to develop a slight bulge as a result of the rollup of the inner streak. Consequently, the average boundary layer thickness across the region of dominant 180 kHz fluctuations at $X/L = 0.345$ is expected to be somewhat higher than the boundary layer thickness within the inboard region. However, as X/L increases from $X/L = 0.345$ to $X/L = 0.445$, the boundary layer thickness within the inboard region also increases, causing the location of peak fluctuations at $f = 180$ kHz to shift inboard, consistent with the tuning of Mack mode disturbances to the local thickness of the boundary layer. Yet, the wide enough azimuthal signature of the Mode I disturbance continues to include the inner streak. As the inner streak strengthens farther downstream, the evolution of the 180 kHz disturbance follows a similar trend as that seen during the earlier works devoted to the secondary instability of both quasi-periodic and azimuthally compact patterns of stationary crossflow vortices over a yawed circular cone at Mach 6 [24, 25]. In particular, this Mode I disturbance gains in strength along the shear layer at the top of this streak, and thereafter, amplifies as a purely shear layer instability for $X/L \geq 0.545$. Throughout the region of computation, the azimuthal signature of this mode remains mostly confined to the inner streak; however, eventually, the fluctuations associated with this mode spread to the outer streak as well at $X/L = 0.745$. Because the peak fluctuations associated with this mode are confined to the top of the shear layer where the boundary layer thickness is the largest, the azimuthal location of the peak velocity fluctuations is likely to be shifted away from the location of a hot streak. It would be interesting to analyze the $|p'|$ fluctuations associated with the 180 kHz mode in order to identify the azimuthal locations of peak pressure fluctuations in relation to the hot streak, because, at times, the locations of the surface pressure sensors in wind tunnel experiments happen to coincide with the azimuthal locations of the hot streaks [13].

In passing, we also note that the dominant mode from Family V ($f = 40$ kHz) continues to exhibit the inverted-V structure throughout the region of its amplification, displaying a dual concentration within the stem and the inboard cap of the inner mushroom structure. The phase speed of this disturbance is supersonic at $X/L = 0.573$ and is expected to remain that way through most of its region of amplification. The dominant mode from Mode Family IX is mostly a mushroom cap mode that is concentrated within the outer part of the cap.

V. Summary and Concluding Remarks

This paper described a numerical study of the modal instabilities of boundary layer streaks within the region of boundary layer thickening on either side of the center plane of the BOLT model configuration, thereby extending the similar analysis in our previous work that was focused on the HIFiRE-5b flight experiment [7]. The present application of the state of the art tools in multidimensional stability analysis also complements the other ongoing research related to the BOLT flight campaign.

The basic state computations for a $1/3^{\text{rd}}$ -scale BOLT configuration were shown to be in satisfactory agreement with the heat flux measurements in the Purdue quiet tunnel and also with the off-surface velocity field from a previous DNS study at the University of Minnesota. The range of peak frequencies associated with the predicted amplification spectra indicates encouraging agreement with the measured surface pressure spectra in the TAMU facilities under both quiet and noisy conditions (Figs. 26(a) and 26(b) from Ref. [11] and Figs. 13(a)-13(b) from Ref. [12]), which had indicated a seemingly modal growth across a lobe with very similar frequencies. The Purdue quiet tunnel data did not reveal modal growth of the type observed in the TAMU experiments. While the reasons behind this apparent absence of modal growth in the centerline region are not clear at present, the agreement with the TAMU data highlights the benefits of a collaborative research approach involving multiple redundant measurements and analyses. The need for redundant, collaborative measurements of transition phenomena was first emphasized by the Transition Study Group [42], was fully embraced under the HIFiRE series of flight experiments [43], and has been continued under the subsequent BOLT series.

The agreement with the TAMU data suggests that streak instabilities identified in the present study are likely to play a significant role during the transition process near the symmetry plane. At the low Reynolds number condition of the wind tunnel experiments ($Re_L = 2.85 \times 10^6$), the peak N-factor of the streak instabilities is equal to approximately five. This modest value of N_{max} may explain why transition would not be observed under quiet flow conditions, but why there is a chance of transition onset within a conventional, noisy facility.

Computations were also performed for the full-scale BOLT configuration at two selected flight conditions from the expected flight trajectory. The predicted N-factor curves for a time instant of $t = 26.98$ seconds during the planned flight experiment ($Re_L = 5.72 \times 10^6$) indicate a strong amplification of streak instabilities across the partial length computational domain ($N_{\text{max}} \approx 12.5$ at $X/L = 0.77$). Peak N-factor values of significantly greater than 15 are likely to be observed at the end of the model length, suggesting that nonlinear breakdown due to these streak instabilities would be a potential scenario for transition adjacent to the center plane at this time. On the other hand, a lower value of N_{max}

between 11.5 and 12.0 is predicted at the later time of $t = 28.88$ seconds ($Re_L = 3.68 \times 10^6$). If the N-factor correlation based on a single data point from the HIFiRE-5b configuration ($N \approx 15$) also happens to apply to the BOLT configuration, then the streak instabilities would be unlikely to cause transition at the latter time.

The dominant disturbance types are found to be different between the wind tunnel and the flight cases, respectively, at $\alpha = \beta = 0$ degrees. For the wind tunnel case, the peak fluctuations associated with the most amplified disturbance are concentrated within the stem region of the mushroom-shaped feature in the basic state velocity contours, whereas for the flight cases, it appears to originate as a Mack mode disturbance and subsequently transition to a shear layer instability within the cap region of the mushroom-shaped contours. Both types of modal evolutions have been observed earlier in the case of the secondary instabilities of stationary crossflow vortices over a 7 degrees cone at 6 degrees angle of incidence (or yaw) and the centerline instability of the HIFiRE-5b flight configurations. As such, the above features are believed to be associated with streak instabilities within hypersonic flows.

To our knowledge, the present study is the first to quantify the amplification of streak instabilities in the vicinity of the center plane and to present a direct comparison of the predicted amplification factors with wind tunnel measurements. Several extensions of the stability analysis described in the present paper are currently in progress, and the results of those computations will be reported in the near future.

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