

# Method for Extracting Aerodynamic Coefficients and Winds On-the-Fly Using Real-Time GPS Data

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The landing accuracy of an autonomous guided parafoil is influenced by its ability to predict the trajectory, which is shaped by the parafoil's aerodynamic characteristics and atmospheric winds. The parafoil is subjective to imperfections from manufacturing, damage during deployment, lateral asymmetry, and so forth, leading to discrepancies between the actual aerodynamic properties and the onboard aerodynamic model. Thus, by acquiring an accurate knowledge of the aerodynamic properties and real-time winds, the onboard simulation can reduce the uncertainties in the dynamics modeling for trajectory propagation to improve the landing accuracy. For that purpose, a non-linear least-squares method is developed to calculate differential corrections for optimizing and extracting the parafoil aerodynamic coefficients and winds on-the-fly using real-time data from the Global Positioning System (GPS). Post-flight analyses from two test flights have shown that the method provides improved simulations that closely match the GPS trajectories.

## Nomenclature

|                       |                                                                     |
|-----------------------|---------------------------------------------------------------------|
| $A$                   | = element of coefficient matrix of linear system of equations       |
| $A_{ref}$             | = aerodynamic reference area                                        |
| $\alpha$              | = angle-of-attack                                                   |
| $B$                   | = constant on right-hand-side of linear system of equations         |
| $\beta$               | = sideslip angle                                                    |
| $C$                   | = variable to be extracted, e.g. aerodynamic coefficients and winds |
| $C_D$                 | = drag coefficient                                                  |
| $C_{D0}$              | = drag coefficient at zero angle-of-attack and deflections          |
| $C_{D\alpha^2}$       | = slope of $C_D$ versus $\alpha^2$                                  |
| $C_{D\delta_{left}}$  | = slope of $C_D$ versus $\delta_{left}$                             |
| $C_{D\delta_{right}}$ | = slope of $C_D$ versus $\delta_{right}$                            |
| $C_L$                 | = lift coefficient                                                  |
| $C_{L0}$              | = lift coefficient at zero angle-of-attack and deflections          |
| $C_{L\alpha}$         | = slope of $C_L$ versus $\alpha$                                    |
| $C_{L\delta_{left}}$  | = slope of $C_L$ versus $\delta_{left}$                             |
| $C_{L\delta_{right}}$ | = slope of $C_L$ versus $\delta_{right}$                            |
| $C_Y$                 | = side force coefficient                                            |
| $C_{Y0}$              | = side force coefficient at zero sideslip and deflections           |
| $C_{Y\beta}$          | = slope of $C_Y$ versus $\beta$                                     |
| $C_{Y\delta_{left}}$  | = slope of $C_Y$ versus $\delta_{left}$                             |
| $C_{Y\delta_{right}}$ | = slope of $C_Y$ versus $\delta_{right}$                            |
| $\Delta C$            | = differential correction                                           |
| $\delta_{left}$       | = left deflection                                                   |
| $\delta_{right}$      | = right deflection                                                  |
| $g$                   | = Earth gravitational acceleration                                  |
| $\gamma$              | = flight path angle                                                 |
| $m$                   | = vehicle mass                                                      |

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|               |   |                                                |
|---------------|---|------------------------------------------------|
| $n_{data}$    | = | number of data points in sample                |
| $n_{unknown}$ | = | number of unknowns                             |
| $\bar{q}$     | = | dynamic pressure                               |
| $\rho$        | = | atmospheric density                            |
| $t$           | = | time                                           |
| $x, y, z$     | = | north, east, down distances relative to target |
| $v$           | = | speed                                          |
| $\psi$        | = | azimuth angle                                  |
| $\infty$      | = | freestream                                     |

## I. Introduction

THE landing accuracy of an autonomous guided parafoil is influenced by its ability to predict the trajectory, which is shaped by the parafoil's aerodynamic characteristics and atmospheric winds. The parafoil is subjective to imperfections from manufacturing, damage during deployment, and so forth, leading to discrepancies between the actual aerodynamic properties and the onboard aerodynamic model. Lateral asymmetry may also exist in the actual aerodynamic properties. Thus, by acquiring an accurate knowledge of the aerodynamic properties and real-time winds, the onboard simulation can reduce the uncertainties in the dynamics modeling for trajectory propagation to improve the landing accuracy. For that purpose, a method is developed at the NASA Johnson Space Center for extracting the parafoil aerodynamic coefficients and winds on-the-fly using real-time data from the Global Positioning System (GPS). In this NASA code, the non-linear least-squares method is used to calculate differential corrections for optimizing the aerodynamic coefficients and winds such that the onboard propagator would produce a simulated trajectory that is the best match to the GPS measured trajectory.

Five test flights were conducted on July 29, 2015 to test the guidance, navigation, and control (GN&C) system under design. The NASA code was not executed during the flights due to insufficient time for a full implementation. The partial implementation of the NASA code produced the GPS data files which were used to perform post-flight analyses. This report describes the NASA code and presents the results from the post-flight analyses.

## II. Calibration maneuvers

The onboard simulation is six degrees-of-freedom (DOF) for translational and rotational motion. The measured quantities from the GPS are the position components. Therefore, only the translational coefficients are extracted. The parafoil is maneuvered by trailing edge deflections ( $\delta_{left}$ ,  $\delta_{right}$ ). The aerodynamics model used for translational motion is given below, and the coefficients to be extracted are highlighted in red. Note that  $C_{Y0}$  is included to allow asymmetry in the parafoil aerodynamics at zero deflection.

$$\begin{aligned}
C_D &= \textcolor{red}{C_{D0}} + C_{D\alpha^2} \alpha_{aero}^2 + \textcolor{red}{C_{D\delta_{left}}} \delta_{left} + \textcolor{red}{C_{D\delta_{right}}} \delta_{right} \\
C_Y &= \textcolor{red}{C_{Y0}} + C_{Y\beta} \beta_{aero} + \textcolor{red}{C_{Y\delta_{left}}} \delta_{left} + \textcolor{red}{C_{Y\delta_{right}}} \delta_{right} \\
C_L &= \textcolor{red}{C_{L0}} + C_{L\alpha} \alpha_{aero} + \textcolor{red}{C_{L\delta_{left}}} \delta_{left} + \textcolor{red}{C_{L\delta_{right}}} \delta_{right}
\end{aligned} \tag{1}$$

To calibrate the aerodynamic coefficients and winds, three maneuvers are performed in order to de-couple the effects of the aerodynamic coefficients. The first maneuver consists of no deflection, e.g.  $\delta_{left} = \delta_{right} = 0$ , reducing Eq. (1) to only two terms on the right-hand-side. The terms containing  $\alpha_{aero}$  and  $\beta_{aero}$  are obtained from the simulation, thus influenced by the fidelity of the models for rotational motion. The second and third maneuvers consist of a left and right deflection, respectively, fixed at 50% of the maximum deflection and simplifying Eq. (1) to three terms on the right-hand-side.

The time allotted for calibrating the aerodynamic coefficients and winds are minimized in order to maximize the time for the autopilot to perform targeting maneuvers. It is determined that 5 sec of sampling data from the GPS is adequate for the first maneuver without deflection, and 10 sec for maneuvers 2 and 3 each with deflections. The two maneuvers with left and right deflections are separated by 5 sec of no deflection for the transient effects to settle.

The maneuvers are summarized in Table 1. In the processing of the sample data collected from the maneuvers, a limit of 180° sweep in azimuth within the 10 sec of fixed deflection is applied to ensure that the least-squares method would converge with good solutions; any sample data beyond the limit are truncated.

**Table 1. Calibration Maneuvers**

| Maneuver | Time (sec) | Description                                   |
|----------|------------|-----------------------------------------------|
| 1        | 0 – 5      | No deflection                                 |
| 2        | 5 – 15     | Left deflection at 50%<br>No right deflection |
| –        | 15 – 20    | No deflection for transient effects to settle |
| 3        | 20 – 30    | Right deflection at 50%<br>No left deflection |

Although maneuver 1 has no deflection, it can still produce lateral motion and thus a slight curvature in the ground track due to the effects of winds and the parafoil aerodynamic asymmetry. However, the lateral motion does not provide sufficient information to de-couple the effects of the winds and the aerodynamic asymmetry. Thus, the preliminary estimate of the wind is set as a known variable, and the  $C_{D0}$ ,  $C_{Y0}$ , and  $C_{L0}$  are the only unknowns solved for maneuver 1. The solutions are prerequisites for solving the  $\delta_{\text{left}}$  and  $\delta_{\text{right}}$  derivatives. Subsequently, the  $C_{D\delta_{\text{left}}}$ ,  $C_{Y\delta_{\text{left}}}$ ,  $C_{L\delta_{\text{left}}}$  and winds are calculated from maneuver 2, where  $\delta_{\text{right}} = 0$ . Similarly,  $C_{D\delta_{\text{right}}}$ ,  $C_{Y\delta_{\text{right}}}$ ,  $C_{L\delta_{\text{right}}}$  and winds are calculated from maneuver 3, where  $\delta_{\text{left}} = 0$ . The unknowns solved in the order of the maneuvers are summarized in Table 2.

**Table 2. Unknowns Solved for Each Maneuver**

| Maneuver | Unknowns                                                                                                                                |
|----------|-----------------------------------------------------------------------------------------------------------------------------------------|
| 1        | $C_{D0}$ , $C_{Y0}$ , $C_{L0}$                                                                                                          |
| 2        | $C_{D\delta_{\text{left}}}$ , $C_{Y\delta_{\text{left}}}$ , $C_{L\delta_{\text{left}}}$ , $v_{\text{wind}2}$ , $\psi_{\text{wind}2}$    |
| 3        | $C_{D\delta_{\text{right}}}$ , $C_{Y\delta_{\text{right}}}$ , $C_{L\delta_{\text{right}}}$ , $v_{\text{wind}3}$ , $\psi_{\text{wind}3}$ |

### III. Method of Calculating Differential Corrections

The measured quantities from GPS are the north, east, down (x, y, z) position components versus time (t). The difference between the measured and simulated data is the residual, and the sum of the squares of the residuals (SSR) can be calculated. For example, for the north component:

$$SSR_x = \sum_{i=1}^{ndata} [x_{\text{meas}}(t_i) - x_{\text{sim}}(t_i)]^2 \quad (2)$$

where ndata is the number of sample data. A linear model for the simulated position can be approximated by a first-order Taylor series expansion about the unknowns:

$$x_{\text{sim}}(t_i) = x_{\text{sim}}(t_i)_0 + \sum_{j=1}^{nunknown} \left( \frac{\partial x_{\text{sim}}}{\partial C_j} \right) \Delta C_j \quad (3)$$

where the unknowns,  $C_j$ , are the aerodynamic coefficients and winds to be extracted.  $\Delta C_j$  is the differential correction to be applied to the current value of the j-th unknown to provide a better fit. Substituting into Eq. (2) yields:

$$SSR_x = \sum_{i=1}^{ndata} \left[ x_{meas}(t_i) - x_{sim}(t_i)_0 - \sum_{j=1}^{nunknown} \left( \frac{\partial x_{sim}}{\partial C_j} \right)_i \Delta C_j \right]^2 \quad (4)$$

In the non-linear least-squares method, the  $\Delta C_j$  are solved by minimizing the SSR. Take the derivative of Eq. (4) with respect to  $\Delta C_j$  and set to 0:

$$-2 \sum_{i=1}^{ndata} \left( \frac{\partial x_{sim}}{\partial C_k} \right)_i \left[ x_{meas}(t_i) - x_{sim}(t_i)_0 - \sum_{j=1}^{nunknown} \left( \frac{\partial x_{sim}}{\partial C_j} \right)_i \Delta C_j \right] = 0 \quad (5)$$

$k = 1, \dots, nunknown$

Rearranging the gradient equation yields:

$$\sum_{i=1}^{ndata} \sum_{j=1}^{nunknown} \left( \frac{\partial x_{sim}}{\partial C_k} \right)_i \left( \frac{\partial x_{sim}}{\partial C_j} \right)_i \Delta C_j = \sum_{i=1}^{ndata} \left( \frac{\partial x_{sim}}{\partial C_k} \right)_i (x_{meas}(t_i) - x_{sim}(t_i)_0) \quad (6)$$

$k = 1, \dots, nunknown$

Similar equations are obtained for the y and z components. This is of the form:

$$[A]\{\Delta C\} = \{B\} \quad (7)$$

where  $\{\Delta C\}$  can be solved via a numerical method, e.g. Gauss-Jordan elimination. Given the three measured quantities (x,y,z), the A and B elements are then:

$$A_{jk} = \sum_{i=1}^{ndata} \frac{\partial x_i}{\partial C_j} \frac{\partial x_i}{\partial C_k} + \frac{\partial y_i}{\partial C_j} \frac{\partial y_i}{\partial C_k} + \frac{\partial z_i}{\partial C_j} \frac{\partial z_i}{\partial C_k} \quad (8)$$

$$B_k = \sum_{i=1}^{ndata} \frac{\partial x_i}{\partial C_k} (x_{meas} - x_{sim0})_i + \frac{\partial y_i}{\partial C_k} (y_{meas} - y_{sim0})_i + \frac{\partial z_i}{\partial C_k} (z_{meas} - z_{sim0})_i$$

$j = 1, \dots, nunknown \quad k = 1, \dots, nunknown$

A system response approach is used to determine the partial derivatives, or Jacobians, in Eq. (8). First, the simulation is performed using the current values of the unknowns to generate the nominal, unperturbed trajectory. Subsequently, the unknowns are perturbed one at a time to generate the perturbed trajectories for determining the system response. The Jacobians are then calculated as a function of the differences in the positions of the perturbed and unperturbed trajectories at each sample time.

$$\frac{\partial x}{\partial C_j} = \frac{x_{pert} - x_{nom}}{C_{j,pert} - C_{j,nom}} \quad (9)$$

The kinematics and dynamics modeling used in the integrator are described in [3]. To carry out the integrations to determine the nominal and perturbed trajectories, the initial conditions and initial values of the unknowns are needed. The initial conditions are obtained from the GPS data. The aerodynamic coefficients are initiated with the nominal values assumed by the onboard guidance, and the preliminary wind vectors are estimated.

The parafoil is maneuvered by trailing edge deflections actuated by pulling or releasing the right or left control lines. Assuming a constant wind, a fixed deflection would produce a sinusoidal speed profile where the amplitude is the speed change due to the *effects* of the wind. However, for the preliminary estimate, this amplitude is assumed to be the wind speed and is found to be an adequate estimate. The wind direction is the parafoil azimuth at the maximum speed.

Once the differential corrections are obtained from solving Eq. (7), they are applied to the unknowns:

$$C_j = C_{j,0} + \Delta C_j \quad (10)$$

This procedure is repeated to refine the unknowns iteratively until either (a) the error tolerance is met, or (b) the limit on the number of iterations is exceeded, which prevents the simulation from getting into an infinite loop.

In the development of an extraction method for the aerodynamic coefficients and winds, another approach was studied where the parafoil acceleration profile was calculated from the GPS data via finite difference. The acceleration profile was then used to determine the aerodynamic coefficients. The advantage of this approach is that the solutions could be obtained quickly without iterations. However, the solutions do not have sufficient accuracy because the small errors and noise that existed in the GPS data get propagated in the derivation of the acceleration. Although this method is not used in the NASA code, a possible application may be to supply the initial estimates of the aerodynamic coefficients, which at the present simply take on the values assumed by the onboard guidance as mentioned previously. A description of this GPS-derived acceleration method is provided in Appendix – GPS-Derived Acceleration Method.

There is an advantage in applying the system response approach to calculate the partial derivatives in Eq. (8) for a parafoil in operation. This approach does not necessarily produce the true parafoil aerodynamic coefficients and winds per se. Rather, it determines the coefficients and winds that the simulator should use in order to produce the best match of the GPS trajectory. Therefore, if the integrator had small errors or low-fidelity models, they would be compensated for and get absorbed into the solutions obtained by this approach, enabling the onboard guidance to improve its predictions of the trajectory. In general, the application of this method is broad and not limited to the extraction of force coefficients and winds, and the measured quantities are not limited to GPS data.

It should be noted that the system response approach did not work well when employed by the researchers in Ref. [1], who experienced some issues. The difficulty in choosing the size of the perturbations to produce accurate derivatives at both the beginning and the end of the integration was cited. Also, convergence occurred only about half of the time using fixed perturbations, and the solutions were somewhat dependent on the size of the perturbations. Those researchers were able to find better solutions consistently by using parametric differentiation to obtain the derivatives. The parametric differentiation approach was also used by other researchers for extracting aerodynamic coefficients [2]. For this research, however, the system response approach worked well under the design conditions. The integration was for short time intervals, e.g.  $\leq 10$  sec, such that derivatives of adequate accuracy were produced throughout the integration. It was found that as long as the measured translational data (e.g. x,y,z) were used only to extract the force coefficients for translational motion, the least-squares method almost always converged. If the moment coefficients for rotational motion were introduced as unknowns, the least-squares method would often fail to converge or produced poor solutions. Therefore, only force coefficients were extracted. To extract moment coefficients, attitude measurements would be needed. Failure to converge also occurred when the initial values assumed for the unknowns contained too large of a deviation from the solutions. As mentioned previously, a resolution for this would be to use calculated, rather than assumed, initial values for the force coefficients as discussed in Appendix – GPS-Derived Acceleration Method. Lastly, failure to converge could occur if the assumed moment equations and coefficients (e.g.  $C_l$ ,  $C_m$ ,  $C_n$ ) did not sufficiently simulate the rotational motion, although that could not be fully assessed without measured rotational data. An alternative would be to perform only 3-DOF trajectory simulation.

#### IV. Test Flights

Five test flights were flown on July 29, 2015 using small commercial parafoils. In the test flights, the three calibration maneuvers were flown first, followed by a box pattern about the target, and then the guided phase, terminating at a target altitude. The files containing the GPS data were created during the test flights to provide inputs to the NASA code in the order that the maneuvers were performed. In three of the five flights, the GPS data

files became corrupted after the flights were completed. Therefore, post-flight analyses were performed only for the two flights with the available GPS data: 2015\_0729\_Unit152 and 2015\_0729\_Unit155.

For these two flights, the GPS data files were successfully created, but the data were arranged incorrectly in the order of 2-1-3 for the maneuvers, of which the data for maneuver 1 came from the interval intended for the transients to settle. This was mitigated by manually shifting the block of data for maneuver 1 ahead of maneuver 2 and subtracting 20 sec from its sample times since the NASA code assumed that the time was increasing for the maneuvers. The modified GPS data files were sufficient for performing the post-flight analyses. The columns in the GPS data files consisted of ten variables: time, north-east-down position components relative to the target, north-east-down velocity components (derived from position), the left and right control line displacements, and a guidance gain that was set to zero for the NASA code.

## V. Post-Flight Analysis Results

The post-flight analysis results are presented in the following subsections. To demonstrate the effectiveness of the NASA code in calibrating the aerodynamic coefficients and winds, outputs of the simulation data for the three maneuvers are generated (a) by using the original aerodynamic coefficients and initial estimate of the winds and (b) by using the extracted coefficients and winds. These trajectory data are plotted against the GPS data for comparison.

### A. Test Flight 2015\_0729\_Unit152

The actual ground track of the test flight from the GPS is given in Figure 1. The sequence of 0-left-0-right deflections in the calibration maneuvers traced an s-shaped ground track at the beginning of the flight. The guided phase used the original aerodynamic coefficients and wind model. The guidance was able to direct the parafoil close to the target.

The co-plots of the GPS data, the trajectory obtained using the original aerodynamic coefficients and initial estimate of winds, and the trajectory obtained using the calibrated coefficients and winds are given in Figure 2. The trajectory using the original coefficients provided a good prediction of the descent rate and turn rate. However, the predicted speed was too high, causing the ground track to be stretched out compared to the actual. The calibration was able to resolve the over-prediction and allowed the calibrated trajectory to coincide with the actual for the three maneuvers.

The solutions for the coefficients and winds from the NASA code are given in Table 3.

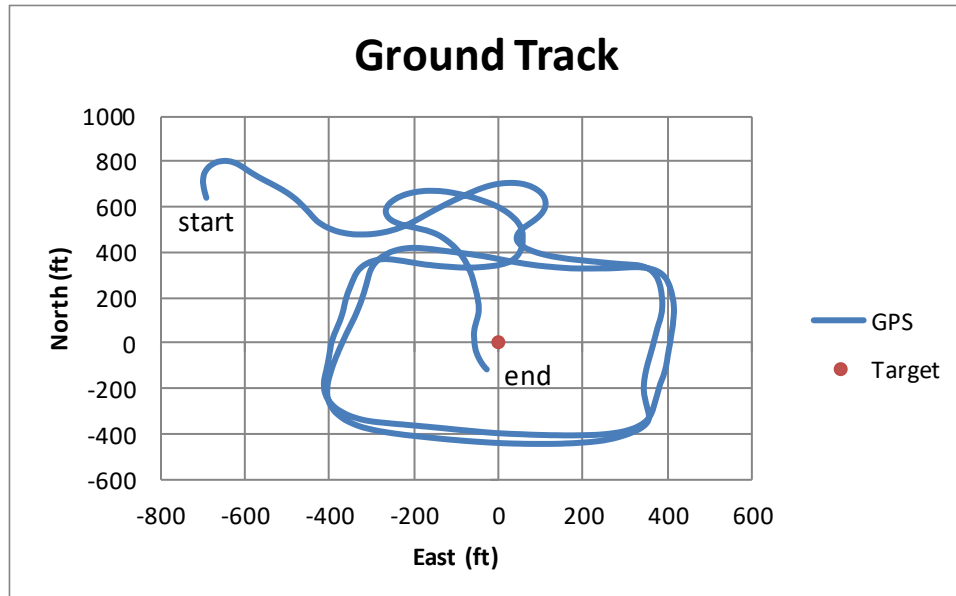


Figure 1. Ground Track for 2015\_0729\_Unit152

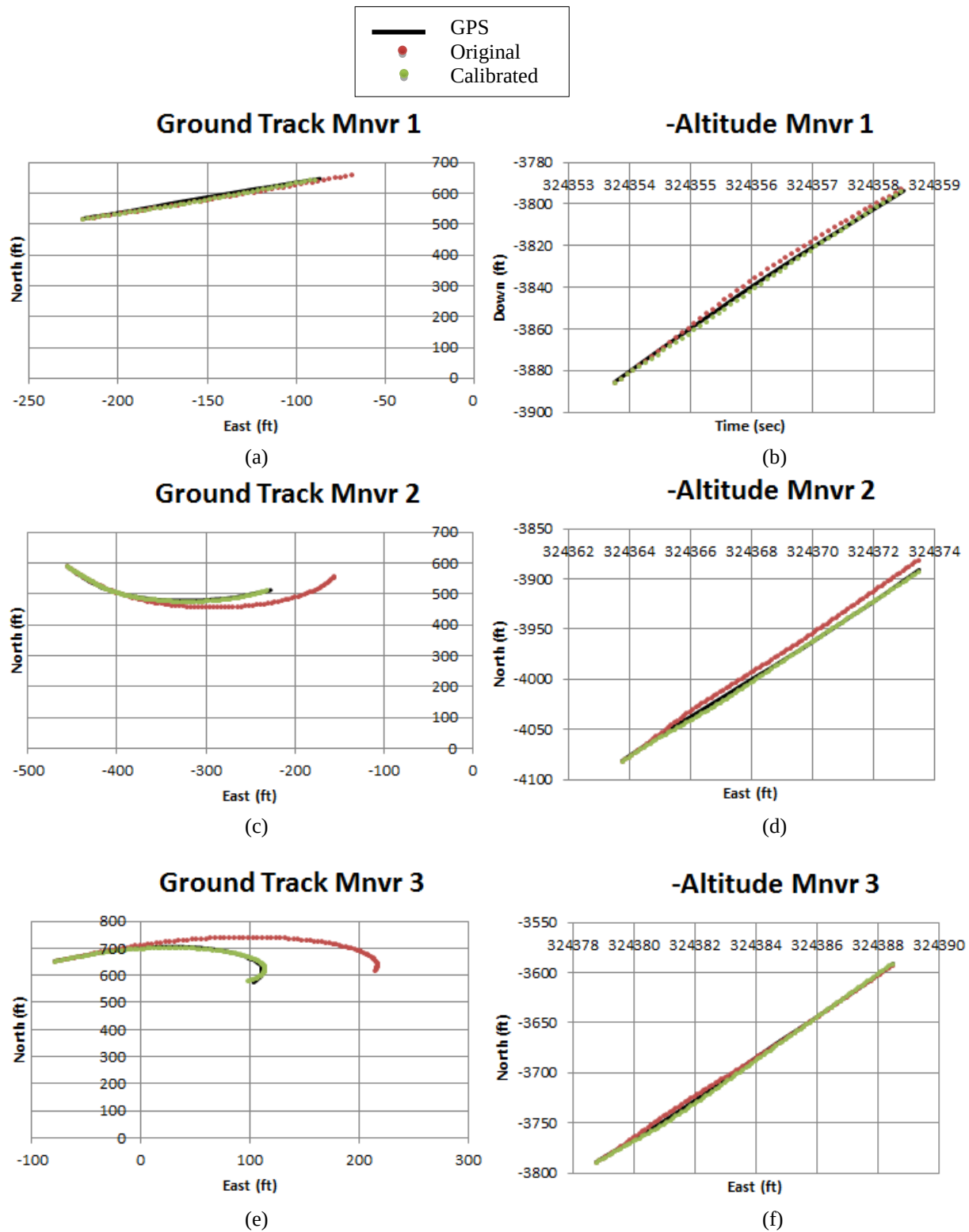


Figure 2. Co-plots for 2015\_0729\_Unit152

**Table 3. Extracted Aerodynamic Coefficients and Winds for 2015\_0729\_Unit152**

|                           |                |
|---------------------------|----------------|
| Wind speed, maneuver 2    | 4.8797511 ft/s |
| Wind azimuth, maneuver 2  | 0.8622330 rad  |
| Wind speed, maneuver 3    | 2.0405775 ft/s |
| Wind azimuth, maneuver 3  | 0.1731459 rad  |
| $C_{D0}$                  | 0.2481731      |
| $C_{D\delta\text{left}}$  | 0.2881560      |
| $C_{D\delta\text{right}}$ | 0.1056907      |
| $C_{L0}$                  | 0.0693210      |
| $C_{L\delta\text{left}}$  | 0.0010078      |
| $C_{L\delta\text{right}}$ | -0.0521514     |
| $C_{Y0}$                  | -0.0023974     |
| $C_{Y\delta\text{left}}$  | 0.0146468      |
| $C_{Y\delta\text{right}}$ | 0.0330747      |

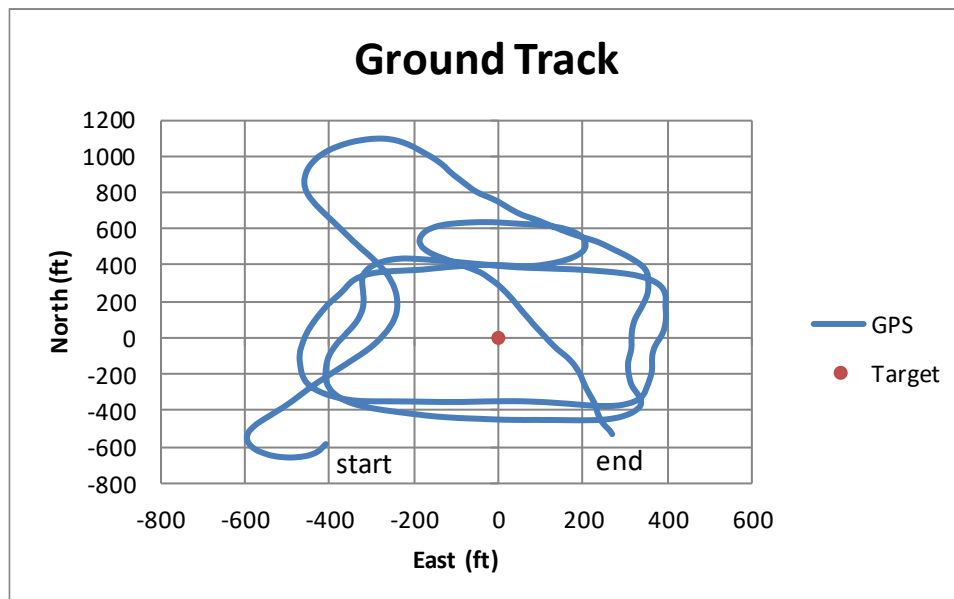
**B. Test Flight 2015\_0729\_Unit155**

The actual ground track of the test flight from the GPS is given in Figure 3. In this case, the parafoil overshoot the target and was able to get only a 16% reduction of the initial distance to the target.

The co-plots of the GPS data, the trajectory obtained using the original aerodynamic coefficients and initial estimate of winds, and the trajectory obtained using the calibrated coefficients and winds are given in Figure 4. The predicted trajectory using the original coefficients showed a slower descent rate, slower speed, and faster turn rate than the actual. Also, there was an asymmetry where the right deflection in maneuver 3 produced a higher turn rate than the left deflection in maneuver 2. The asymmetry was not captured in the original coefficients since the original model assumed lateral symmetry. These differences were mostly reconciled by the calibrated coefficients and winds, except for the altitude profile in maneuver 1. As inferred, there is room for improvement for the solutions. One simple option that has been shown to be effective is to use the calibrated values as initial values and execute the NASA code a second time.

The solutions for the coefficients and winds from the NASA code are given in Table 4. Note that the negative wind for maneuver 2 can be made positive by adding  $180^\circ$  to the wind azimuth angle.

The results from the two flight tests support the understanding that the ability to predict the trajectory accurately has an impact on the targeting capability of the parafoil.

**Figure 3. Ground Track for 2015\_0729\_Unit155**

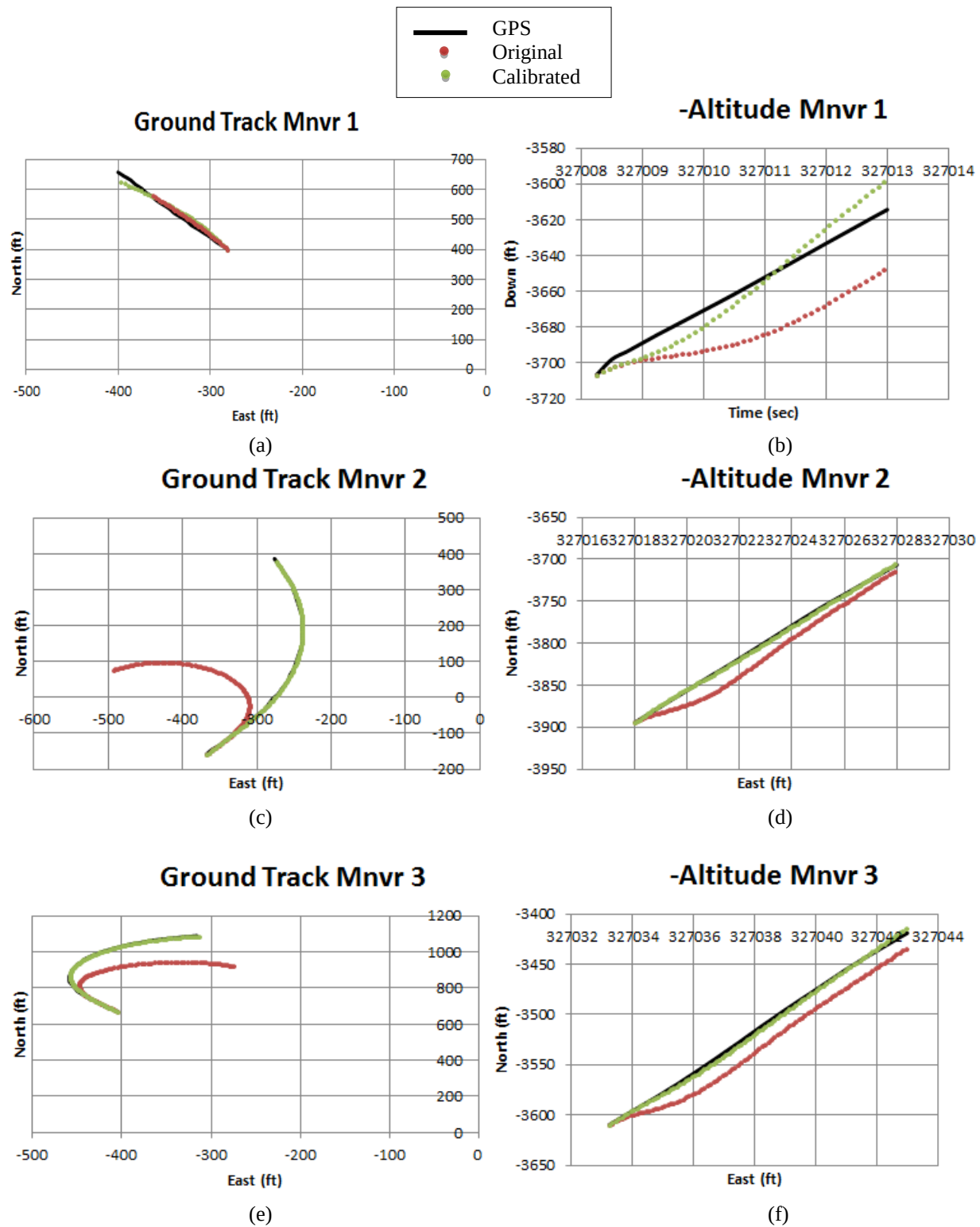


Figure 4. Co-plots for 2015\_0729\_Unit155

**Table 4. Extracted Aerodynamic Coefficients and Winds for 2015\_0729\_Unit155**

|                           |                  |
|---------------------------|------------------|
| Wind speed, maneuver 2    | -30.8010985 ft/s |
| Wind azimuth, maneuver 2  | 3.1624039 rad    |
| Wind speed, maneuver 3    | 21.9220014 ft/s  |
| Wind azimuth, maneuver 3  | 6.1716919 rad    |
| $C_{D0}$                  | 0.0576745        |
| $C_{D\delta\text{left}}$  | 0.7967171        |
| $C_{D\delta\text{right}}$ | 0.3549609        |
| $C_{L0}$                  | 0.0329507        |
| $C_{L\delta\text{left}}$  | -0.6782196       |
| $C_{L\delta\text{right}}$ | 0.0050170        |
| $C_{Y0}$                  | -0.0004873       |
| $C_{Y\delta\text{left}}$  | 0.0726473        |
| $C_{Y\delta\text{right}}$ | -0.0128472       |

## VI. Conclusion

A NASA code for extracting aerodynamic coefficients and winds on-the-fly using real-time GPS data have been developed for application to an autonomous guided parafoil. The code uses the non-linear least-squares method and calculates differential corrections from system response to small perturbations for calibrating the aerodynamic coefficients and winds. The result is an improved simulation that is a close match to the GPS trajectory.

Post-flight analyses from two test flights have shown that the original simulation produced trajectories that exhibited some discrepancies from the actual GPS trajectories in terms of parafoil speed, turn rate, and descent rate. Using the calibrated aerodynamic coefficients and winds generated by the NASA code, the differences between the simulations and the actual trajectories were mostly reconciled.

It was observed that the ability to accurately predict the trajectory has an impact on the landing accuracy. For future work, studies could verify whether the calibration of the simulation in-flight would actually improve the landing accuracy. To further improve the overall accuracy of the simulation, future work can include the calibration of moment coefficients from the sampling of rotational data as measured quantities, e.g. roll, pitch, and yaw.

## Appendix – GPS-Derived Acceleration Method

The aerodynamic force coefficients describe the translational acceleration characteristics of the parafoil. Therefore, to calculate these coefficients, the parafoil acceleration is needed. Given the GPS position profile, the parafoil velocity can be calculated via finite difference:

$$\begin{aligned}
 v_{north}(t_i) &= \frac{x_{meas}(t_{i+1}) - x_{meas}(t_i)}{t_{i+1} - t_i} \\
 v_{east}(t_i) &= \frac{y_{meas}(t_{i+1}) - y_{meas}(t_i)}{t_{i+1} - t_i} \\
 v_{down}(t_i) &= \frac{z_{meas}(t_{i+1}) - z_{meas}(t_i)}{t_{i+1} - t_i}
 \end{aligned} \tag{11}$$

The velocity components in spherical coordinates are obtained by:

$$\begin{aligned}
v &= \sqrt{v_{north}^2 + v_{east}^2 + v_{down}^2} \\
\psi &= \tan^{-1} \frac{v_{east}}{v_{north}} \\
\gamma &= -\sin^{-1} \frac{v_{down}}{v}
\end{aligned} \tag{12}$$

The acceleration components  $\dot{v}$ ,  $\dot{\psi}$ , and  $\dot{\gamma}$  are then calculated from Eq. (12) via finite difference:

$$\begin{aligned}
\dot{v}(t_i) &= \frac{\dot{v}(t_{i+1}) - \dot{v}(t_i)}{t_{i+1} - t_i} \\
\dot{\psi}(t_i) &= \frac{\dot{\psi}(t_{i+1}) - \dot{\psi}(t_i)}{t_{i+1} - t_i} \\
\dot{\gamma}(t_i) &= \frac{\dot{\gamma}(t_{i+1}) - \dot{\gamma}(t_i)}{t_{i+1} - t_i}
\end{aligned} \tag{13}$$

The equations of motion in spherical coordinates are:

$$\begin{aligned}
\dot{v} &= -g \sin \gamma - \frac{\bar{q}_{aero} A_{ref} C_D}{m} \\
\dot{\psi} &= \frac{\bar{q}_{aero} A_{ref} C_Y}{mv \cos \gamma} \\
\dot{\gamma} &= -\frac{g \cos \gamma}{v} + \frac{\bar{q}_{aero} A_{ref} C_L}{mv}
\end{aligned} \tag{14}$$

The dynamic pressure,  $\bar{q}_{aero}$ , takes into account the winds. The vertical component of the wind is not modeled.

$$\bar{v}_{aero} = \left\{ \begin{array}{c} v_{north} - v_{wind} \cos \psi_{wind} \\ v_{east} - v_{wind} \sin \psi_{wind} \\ v_{down} \end{array} \right\} \tag{15}$$

$$v_{aero} = \sqrt{v^2 + v_{wind}^2 - 2v_{north} v_{wind} \cos \psi_{wind} - 2v_{east} v_{wind} \sin \psi_{wind}} \tag{16}$$

$$\bar{q}_{aero} = \frac{1}{2} \rho_{\infty} v_{aero}^2 \quad (17)$$

Substituting Eq. (1) into Eq. (14), rearranging, and setting  $\delta_{left} = \delta_{right} = 0$  for maneuver 1, the aerodynamic coefficients  $C_{D0}$ ,  $C_{Y0}$ , and  $C_{L0}$  are obtained.

$$\begin{aligned} C_{D0} &= -C_{D\alpha^2} \alpha_{aero}^2 - \frac{m}{\bar{q}_{aero} A_{ref}} (\dot{v} + g \sin \gamma) \\ (maneuver\ 1) \quad C_{Y0} &= -C_{Y\beta} \beta_{aero} + \frac{mv \cos \gamma}{\bar{q}_{aero} A_{ref}} \dot{\psi} \\ C_{L0} &= -C_{L\alpha} \alpha_{aero} + \frac{mv}{\bar{q}_{aero} A_{ref}} \left( \dot{\gamma} + \frac{g \cos \gamma}{v} \right) \end{aligned} \quad (18)$$

For maneuver 2,  $\delta_{right}$  is set to 0 to obtain  $C_{D\delta_{left}}$ ,  $C_{Y\delta_{left}}$ , and  $C_{L\delta_{left}}$ .

$$\begin{aligned} C_{D\delta_{left}} &= -\frac{1}{\delta_{left}} \left[ \frac{m}{\bar{q}_{aero} A_{ref}} (\dot{v} + g \sin \gamma) + C_{D0} + C_{D\alpha^2} \alpha_{aero}^2 \right] \\ (maneuver\ 2) \quad C_{Y\delta_{left}} &= \frac{1}{\delta_{left}} \left[ \frac{mv \cos \gamma}{\bar{q}_{aero} A_{ref}} \dot{\psi} - C_{Y0} - C_{Y\beta} \beta_{aero} \right] \\ C_{L\delta_{left}} &= \frac{1}{\delta_{left}} \left[ \frac{mv}{\bar{q}_{aero} A_{ref}} \left( \dot{\gamma} + \frac{g \cos \gamma}{v} \right) - C_{L0} - C_{L\alpha} \alpha_{aero} \right] \end{aligned} \quad (19)$$

For maneuver 3,  $\delta_{left}$  is set to 0 to obtain  $C_{D\delta_{right}}$ ,  $C_{Y\delta_{right}}$ , and  $C_{L\delta_{right}}$ .

$$\begin{aligned} C_{D\delta_{right}} &= -\frac{1}{\delta_{right}} \left[ \frac{m}{\bar{q}_{aero} A_{ref}} (\dot{v} + g \sin \gamma) + C_{D0} + C_{D\alpha^2} \alpha_{aero}^2 \right] \\ (maneuver\ 3) \quad C_{Y\delta_{right}} &= \frac{1}{\delta_{right}} \left[ \frac{mv \cos \gamma}{\bar{q}_{aero} A_{ref}} \dot{\psi} - C_{Y0} - C_{Y\beta} \beta_{aero} \right] \\ C_{L\delta_{right}} &= \frac{1}{\delta_{right}} \left[ \frac{mv}{\bar{q}_{aero} A_{ref}} \left( \dot{\gamma} + \frac{g \cos \gamma}{v} \right) - C_{L0} - C_{L\alpha} \alpha_{aero} \right] \end{aligned} \quad (20)$$

The aerodynamic coefficients are calculated at each sample time and an average is taken for the maneuver.

It is challenging for this method to produce accurate solutions because the small errors and noise in the GPS data will get propagated and grow; first, in Eq. (11) for the derivation of the velocity, and further in Eq. (13) for the acceleration.

### References

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<sup>3</sup>Gorman, C., and Slegers, N., “Comparison and Analysis of Multi-Body Parafoil Models with Varying Degrees of Freedom”, *21<sup>st</sup> AIAA Aerodynamic Decelerator Systems Technology Conference and Seminar*, 2011, 10.2514/6.2011-2615.