

SPIN-ORBIT RESONANCE AND STABILITY IN ECCENTRIC, LOW ALTITUDE MARS ORBITS

Andres Dono,^{*} Jose Alvarellos,^{*} Laura Plice,[†] Ted Hendriks[‡],
and Ron Evans[§]

Low altitude orbit perturbations derive from mass concentrations and show high sensitivity to spin-orbit resonance conditions and initial configuration relative to the central body. Here we focus on low, polar, non-circular orbits of Mars. The method uses analytical derivations to identify regimes of interest and computational modeling and graphical design aids for detailed investigation. Evaluation of spin-orbit resonance and gravity perturbation effects are important in mission design applications. The obtained results indicate the presence of unstable regions to avoid for potential future Mars orbiters.

INTRODUCTION

This paper presents the study of a particular regime of spin-orbit resonances that exist at low Martian altitudes, specifically, placing the focus on the parametric exploration of stability in resonant, eccentric, polar orbits. We compute theoretical semi-major axis where the resonance occurs, and study the long-term effects when the orbit remains locked in such a regime. The results show that orbits in a low spin-orbit resonance regime with a small number of repeating ground tracks, have a different behavior than orbits with higher resonances, i.e., either greater number of rotations or greater number of passes. These effects may influence the overall stability of the orbit, translating into higher risks for mission concepts, such as undesired orbital decays.

Low altitude Mars orbits are subject to gravitational perturbations that may be exacerbated by repeating passes over certain mass concentrations. The simplest repeating ground tracks for feasible low altitude orbits associate with 12, 13, or 14 complete passes for each Mars rotation. Mission design activities may also need to address such factors as atmospheric drag, ground coverage, insertion accuracy, and communications lines of sight, among others.

The precise altitudes at which spin resonance occurs have multivariate dependence on particularly two orbital elements: eccentricity and inclination. However, the initial longitude of the ascending node (LAN), and the argument of periapsis (ω) also play an important role in the evolution

^{*} Flight Dynamics Analyst, Millennium Engineering and Integration Services NASA Ames Research Center, Moffett Field, Mountain View, CA 94035.

[†] Flight Dynamics Lead, Metis Technology Solutions, NASA/Ames Research Center, Mission Design Center, Moffett Field, Mountain View, CA 94035.

[‡] Flight Dynamics Analyst, Metis Technology Solutions, NASA/Ames Research Center, Mission Design Center, Moffett Field, Mountain View, CA 94035

[§] Flight Dynamics Programmer, Millennium Engineering and Integration Services NASA Ames Research Center, Moffett Field, Mountain View, CA 94035.

and stability of the orbit. Our approach is to study the immediate effects of the initial orbital elements selection in the orbit propagation for a given amount of time. We observe that for orbits located in the low spin-orbit regime, periodic evolution of the orbital elements may differ depending on the particular initial configurations. Periapsides located at the poles tend to indicate stable situations. On the other hand, periapsides clustered around the equator may yield chaotic and fast-paced decay, which can be potentially dangerous to some mission concepts.

This paper builds upon a previous publication analyzing the effects of the spin-orbit resonance for the stability of circular orbits. Our previous paper introduced the idea and presented initial results of the observed orbit decay for certain initial conditions. In this publication, we plan to expand our analytical understanding of the coupling of these two natural phenomena, and to explore a wider array of cases. We introduce more innovative methods of representing the evolution of the orbital elements, and we place the focus on eccentric orbits that show meaningful effects with carefully selected initial conditions in their orientation, meaning the ω and the LAN.

GENERALIZED SPIN-ORBIT RESONANCE

In our previous paper¹, we used an equation from Klokocnik et al. (2002)² that relates the averaged mean motion n' for circular orbits of arbitrary inclination to a couple of integers α, β representing the number of rotations and the number of draconic revolutions of a spacecraft around a planet, respectively (please see Klokocnik et al. 2002 for exact definitions):

$$n' = \frac{\beta}{\alpha} \dot{\theta} - \frac{3}{4} J_2 n' \left(\frac{R}{a'} \right)^2 \left[5 \cos^2 i - 2 \frac{\beta}{\alpha} \cos i - 1 \right] \quad (1)$$

In Equation 1 planetary oblateness is represented by J_2 , and $\dot{\theta}$ is the planetary spin-rate, R is the planetary radius, a' is the mean semi-major axis (related to n' by Kepler's Third Law) and i is the inclination. Based on the original derivation of the above equation but generalizing for arbitrary eccentricities we find that

$$n' = \frac{\beta}{\alpha} \dot{\theta} - \frac{3}{4} J_2 n' \left(\frac{R}{p'} \right)^2 \left[5 \cos^2 i - 2 \frac{\beta}{\alpha} \cos i - 1 \right] \quad (2)$$

where $p' = a(1-e'^2)$. Therefore, in the most general case, with arbitrary eccentricity and inclination, the ratio to apply is radius over the semi-latus rectum, rather than radius over semi-major axis.

PARAMETRIC OVERVIEW OF LOW, PERTURBED MARS ORBITS

Orbit propagation used the General Mission Analysis Tool (GMAT) developed by Goddard Space Flight Center. Extensive benchmarking was necessary to reduce the computation time due to the large number of cases. A standalone tool automated execution with optimal inputs. Tool configuration allows parametric runs based on any of the six Keplerian orbital elements, as well as the gravity model fidelity. We used the MRO110C gravity model for the high fidelity propagator.

Our approach is to compute analytically the theoretical resonances occurring in low altitude and low eccentricity Mars orbits. Once we identify cases that yield a small number of repeating ground tracks, β , over a short period of time, or planetary rotations, α , we proceed to study them numerically. The process of propagating the orbit with high fidelity simulations allows us to identify patterns in the orbital elements evolution. Therefore, we study how the eccentricity and the argument of periapsis evolve over time, even achieving periodic behaviors in some cases, which typically indicate the presence of frozen or quasi-frozen orbits.

Our premise include the coupling of these two natural effects: spin-orbit resonance and stability. We have observed that for certain initial conditions, which occur close to low order resonance

regions, stability can be compromised. In order to study a wide array of orbits, we have built custom tools that allow us to plot the orbit evolution over time, as well as the minima and maxima of several orbital elements. Our main objective becomes to perform parametric studies in the areas of interest, previously found by our analytical methods that generate survey plots as in Figure 1. In this graph, we can observe theoretical resonances exist for low altitude orbits. Our main area of interest focuses on the left of the graph, where low resonances are present, as small as 14:1 or 27:2, indicating the presence of that number of repeating ground tracks every day or every two days respectively.

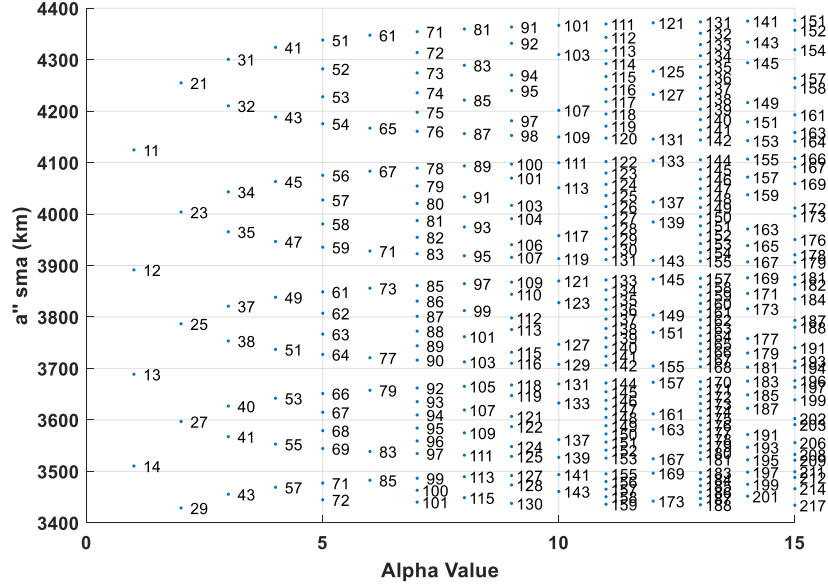


Figure 1. A spin-orbit resonance plot that indicates the location of the analytical solutions for specific resonances. The y-axis indicates the semi-major axis at which the resonances expressed by their alpha and beta values occur. Alpha represents the amount of Mars rotations or days, in the x-axis; while beta is the number of passes or repeating tracks for a given alpha, expressed in the small number to the right of each data point.

An example parametric investigation appears in Figure 2, where a selected spin-orbit resonant case fixes initial semi-major axis of 3512 km, eccentricity of 0.01, and inclination of 85 deg to examine orbit stability dependence on initial conditions of orbit orientation relative to the central body in terms of longitude of ascending node and argument of periapsis. The x- and y- axes form an abstract design space of all possible orbit configurations for the selected case. The z-axis appears as a heat map to display the changes in orbit shape, specifically periapsis altitude in the figure, expressed as minimum altitude during a propagation time of one Earth year. The graphical display aid illustrates the stability resulting from the placement of the periapsis at the South Pole ($\omega = 270^\circ$ on the y-axis).

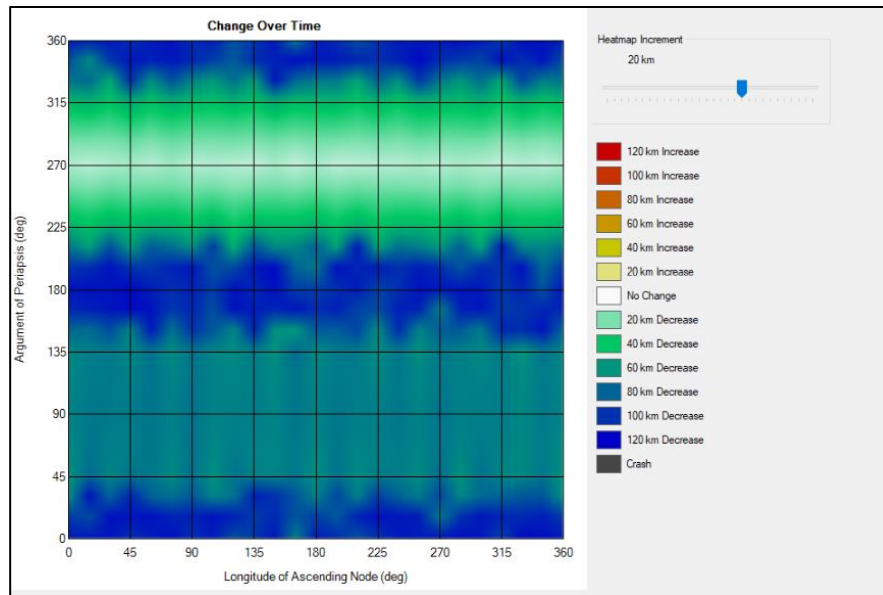


Figure 2. Example graphical output for parametric examination of spin resonant cases for a period of time of one Earth year. In this image, the effects of the spin-orbit resonance and stability coupling are noticeable by observing the sharp maximum altitude decay for cases that have an initial ω in the equator. Cases that include an initial ω in the poles, especially the South Pole, exhibit greater stability

NUMERICAL MODELING APPROACH AND TOOLS

To create the orbit propagation data, a data generation module saves each satellite run ephemeris to its own file once each simulation is finished. Folders collect simulations that hold most variables fixed with LAN and ω cycling from 0 to 360 degrees in 15 degree increments, creating a 24 by 24 matrix of files for each run. File names include metadata to identify initial conditions.

Post-processing to extract local minima for the radius magnitude decreases the file size and improves read times. For the current display tool, X is LAN, Y is ω , and Z is a user-selected variable related to an orbital element of the satellite such as apsis radii or the eccentricity.

MISSION DESIGN APPLICATIONS

This study is particularly interesting for potential orbiting missions around Mars that require low altitudes to meet their science requirements. In some cases, such as planet imaging or remote sensing applications, the science objectives may require the optimization of either the global coverage and/or the revisit times at a particular location. In order to achieve identified ground coverage goals, and depending on the sensor on board, spacecraft typically use stable orbits to first avoid any risk of decay, and second to obtain consistent measurements. Spin-orbit resonance plays a fundamental role in this equation since low resonances can translate into just a few repeating ground tracks, neglecting the idea of global coverage, and can be utilized as an advantage as well for missions that require observing a particular location numerous times over short time periods. In addition, as with the case of the Moon, the gravity field of Mars has irregularities that can make an effect on the stability of an orbiting spacecraft, especially at lower altitudes.

Studying the consequences of the selection of particular initial orbital elements in this environment has been an important research topic for lunar satellites, and is starting to raise attention in the case

for Mars, as more missions become a reality. Different initial conditions may translate in a drastic difference in the evolution of the orbit propagation, in some cases even leading to an undesired orbit decay. This raises even more the importance of accurate orbit insertion in these regimes. In this paper, we analyze two low altitude orbiting mission concepts may benefit from the careful study of the spin-orbit resonance and stability coupling.

CONCLUSION

We have expanded our previous analysis with further derivation of the analytical approach to consider the terms with eccentricity. That allowed us to study eccentric orbits and to introduce a new free variable that plays a fundamental role in the orbit stability, the initial argument of periaapsis. We conducted parametric studies that computed numerically the cases of interest with a high fidelity Mars orbit propagator consisting of the MRO gravity model. Our method to plot and present the results has been refined, introducing dynamic animations that helped finding the distinct features of stable and unstable orbits. These methods also include contour plots to help explaining the results of the coupling nature of spin-orbit resonance and orbit stability for eccentric orbits. We applied our findings to study two case studies of Mars orbiting spacecraft that benefit from the careful selection of the initial orbital elements to optimize their mission requirements.

ACKNOWLEDGMENTS

The authors would like to thank the NASA Ames Mission Design Division management support, in particular Ryan Vaughan and Sally Cahill. As always, enduring gratitude goes to the late Professor Harm Buning, whose many students of orbital mechanics included Mercury, Gemini, and Apollo astronauts.

REFERENCES

¹ Dono Perez, A., and L. Plice, 2019. “Stability and spin-orbit resonance analysis of low altitude Martian orbits,” AAS/AIAA 29th AAS/AIAA Space Flight Mechanics Meeting; January 13-17, 2019, Ka’anapali, HI. AAS 19-379.

² Klokočník, J., Kostecký, J., & Gooding, R. 2003, *Journal of Geodesy*, 77 30