

Predicting Quadcopter Noise With The Lattice-Boltzmann Method

Francois Cadieux*, Michael F. Barad†, James C. Jensen‡, and Cetin C. Kiris§
NASA Ames Research Center, Moffett Field, CA, 94035

I. Nomenclature

c_t = blade tip chord length
 R = blade radius
 Δt = time step
 Δx = Cartesian cell size

II. Introduction

From small delivery Unmanned Aircraft Systems (UAS) to short-haul building-to-building aircraft, urban air mobility is gaining traction as a new approach to tackle the problem of transportation in densely populated areas. NASA’s vision for vertical lift vehicles is to capitalize on and improve unique capabilities to greatly benefit the United States’ growing civil flight requirements. This vision is embodied in the Revolutionary Vertical Lift Technology (RVLT) project [1]. Beyond safety, one of the chief concerns of communities where drones are becoming more popular is the noise they generate. Noise will undoubtedly be one of the major obstacles to public acceptance of any new urban air mobility technology. The ability to predict the acoustics of new conceptual aircraft with multiple rotors and complex fuselages is critical to enable the creation of quieter designs. The objective of this research is to build up a better physical understanding of the noise generated by a typical quadcopter drone and what it takes to predict it from first principles using computational fluid dynamics (CFD) with the Lattice-Boltzmann method (LBM). The specific goals are to establish best practices to predict multi-rotor and vehicle interaction noise with LBM, validate these predictions by comparing to wind tunnel measurements, and assess the computational cost necessary to obtain accurate predictions.

A. Problem Description

Zawodny and Pettingill [2] have recently measured the performance, and characterized the acoustics of a small quadcopter UAS in forward flight inside the Low Speed Aeroacoustic Wind Tunnel (LSAWT) at the NASA Langley Research Center. Since small multi-copter UAS are rapidly gaining popularity for a number of purposes, this case is a relevant and representative starting point for package delivery drones and future urban air mobility rotor-craft. This experimental validation case will be used to assess the noise prediction accuracy and computational cost of the Cartesian adaptive mesh refinement (AMR) LBM approach. The UAS is stationary at an angle of attack of $\alpha_v = -10^\circ$ with respect to the oncoming Mach 0.045 flow. Each blade rotates at the mean measured rotation rate from the experiment detailed in [2], and are reproduced in figure 1 for reference. The goal is to predict the noise generated by the UAS, focusing on the tonal noise associated with the blade passing frequency (BPF) of the front and rear rotor pairs, with an eye toward capturing broadband noise due to rotor-rotor interaction as well as rotor-airframe interaction.

III. Methodology

A. Numerical methods

The Lattice-Boltzmann solver uses the Launch, Ascent, and Vehicle Aerodynamics (LAVA) Cartesian AMR infrastructure described in Kiris et al. [3] but solves simplified LBM kinetic equations that retain just enough detail to

*Science & Technology Corporation, AIAA member, francois.cadieux@nasa.gov

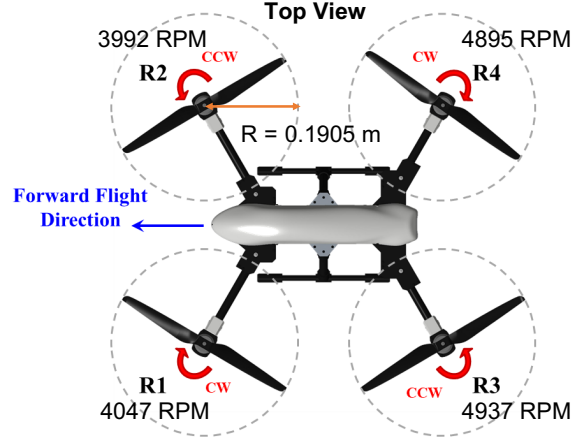
†Computational Aerosciences, AIAA Senior Member, michael.f.barad@nasa.gov

‡Computational Aerosciences, AIAA Member, james.c.jensen@nasa.gov

§Computational Aerosciences, AIAA Senior Member, cetin.c.kiris@nasa.gov



(a) Experimental setup in the LSAWT facility [2]



(b) Top view detailing rotation rates of reach rotor

Fig. 1 Problem description

satisfy the desired macroscopic equations of fluid motion (weakly compressible, isothermal Navier-Stokes equations in the present context). The Entropic Multiple-Relaxation Time (EMRT) collision model is used in this work [4] for its robustness and targeted dissipation that is large enough to avoid the necessity of subgrid-scale modeling. The details of the implementation are available in Barad et al. [5].

B. Geometry Representation

The quadcopter fuselage is represented via discrete watertight triangulation generated from the high fidelity as-tested CAD representation without any simplifications other than to exclude internal components. Pictures of the surface mesh highlighting the level of detail represented are shown in figure 2. The fuselage triangulation contains 356,478 triangles. For lifting devices, our best practice is to use scanned geometry when it is available to ensure that the simulations are representative of the as-built and as-tested model, as opposed to the as-designed model. However, we chose to deviate from this approach in this case due to uncertainties associated with the available scanned data. The scanned version was unreliable near the leading edge, trailing edge, and tip of the blade. The geometry of the blade was thus described using a number of discrete airfoil sections and twist angles at corresponding spanwise stations. From this description, each airfoil section is splined and the points on its curve redistributed to ensure that all sections can be joined into a single grid. The airfoil sections are terminated in a blunt trailing edge, whereas pictures and scanned data both show a rounded and fairly sharp trailing edge. To ensure that the grid would be representative of the real geometry, a spline was fit between the two points forming the blunt trailing edge by matching the upper and lower surface tangents. The blades were then discretized with triangles following best practices for airfoils (roughly 50 points in the chord direction with strong clustering in the leading and trailing edge), and then spacing no greater than 40% of c_t in the spanwise direction. The airfoil section based reconstruction of the blade was slightly better than the scanned version, but had some non-smooth spanwise curves which led to spurious cross-flow vortices being shed off of the trailing edge away from the tip in initial simulations. The spanwise curves were thus re-splined and smoothed. The tip definition was also missing compared to pictures of the blade, so one was created – see figure 3c. An extra section made from a scaled down version of the outermost airfoil section was added to make the recreated blade have a similar rounded tip to the actual blade seen in figure 1a. Each blade was assumed to be identical and contains 60,972 faces.

C. Boundary Conditions

A subsonic inflow boundary condition is applied at $x = -172R$, where the freestream velocity is specified and density is extrapolated from the interior. Subsonic outflow is applied at $x = 172R$ where the density is set to the reference conditions and the velocity is extrapolated from the interior. We used a shear-free extrapolation condition in the y and z directions. These far-field boundary conditions are applied by filling in domain ghost cell layers with $f = f_{eq}$ where f_{eq} is computed from the conditions described above.

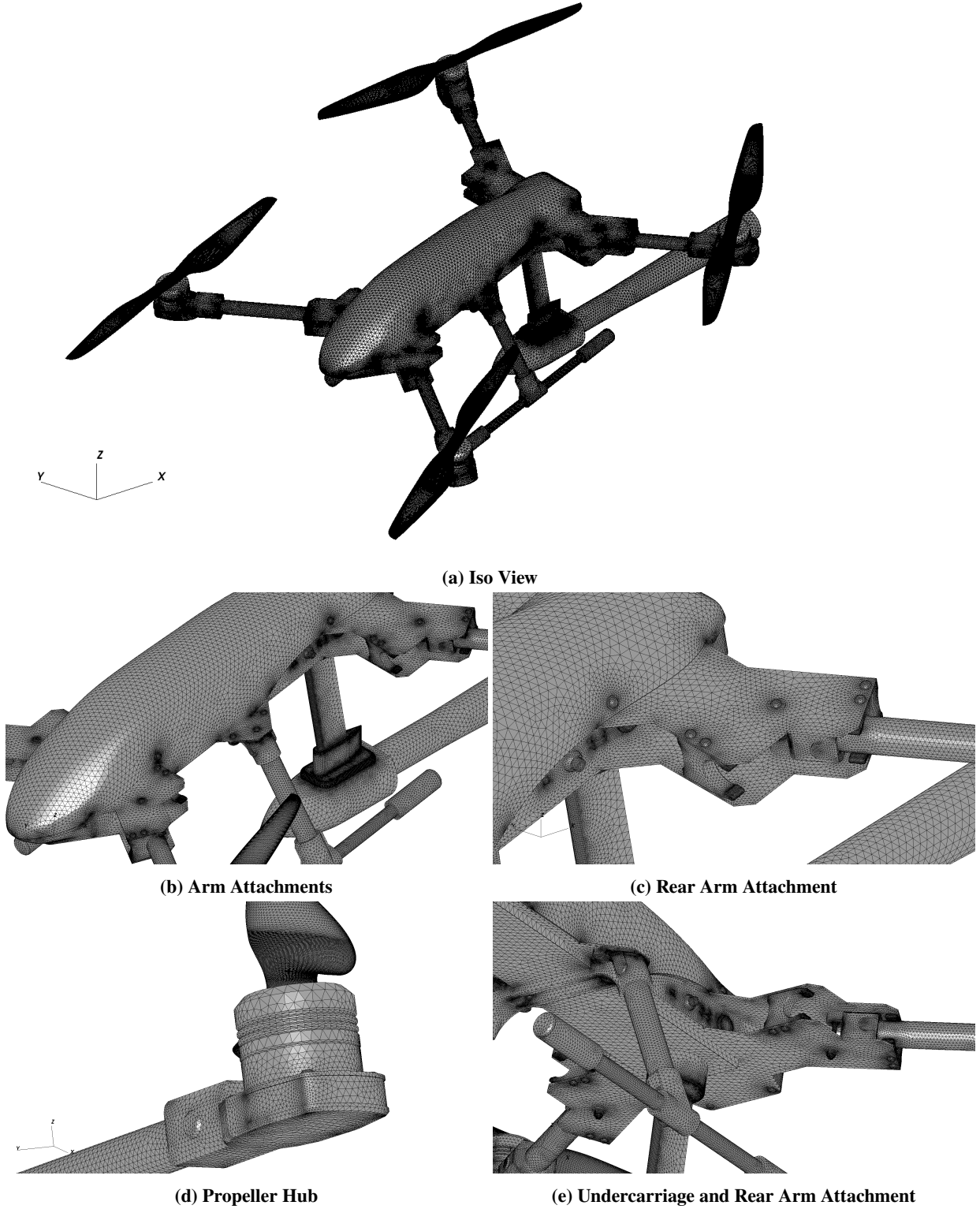


Fig. 2 Quadcopter Surface Mesh

Arbitrary relative motion is handled seamlessly by physically moving the blades in space and making use of in-out testing and ray-casting to determine the set of cells next to the object's new position at the new time t_{n+1} . Given the low

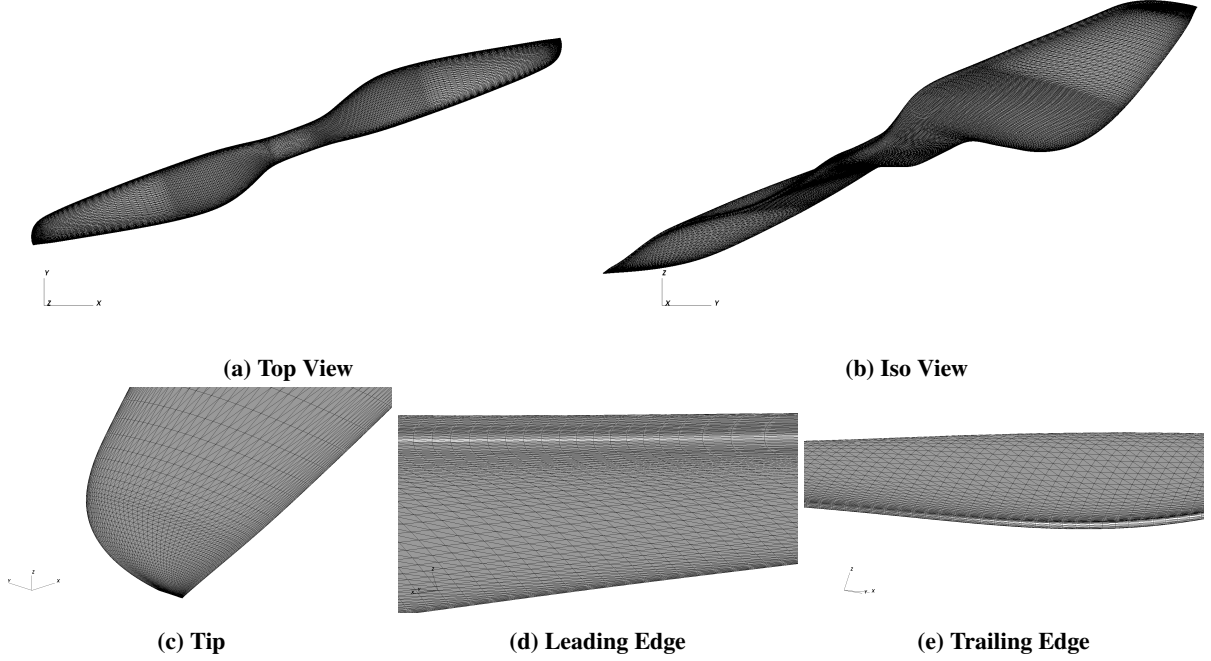


Fig. 3 Propeller Blade Surface Mesh

chord Reynolds number of the blades: $Re_c = \rho c_t 2\pi R \Omega / \mu \approx 50,000$, the boundary layer on the blades is most likely laminar and attached. Moreover, from previous curvilinear body-fitted unsteady RANS simulations of a single one of these blades in hover at a similar RPM, the measured forces on the blades are dominated by inviscid effects, which are roughly 3 orders of magnitude higher than the viscous forces. Given that the LBM is limited to Cartesian isotropic cells, it would be computationally intractable to resolve the razor thin laminar boundary layers on the blade. With these facts in mind, the rotors and the fuselage are treated as slip walls in an otherwise viscous scale-resolving simulation.

There are many ways to implement slip boundary conditions in the LBM for immersed boundaries. The approach employed here is to correct the information that was streamed from inside or across the thin blades after the streaming step. Broken links are identified along with their intersections on the geometry, the geometry velocity vector at the intersection point, and the normal to the surface at the intersection point $\hat{n}$. With this information, the density is obtained from the opposite (valid) link, while the velocity is interpolated using the full velocity vector at the neighbor U_p along the link along with a reconstructed wall velocity $U_w = (U_{body} \cdot \hat{n})\hat{n} + (U_p - (U_p \cdot \hat{n})\hat{n})$. The probability density functions in the opposite directions of the broken links are then set to the equilibrium distribution which most closely matches the density and velocity computed above. This provides shear-free treatment of walls that is robust and does not require information beyond direct neighbors. However, this approach can cause dipole noise sources in regions of high curvature that are under-resolved by the Cartesian cells due to intersections lying above and then below the geometry in close succession in the spanwise direction. This pitter-patter that is most prominent when the blade is aligned with the Cartesian directions is mostly avoided here because the vehicle is in forward flight with a -10 degree angle of attack and no attempt was made to rotate the grid with respect to the vehicle – the grid is aligned with the Cartesian coordinate system. Development of a quieter slip boundary condition is ongoing, alongside efforts to develop a robust and adaptive wall-model which would reduce to slip on highly under-resolved boundary layers.

D. Grid

A grid study where the fine voxel size Δx was refined by factors of 2 from 40% to 2.5% c_t was performed to determine the level of resolution required to capture loads, as well as tonal and broadband noise. Not knowing ahead of time the nature of the solution, a new adaptive mesh refinement (AMR) procedure was developed. A base level of resolution required to capture the highest frequency (20 kHz) of interest is computed ($\Delta x / c_t \approx 40\%$), and specified within the volume of our FWH surface to ensure all acoustic content up to this frequency would reach it. We also specify four cylinders encompassing the rotors to be covered with the finest voxels to ensure the blade geometry

is well-represented. Then, based on on-the-fly statistics, we compute the time-averaged turbulent kinetic energy ($u'_i u'_i = \langle u_i u_i \rangle - \langle u_i \rangle \langle u_i \rangle$) and the pressure fluctuations ($p' = \langle p p \rangle - \langle p \rangle \langle p \rangle$), normalize each of them by their respective global maximums, and use their sum to tag cells for refinement or coarsening. All regions with non-zero values are refined until the specified maximum number of cells per level is reached, which is a user input in order to avoid large meshes that would exceed the computational resources allotted for the problem. The purpose of this approach is to reach an optimal grid that will not change in time after a few rotor revolutions. This is important because we have noted in the past that grid refinement and coarsening changes in time can affect acoustic wave propagation, especially at frequencies marginally resolved by the coarser mesh. Figure 4 shows the three finest level of mesh on the left half of the vehicle after 4 revolutions and the mesh has converged and no longer changes for the 5% c_t case, and isosurfaces of Q-criterion colored by vertical velocity are shown on the right, giving some context to the distribution of the voxels.

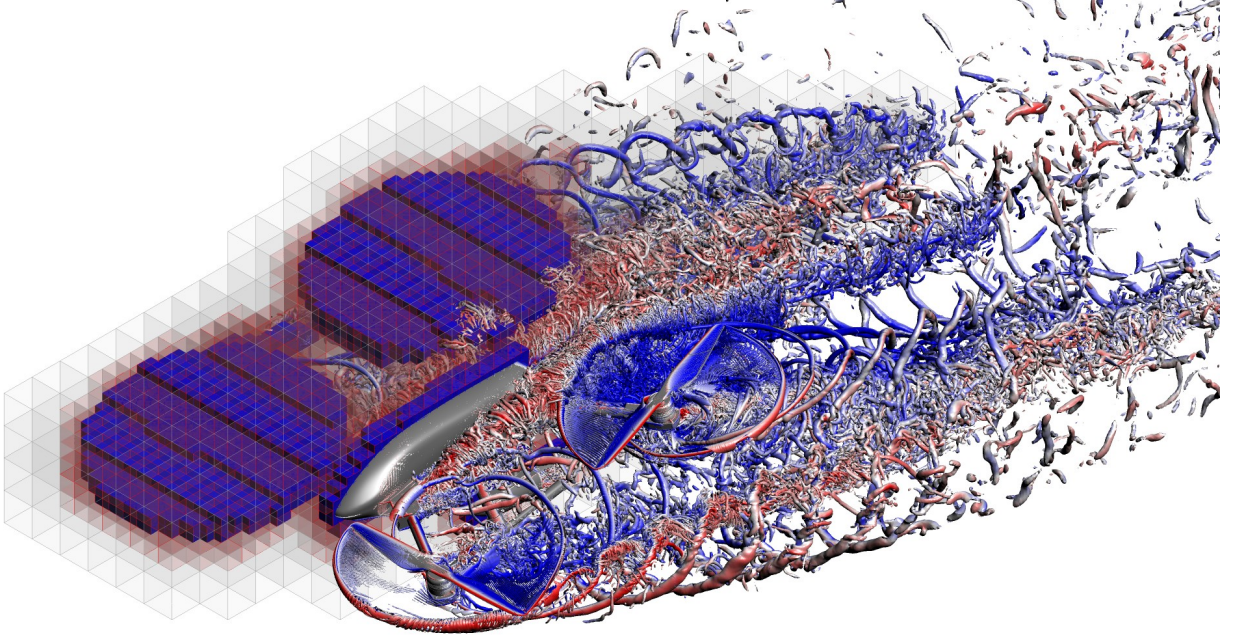


Fig. 4 Snapshot of finest three levels of voxels in the 5% c_t simulation and their placement on the left of the vehicle, along with isosurfaces of Q-Criterion colored by vertical velocity.

E. Acoustics

The simulation is impulsively started with a constant RPM for each rotor, and runs for 4 rotations of the slowest rotor before forces and acoustic data are recorded. The physics of the problem dictates the lowest frequency of interest through the lowest blade passing frequency of 133.1 Hz, whereas the range of most human hearing dictates the highest frequency considered of 20 kHz. Simulations are run for a total of 30 of the slowest rotor revolutions which corresponds to roughly 0.45 seconds. Pressure and velocity are interpolated to the FWH triangulation at regular intervals with a sampling frequency of 36 kHz, to give a maximum resolved frequency of 18 kHz. The triangle edge length of the FWH surface is equal to that of the CFD mesh in that region at 40% of c_t and contains a total of 1,371,960 triangles. Its shape and location is detailed in figure 5. The fully parallelized far-field noise propagation module in LAVA is used to obtain narrow-band spectra at the microphone locations described in the experiment. Hann windowed and energy corrected spectra are obtained for each segment of 0.2 seconds with 75% overlap contained in the 0.4 seconds of FWH surface data. The details of this procedure are chosen to match those used in Zawodny and Pettingill [2] to analyze the experimental data.

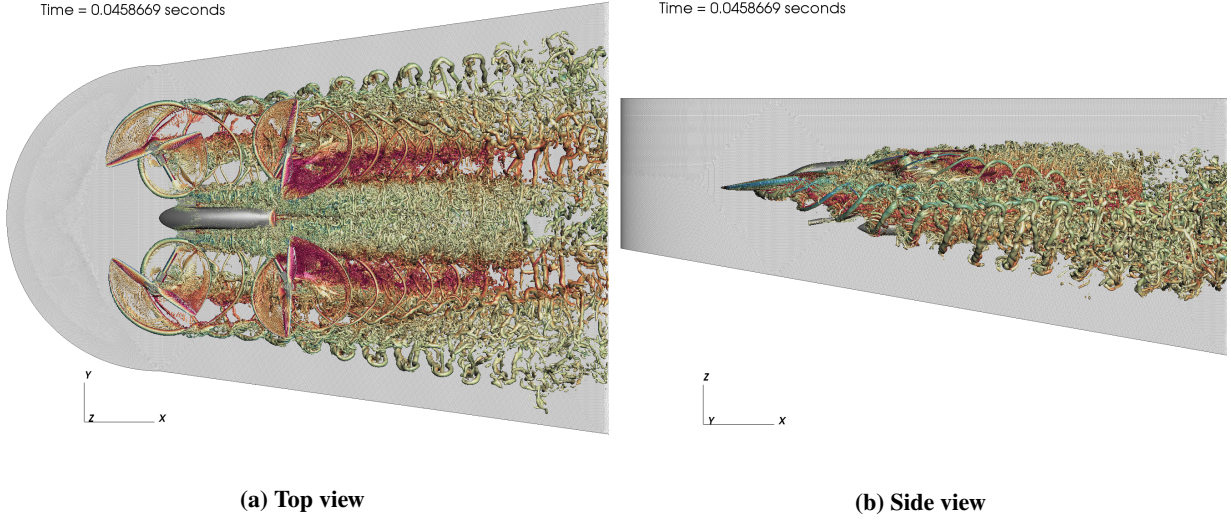


Fig. 5 Size, shape, and placement of the open FWH surface used for far-field noise propagation. Vehicle is shown with isosurfaces of Q-criterion colored by vertical velocity for a snapshot during the 5% c_t simulation.

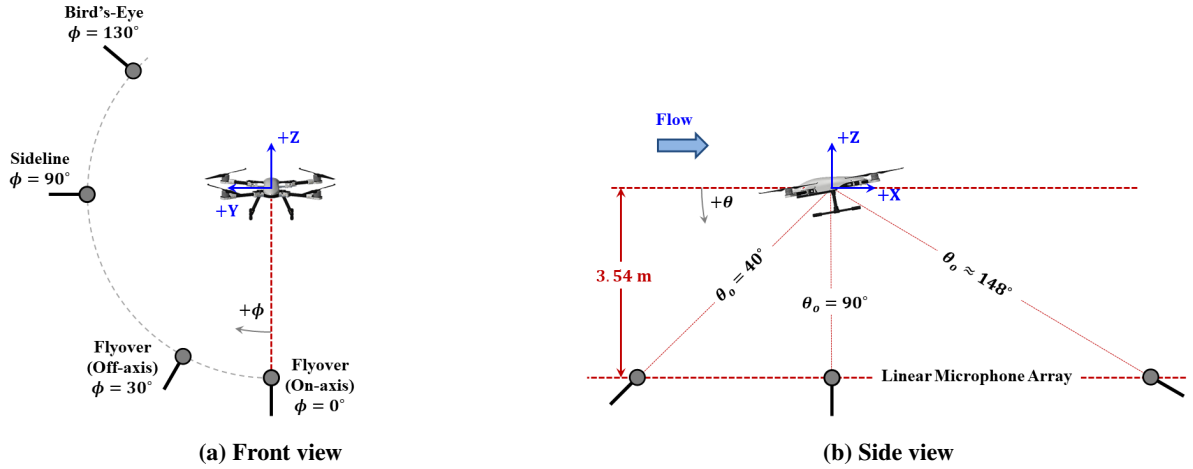


Fig. 6 Location of microphones with respect to vehicle [2].

IV. Results

Figure 7 details the convergence of the time-averaged thrust recorded on the vehicle with respect to the finest voxel size. It also displays the total number of cells, and core-hours necessary to simulate 30 revolutions. It is clear from this figure that LBM is not the most efficient way to obtain accurate average loads. Nevertheless, past results indicate that Cartesian-based approach can predict tonal noise accurately without matching the forces on the blade to the same accuracy as the body-fitted approaches [6].

Figure 8 shows a snapshot demonstrating the level of detail captured in these simulations in terms of turbulent structures identified by the Q-criterion. It also makes acoustic perturbations stand out on cut planes to the left of and below the vehicle through the magnitude of the gradient of pressure. The use of the logarithm helps to surface the smaller amplitude pressure waves and highlights the minimum wavelength that can be captured on each of the mesh resolution levels. It is particularly satisfying to see that the waves are advected with minimal numerical dissipation and without reflections through successive fine-to-coarse transitions, which is difficult to accomplish in a Navier-Stokes setting. We owe this to the coarse-fine interface treatment that conserves both mass and momentum to machine precision [5]. This figure also highlights one of the likely sources of broadband noise beyond self-noise and rotor-rotor non-linear

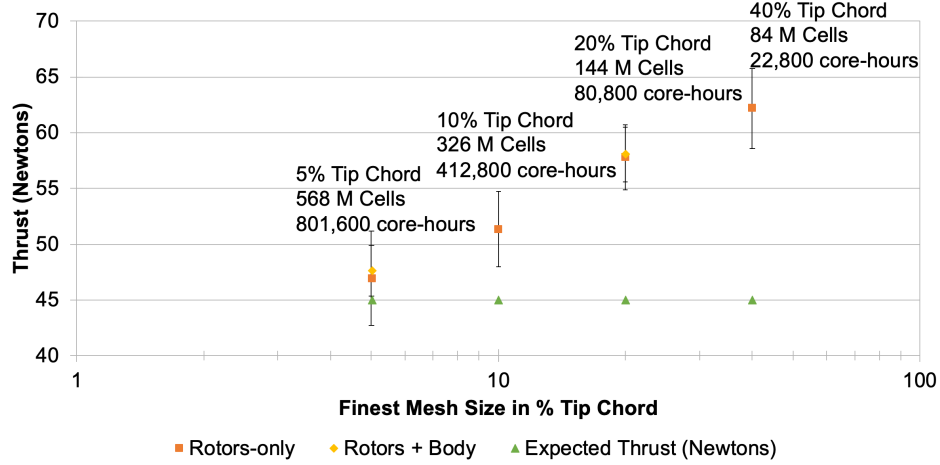


Fig. 7 Time-averaged thrust with respect to cells size. Total number of cells for each case is also noted, along with the computational resources required to simulate 30 revolutions.

interactions: the rear rotors cut through the turbulent wake of the arm attachment location on the fuselage. We also see evidence of rotor-rotor non-linear interaction noise through instabilities in the Q-criterion along the line which connects the outboard section of the front rotors to the outboard section of the rear rotors. This phenomenon is often under-resolved and dissipated which leads to the appearance of so-called “super” tip vortices linking the front to the back rotors in a single vortex tube.

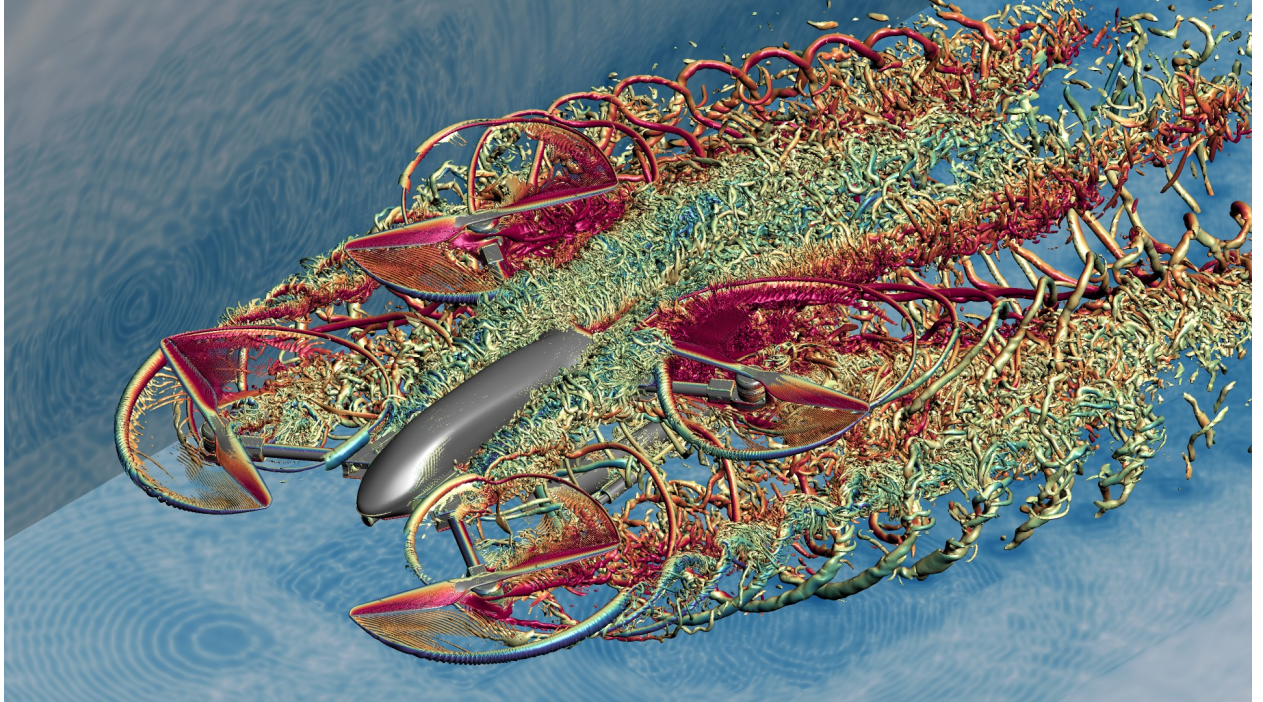


Fig. 8 Snapshot of 5% c_t simulation showing isosurfaces of Q-Criterion colored by vertical velocity, along with cut planes showing the logarithm of the magnitude of the pressure gradient to identify pressure waves.

Results from far-field acoustic propagation using permeable FWH in frequency domain are shown in figure 9 for the on axis fly-over location ($\phi = 0^\circ$) and slightly upstream of the vehicle ($\theta = 70^\circ$) and compared to wind tunnel measurements. The blade passing frequency (BPF) of rotors R3 and R4 ($R3 + R4$) whose peak falls on 165 Hz due

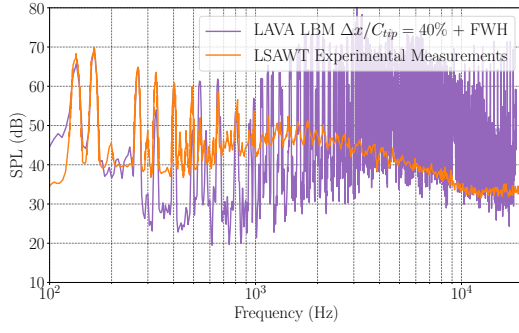
to the $\Delta f = 5$ Hz is predicted in perfect agreement with experiment regardless of resolution. This is to be expected because the physical source of the first BPF is predominantly the blade loading and displacement. So any similarly sized object with the right metal angle (causing flow turning) and thickness at that rotation rate would likely be predicted to have this sound pressure level (SPL). Similarly, all of the simulations predict the same SPL for BPF $R1 + R2$, but it is 3 dB lower than that measured in the experiment. This is due to the fact that each rotor has a different revolution rate causing a beating phenomenon with its neighbor. The front rotor pair has beating period of approximately 1.1 seconds, while the rear rotor pair completes a beating cycle every 1.4 seconds. We did not obtain an accurate picture of the full acoustics because we simulated only 0.4 seconds and thus did not capture the full effects of the beats constructive and destructive interference compared to the experiment's 12 seconds which capture multiple beating cycles.

The 40 and 20% simulations predict the SPL at the frequency associated with the second harmonic of BPF $R1 + R2$ to be 65 dB, perfectly in line with the experimental measurements. However, the 10 and 5% simulation results then drop to 61 dB for that frequency, 4 dB lower than the experiment. The finer mesh results are in line with expectations due to the under-prediction of the first $R1 + R2$ tone. We also expect the discrepancy to get larger because the difference between the harmonics of rotor $R1$ to that of $R2$ grows proportionally to n for the n^{th} harmonic, and will be greater than Δf for $n \geq 3$. This is consistent with the under-prediction of the higher harmonics at 405 Hz, 540 Hz and 675 Hz in the finest simulation which land roughly 7 dB lower than the experiment. Conversely, it is not entirely surprising that the two coarsest simulations predict higher SPL for the higher harmonics because the mesh cannot support more complex difference interactions. We see much lower levels between the harmonics, meaning that no energy was transferred to the interactions like $((2 \times 165) - 135 = 195 \text{ Hz})$ corresponding to destructive interference between harmonics of different rotors. Instead the energy keeps piling up in the n^{th} harmonics of individual rotors. This phenomenon is especially visible at frequencies beyond 1 kHz for the 40% and 20% c_t simulations where the SPL reach spuriously high values of over 60 dB.

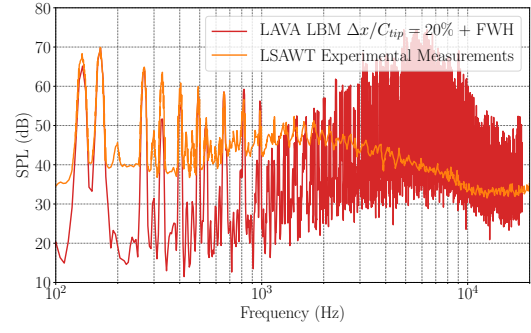
As the mesh gets finer, we start to resolve these interactions. For example, the content at 195 Hz goes from 30 to 43 dB. Indeed, there are other interaction tones at 225 and 240 Hz that explain the broad bump seen in the 5% simulation results from 195 to 250 Hz. But these interactions at 225 and 240 Hz involve higher harmonics of $R1 + R2$ which we know are directly affected by the beating phenomenon, so we expect their RMS to be lower the experiment's. This is consistent with the difference between the 5% c_t simulation results and the experimental values at 225 and 240 Hz, as well as 360 and 375 Hz. The experimental results show a strong constructive interference peak at BPF($R1 + R2$) + BPF($R3 + R4$) of 48 dB at 295 to 300 Hz, whereas the SPL hovers around 40 dB in the simulation results which is again explained by our short simulation period.

The SPL associated with the second harmonic of BPF $R3 + R4$ at 330 Hz is consistently under-predicted across all resolutions despite the first harmonic being predicted rather accurately. The SPL at this frequency is predicted to be roughly 54, 52, 47.5, 48 dB for the 40, 20, 10, and 5% c_t simulations respectively. The fact that the last two simulations see no significant change could mean that the value is mesh converged. One could argue that this is again due to the beating phenomenon and that we have captured such a small and uneven portion (57%) of the rear rotor pair's second interference cycle that this level of disagreement is not entirely surprising. The higher harmonics of BPF $R3 + R4$ at 495, 660, 825, 990 Hz move closer to the experimental measurements with differences shrinking from 8 to 4 to 2 and to under 2 dB respectively. This is consistent with their interference cycle diminishing and multiple periods being included in each 0.2 second segment.

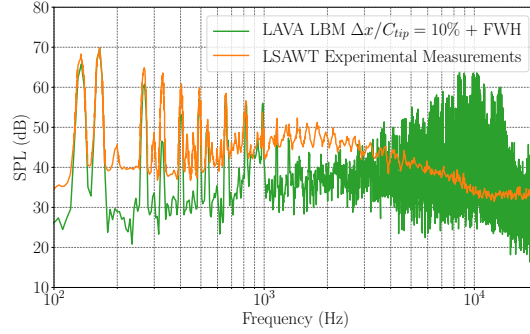
Beyond 1 kHz, the spectrum is dominated by broadband interaction noise [2]. Much like turbulence, broadband noise is often driven by non-linear interactions between all wavelengths. This unfortunately means that the beating phenomenon continues to be a source of discrepancy between our results and the experiment's. However, qualitatively, the trends are well represented in the 5% c_t simulation results, which is highly encouraging. The maximum SPL track the experimentally measured levels quite well until roughly 8 kHz. Beyond this range we are likely seeing the effects of aliasing because of our relatively infrequent sampling compared to our time step. There is a factor of 4 between how often we interpolate the onto the FWH surface and save it to disk, versus how often we perform a time step on that level in the subcycling algorithm. This approach was chosen to minimize the amount of disk I/O, which increases significantly the cost of the time step at which writing is performed since the LBM approach is so fast. In retrospect, this hit in performance is probably worthwhile for this problem to avoid such issues where the spectrum shows large spurious oscillations in amplitude from one frequency to the next beyond 8 kHz. Another cause for the smaller but still noticeable oscillations in amplitude from one frequency to the next in the 1 kHz to 8 kHz range beyond the log scale of the axis is the relatively short integration time. As more windows of 0.2 seconds are averaged, the oscillations in the broadband segment of the spectrum should tend to average out from one realization to the next. This is one of the features of broadband noise, as opposed to tonal noise, which should remain constant from one realization to the next given a long



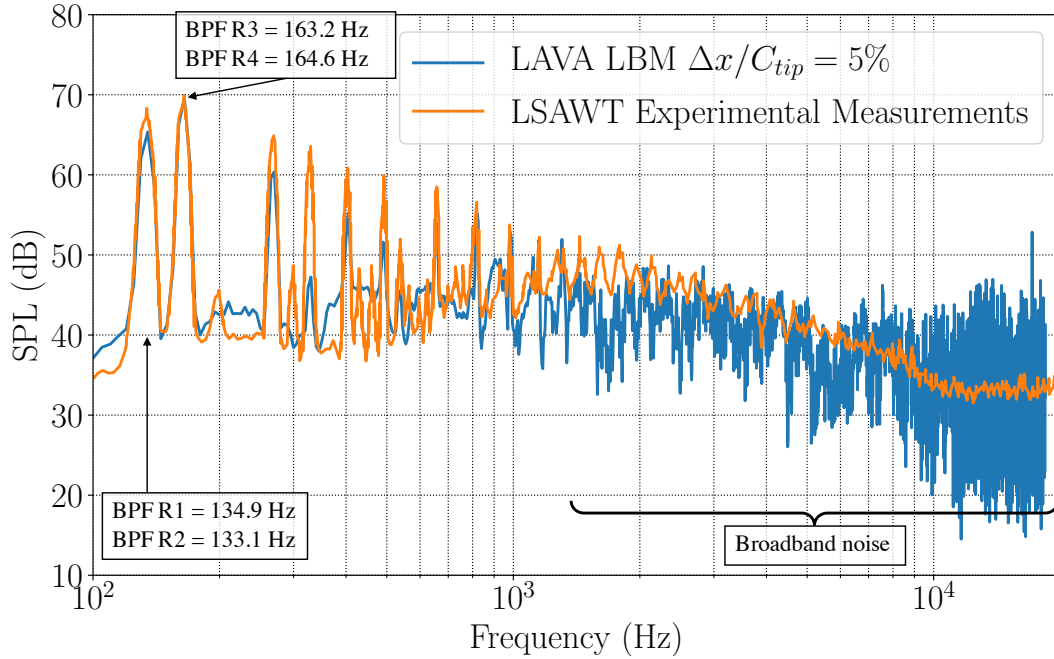
(a) 40% c_t



(b) 20% c_t



(c) 10% c_t



(d) 5% c_t

Fig. 9 Narrow-band spectrum at $(\phi = 0^\circ, \theta = 70^\circ)$ for different mesh resolutions.

enough period. This is not the case here: the tonal noise amplitudes in each 0.2 second segment vary significantly because of the long period of each rotor pair interference (beats). This could be alleviated by choosing a window length on the order of the longest interference cycle of 1.4 seconds, at the cost of increasing statistical uncertainty at each frequency (fewer segments for averaging). The high frequency spectrum could also be further smoothed by adding more observers (fictitious microphones) in the vicinity and averaging them: e.g. $\phi = [-2^\circ, +2^\circ]$ and $\theta = [68^\circ, 72^\circ]$.

V. Discussion

The major obstacle for Cartesian AMR approaches to simulate thin low Reynolds number propellers is the inability to efficiently resolve boundary layers due to non body-fitted isotropic cells. Even when we ignore the boundary layer as is done in this study, the Cartesian approach requires significantly larger number of cells to capture the curvature of the blade and thus the forces acting on it accurately. This leads to longer turn-around times due in part to the explicit nature of the method – adding an extra level of resolutions means reducing the time step size by a factor of 2 and correspondingly increasing the number of time steps to reach 0.45 by a factor of 2. Subcyclng significantly reduces this expense by only advancing the coarse levels when they are synchronized with the finest. Nevertheless, if the finest levels is where the bulk of the cells are, which usually holds true, then the computational cost goes up by a factor between 2 and 4 as seen in figure 7. Thus although LBM may be 10 times faster than a body fitted Navier-Stokes solver per time step, it may have to do many more time steps on many more cells, and so the turnaround time may be similar or worse for LBM. Its main advantage over traditional Navier-Stokes approaches then becomes its unambiguously better effective resolution for an algorithm that only needs neighbor information, which is a big advantage for scale-resolving simulations focused on acoustics like this one.

Fine-to-coarse transitions in the Cartesian AMR mesh paradigm can be particularly problematic for acoustics. Acoustic or vortical waves that are resolved on the fine mesh but have a wavelength shorter than 2 to 4 times the adjacent coarse mesh cell size simply cannot be represented on the coarse mesh. In a Navier-Stokes setting, this can lead to reflections at such boundaries, and how to best handle fine-to-coarse interfaces in scale-resolving simulations is still an area of research. This problem of fine-to-coarse interfaces is better posed in the LBM, where mass conservation is guaranteed, but it still remains an active topic of research because the level of numerical dissipation or SGS dissipation is generally linked to the cell size and both vortical waves and turbulence are known to be sensitive to any abrupt changes in these quantities. Acoustic waves were not observed reflecting off of fine-to-coarse transitions in these simulations as can be gleaned from the cut planes in figure 8. The content that cannot be resolved on the coarse meshes seems to be dissipated relatively smoothly by the EMRT scheme.

VI. Summary

We performed a number of Lattice-Boltzmann simulations of a quadcopter in forward flight to explore grid sensitivity in loads and acoustic predictions. Thrust was predicted within 4.5% error at 5% tip chord resolution which is disappointing given the large computational expense. There is hope that this agreement can be improved with a better slip boundary condition treatment. At this resolution, when the LBM is coupled with the FWH far-field propagation module within LAVA, the tonal noise predicted is in reasonably close agreement with the experimental microphone data of Zawodny and Pettingill [2]. Differences in tonal noise between simulation and experiment are explained by the beating phenomenon caused by the slight variation in rotation rate between front and back rotor pairs, and the short duration of the simulations compared to the experiment. Despite these differences, the trends in broadband noise levels at frequencies between 1 kHz and 8 kHz are well predicted compared to the levels measured in the experiment. To the authors' knowledge, this level of agreement for broadband noise in a complex quadcopter configuration in forward flight is unprecedented.

In the final paper, we will report the results for a 2.5% c_t mesh resolution for loads and acoustics, and present acoustic results from other microphone locations.

Acknowledgments

This work is funded by the NASA RVLt and Transformational Tool and Technologies (T³) projects. Computer resources were provided by the NASA Advanced Supercomputing (NAS) Pleiades facility. The authors thank Nikolas Zawodny for providing the quadcopter UAS as-tested geometry, experimental data, and for valuable discussions. We would also like to thank Gerrit Stich and Jeffrey Housman for sharing their expertise in FWH methodology, Gaetan

Kenway for helping us create a smoother and more realistic blade geometry, and Aditya Ghate for helping us understand and improve the slip boundary condition formulation in LBM.

References

- [1] Dunbar, B., “Revolutionary Vertical Lift Technology (RVLT) Project,” , Aug. 2017. URL <https://www.nasa.gov/aeroresearch/programs/aavp/rvlt>.
- [2] Zawodny, N., and Pettingill, N., “Acoustic wind tunnel measurements of a quadcopter in hover and forward flight conditions,” *INTER-NOISE and NOISE-CON Congress and Conference Proceedings*, Vol. 258, Institute of Noise Control Engineering, 2018, pp. 487–500.
- [3] Kiris, C. C., Housman, J. A., Barad, M. F., Brehm, C., Sozer, E., and Moini-Yekta, S., “Computational framework for launch, ascent, and vehicle aerodynamics (LAVA),” *Aerospace Science and Technology*, Vol. 55, 2016, pp. 189–219.
- [4] Bösch, F., Chikatamarla, S. S., and Karlin, I. V., “Entropic multirelaxation lattice Boltzmann models for turbulent flows,” *Physical Review E*, Vol. 92, No. 4, 2015, p. 043309.
- [5] Barad, M. F., Kocheemoolayil, J. G., and Kiris, C. C., “Lattice Boltzmann and Navier-Stokes Cartesian CFD Approaches for Airframe Noise Predictions,” *23rd AIAA Computational Fluid Dynamics Conference*, 2017, p. 4404.
- [6] Brehm, C., Barad, M. F., and Kiris, C. C., “Open Rotor Computational Aeroacoustic Analysis with an Immersed Boundary Method,” *54th AIAA Aerospace Sciences Meeting*, 2016, p. 0815.